

**Day-Ahead Electricity Spot Prices -  
Fundamental Modelling and the Role of Expected Wind  
Electricity Infeed at the European Energy Exchange**

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The President:

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# Summary

With the liberalization of global power markets, modelling of exchange traded electricity contracts has attracted significant attention of both, academy and industry. While to date, most modelling and forecasting techniques have been built on purely stochastic models carried over from classical financial markets, forecast models accounting for fundamental factors are still rarely applied. However, transparency efforts by governments and authorities make the use of fundamental variables increasingly attractive. In the thesis at hand, several fundamental models are applied taking into account a rich variety of demand and supply side related fundamental variables in order to forecast hourly prices of day-ahead electricity contracts for the German power market. In doing so, we want to cope with distinct characteristics of electricity price patterns such as seasonality, high and clustered volatility, or extreme price observations. Given the increasing promotion of electricity from renewable sources and according amendments in the German renewable energy law, special attention is paid to the impact of expected wind electricity infeed and its explanatory power in forecasting electricity spot prices. To start, a GARCH regression model which combines a linear multiple regression relationship with a conditional variance specification will be estimated. Afterwards, a threshold regression model which allows for asymmetric dependency patterns will be introduced. In order to allow for seasonal price sensitivities of electricity spot prices towards fundamental variables, a time-varying parameter regression model will be applied and estimated via a Kalman filter algorithm. The latter model will prove to be particularly powerful in considering the adaptive nature of German electricity spot prices and in providing short-term price forecasts. In order to gain a deeper insight into the impact of intermittent supply from renewable sources (i.e. wind energy) and the latest amendments in the German renewable energy law on the risk behavior of market participants, the thesis will be completed by a forward market analysis. Risk premia on contracts with a short time-to-maturity will constitute the focal point of the analysis.



# Zusammenfassung

Im Zuge der Liberalisierung der globalen Strommärkte hat die Modellierung von börsengehandelten Elektrizitätskontrakten sowohl in der Lehre als auch in der Praxis an Beachtung gewonnen. Bis heute werden hierzu vorwiegend rein stochastische Methoden angewendet, welche von klassischen Finanzmärkten übernommen wurden. Modelle, welche fundamentale Faktoren berücksichtigen, sind nach wie vor wenig verbreitet. Anstrengungen zur Verbesserung der Transparenz von Seiten der Regierungen machen fundamentale Informationen des Strommarktes allerdings zunehmend zugänglich und die Anwendung fundamentaler Prognosemodelle attraktiver. In der vorliegenden Arbeit werden verschiedene fundamentale Prognosemodelle für die Preise von stündlichen Day-Ahead Elektrizitätskontrakten auf den deutschen Markt geschätzt und getestet. Dabei wird ein breites Spektrum an fundamentalen Variablen berücksichtigt, um den spezifischen Charakteristiken von Strompreisen gerecht zu werden. Aufgrund der zunehmenden Bedeutung von alternativen Energien im deutschen Markt wird der Elektrizität aus Windkraftwerken besondere Beachtung geschenkt. Insbesondere interessiert dabei der Einfluss der prognostizierten Windeinspeisung auf die Spot Preise sowie ihr Beitrag zur Verbesserung der Prognosegüte. Zu Beginn wird ein GARCH-Regressionsmodell geschätzt, welches eine multiple lineare Abhängigkeit mit einer bedingten Varianz kombiniert. Anschliessend wird ein Threshold-Regressionsmodell eingeführt, welches asymmetrische Effekte in den Abhängigkeiten erlaubt. Schliesslich wird ein Regressionsmodell mit zeitvariierenden Parametern angewendet, um saisonale Abhängigkeiten von Strompreisen gegenüber verschiedenen fundamentalen Variablen zu modellieren. Es wird sich zeigen, dass sich insbesondere die letzte Modellklasse dazu eignet, den adaptiven Charakter der Spot Preise zu berücksichtigen und akkurate Preisprognosen zu erstellen. Den Abschluss dieser Arbeit bildet eine Analyse des Terminmarktes für Strom. Diese soll Aufschluss über den Einfluss der erwarteten Windeinspeisung sowie der jüngsten Änderungen im Erneuerbaren-Energien-Gesetz auf das Risikoverhalten der Marktteilnehmer geben.





# Chapter 1

## Introduction

### 1.1 Motivation & Research Questions

Global markets for electricity have undergone a radical transformation process over the last decade. In the mid-nineties electricity markets in most countries were characterized by monopolistic price setting power. They have evolved into deregulated markets in most regions of the western world to date. Parallel to liberalization efforts, industrialized societies have increasingly shifted their focus away from fossil fuels towards energy from renewable sources, initiated by arising discussions on global warming and the greenhouse effect. For this reason, laws have been enacted with the aim of setting effective incentives and guidelines. Germany, where renewable energy sources have historically enjoyed a lot of attention, is a prominent example for these developments. With its first renewable energy law dating back to 1991, the country was an early mover in steering its domestic electricity market in a more environmentally friendly direction and has stimulated the production of green electricity mainly from wind, but also other sources, through various incentive systems.

As one of the latest directives, the Equalization Mechanism Ordinance (ger.: Verordnung zur Weiterentwicklung des bundesweiten Ausgleichsmechanismus, abbr.: AusglMechV) was enacted and became effective as of January 1, 2010. In its core, the directive redefines the way of how electricity from renewable sources is to be marketed. As a result, renewable energies, and among them wind in particular, have taken an even more important role in the electricity spot price formation process. Nevertheless, so far, not much research has been conducted on how the increased importance of wind energy affects price dynamics and translates into the ability to forecast electricity spot prices.

Along the liberalization process and the resulting formation of dedicated public

trading venues such as the European Energy Exchange (EEX) in Leipzig (Germany), researchers from several disciplines have begun to build methodologies to model and forecast electricity spot prices and their distinctive dynamics. Until today, stochastic models carried over from classical financial markets such as equities or interest rates have remained the most widespread techniques to model electricity prices. Despite known weaknesses of these models, which mainly originate from the unique characteristics of electricity prices, other approaches have not established themselves so far. However, as international and national laws enforce more transparency, fundamental data such as renewable energy generation forecasts or plant availability are increasingly becoming available. As a result, in markets such as the United Kingdom, where the level of transparency is comparatively high, researchers have started to model electricity spot prices by means of fundamental market variables over recent years. To all our knowledge, there have been no efforts to create extensive fundamental models for the German market so far. In particular, we see a research gap with respect to the role and explanatory power of information on electricity production from renewable sources when it comes to the modelling and forecasting of electricity day-ahead spot prices.

The fact that electricity is a non-storable commodity makes the application of common forward pricing approaches based on the well-established theory of storage impossible. Therefore, from a methodical perspective, differences between spot and forward electricity prices are often considered as pure risk premia. By defining day-ahead electricity contracts traded in Vienna as the last possibility for traders to hedge their positions before the German auction takes place, risk premia on these contracts seem an appealing research field to investigate the influence of wind energy on very short-term forward prices. As far as we know, this aspect has not yet been touched in the existing literature. Furthermore, the latest renewable energy ordinance has considerably changed the way of how green electricity is marketed in the spot and forward market. Therefore, it remains to be analyzed how risk premia on short-term futures contracts have reacted to the regulatory changes.

Given the current stage of research and latest regulatory developments, this thesis aims to contribute to the existing literature by investigating the following main questions:

- How are hourly electricity day-ahead spot prices in Germany affected by various fundamental factors on the demand and the supply side of the market?
- What is the particular role of wind energy in the price formation process and can the forecast accuracy for hourly spot prices be increased by considering expected wind electricity generation?

- How does wind energy affect exchange-traded electricity prices jointly with other exogenous factors such as fuel prices, expected demand, or the expected availability of production capacities?
- Which class of models is suitable to forecast day-ahead electricity spot prices by means of fundamental variables in general and expected wind (electricity) infeed particularly?
- How have risk premia on short-term forward contracts been affected by the introduction of the latest renewable energy ordinance and what is the role of expected wind infeed?

By answering these questions, the thesis intends to expand existing research on electricity spot price modelling and especially give a better understanding about the impact of expected wind energy production. To us, the latter seems particularly attractive as findings may be translated to electricity from other renewable sources that are currently gaining importance, such as photovoltaics. We also see our contribution in providing evidence on how extensive fundamental models, covering aspects from the demand as well as the supply side of the market, perform in forecasting electricity spot prices in the German market. The extension of our analysis to short-term forward markets shall additionally help to better understand the impact of wind energy in combination with the latest regulatory changes.

From a methodical perspective, we shall use a set of econometric methods which constitute more sophisticated extensions of the standard regression model to forecast electricity spot prices. We will introduce these models in detail before applying them in the respective chapters of this thesis.

The following section provides an overview on how the thesis will be structured in order to answer the aforementioned research questions as well as on applied methodologies.

## 1.2 Structure & Methodology

To start, *chapter 2* will provide an overview on the liberalization process in the German electricity market and the current market setting. We will explain the functioning of exchange-based electricity trading and the auction mechanism for German electricity contracts. Furthermore, we will discuss most relevant stages in the history of laws and policies on renewable energy in Germany.

In *chapter 3* we will discuss distinct characteristics of electricity spot prices using prices for German day-ahead electricity contracts which will be investigated in this

thesis. We will define the dataset our spot price models will be applied to. Moreover, we will motivate the use of normal prices in our models rather than their returns and/or logarithms which is still the most favoured approach in the existing literature. The chapter will be completed by an overview on existing research on spot price modelling in global electricity markets.

In *chapter 4* we will provide an exhaustive introduction to fundamental drivers of the supply and the demand side of the market for electricity. We will derive variables representing or approximating these drivers which shall be incorporated as exogenous factors into our spot price models in chapters 5, 6, and 7. Among others, we will introduce and estimate our own demand forecast model as there exists no dedicated number for expected total electricity demand in Germany.

In *chapter 5* we will introduce GARCH regression models which combine a standard multi regression model with a conditional volatility process as pioneered by Bollerslev (1986). After a theoretical introduction we will test the model parameters for statistical significance on an in-sample dataset which is composed of a time series starting on January 1, 2010 and ending on December 31, 2011. We will then apply 24 calibrated models (one dedicated model for each hour of the day) to an out-of-sample dataset including data from January 1, 2012 until April 30, 2012 in order to test their ability to forecast hourly electricity spot prices. To find out about the explanatory power of expected wind infeed in GARCH regression models we will estimate all models on the in-sample dataset including and excluding the wind variable and investigate differences in the results. Additionally, an in-depth analysis on regression parameters will allow us to gain more insight into the dynamics and seasonality patterns of fundamental price formation across the day. Besides the conditional mean we will also look at the conditional variance process of the models and show whether the remaining variance can be reduced by incorporating a fundamental variable for expected electricity supply from wind power plants.

Following GARCH regression models we will estimate threshold regression models in *chapter 6* according to the approach introduced by Hansen (2000). These models provide more flexibility as they allow the linear relation between exogenous variables and the dependent variable to alternate between two regimes which are defined by the estimated value of the threshold variable which we decide to be expected wind infeed. Again, we will apply models for single hours to in-sample as well as to out-of-sample data.

As a third class of spot price models we will introduce time-varying parameter (TVP) regression models in *chapter 7*. These models assume regression parameters to be unobserved states following a random walk process. Regression parameters are estimated by means of a filter provided by Kalman (1960). We will estimate TVP

regression models for all 24 hours and in addition to the forecasting accuracy we will elaborate on the seasonality of regression parameters which becomes apparent as a result of the estimation routine of the filter. Finally, we will compare conditional volatility (which is also a side-product of the filter process) of TVP regression models with conditional volatility we obtained when estimating GARCH regression models in chapter 5 and discuss differences.

While the preceding part of the thesis concentrates on spot prices, *chapter 8* provides an analysis on short-term forward prices in order to shed more light on the role of expected wind infeed as well as possible impacts caused by the introduction of the ordinance on the trading of electricity from renewable sources. Motivated by the non-storability feature of electricity we will base our analysis on (ex post) risk premia. The main part of our forward market analysis is dedicated to the investigation of premia on day-ahead electricity contracts traded in Austria which will be defined as the nearest forward contracts for EEX day-ahead contracts. Besides risk premia we will estimate and analyze the market price of risk as introduced by Kolos & Ronn (2008). Finally, we will also look at front-month futures contracts traded at the European Energy Exchange (EEX).

*Chapter 9* will conclude by providing a summary of our findings and by identifying possible future research opportunities.



## Chapter 2

# Economic & Regulatory Background

### 2.1 The German Electricity Market

#### 2.1.1 Liberalization Process & Market Setting

Liberalization processes in electricity markets can typically be split into three stages as explained by Stender (2008). In a first step, any hurdles that impede the entrance of potential participants into the market are abolished. This opens the different parts of the industry's value chain such as production, trade, and distribution to new companies. As there may be room for competition discrimination, in a second step, integrated power companies are forced by law to unbundle their value chain to a level that allows fair competition. In a third step, the enhanced transparency resulting from the previous two actions enables an increase in cost reduction potential in areas of the electricity business which are natural monopolies.

The liberalization of electricity markets in the European Union was initiated by the enacting of an EU directive (Directive 96/92/EC) in December 1996 which aimed at lower electricity and gas prices for end consumers by regulating existing monopolies and promoting market competition.<sup>1</sup> To transform the European directive into national law, a new German energy law (ger.: Energiewirtschaftsgesetz, abbr.: EnWG) was established in 1998 ushering in the liberalization of the German electricity market. Ever since, the supply side of the German electricity market has undergone a structural transformation process characterized by mergers, cooperations, and strategic partnerships. The most intense concentration took place at the very beginning of

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<sup>1</sup>See A. T. Kearney (2007).



the liberalization process when eight former power supply firms were merged into four large companies, namely RWE, E.ON, Vattenfall Europe, and EnBW Energie Baden-Wuerttemberg. In sum, these four new firms were in possession of more than 80 percent of the domestic production capacities.<sup>2</sup> As the European Union considered liberalization progressions to be too slow, an accelerating directive was enacted in 2003 (Directive 2003/54/EC) which was transformed into national law in Germany in 2005. Among others, the new act required that vertically integrated companies with more than 100,000 clients had to operationally and legally demerge their areas of production, main operations, and distribution.<sup>3</sup> To further advance the liberalization process, an incentive based regulation framework was introduced by the government in early 2009.<sup>4</sup>

Today, more than 15 years after first liberalization efforts in Germany, discussions about their impact and the level of competition are omnipresent. The two mostly discussed points are, first, the fact that electricity generation capacities are still owned by a very low number of companies<sup>5</sup> and, second, prices for end consumers which, over the last decade, have rather increased than decreased. Research on market power in the German electricity market can be found in Ellersdorfer (2005), Muesgens (2006), Schwartz & Lang (2006), or von Hirschhausen et al. (2007). Overall, the studies conclude that electricity spot prices are not completely determined by fundamental factors but partly influenced by the exertion of market power by producing companies.

Technically, the German electricity market consists of a multilayer grid system. The high voltage grid, to which most power plants are directly connected, divides into four control areas that are operated by the respective transmission system operators (TSOs) TenneT TSO GmbH, Amprion GmbH, 50Hertz Transmission GmbH, and TransnetBW GmbH which are former affiliates of the four aforementioned large energy companies. The main duties of the TSOs are the maintenance of the transmission grid and ensuring a balanced grid (i.e. equilibrium between power supply and consumption) which is required by the non-storability of electricity. This task is accomplished by the TSOs via three levels of reserve control as explained by Konstantin (2007). Primary control is provided by a so-called spinning reserve which is unused capacity held by power plants for exactly this purpose to an extent of 3-5% of

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<sup>2</sup>See Krisp (2008).

<sup>3</sup>See Konstantin (2007).

<sup>4</sup>See Stender (2008).

<sup>5</sup>As Weigt & von Hirschhausen (2008) report, the big four had a combined market share of 85% in 2006.



their total generation capacity. Primary control must be available within 15 to 30 seconds. It is then scheduled that within 30 seconds, primary control is automatically replaced by secondary control which is provided by power plants that operate in part load mode. Secondary control must be held up for at least 1 hour after the imbalance occurs. Finally, tertiary reserve is scheduled to replace secondary reserve after 15 minutes at the latest and is manually activated through pump storage and gas turbines power plants mainly. In case that negative balancing energy is required (i.e. short-term supply exceeds short-term demand), power plants which are scheduled to deliver reserve energy have to reduce their production. Erdmann & Zweifel (2008) note that in liberalized electricity markets, TSOs are obliged to facilitate reserve control in a transparent, non-discriminating, and market-oriented way.

One level below the TSOs, distribution system operators (DSOs) are responsible for the operation of the medium and low voltage grid. As such, they are in charge of the distribution of power to electricity supply companies (ESCs) which transmit it to the end customers.

Together, Germany and Austria form a single market area. This means that contracts traded for this area can be settled at any point of the German as well as the Austrian transmission grid. Today, there are hardly electricity flow restrictions or bottlenecks between the two countries. While the German part consists of the four mentioned transmission grid zones, Austria is divided into three different grid zones.

### **2.1.2 Auction Mechanism & The European Energy Exchange**

As an integrative part of the German electricity market and its liberalization process, the European Energy Exchange AG (EEX) in Leipzig has facilitated power trading since it was formed through the merger of the Leipzig-based Power Exchange (LPX) and the Frankfurt-based EEX in 2002. In 2009, the EEX agreed to collaborate more closely with Powernext SA in Paris which is the exchange place for the French area. The two companies set up a joint venture named European Power Exchange Spot SE (EPEX) based in Paris (with an operating branch in Leipzig for the German market) into which they transferred their spot trading activities. This corporate action, however, did not have an economic impact on the trading of electricity contracts which is still handled separately for different market areas.<sup>6</sup> Czakainski et al. (2010) note that, together with the Scandinavian market place, the EEX has been the main driver of the fast development of exchange-based power trading in Europe.

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<sup>6</sup>See Kroneberg & Boehnke (2010).

For the German/Austrian market area, the EPEX offers two main types of spot trading which are (i) the day-ahead auction and (ii) the intraday auction. Although intraday trading would be the 'real' spot market, its liquidity is rather low and thus the day-ahead auction is usually considered the spot market for electricity contracts. The day-ahead auction for hourly delivery in the German and Austrian TSO zones takes place every day at 12:00 noon (including weekends and holidays). Until then, market participants can anonymously submit their bids with a minimum volume of 0.1 megawatt (MW) for individual hours and blocks and a minimum price change of EUR 0.1 per megawatt hour (MWh). Moreover, prices for hourly contracts must remain within the range between EUR -3,000 and EUR 3,000 per MWh. After all bids have been collected, the market clearing price, which applies to all transactions, is determined and published after 12:40 pm. The delivery takes place during the respective hour of the following day (which reasons the term 'day-ahead'). Besides contracts with hourly deliveries, block contracts with delivery during peak hours (from 08:00 am to 08:00 pm) as well as block contracts with delivery during base hours (including all 24 hours of the day) are traded in the day-ahead market. Their price is calculated as the arithmetic mean price of the underlying hourly contracts. Hourly and block contracts are summarized under the term Physical Electricity Index (abbr.: Phelix) and serve as price signals for contracts that are negotiated outside the energy exchange (OTC).<sup>7</sup>

Trading for derivatives is facilitated by the EEX Power Derivatives GmbH of which the EEX is the majority shareholder. For the German/Austrian market area, futures (all financially settled exclusively) with delivery during the current and the following four weeks, the current and the next nine months, the next eleven quarters, and the next six years are continuously traded on every working day between 08:00 am and 06:00 pm. Unlike spot market contracts, futures are only available for peak, off-peak, and base blocks. The daily delivery period for peak contracts is between 08:00 am and 08:00 pm, the delivery period for off-peak contracts is between midnight and 08:00 am (off-peak I) as well as between 08:00 pm and midnight (off-peak II), and the delivery period for base contracts is across all 24 hours of the day.

When discussing and modelling electricity spot prices in chapters 5, 6, and 7 in this thesis, we are considering hourly day-ahead contracts. For the forward market analysis in chapter 8, we will also look at peak and base month futures contracts.

To avoid confusion, note that in the remainder of this thesis, for the sake of simplicity, the term 'EEX prices' will refer to prices for contracts on the German/Austrian

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<sup>7</sup>See European Energy Exchange AG (2012).

market area traded in Leipzig, regardless of whether trading is (legally) facilitated by the EEX (futures contracts) or by the EPEX (spot contracts).

## 2.2 Renewable Energy Legislation

Along with the worldwide increasing debate on global warming and accompanying political efforts towards the end of the last century, energy from renewable sources has gradually gained attention. In 1991, the first feed-in act (ger.: Stromeinspeisegesetz fuer Erneuerbare Energien, abbr.: StrEG) was introduced which supported the construction and operation of wind power plants by smaller suppliers.<sup>8</sup> The feed-in act, which was revised in 1998 (Law of the Energy Industry, abbr.: EnWG), can be considered the predecessor of the later enacted new energy laws. In 2000 the first Renewable Energy Law (ger.: Erneuerbare-Energien-Gesetz, abbr.: EEG) became effective, followed by an amended version in 2004. These laws obliged local grid operators to accept all electricity from renewable sources, pay a minimum price for it, and to pass it to the transmission system operators who were then responsible to balance the obtained electricity. Finally, renewable electricity was delivered to electricity supply companies (ESCs) which had to accept it up to a certain amount relative to their overall turnover<sup>9</sup> and which distributed it to the end customer.

In 2009 another era in German electricity politics was ushered in by the reformation of the Renewable Energy Law. The main objective was to reduce dependence on fossil fuels and to increase the share of renewable energy in total energy consumption to 30 percent by 2020.<sup>10</sup> A central point of the new EEG was the ruling regarding the processing of electricity from renewable sources by the transmission system operators. Until December 31, 2009, operationally, the delivery of renewable electricity from transmission system operators to energy supply companies was facilitated through so-called monthly bands. Concretely, the amount of renewable electricity delivered was specified based on forecasts by transmission system operators in the preceding month. These monthly bands can economically be understood as forward contracts with a delivery period of one month. As most renewable electricity (e.g. wind, photovoltaics) is not fully predictable, transmission system operators had to balance resulting gaps which was mainly done through the day-ahead spot market.<sup>11</sup>

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<sup>8</sup>See Stroebele et al. (2010).

<sup>9</sup>See Konstantin (2007).

<sup>10</sup>See EEG, Federal Republic of Germany (2008).

<sup>11</sup>See for example Fraunhofer Institute for Systems and Innovation Research et al. (2007).

This mechanism was flawed in various respects and was replaced as a side-product of the new EEG by a federal ordinance (ger: Verordnung zur Weiterentwicklung des bundesweiten Ausgleichsmechanismus, abbr.: AusglMechV) which became effective as of January 1, 2010. With the new equalization mechanism ordinance, transmission system operators have to market renewable electricity in the day-ahead or the intraday spot market in a transparent and non-discriminating way.<sup>12</sup> Given the low market activity on the intraday market one can assume that EEG electricity is mainly marketed in the day-ahead market. Transmission system operators are no longer forced to transmit renewable energy electricity physically to electricity supply companies and the latter are given the freedom of covering their power requirement independently of EEG electricity production and thus with a higher planning reliability at the exchange.<sup>13</sup>

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<sup>12</sup>See §1 and §2 AusglMechV, German Bundestag (2009).

<sup>13</sup>See German Bundestag - Research Services (2010).

## Chapter 3

# Electricity Spot Prices

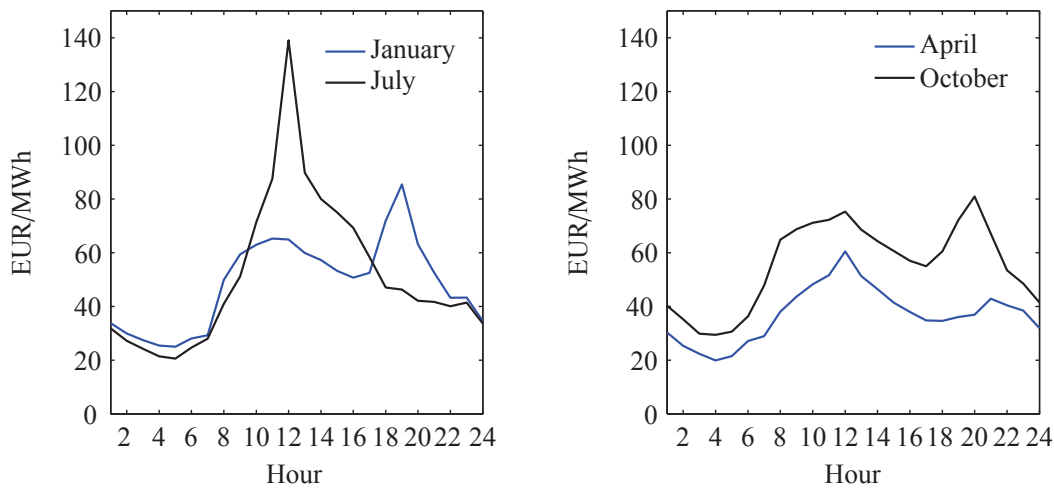
This chapter focuses on the particularities of electricity spot prices and established methodologies to model and forecast them. First, we will discuss distinct price patterns that make electricity a unique commodity. Afterwards, we will describe the dataset of electricity prices used in this thesis and motivate why we shall exclusively work with normal prices rather than their logarithms. To conclude, an overview on existing literature of spot price modelling will be presented.

### 3.1 Stylized Facts of Electricity Spot Prices

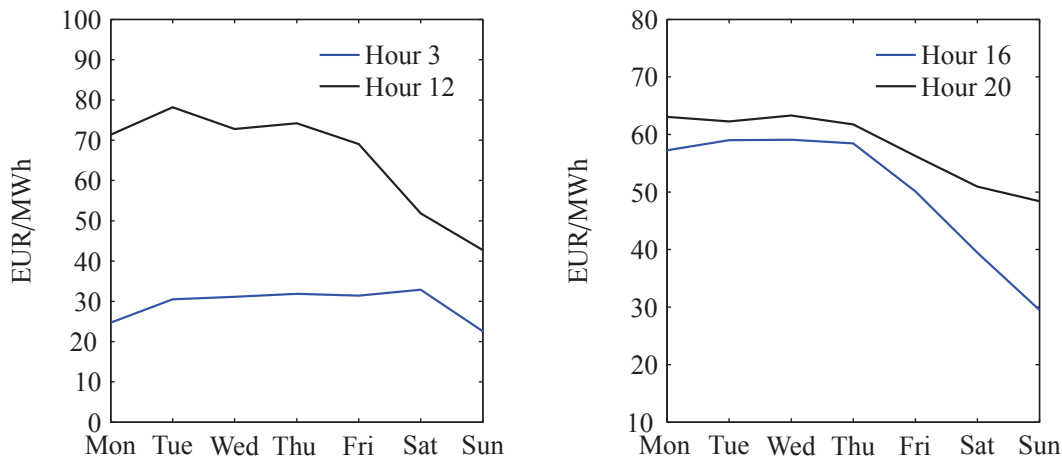
Electricity spot prices display distinct characteristics which are different from properties of other financial assets in general as well as other commodities. To a large extent, these unique price patterns are due to the seasonality of underlying demand, the non-storability feature, and (closely related to the latter) the fact that the power grid has to be balanced at every point in time. The most obvious characteristics are:

- Mean reversion
- Seasonality
- Extreme prices
- High and clustered volatility

Over time, electricity spot prices tend to *revert towards the mean* which is a characteristic they share with many other commodities. Whereas long-term sustainable changes in price levels can be caused by structural market changes or governmental interventions, mean-reversion patterns usually hold for the short to medium term.



**Figure 3.1:** *Intraday seasonality illustrated by average hourly day-ahead spot prices for different months calculated using data between January 1, 2010 and December 31, 2011. For example, hourly prices for January are computed as the arithmetic mean of hourly prices in January 2010 and January 2011. All weekdays are included.*



**Figure 3.2:** *Intraweek seasonality illustrated by daily average day-ahead spot prices for different hours of the day calculated using data between January 1, 2010 and December 31, 2011.*

Electricity spot prices display pronounced *seasonal patterns* with respect to yearly seasons, weekdays, and time of day. Overall, seasonal structures are mainly driven by demand which will be discussed in detail in section 4.1. Figure 3.1 depicts hourly average spot prices for four different months representing different seasons. We can observe distinct price peaks at noon as well as in the evening around 06:00 pm. Whereas in summer the noon peak is more pronounced than the evening peak, a reverse pattern is apparent for wintertime. During early morning and late evening hours when demand is reduced, prices are lower across all seasons. Looking at figure 3.2 we ob-

serve that prices do also vary over different week days. The most apparent pattern is a sharp decrease in prices over the weekend when demand from industry disappears. The least price difference between working and weekend days can be observed for early morning hours. Across working days we observe modest fluctuations for some hours, but in general prices prove to be rather uniform. It can be concluded that most price variation is with regard to different hours. Electricity prices of different hours are therefore, from an economic perspective, often handled as different commodities.<sup>1</sup>

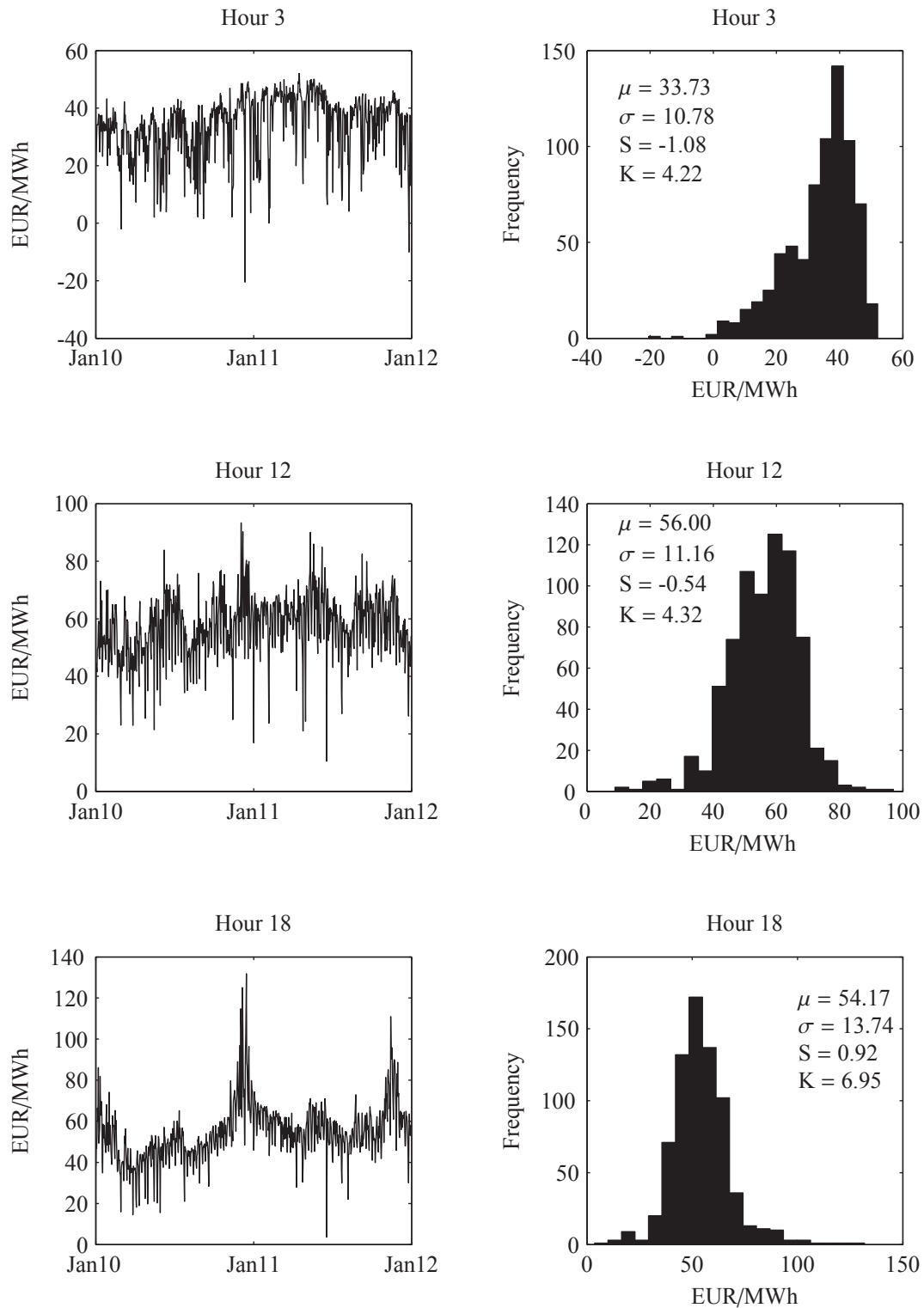
Another distinct characteristic of electricity spot prices are *extreme price formations*. Unlike stocks which regularly display jumps in their price evolutions, electricity prices are governed by price spikes which are usually short-lived and much more extreme in magnitude. The line plots in figure 3.3 show that spike characteristics of spot prices vary significantly across the day. During the night (hour 3) spikes are negative and jumps to the upside are rarely observed. Negative prices at these times are mainly driven by the intermittent infeed of renewable electricity (wind) which creates temporary excess supply on the market which can sometimes even lead to negative prices as all available electricity has to be balanced. The occurrence of negative spikes is also reflected in the distributional characteristics of spot prices. The probability distribution of spot prices for hour 3 displays negative skewness. For hours when demand is at high or very high levels, positive spikes become more probable. This is reflected in the line chart for hour 12 in figure 3.3 as well as in the related price distribution skewness which is closer to zero. Looking at hour 18, we see, similar to hour 12, negative as well as positive price spikes. To sum up, not only average price levels but also price spikes are subject to seasonal patterns.

*Volatility* of electricity spot prices is not constant but highly variable and clustered. As an example we can look at prices for hour 18 in figure 3.3 where a temporary increase during winter 2010/2011 or towards the end of 2011 is apparent.

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<sup>1</sup>See Nan et al. (2010).





**Figure 3.3:** Electricity spot prices for hours 3, 12, and 18 and corresponding frequency distributions. The underlying dataset starts on January 1, 2010 and ends on December 31, 2011. All weekdays are included.



## 3.2 Dataset

We will work with two main datasets when estimating and testing different spot price models. The in-sample dataset includes prices starting on January 1, 2010 and ending on December 31, 2011 whereas the out-of-sample dataset includes prices from January 1, 2012 until April 30, 2012. There are several reasons why we do not use data before 2010. The main reason is that at the beginning of 2010, the latest significant regulatory change (AusglMechV) became effective. Another reason is that some data we will use to reflect the fundamentals driving spot prices are not available or incomplete for previous time periods.

There are different approaches how outliers in electricity spot prices are handled. On the one hand, there is reason to keep them in the dataset in order to estimate under as much realistic conditions as possible. On the other hand, researchers are tempted so smooth spiky data in order to obtain more robust results. To us, the main motivation to eliminate most extreme prices is that they are often the result of technical system failure rather than a normal price formation process<sup>2</sup> and thus usually non-repetitive<sup>3</sup>. We choose to exclude observations with a spot price outside a 3-sigma band around its mean which results in the elimination of 1 to 9 data points, depending on the hour, which is less than 2% of all observations for the in-sample dataset. Similar approaches have been chosen before. For example Clewlow & Strickland (1999) eliminate as well data beyond the 3-sigma band (eliminating 1.48-2.11% of all data) while Thomas & Mitchell (2007) choose a 4-sigma band to be appropriate (eliminating 0.65-1.05% of all data in the in-sample dataset and 1.20-3.60% in the out-of-sample dataset).

Table 3.1 summarizes descriptive statistics for the in-sample electricity spot price dataset (after outliers correction) we will use in this thesis and table 3.2 provides the same figures for the out-of-sample dataset.

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<sup>2</sup>See O'Mahoney & Denny (2011).

<sup>3</sup>See Guirguis & Felder (2004).

<b>Hour</b>	<b>1</b>	<b>2</b>	<b>3</b>	<b>4</b>	<b>5</b>	<b>6</b>	<b>7</b>	<b>8</b>
Observations	486	487	486	491	489	484	489	492
Outliers	7	6	7	2	4	9	4	1
Min	17.55	11.49	6.52	1.88	5.05	16.07	28.88	31.52
Max	57.32	52.74	50.06	50.15	51.49	53.46	68.49	84.99
Mean	40.92	37.91	35.19	32.43	33.59	39.37	48.42	58.37
Standard Dev.	7.12	8.08	8.92	10.06	9.49	6.78	6.49	8.87
Skewness	-0.90	-0.99	-0.88	-0.74	-0.74	-0.63	0.27	0.12
Kurtosis	3.74	3.73	3.29	2.94	3.04	3.26	2.91	2.63
<b>Hour</b>	<b>9</b>	<b>10</b>	<b>11</b>	<b>12</b>	<b>13</b>	<b>14</b>	<b>15</b>	<b>16</b>
Observations	491	491	490	490	492	492	492	491
Outliers	2	2	3	3	1	1	1	2
Min	39.20	39.93	39.97	41.63	39.28	38.52	35.05	33.83
Max	84.19	84.85	83.95	86.00	82.92	80.92	78.49	77.94
Mean	60.86	60.74	60.59	61.41	59.29	57.51	55.44	53.52
Standard Dev.	8.74	8.20	7.79	8.06	7.80	7.84	8.02	8.01
Skewness	0.07	0.00	-0.05	-0.01	-0.05	-0.02	0.15	0.30
Kurtosis	2.57	2.69	2.75	2.92	2.93	2.83	2.97	3.11
<b>Hour</b>	<b>17</b>	<b>18</b>	<b>19</b>	<b>20</b>	<b>21</b>	<b>22</b>	<b>23</b>	<b>24</b>
Observations	487	486	484	489	492	491	488	490
Outliers	6	7	9	4	1	2	5	3
Min	33.82	34.98	38.66	39.93	39.34	35.27	34.97	23.58
Max	79.93	97.02	100.46	91.67	79.93	71.95	66.05	60.35
Mean	53.02	57.07	60.63	59.89	56.15	50.98	49.87	44.80
Standard Dev.	8.33	11.18	11.57	10.47	8.87	7.07	5.88	5.83
Skewness	0.46	0.84	0.66	0.25	0.25	0.53	0.19	-0.38
Kurtosis	3.28	4.15	3.54	2.14	2.08	3.12	2.95	3.39

**Table 3.1:** Descriptive statistics of the in-sample dataset starting on January 1, 2010 and ending on December 31, 2011. The dataset is corrected for weekends, holidays, and bridge days as well as for outliers, i.e. price observations outside a 3-sigma band around the mean.

<b>Hour</b>	<b>1</b>	<b>2</b>	<b>3</b>	<b>4</b>	<b>5</b>	<b>6</b>	<b>7</b>	<b>8</b>
Observations	81	80	81	82	82	81	82	82
Outliers	2	3	2	1	1	2	1	1
Min	11.17	11.71	0.04	-5.03	-1.48	10.99	20.44	37.39
Max	55.07	51.80	46.96	45.13	46.20	49.57	70.94	120.03
Mean	37.11	35.03	33.10	31.40	32.04	36.22	46.49	58.70
Standard Dev.	8.40	7.99	8.58	9.33	8.73	7.00	8.48	15.87
Skewness	-1.04	-0.94	-1.32	-1.47	-1.27	-1.26	0.31	2.08
Kurtosis	4.75	4.05	5.01	5.84	4.97	5.21	4.15	7.79
<b>Hour</b>	<b>9</b>	<b>10</b>	<b>11</b>	<b>12</b>	<b>13</b>	<b>14</b>	<b>15</b>	<b>16</b>
Observations	81	80	82	81	80	81	81	81
Outliers	2	3	1	2	3	2	2	2
Min	37.19	39.96	37.03	35.26	25.77	25.12	24.55	24.47
Max	120.00	115.00	107.52	95.93	85.82	84.27	73.97	73.78
Mean	62.05	57.91	55.64	53.07	49.45	48.26	46.17	46.33
Standard Dev.	16.34	13.60	15.49	12.69	10.84	10.28	8.37	7.84
Skewness	1.86	1.87	1.79	1.38	0.99	1.20	1.06	1.04
Kurtosis	6.73	7.78	6.00	4.99	4.36	5.25	5.40	6.08
<b>Hour</b>	<b>17</b>	<b>18</b>	<b>19</b>	<b>20</b>	<b>21</b>	<b>22</b>	<b>23</b>	<b>24</b>
Observations	81	82	80	82	81	82	82	81
Outliers	2	1	3	1	2	1	1	2
Min	25.13	35.96	41.56	42.05	35.17	28.62	28.44	16.31
Max	84.98	110.41	142.91	127.13	89.76	71.59	65.27	57.94
Mean	46.59	55.08	63.73	64.80	54.51	48.99	47.05	41.60
Standard Dev.	8.63	15.16	19.48	18.66	9.42	8.03	7.15	7.25
Skewness	1.69	2.12	2.42	1.73	1.41	0.44	0.24	-0.42
Kurtosis	8.29	7.48	9.22	5.85	6.52	3.61	3.72	4.57

**Table 3.2:** *Descriptive statistics of the out-of-sample dataset starting on January 1, 2012 and ending on April 30, 2012. The dataset is corrected for week-ends, holidays, and bridge days as well as for outliers, i.e. price observations outside a 3-sigma band around the mean.*

### 3.3 Normal Prices versus Log Prices

In financial markets, researchers normally work with logarithmized prices and corresponding returns. However, for (physical) commodities in general, and electricity spot prices specifically, the situation looks somewhat different and researchers do not uniformly work in the logarithmized world. In this thesis we will consistently work with normal prices instead of following the more often applied approach of using log prices or log returns. The motivation for this approach shall be set forth subsequently.

There is a number of reasons why, especially in classical finance, researchers usually do not work with absolute prices but instead use their logarithms or logarithmized returns. First of all, econometric models often require underlying time series to be stationary meaning that they possess constant distributional properties over time.<sup>4</sup> One of the most prominent approaches to transform a non-stationary series into a stationary process is the application of the natural logarithm function and taking first differences. While it is obvious that equity price series have a unit root, series of electricity prices are often found to be stationary.<sup>5</sup>

Another frequently mentioned argument is that working with log prices ensures positivity of predicted prices as the log function is not defined for the negative space  $\mathbb{R}^-$ . However, negative prices in electricity markets can be economically reasoned by the non-storability characteristic of the underlying commodity<sup>6</sup> and thus do not have to be avoided necessarily. Insofar as negative prices are feasible given the regulations of the relevant trading venue, allowing for negative prices can even be a requirement when striving for reasonable price forecasts. In the German electricity market, negative prices have increasingly been observed along with the promotion of renewable energies and after the EEX removed restrictions on negative prices in 2009.<sup>7</sup> It is therefore our opinion that restricting ourselves to positive prices would be misleading.

As electricity prices feature pronounced spike characteristics as well as high volatilities, researchers often apply a log transformation in order to obtain series with more stable variances which is a desired feature when using certain quantitative models.<sup>8</sup> As opposed to this, Karakatsani & Bunn (2010) mention that when investigating the variability of electricity prices, efforts to stabilize the variance of the original series

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<sup>4</sup>For an introduction to stationarity and details on different types of stationarity, see for example Verbeek (2008) or Kirchgassner & Wolters (2008).

<sup>5</sup>See for example Lucia & Schwartz (2002).

<sup>6</sup>See Meyer-Brandis & Tankov (2008).

<sup>7</sup>See for example Brandstaett et al. (2011).

<sup>8</sup>See for example Weron (2006) or Nan et al. (2010).

are not in the researcher's interest as they conceal detailed statistical properties and also lead to multiplicative error effects. In the thesis at hand we intend to explain price variability by the introduction of fundamental variables. Smoothing the underlying time series would therefore not be an appropriate measure.

A practical point which can be raised is that when investigating physical commodity markets, traders are mainly concerned about actual prices rather than their (log) returns.<sup>9</sup>

Researchers in electricity markets who worked with normal prices instead of their logarithms are for instance Escribano et al. (2002), Meyer-Brandis & Tankov (2008), or Karakatsani & Bunn (2010).

### 3.4 Spot Price Modelling: Literature Overview

Along with the liberalization process of global electricity markets, research on forecasting spot prices has increasingly been conducted. There is a tremendous amount of research especially for markets which initialized liberalization earlier, such as the US, the Nordic, or the British market. On the contrary, there are fewer analyses available for younger markets such as Germany or Austria.

The majority of existing research is based on stochastic modelling methodologies that have been carried over from classical financial markets. First attempts to model and forecast electricity spot prices were mainly based on the mean-reverting characteristic and derived from the general commodity price model introduced by Schwartz (1997) with an underlying Ornstein-Uhlenbeck process.<sup>10</sup> Mean-reverting models allowing for one or two factors seem to be attractive mainly because of their ability to catch autocorrelation characteristics of electricity prices.<sup>11</sup> A prominent method is the 2-factor model introduced by Pilipovic (1997) which consists of a short-term spot price factor and a long-term equilibrium price factor. Applications of classical mean-reverting models were done by Lucia & Schwartz (2002) who investigated the Nordic market with one and two factor models or by Barlow et al. (2004) who analyzed the Australian Market estimating the Pilipovic model with a Kalman filter based algorithm.

A drawback of pure mean-reverting and classical commodity models is that their

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<sup>9</sup>See Pilipovic (2007).

<sup>10</sup>See Uhlenbeck & Ornstein (1930).

<sup>11</sup>See Pilipovic (2007).

ability to mimic regularly observed extreme prices is very limited.<sup>12</sup> Price spikes are often caused by the non-storability of electricity and are therefore a unique characteristic which is not observed for most other assets. One branch of research has therefore tried to better model fat tails and volatility clusters by specifying a conditional variance via General Autoregressive Conditional Heteroskedasticity (GARCH) processes as introduced by Bollerslev (1986). Examples are Garcia & Contreras (2005) who applied GARCH models to predict day-ahead prices in the Californian and the Spanish market, Mugele et al. (2005) estimating ARMA-GARCH models on different European markets, including Germany, or Bowden & Payne (2008) modelling prices in US markets with ARIMA-EGARCH models allowing for asymmetric volatility.

However, as noted by Duffie et al. (1998) or Escribano et al. (2002), jumps and spikes may cause severe problems and lead to erroneous results when estimating GARCH-type models. An alternative approach is the expansion of classical mean-reverting models by jump diffusion processes which are common means to model the discontinuous behavior of other financial assets such as stocks or interest rates. Analyses incorporating jump diffusion processes have been provided by Cartea & Figueroa (2005) who apply a mean-reverting jump diffusion model to the market in Great Britain, Geman & Roncoroni (2006) introducing various discontinuous processes with a jump component for the Nordic market, Hambly et al. (2009) expanding an Ornstein-Uhlenbeck process with a mean-reverting jump process, or Weron & Misiorek (2006) who calibrate an autoregressive price model with mean-reversion and jump diffusion extension to data from the Californian and the Nordic market.

There are some deficiencies which are often associated with jump processes that are carried over from classical financial markets. Looking at electricity spot prices it can be easily observed that extreme price moves normally appear in the form of spikes rather than jump processes. While stock prices often continue to evolve from the new price level, electricity prices are usually pushed back to their starting level after only a few extreme price observations. In addition, Huisman & Mahieu (2003) note that stochastic jump processes can distort the specification of the true mean-reversion characteristic of the spot price process. Regime-switching models are an established approach to tackle the shortcomings of pure jump processes. Introduced by Hamilton (1988 & 1989) they are a wide-spread alternative to model processes with heteroskedasticity, structural breaks, or distinct patterns in different market situations.<sup>13</sup> Regime-switching models for electricity prices have been implemented

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<sup>12</sup>See Hambly et al. (2009).

<sup>13</sup>See Kim & Nelson (1999).



by Huisman & Mahieu (2003) for the Dutch, the German, and the UK market, by de Jong (2005) to various markets including Germany, by Weron & Misiorek (2006) and Haldrup & Nielsen (2006) for the Nordic market, and by Bierbrauer et al. (2007) and Bloechlinger (2008) for the German market. Overall, the mentioned studies conclude that among purely stochastic models, regime-switching methods provide the best fit when forecasting electricity spot prices.

More recently, models including exogenous variables have increasingly been introduced to electricity spot prices, motivated by the assumption that characteristic price patterns are the result of the joint behaviour of fundamentals. However, the range of variables used is still rather limited. Besides dummies to cover seasonal effects, weather data and the level of demand are often used as explanatory variables. Knittel & Roberts (2005) estimate an ARMAX model with temperature data as an external regressor on the Californian market, Torro (2008) defines an ARIMAX model with temperature, precipitation, reservoir levels, and the difference between current futures and spot prices (basis) as external variables for the Nordic market, and Cartea & Villaplana (2008) explain spot prices in the US, England/Wales, and the Nordic area by fundamental variables representing demand and capacity. Richer specifications are used by Karakatsani & Bunn (2010) who investigate spot prices with different time series models such as GARCH or time-varying parameter regression models. They examine a broad set of exogenous variables such as characteristics of the demand curve, generation capacity, or market agents' behavior and find especially time-varying regression models to be a superior choice when analyzing the British electricity market. A similar approach is chosen by Nan et al. (2010) to estimate several spot price models for the same market.





## Chapter 4

# Fundamental Drivers of Spot Prices & Variable Selection

In this chapter we will elaborate on the fundamental factors which influence the evolution of electricity spot prices in the short and in the long term. Besides giving a deeper understanding of the price building dynamics, this chapter shall motivate the selection of specific variables we will apply in the various models. We have chosen to categorize the variables into demand and supply side influencing factors. Variables that can neither be allocated to the supply nor to the demand side are pooled in a third category.

### 4.1 Demand

The demand for electricity which is relevant for the daily spot price formation is not available in the form of a dedicated number. Some of the transmission system operators provide approximated time series which are derived from the sum of all infeeds by power plants into the transmission grid. However, this data is often incomplete and not comprehensively provided. For our analysis we will represent hourly electricity demand by means of the vertical system load which is defined as the appropriately signed sum of all power which is transferred from the high voltage transmission grid to the next lower level, which is the distribution grid. This approach of representing demand is the usually followed method in power prices research. The total vertical system load for the German market area is provided by the four transmission grid operators. However, not all of them do also provide a corresponding day-ahead forecast

figure which is why we will generate our own forecast time series for all single hours of the day. Subsequently, we will first summarize the most important characteristics and drivers of electricity demand and established modelling approaches. We will then introduce a forecast model and present empirical in-sample as well as out-of-sample results.

### **4.1.1 Drivers of Electricity Demand**

Demand for electricity mainly comes from manufacturing industry, households, and the service industry.<sup>1</sup> In the short run, industrial production is primarily driven by the hour of the day whereas when looking at longer time horizons, business activity becomes more relevant. During holidays, weekends, and the night, when industrial production is switched off, the need for electricity is substantially lower. Over the span of several months and quarters, a possible economic boom or recession impacts demand significantly. Over the year, school holidays in summer or usual plant holidays around Christmas lead to lower demand.

As for electricity demand stemming from private consumers, the daytime is a relevant driver as well. Around noon or in the evening between 05:00 pm and 07:00 pm when people prepare lunch and dinner, demand is increasing. On the contrary, during the night when most people sleep, the need for electricity is relatively low. Along with the hour of the day, private demand is heavily driven by environmental factors which for example determine heating and cooling activities or the use of electrical light. Distinct patterns can also be observed over the course of the year. During summer time, the peak in demand for noon and afternoon hours is more pronounced as air condition runs. By the same token, the evening peak is more explicit during winter months as people increase heating at the time they return from work.

### **4.1.2 Modelling Approaches & Literature Overview**

Modelling electricity load has a long history. The ambitions to obtain precise load forecasts originate from different reasons. Research has shown that extreme loads have a significant impact on the probability of observing price spikes.<sup>2</sup> Moreover, an exact load forecast reduces the risk of market participants such as distributors and

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<sup>1</sup>See Burger et al. (2007).

<sup>2</sup>See Christensen et al. (2012).

generators when entering contracts.<sup>3</sup> Nowicka-Zagrajek & Weron (2002) state that although mean absolute percentage errors<sup>4</sup> of 10 percent and less are attained rather easily, financial costs triggered by erroneous forecasts can be so immense that research strives to improve estimation accuracy even by only a few percentage points. Moreover, the non-storability characteristic makes demand forecasting essential since electricity has to be consumed as it is produced. For the buy side, forecasts are imperative due to the fact that electricity, unlike many other goods, cannot be substituted easily.

Load forecasts can be categorized according to their time horizon which is either short-term (one hour to one week), medium-term (one month to one year or sometimes even up to three years), or long-term (over three years).<sup>5</sup> Among the three categories, short-term forecasts have experienced most attention from researchers.

The largest part of load forecasting work (especially for short-term horizon) is represented by seasonal regression models (mainly of linear nature), led by Autoregressive Integrated Moving Average (ARIMA) and state space models which have their origins in the 1980s.<sup>6</sup> Besides dummy-type and sinusoidal variables to model seasonality, these models usually include exogenous regressor variables to take environmental changes into account.<sup>7</sup> Early results in this category were provided by Ackermann (1985), Gupta (1985), Schneider et al. (1985), or Engle et al. (1992). At a later stage, Ramanathan et al. (1997) tested rich linear regression models in a Northamerican area for single hours by accounting for deterministic (i.e. seasonal), meteorological, current load, and past errors information. They report mean absolute percentage errors (MAPE) between 3.84% and 5.66%. Linear regression models were also used by Feinberg et al. (2003) including characteristic data on temperature, humidity, wind speed, sky cover, and sunshine. Analyzing several areas in the north-eastern part of the US they report an  $R^2$  of 0.87 to 0.97. Nowicka-Zagrajek & Weron (2002) applied ARMA models to deseasonalized data of the Californian market ending up at a low MAPE of slightly below 1.7%. Quite recently, Hinman & Hickey (2009) tested ARMAX and ARMA models with exogenous weather variables on an area in the US Midwest reporting a MAPE of 3.27-4.54%.

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<sup>3</sup>See Soares & Souza (2006).

<sup>4</sup>The mean absolute percentage error (MAPE) is defined as  $\frac{1}{N} \sum_{n=1}^N \left| \frac{y_n - \hat{y}_n}{y_n} \right|$ .

<sup>5</sup>See Willis (1996).

<sup>6</sup>See Taylor et al. (2006).

<sup>7</sup>Models including exogenous regressors are labelled by adding 'X' to the abbreviation of the model type (e.g. ARX, ARMAX, ARIMAX).

While most load forecasting efforts have been done on US markets, there is much less published work on the German electricity market so far. Viehmann (2011) applied an ARMA model to predict load reporting a MAPE of 2.3-5.3% for the time period between October 2005 and September 2008.<sup>8</sup>

Besides regression models, neural networks have often been used to predict load.<sup>9</sup> The purpose of this method, which was increasingly introduced in the 1990s, is to better incorporate nonlinear dependencies in load data.<sup>10</sup> In terms of comparative analyses, Taylor et al. (2006) have tested neural networks and five other univariate methods, including ARMA and other regression models, for load in Rio de Janeiro (observation period in 1996) and Wales/England (observation period in 2000). They conclude that overall, ARMA models perform better than neural networks although the latter are capable to account for highly sophisticated non-linear relationships. This finding is confirmed by Darbellay & Slama (2000) who state that often, linear representations are sufficient to describe load dynamics. They perform their analyses on the Czech electricity market.

Although accuracy of forecasts in existing research is often at rather impressive levels, it has to be considered that in many cases, very short time frames are used and only in-sample estimates are reported. Nevertheless, concluding from existing research, an appropriately calibrated ARMAX-type model should deliver a forecast accuracy within a MAPE range of 4 to 8 percent for out-of-sample data.

### 4.1.3 Preliminary Data Analysis

The total vertical system load in Germany does not include wind energy since electricity from wind power plants is directly fed into the medium voltage grid. To have an as precise as possible approximation for electricity demand, we add total wind infeed to the vertical system load reported by the four transmission system operators.<sup>11</sup> Thus, we can formulate the demand variable as follows:

$$\text{Demand}(t) = \text{Total Vertical System Load}(t) + \text{Total Wind Infeed}(t) \quad (4.1)$$

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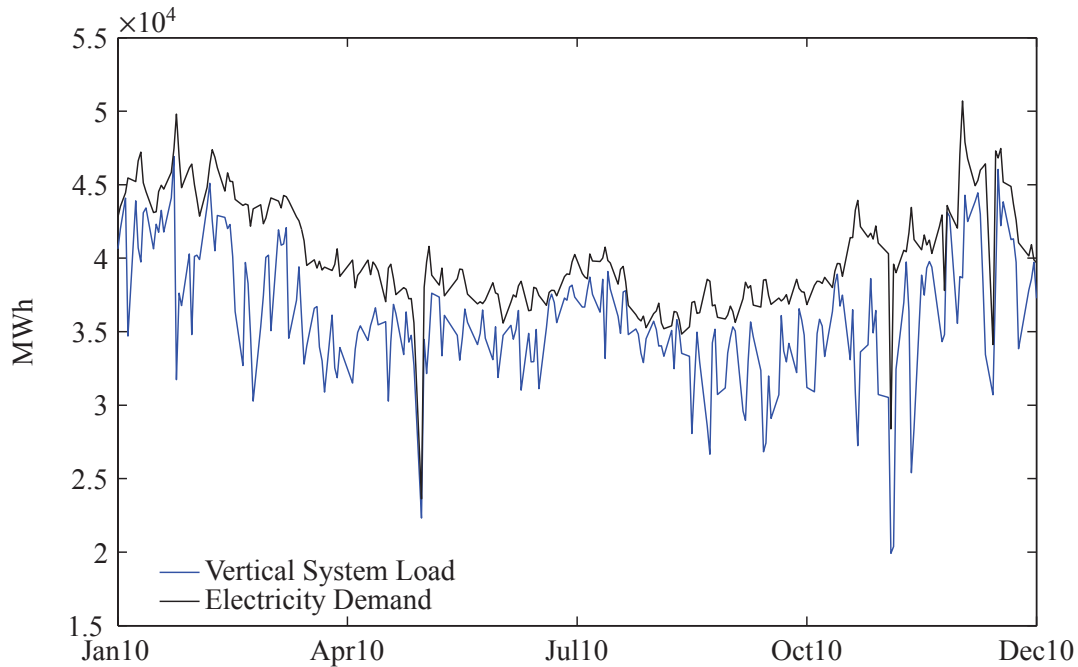
<sup>8</sup>Note that the main purpose of the paper was not to develop or test a load forecast model.

<sup>9</sup>See for example Darbellay & Slama (2000), Hippert et al. (2001), or Carpinteiro et al. (2004).

<sup>10</sup>See Darbellay & Slama (2000).

<sup>11</sup>Electricity from solar power plants is fed into the medium voltage grid as well. However, in this case there is no consistent dataset available for the entire investigated period. Given the increasing importance of photovoltaics, a consideration in future work would be advisable.

Figure 4.1 illustrates the total vertical system load and electricity demand as defined in (4.1) for hour 12 during 2010.<sup>12</sup>



**Figure 4.1:** Total vertical system load (of all four TSO grid zones) and electricity demand between January 1, 2010 and December 31, 2010 for hour 12. Weekends, holidays, and bridge days are excluded.

To gain more insight into the structural properties of electricity demand data we look at the autocorrelation function (ACF)

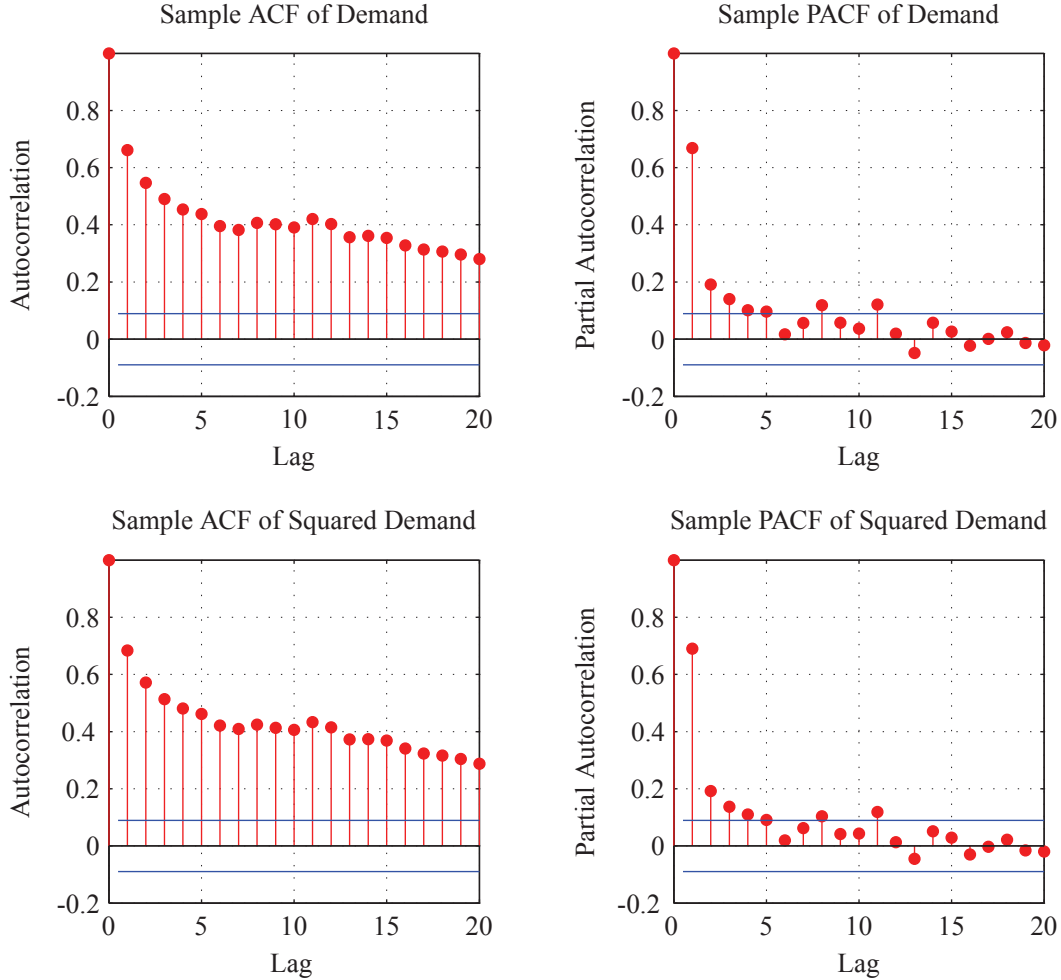
$$\rho_k = \frac{\text{cov}\{D_t, D_{t-k}\}}{\text{var}\{D_t\}} = \frac{\gamma_k}{\gamma_0} \quad (4.2)$$

for lags  $k = 1, \dots, 20$ . In addition to this, we investigate the partial autocorrelation function (PACF) which examines the correlation between  $D_t$  and  $D_{t-k}$  after adjusting for correlation effects caused by intermediate values  $D_i, 0 < i < k$ .<sup>13</sup> The ACF and PACF of demand  $D_t$  and squared demand  $D_t^2$  up to lag 20 for hour 12 are plotted in figure 4.2. The plots indicate clear persistence in both, first and second moment,

<sup>12</sup>Descriptive statistics for all hours can be found in table A.1 in the appendix.

<sup>13</sup>For a detailed introduction see for example Verbeek (2008).

speaking in favor of an ARMA/GARCH structure to model conditional mean and variance.



**Figure 4.2:** Sample autocorrelation function (ACF) and sample partial autocorrelation function (PACF) of demand and squared demand for hour 12 computed in a preliminary step before the calibration of the demand model. The observation period is the in-sample dataset starting on January 1, 2010 and ending on December 31, 2011. The horizontal lines indicate 95% confidence bounds.

In addition to graphical analyses we perform a Ljung-Box Q-Test with test statistic

$$Q = n(n+2) \sum_{k=1}^h \frac{r_k^2}{(n-k)} \quad (4.3)$$

where  $n$  is the sample size,  $h$  denotes the number of autocorrelation lags considered,

and  $r_k^2$  is the squared sample autocorrelation of demand at lag  $k$ .<sup>14</sup> In case test-statistic  $Q$  is above the applicable critical value (derived from a  $\chi^2$ -distribution), the null hypothesis of no autocorrelation is rejected. We test lags 1, 5, 10, 15, and 20 for all hours. Test results<sup>15</sup> confirm the presence of autocorrelation in the data sample at all chosen lags uniformly for all hours. In all cases, the null hypothesis of zero autocorrelation can be rejected at the 99% significance level.

As a formal test for heteroskedasticity we perform Engle's ARCH test where squared demand is regressed on its  $h$  lags:

$$D_t^2 = c + \sum_{k=1}^h \alpha_k D_{t-k}^2 \quad (4.4)$$

To examine the null hypothesis of no heteroskedasticity, all  $\alpha_k, k = 1, \dots, h$  are tested to be jointly zero using a Lagrange multiplier (LM) test statistic which is derived from a maximum likelihood optimization problem and asymptotically  $\chi^2$ -distributed with  $h$  degrees of freedom.<sup>16</sup>

For all lags (1, 5, 10, 15, 20) and all hours of the day we can reject the null hypothesis of i.i.d. disturbances at the 99% significance level.<sup>17</sup> In addition to the graphical analysis, this supports a GARCH representation for volatility.

To conclude the preliminary data analysis we test whether hourly demand is stationary or whether it possesses a unit root. Therefore, we perform an augmented Dickey-Fuller test (ADF test) with one lag (motivated by the below applied AR(1) structure).<sup>18</sup> Results are presented in table A.6 in the appendix. For all hours, we can reject the null hypothesis of a unit root at a 95% significance level meaning that the data is stationary. Also when testing with five lags (motivated by the weekly cycle, results are not reported) we can reject the null hypothesis of non-stationarity for the majority of all hours.

#### 4.1.4 Model & Variable Selection

Based on the results of the existing literature as well as on preliminary data analysis we decide to apply an Autoregressive Moving Average model with exogenous regressors

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<sup>14</sup>See Box & Pierce (1970) and Ljung & Box (1978).

<sup>15</sup>See table A.2 in the appendix.

<sup>16</sup>See Engle (1982) and Hamilton (1994b).

<sup>17</sup>See table A.3 in the appendix.

<sup>18</sup>For details on the ADF testing procedure see Fuller (1976) and Dickey & Fuller (1979).



(ARMAX) to forecast hourly load in the German market. Given our findings from the preceding section, we additionally allow for a conditional variance by introducing a GARCH(1, 1) structure.

Our comprehensive model to estimate electricity demand reads as follows:

$$D_t = \alpha + \rho D_{t-1} + x_t' \beta + \varphi \epsilon_{t-1} + \epsilon_t \quad (4.5)$$

with  $\epsilon_t = \sqrt{\sigma_t^2} u_t$ ,  $u_t \sim N$  and a conditional variance process<sup>19</sup>

$$\sigma_t^2 = \omega + \phi \epsilon_{t-1}^2 + \psi \sigma_{t-1}^2 \quad (4.6)$$

In (4.5) vector  $x_t$  contains exogenous variables which will be introduced below. By estimating 24 individual load forecast models we account for intraday seasonality. To catch seasonal patterns over different weekdays and months we introduce dummy variables with values 1 or 0 for four out of five weekdays and for eleven out of twelve months. Environmental variables are used to cover influences caused by varying climatic conditions. We source all climatic data from the German Weather Service<sup>20</sup> which provides a rather comprehensive range of environmental figures at no charge. Climatic data in Germany is gathered by authorities at about 80 observation stations across the country. To obtain reasonable information on German electricity demand, we source and average data from four observation stations in Hamburg, Berlin Tempelhof, Duesseldorf, and at Munich Airport. We consider this choice a good combination to represent the biggest congested areas and, at the same time, geographically diverse regions in Germany. Specifically, we utilize the following environmental variables:

**Sunshine Duration** This variable denotes the average duration of sunshine in hours, measured 24 times across the day.

**Mean Degree of Cloud Cover** The mean degree of cloud cover is defined as the number of eighths to which the sky is covered by clouds, computed as the average of 24 measurements across the day.<sup>21</sup>

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<sup>19</sup>For a more detailed introduction to GARCH models, their formulation, and relevant conditions we refer to chapter 5.

<sup>20</sup>See Federal Ministry of Transport, Building and Urban Development (2012).

<sup>21</sup>For the mean degree of cloud cover, data from Berlin Tempelhof was replaced by data from the nearby observation station Berlin Tegel since the former stopped reporting respective information as of November 1, 2008.



**Maximum Air Temperature** This variable represents the average temperature in centigrade two meters above ground based on 24 measurements across the day.

**Mean Relative Humidity** Relative humidity is expressed as a percentage and computed as the average of 24 measurements across the day.

**Cooling Degree Days** The concepts of *cooling-degree-days* (CDD) and its counterparty *heating-degree-days* (HDD) are widely used measures which indicate the need of cooling and heating using outdoor temperatures. They are, among others, relevant quantities for temperature futures trading. Mathematically, CDD for a certain day  $t$  are defined as

$$CDD_t = \max(T_t - c, 0) \quad (4.7)$$

with  $T_t$  denoting the mean temperature on the relevant day computed as the average of the observed maximum and minimum temperature over a 24 hours cycle.<sup>22</sup> The comfort level  $c$  denotes the threshold above and below which cooling and heating activities are assumed to start. For the central European area a reasonable value for  $c$  is 18.3 centigrade.<sup>23</sup> HDD are accordingly defined as

$$HDD_t = \max(c - T_t, 0). \quad (4.8)$$

Often, cooling-degree-days and heating-degree-days are summed up resulting in a combined, more general temperature measure called *energy-degree-days* (EDD).

The following two variables have been excluded after preliminary data analyses:

**Heating Degree Days** Correlation analyses on the regressor variables have revealed a high correlation (close to 1 and -1, respectively) of heating-degree-days and maximum and minimum air temperature. This gives rise to multicollinearity which, as we are primarily interested in forecasting results and not in the interpretation of coefficients, is not a serious issue though. However, nearly perfect negative or positive correlations also imply that most (or nearly all) of the information incorporated in the variable is already provided by other factors. In the case at hand this seems obvious which is why we omit the variable.

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<sup>22</sup>See Benth et al. (2008).

<sup>23</sup>See for example Bloechlinger (2008).

**Minimum Air Temperature** Similar to heating degree days, minimum air temperature is excluded due to very high correlations with the mentioned variables. Among the two variables maximum and minimum air temperature, the first proves to have the higher explanatory power over different hours of the day which is the reason why it is kept in the model.

For each hour of the day we calibrate a dedicated model. In a preliminary step we have eliminated exogenous variables (dummies and climatic variables) which were not significant on a 90% level. Accordingly, we have a different set of explanatory variables for every model.

### 4.1.5 Empirical Results

Tables 4.1 and 4.2 depict mean absolute percentage errors of the estimated models. The in-sample period spans over entire 2010 and 2011. The out-of-sample period reaches from January 1, 2012 to April 30, 2012. We eliminate all weekends, holidays, and bridge days. As can be seen from the tables, the in-sample MAPE is between 3.0% and 5.0% with a mean of 3.8%. Highest deviations from actual demand figures we note for noon peak hours. Looking at out-of-sample results it is apparent that errors are consistently higher. We note a MAPE range of 2.1% to 8.6% with a mean of 4.8% which corresponds to what has been reported by other researchers applying similar approaches. The errors are highest for hours when demand is higher and more volatile, which is especially over noon. For hours when load is at lower levels and more stable, the model fit proves to be clearly better. Without hours 12 to 15, the mean out-of-sample MAPE amounts to only 4.1%. Figure 4.3 illustrates the out-of-sample model fit for hour 12 where the performance is relatively weak and for hour 18 where the overall performance is rather good.

Our out-of-sample results are not completely out-of-sample in a strict sense. We use environmental data which is only available after the demand of the specific hour is known. However, as with technologies available today, temperature data can be predicted rather accurately 24 hours in advance, we deem this approach justifiable.

Exemplarily, ACF and PACF of the standardized innovations (corresponding to  $u_t$  in the error specification of equation 4.5) and their squares for hour 12 are plotted in figure 4.2. The plot gives evidence that serial correlations are no longer present. As for pre- and post-estimation autocorrelation and partial autocorrelation analysis, we receive similar results for all hours of the day.

To receive additional confirmation that our model is appropriate, we repeat statistical tests for autocorrelation (Ljung-Box Q-test) and heteroskedasticity (Engle's

ARCH test).<sup>24</sup> As for autocorrelation we can no longer reject the null hypothesis for most hours at the first lag whereas at higher lags, there is still evidence for some moderate remaining persistence, especially for noon peak hours. As for Engle's ARCH test results, we can no longer reject the null hypothesis of homoskedasticity for most hours at most lags. The latter results are even clearer than results from autocorrelation tests.

Overall, based on error analysis, graphical analysis, and formal statistical tests we consider the suggested forecast models for demand to be appropriate for our purpose.<sup>25</sup>

	MAPE		MAPE
Hour 1	0.036	Hour 13	0.045
Hour 2	0.039	Hour 14	0.047
Hour 3	0.040	Hour 15	0.050
Hour 4	0.039	Hour 16	0.046
Hour 5	0.039	Hour 17	0.036
Hour 6	0.038	Hour 18	0.031
Hour 7	0.041	Hour 19	0.033
Hour 8	0.042	Hour 20	0.033
Hour 9	0.036	Hour 21	0.033
Hour 10	0.037	Hour 22	0.031
Hour 11	0.038	Hour 23	0.030
Hour 12	0.040	Hour 24	0.032

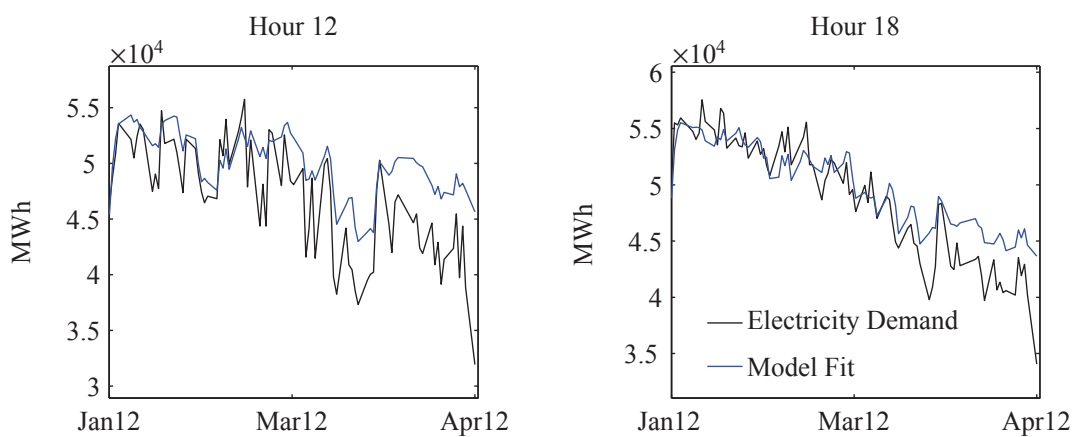
**Table 4.1:** *In-sample results (mean absolute percentage errors) for estimated demand ARMA-GARCH regression models.*

<sup>24</sup>Detailed test results can be found in tables A.4 and A.5 in the appendix.

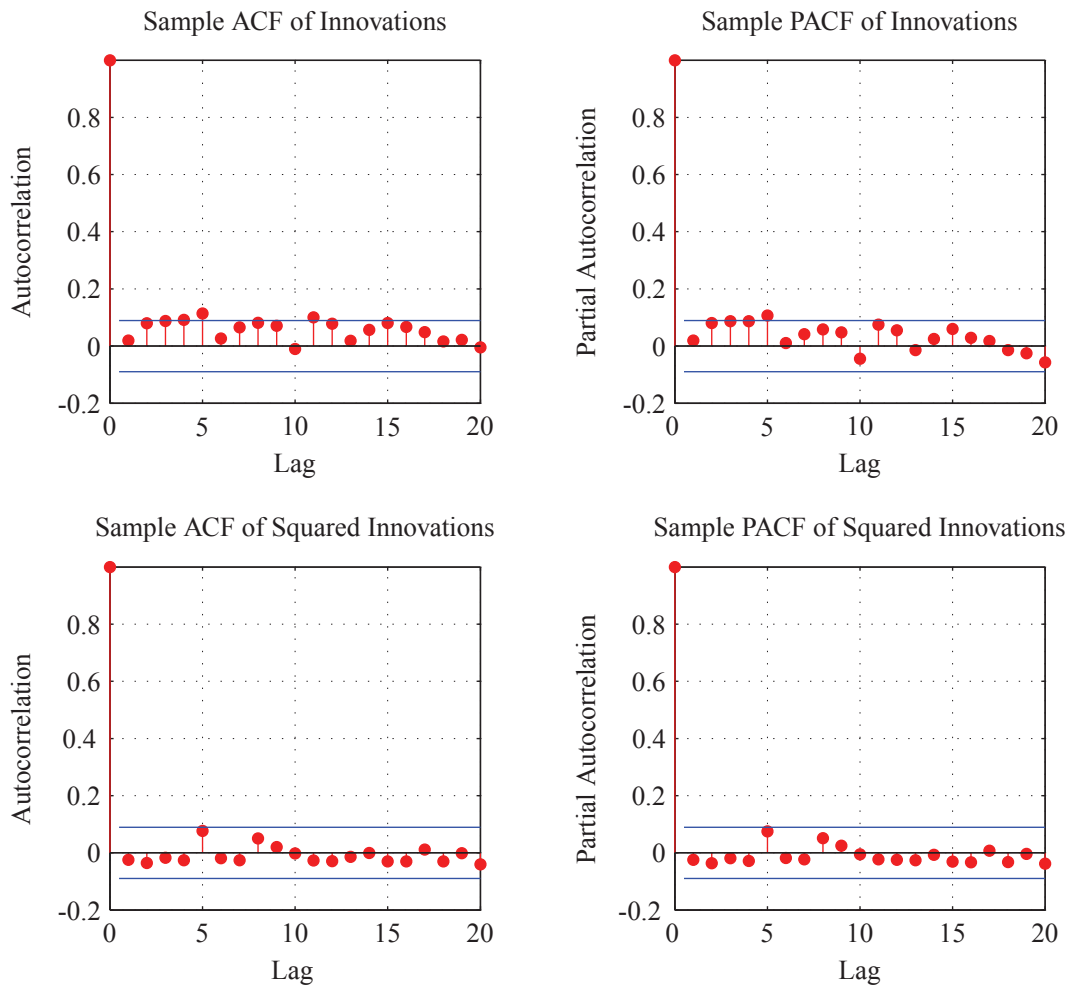
<sup>25</sup>The estimation of the demand forecast model is not the focus of the thesis. We will therefore not report and discuss estimation results of single hours in details.

	MAPE		MAPE
Hour 1	0.039	Hour 13	0.083
Hour 2	0.039	Hour 14	0.086
Hour 3	0.037	Hour 15	0.082
Hour 4	0.037	Hour 16	0.072
Hour 5	0.032	Hour 17	0.057
Hour 6	0.026	Hour 18	0.046
Hour 7	0.021	Hour 19	0.037
Hour 8	0.027	Hour 20	0.031
Hour 9	0.040	Hour 21	0.034
Hour 10	0.056	Hour 22	0.027
Hour 11	0.074	Hour 23	0.038
Hour 12	0.080	Hour 24	0.044

**Table 4.2:** *Out-of-sample results (mean absolute percentage errors) for estimated demand ARMA-GARCH regression models.*



**Figure 4.3:** *Out-of-sample fit of demand ARMA-GARCH regression model for hours 12 and 18.*



**Figure 4.4:** *Sample autocorrelation function and sample partial autocorrelation function of standardized innovations and their squares of the demand ARMA-GARCH regression model for hour 12. The observation period is the in-sample dataset starting on January 1, 2010 and ending on December 31, 2011. Horizontal lines indicate 95% confidence bounds.*

## 4.2 Supply

### 4.2.1 The Merit Order Curve

When discussing electricity markets it is imperative to have an understanding of the merit order concept which plays a central role in the formation process of electricity spot prices. The merit order curve can be defined as the sorted marginal cost curve of electricity production which defines the price of electricity given a certain quantity.<sup>26</sup> A schematical merit order curve is shown in figure 4.5. Different production technologies are ordered according to their marginal costs starting with the least expensive technologies at the very left. The figure depicts a demand curve for a low demand and for a peak demand scenario. The type of production technology in whose area the intersection of the demand and supply (merit order) curve is located constitutes the price setting technology. Its marginal costs determine the price that all suppliers receive (including power plants with much lower production costs). During hours of lower demand, lignite or hard coal usually serve as the price setting technologies whereas during hours when demand is very high, the price is rather set by expensive gas or oil fired plants. Power plants which are located on the left of the merit order curve (except renewable energy plants) are so-called must-run capacities as they never go off-line due to very high ramp-up costs. In addition to the intermittent supply from renewable energy sources, these must-run capacities are the main reason for occasionally occurring negative prices.<sup>27</sup>

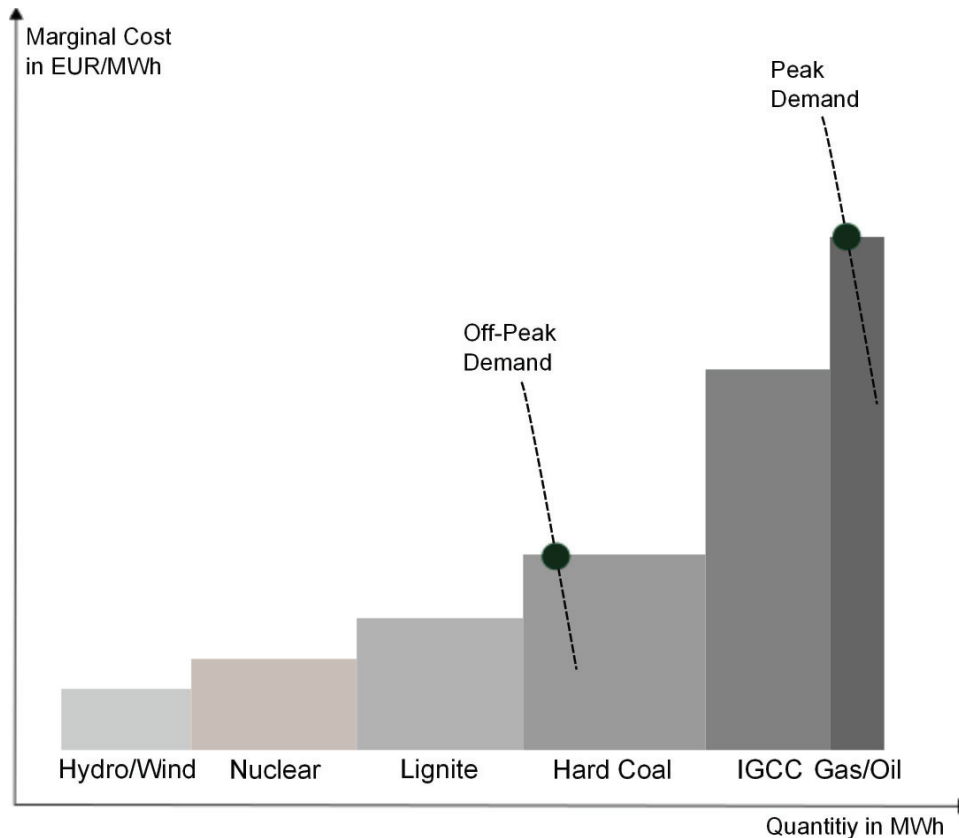
If a power plant located at the lower end of the merit order has to stop production because of technical failure, it needs to be replaced by the next available free capacity at a much higher point of the curve which is one of the reasons for temporary spikes in electricity prices. Sensfuss et al. (2008) mention that the installed capacity of renewable generation, the development of fuel prices, and prices for emission allowances are the factors which have the greatest impact on the current shape of the merit order curve. They discuss different scenarios by simulating changes in these factors.

In practice, the supply curve deviates from the theoretical merit order curve. The main reason for this is that besides production capacity owners, also traders with speculative motivations participate in the market who use to buy production capacities bilaterally or via forward markets in order to sell them at a profit in the day-ahead

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<sup>26</sup>See for example von Roon & Huck (2010).

<sup>27</sup>See Stroebele et al. (2010).



**Figure 4.5:** Simplified illustration of the merit order curve and electricity spot price formation. The two points at the intersections of the dashed demand curves and the merit order (supply) curve represent electricity spot prices (market clearing prices) for two given levels of demand (peak and off-peak). IGCC: integrated gasification combined cycle.

auction.<sup>28</sup>

The different production technologies and their role in the price formation process are discussed in more detail in the subsequent sections.

### 4.2.2 Fuels

The slope of the merit order curve in the German electricity market, and therefore also spot prices, are heavily determined by fuels. Burger et al. (2007) mention that the costs of fuels, which are the main driver of short-run variable power generation costs, can be derived from market fuel prices and transport costs from the extraction place to the power plant. In this section we will briefly discuss the fuels which determine

<sup>28</sup>See von Roon & Huck (2010).

German electricity prices and elaborate on how we will account for them in the models to be estimated.

#### 4.2.2.1 Coal

Coal used for electricity production can be classified into two groups, namely lignite (a.k.a. brown coal) and hard coal (a.k.a. stone coal). *Lignite* is the most prominent primary energy carrier in Germany with a share of 25 percent in 2009 and is exclusively domestically mined.<sup>29</sup> It is a low quality coal with comparatively high specific CO<sub>2</sub> emissions and investment costs but rather low fuel costs (compared to other thermal power plants).<sup>30</sup> As transport costs are high, lignite is usually transformed into electricity at the place where it is mined. It is not traded on markets and its inflation-adjusted costs can be assumed constant.<sup>31</sup> As lignite power plants cannot be operated flexibly and have rather long ramp-up times, they are used to cover basic load.<sup>32</sup> *Hard coal* is another important fuel representing 18 percent of all primary energy carriers in Germany<sup>33</sup> and, unlike lignite, is mainly imported<sup>34</sup>. Burger et al. (2007) report that due to higher CO<sub>2</sub> costs and fuel costs, the relative importance of coal in general has been declining over the last decades. Nevertheless, in times of lower electricity demand, hard coal remains the price setting technology.<sup>35</sup> Although ramp-up times are shorter than for lignite power plants, they are still clearly longer than for gas power plants.<sup>36</sup>

#### *Variable Selection*

As a variable representing the coal price we choose Amsterdam-Rotterdam-Antwerp (ARA) futures contracts. They are daily traded at the EEX with financial settlement and times-to-maturity of up to six months.<sup>37</sup> For every relevant delivery date we choose the current settlement price of the front-month futures contract which we

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<sup>29</sup>See Federal Ministry of Economics and Technology (2011).

<sup>30</sup>See Burger et al. (2007).

<sup>31</sup>See Konstantin (2007).

<sup>32</sup>See Stroebele et al. (2010).

<sup>33</sup>See Federal Ministry of Economics and Technology (2011).

<sup>34</sup>See Konstantin (2007).

<sup>35</sup>See Sensfuss et al. (2008).

<sup>36</sup>See Stroebele et al. (2010).

<sup>37</sup>See European Energy Exchange AG (2010).



consider a reasonable representation of the prevailing price level for hard coal in Germany.

#### 4.2.2.2 Gas

In 2009 gas represented 16 percent of all primary energy carriers in Germany.<sup>38</sup> Due to their high operational flexibility and short ramp-up times, gas power plants are price setting during peak hours when demand for electricity is high.<sup>39</sup> Konstantin (2007) notes that it takes only a few minutes from cool start to maximum power output. Because investment costs are comparatively low, due to the expansion of the continental gas pipeline system, because its relative environmental sustainability, and because the development of more efficient technologies, the importance of gas in electricity production has increased over the last decades.<sup>40</sup>

##### *Variable Selection*

As a variable representing the gas price we use the NCG Day Ahead Natural Gas Spot Price<sup>41</sup> which is the price at the NCG (NetConnect Germany) hub in Southern Germany, serving as the reference gas price for the German market. We use daily last prices sourced from financial data provider Bloomberg. Sensfuss et al. (2008) report that the ratio of coal prices to gas prices is of high relevance as well since it has a significant impact on the slope of the market supply curve. In addition to coal and gas prices, we will therefore introduce the quotient of the two variables.

#### 4.2.2.3 Oil

Compared to the aforementioned fuels, oil has a rather low impact on the merit-order effect.<sup>42</sup> According to Burger et al. (2007), the negligible role of oil in German and European power generation is mainly due to environmental legislation. They mention that in Europe, Italy is the only country where oil fired plants play a major role. The reason why we want to consider this fuel is its impact on transportation costs which is particularly relevant in the case of imported hard coal.

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<sup>38</sup>See Federal Ministry of Economics and Technology (2011).

<sup>39</sup>See Sensfuss et al. (2008).

<sup>40</sup>See Burger et al. (2007).

<sup>41</sup>Bloomberg Ticker: EGTHDAHD Index

<sup>42</sup>See Sensfuss et al. (2008).

### *Variable Selection*

As a variable we use the active ICE Brent Crude futures contract<sup>43</sup> which is the oil price reference in Europe. We use daily closing prices sourced from Bloomberg.

### **4.2.3 Emission Allowances**

As a result of the Kyoto Protocol, which aims to combat greenhouse gas emissions in involved countries, carbon dioxide (CO<sub>2</sub>) producing companies are obliged to buy emission allowances (a.k.a. 'CO<sub>2</sub> certificates'). Starting in 2005, EU emission allowances have been traded at the EEX for more than seven years.<sup>44</sup> Especially coal fired power plants (but also many gas fired plants to some lower extent) belong to the group of companies that require emission allowances for energy production. As they represent a considerable part of the German power plant portfolio, prices of CO<sub>2</sub> certificates have a significant impact on electricity spot prices in general. In times of high prices for emission allowances, a phenomenon described as *fuel switch* can be observed. It is basically a change in the merit order with electricity from more efficient gas power plants becoming cheaper relative to electricity from CO<sub>2</sub> intense coal fired plants.<sup>45</sup>

### *Variable Selection*

To represent current costs of emission allowances we select the EEX Carbon Index (Carbix) which is traded at the EEX. The index price is daily auctioned at 10:30 am and serves as the reference price for emission allowances.<sup>46</sup>

### **4.2.4 Wind**

Among all renewables, wind energy is the one which has experienced highest attention in Germany over the past years and still proves to be the green energy carrier with highest growth rates.<sup>47</sup> Today Germany possesses one of the largest markets for

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<sup>43</sup>Bloomberg Ticker: COA Comdty

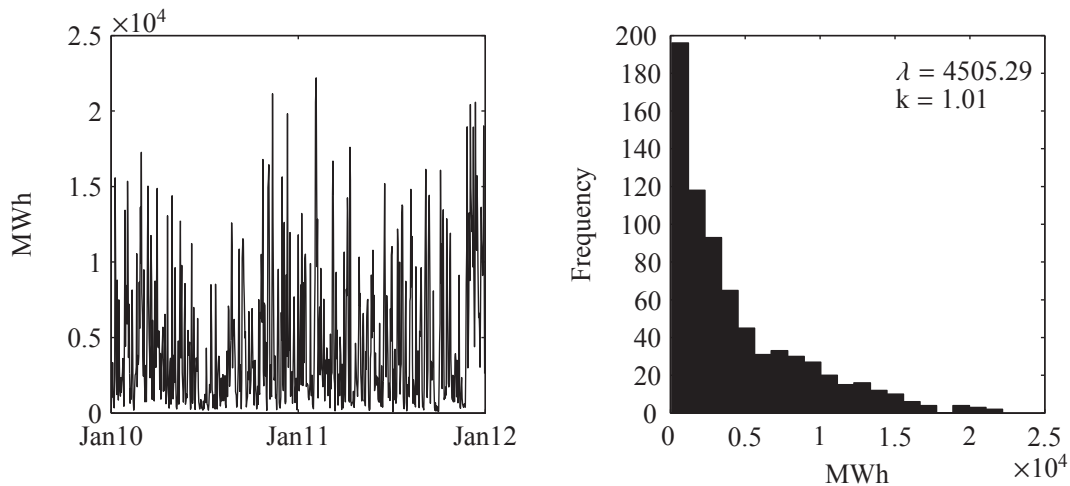
<sup>44</sup>See European Energy Exchange AG (2011a).

<sup>45</sup>See for example Sensfuss et al. (2008) or Liebau & Stroebele (2011).

<sup>46</sup>See European Energy Exchange AG (2011b).

<sup>47</sup>See Stroebele et al. (2010).

wind power globally<sup>48</sup> and electricity generated by wind represented about 40% of all energy from renewable sources in 2009<sup>49</sup>. As a comparison, electricity from water represented 20% and electricity from photovoltaic technology represented nearly 7%. Already in 2007, almost 10% of all electricity production in Germany was represented by wind power.



**Figure 4.6:** Actual wind electricity infeed and corresponding frequency distribution for hour 12 between January 1, 2010 and December 31, 2011. All weekdays are included.

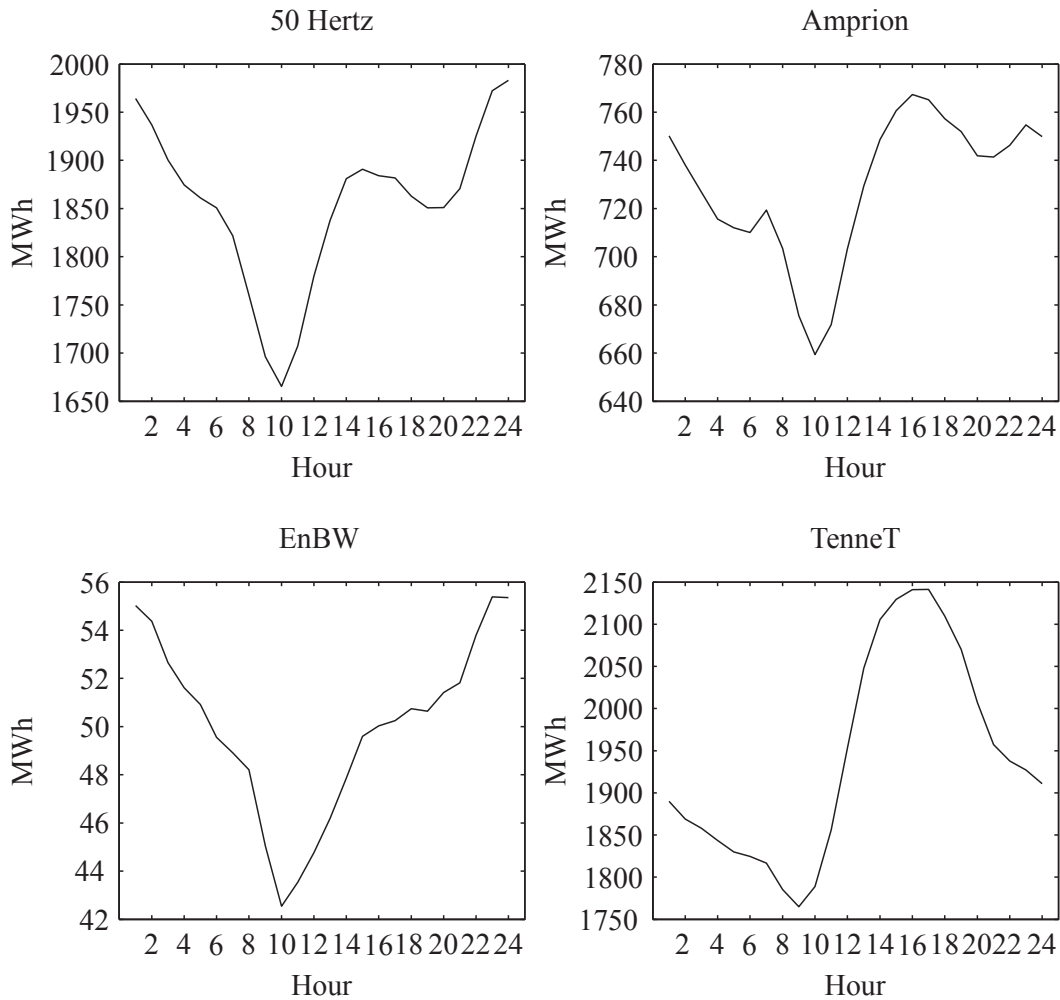
Wind energy has a set of distinct characteristics. Its supply for example is, like energy from photovoltaics, significantly determined by meteorological conditions.<sup>50</sup> Moreover, its share of fixed costs compared to variable costs is different from electricity from thermal power plants. About 80% of total wind energy costs are fixed and principally composed of expenses for turbines, foundations, and grid connection.<sup>51</sup> In contrast to this, variable and thus also marginal costs are comparatively low. Leuthold et al. (2008) compare costs of wind power with other energy sources and report that with marginal generation costs of about EUR 4.00 per MWh, once installed, wind power plants produce at much lower costs than for instance nuclear power plants (~EUR 10 per MWh), lignite (~EUR 15 per MWh), or gas fired plants (~EUR 40 per

<sup>48</sup>See Grothe & Schnieders (2011).

<sup>49</sup>See Wernsmann & Wernsmann (2009).

<sup>50</sup>See Grothe & Schnieders (2011).

<sup>51</sup>See Blanco (2009).



**Figure 4.7:** Average actual wind electricity infeed across the day for all four grid zones. Hourly averages are computed using data between January 1, 2010 and December 31, 2011, including all weekdays.

MWh). From an ecological perspective, another benefit of wind energy is that it is produced nearly carbon free.<sup>52</sup>

German wind power plants can be split into two categories, namely onshore and offshore plants. Whereas generation conditions are better offshore, these plants require significantly higher investments and thus produce at higher costs than onshore plants.<sup>53</sup> Geographically, wind is mainly produced on the coastline and in Eastern Germany.

Wind electricity which is fed into the German grid features certain seasonal patterns due to its dependency on meteorology. Figure 4.7 depicts the average wind electricity

<sup>52</sup>See Traber & Kemfert (2011).

<sup>53</sup>See Blanco (2009).

infeed into the medium voltage grid by the four transmission system operators in their respective areas for 2010 and 2011. In all four cases, infeed tends to be higher in early morning hours and during afternoon hours. For TenneT, which has a high number of wind power plants located at the North Sea coast, the afternoon spike is more pronounced. The levels of TenneT and 50Hertz (which operates the grid in the former German Democratic Republic) show that, as mentioned above, the biggest share of wind electricity is generated on the coastline and in Eastern Germany.

Figure 4.6 depicts some statistical properties of electricity from wind power plants. The left chart presents wind electricity infeed between January 1, 2010 and December 31, 2011. It is obvious that wind infeed has a high volatility. Furthermore, annual seasonality patterns are apparent with wind infeed which tends to be higher during winter months than during summertime. The right chart depicts the frequency distribution for wind infeed for hour 12 over the same time period. Wind infeed proves to be Weibull distributed with a scale parameter  $\lambda$  and a shape parameter  $k$ .<sup>54</sup> The empirical distribution has mass at low levels of wind infeed close to zero and is clearly skewed to the right with occasionally very high wind infeed. In general, these patterns look similar for all hours of the day.

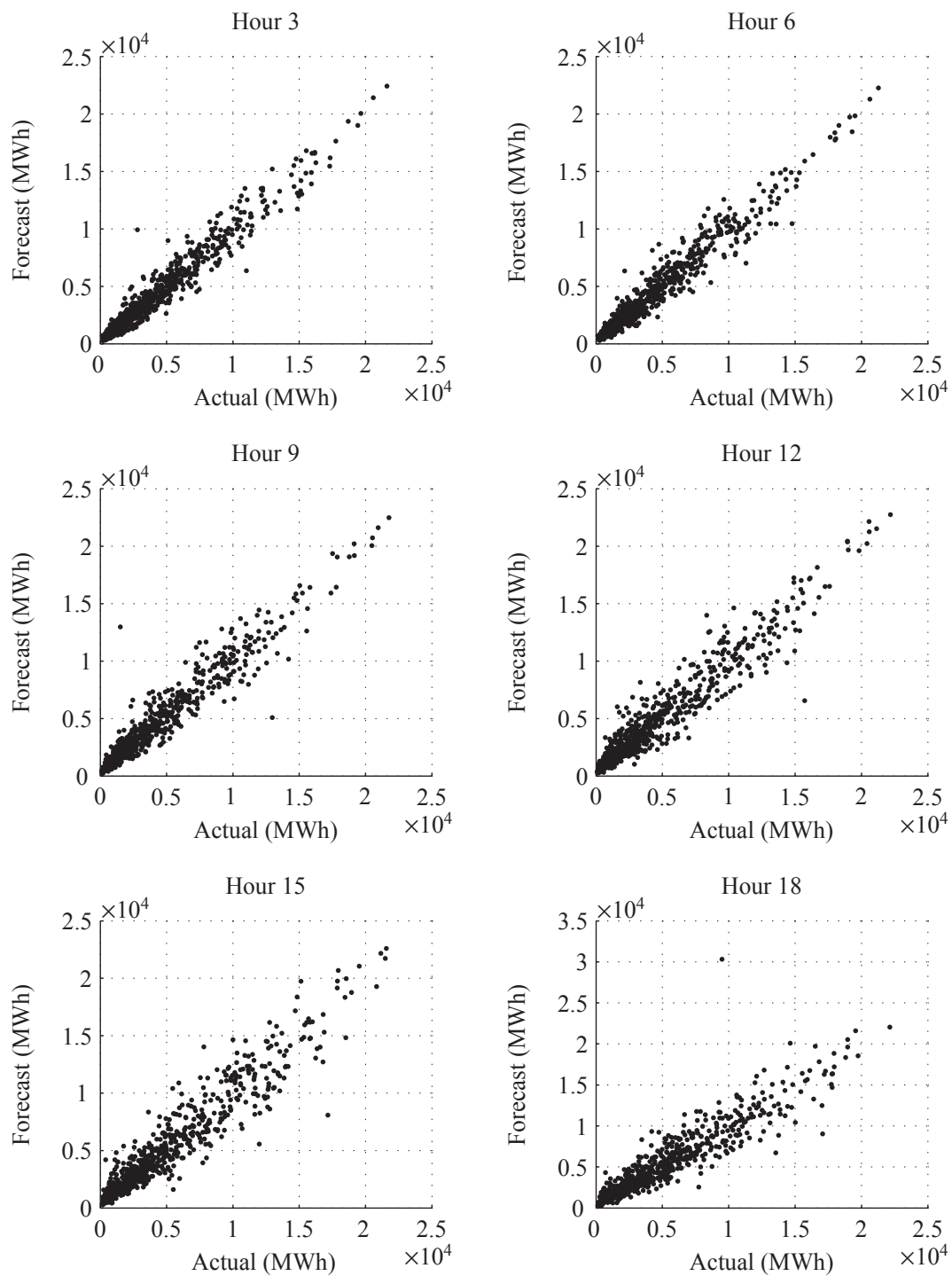
### *Variable Selection*

As a variable representing electricity from wind we incorporate the expected wind (electricity) infeed<sup>55</sup> which is published on the day before the delivery by the four TSOs takes place. We define our wind forecast variable, denoting the total expected wind electricity fed into the German grid, as the sum of the four forecasts. All TSOs publish their forecasts daily in the late afternoon which is after the day-ahead electricity price auction has taken place. Hence, from a practical perspective, the inclusion of this variable could be criticized. However, it is known that market participants use wind electricity forecasts which are offered by a variety of private institutions earlier in the day. In our opinion it is therefore reasonable to use the later in the day published (official) number as a realistic approximation for non-public forecasts which are available before the auction. Our results shall confirm the appropriateness of this approach. Forecasts of wind electricity infeed prove to be rather precise. Figure 4.8 plots forecasts against actual infeed for a selection of six hours across the day.

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<sup>54</sup>For a theoretical introduction into the Weibull distribution see Weibull (1951).

<sup>55</sup>Note that we will use the terms 'expected wind (electricity) infeed' and 'wind forecast' likewise.



**Figure 4.8:** Predicted and actual wind electricity infeed (sum of all four grid zones) for different hours of the day. The observation window starts on January 1, 2010 and ends on December 31, 2011, including all weekdays.

### 4.2.5 Power Plant Availability

Another factor which impacts the supply side is power plant availability. If power plants with representative production volumes have to go off-line due to reparation or maintenance, this can have considerable implications on the price building process depending on which technologies are affected and the current level of demand.

#### *Variable Selection*

The EEX Transparency Platform, which is a joint venture of the EEX and the transmission system operators, publishes, among others, various data on installed and available capacities. Although these publications are voluntary, participating companies have tripled in 2010 and by the end of the year represented 89 percent of all relevant companies.<sup>56</sup> Thus, the numbers provided can be considered a reasonable approximation for the entire market. We use ex ante expected power plant availability as an explanatory variable for our models. This figure is daily published at 10:00 am for a rolling window including the next 365 days.<sup>57</sup>

### 4.2.6 Omitted Variables

#### 4.2.6.1 Nuclear Energy

We do not consider a variable representing electricity supplied by nuclear power plants. Nuclear power plants are located at the very left of the merit order curve given the fact that they produce at rather low marginal costs. Moreover, even during hours of lowest demand levels, load is never below the total installed capacity of nuclear power plants.<sup>58</sup>

#### 4.2.6.2 Other Renewable Energies

The general support of renewables by German politics has not only affected wind energy production but also other sources of green energies. Among them, the most prominent exponents are biomass and photovoltaics. The reason why we do not consider these technologies is mainly because respective data is not available for the entire time period we are interested in and/or because published data is still incomplete.

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<sup>56</sup>See European Energy Exchange AG (2011a).

<sup>57</sup>See EEX Transparency Platform (2012).

<sup>58</sup>See r2b research to business energy consulting (2011).



## 4.3 Others

### 4.3.1 Lag Spot Price

The introduction of the spot price for the same hour on the previous relevant delivery day is motivated by two main reasons. First, information on the current market situation may not entirely be reflected by the chosen fundamental variables. Especially strategic and speculation-based behavior of traders is difficult to catch by fundamental variables. We believe that at least part of such information may be reflected in the latest spot price level. Second, a motivation of using lagged spot prices is to deal with potential autocorrelation problems.

### 4.3.2 Risk Considerations

To reflect risk behavior of market participants we introduce historical spot price volatility as an explanatory variable. We define it as the standard deviation of the prices of the last five relevant delivery days. Doing so, we suppose that traders react and adapt their bidding behavior according to the level of price variability over the past full working week cycle.

### 4.3.3 Seasonality Function

Researchers often work with deseasonalized time series when modelling electricity spot prices meaning that they estimate the (deterministic) seasonal component which is then deducted from the actual spot price in a preliminary step.<sup>59</sup> As an alternative to this, one can allow for seasonality in the models directly. We follow the latter approach and, in doing so, integrate seasonality in two ways. First, seasonality is implicitly incorporated in various fundamental variables (mainly in expected demand and fuel prices). Second, we additionally include a sinusoidal seasonality function into the models which we will find to be highly significant for almost all hours.<sup>60</sup>

$$\cos\left(\frac{2\pi t}{T} - \vartheta\right) \quad (4.9)$$

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<sup>59</sup>See for example Bierbrauer et al. (2007).

<sup>60</sup>We do not include dummy variables, which is regularly done by other researchers, because this would significantly increase the number of regressors and since in preliminary analyses we have found that introducing monthly dummies does only marginally improve results.



We calibrate  $\vartheta$  in a preliminary step using climatic data. Thus, we implicitly assume that the annual seasonal cycle is located according to the outside temperature. In the preliminary estimation we follow an approach developed by Stolwijk et al. (1999). They show that in a cosine function of the form

$$f(t) = \alpha \times \cos\left[\left(\frac{2\pi t}{T}\right) - \vartheta\right], \quad (4.10)$$

the location parameter of the sinusoidal curve  $\vartheta$  can be obtained via a standard regression model. We first need to estimate the following model:

$$f(t) = \beta_1 \times \sin\left(\frac{2\pi t}{T}\right) + \beta_2 \times \cos\left(\frac{2\pi t}{T}\right) \quad (4.11)$$

The authors show that the two extreme values of the cosine function in 4.10 are obtained at the solutions of

$$\tan\left(\frac{2\pi t}{T}\right) = \frac{\beta_1}{\beta_2}. \quad (4.12)$$

In order to find  $t$ , the following equation has to be solved:

$$t = \arctan\left(\frac{\beta_1}{\beta_2}\right) \times \frac{T}{2\pi} \quad (4.13)$$

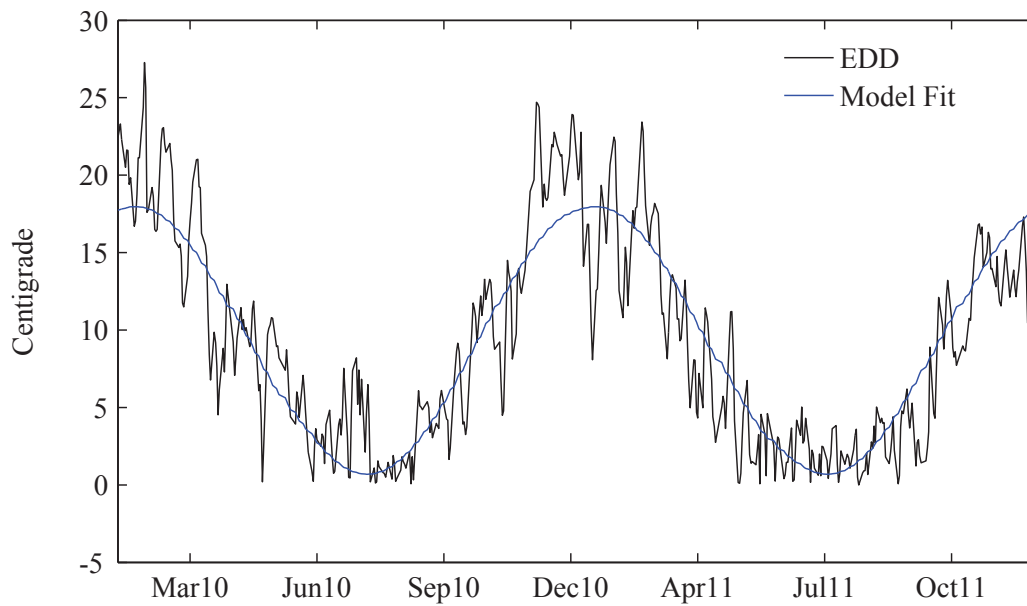
For the case that  $\beta_1/\beta_2$  is positive,  $t$  represents the first extreme and the second extreme is located at  $t + T/2$ . If, however,  $\beta_1/\beta_2$  is zero or negative, the extremes are located at  $t + T/2$  and  $t + T$ . For both cases, if  $\beta_1$  is positive, the first extreme constitutes a maximum,  $t_{max}$ , and the second constitutes a minimum,  $t_{min}$ . If  $\beta_1$  is zero or negative, the opposite applies. Finally, the shifting factor  $\vartheta$  can be obtained through

$$\vartheta = \frac{2\pi t_{max}}{T}. \quad (4.14)$$

If amplitude  $\alpha$  in 4.10 is required, it can be calculated as follows:

$$\alpha = \sqrt{\beta_1^2 + \beta_2^2} \quad (4.15)$$

To obtain the deterministic seasonal trend which we will enter as an exogenous variable into our models, we calibrate  $\vartheta$  on energy degree days (EDD) data as introduced in section 4.1.4. By this, we implicitly assume, as addressed above, that the annual seasonality cycle in electricity prices is driven by temperature (estimation results will confirm that this assumption is reasonable). As temperature data is available with daily granularity, we only need to calibrate one shift factor  $\vartheta$  which shows to be 0.2553. Figure 4.9 depicts EDD data and the calibrated cosine function over the span of the in-sample dataset from January 1, 2010 to December 31, 2011.



**Figure 4.9:** *Fit of the sinusoidal seasonality function to energy degree days (EDD) for the observation window between January 1, 2010 and December 31, 2011. Weekends, holidays, and bridge days are excluded.*

#### 4.4 Overview

Table 4.3 summarizes all fundamental variables and corresponding data sources we will use to model electricity spot prices. Moreover, abbreviations and synonyms which we will regularly use are presented for convenience.

Table 4.4 defines whether the respective variables are available with daily or with hourly granularity. In case of daily granularity, the same data is used in all 24 hourly models.

<b>Variable</b> <i>Abbreviations</i>	<b>Description</b>	<b>Data Source</b>
<b>Lag Spot Price</b> <i>Spot(-1)</i>	Market clearing price for the same hour of the last relevant delivery day	European Energy Exchange: <a href="http://www.eex.com">http://www.eex.com</a>
<b>Average Lag Spot Price</b> <i>Av. Spot(-1)</i>	Average market clearing price across all 24 hours of the last relevant delivery day	European Energy Exchange: <a href="http://www.eex.com">http://www.eex.com</a>
<b>Spot Price Volatility</b> <i>Spot Vol.</i>	Standard deviation of market clearing prices for the same hour on the last five relevant delivery days	European Energy Exchange: <a href="http://www.eex.com">http://www.eex.com</a>
<b>Coal Price</b> <i>Coal</i>	Latest available price (daily auctioned) of the front-month Amsterdam-Rotterdam-Antwerp (ARA) futures contract before the electricity price auction takes place	European Energy Exchange: <a href="http://www.eex.com">http://www.eex.com</a>
<b>Gas Price</b> <i>Gas</i>	Last price of the NCG Day Ahead Natural Gas Spot Price on the day before the electricity price auction takes place	Bloomberg, Ticker: GTHDAHD Index
<b>Oil Price</b> <i>Oil</i>	Last price of the active ICE Brent Crude futures contract on the day before the electricity price auction takes place	Bloomberg, Ticker: COA Comdty
<b>Price for EU Emission Allowances</b> <i>CO<sub>2</sub> Price</i>	Latest available price of the EEX Carbon Index (Carbix), daily auctioned at 10:30 am	European Energy Exchange: <a href="http://www.eex.com">http://www.eex.com</a>
<b>Expected Wind (Electricity) Infeed</b> <i>Wind Forecast, Wind</i>	Sum of expected infeed of wind electricity into the grid, published by German transmission system operators in the late afternoon following the electricity price auction	Transmission system operators: <a href="http://www.50Hertz.com">http://www.50Hertz.com</a> , <a href="http://www.amprion.de">http://www.amprion.de</a> , <a href="http://www.transnetbw.de">http://www.transnetbw.de</a> , <a href="http://www.tennetso.de">http://www.tennetso.de</a>
<b>Expected Power Plant Availability</b> <i>Exp. PPA, Power Plant Av.</i>	Ex ante expected power plant availability for electricity production (voluntary publication) on the delivery day (daily granularity), daily published at 10:00 am	European Energy Exchange & transmission system operators: <a href="ftp://infoproducts.eex.com">ftp://infoproducts.eex.com</a>
<b>Expected Demand</b> <i>Demand, Dem</i>	Demand forecast for the relevant hour on the delivery day as modelled in section 4.1	Own data, German Weather Service: <a href="http://www.dwd.de">http://www.dwd.de</a>
<b>Lag Demand</b> <i>Demand(-1), Dem(-1)</i>	Sum of total vertical system load and actual wind infeed for the same hour on the last relevant delivery day	Transmission system operators: <a href="http://www.50Hertz.com">http://www.50Hertz.com</a> , <a href="http://www.amprion.de">http://www.amprion.de</a> , <a href="http://www.transnetbw.de">http://www.transnetbw.de</a> , <a href="http://www.tennetso.de">http://www.tennetso.de</a>
<b>Seasonality Function</b> <i>Seasonality</i>	Deterministic sinusoidal curve located using temperature data as outlined in section 4.3.3	Own data, German Weather Service: <a href="http://www.dwd.de">http://www.dwd.de</a>

**Table 4.3:** Summary of all fundamental variables we will use as exogenous variables in the various spot price models.

<b>Variable</b>	<b>Daily</b>	<b>Hourly</b>
Lag Spot Price		×
Average Lag Spot Price		×
Spot Price Volatility		×
Coal Price	×	
Gas Price	×	
Oil Price	×	
Price for EU Emission Allowances	×	
Expected Wind Infeed		×
Expected Power Plant Availability	×	
Expected Demand		×
Lag Demand		×
Seasonality Function	×	

**Table 4.4:** *Data granularity of fundamental variables.*

# Chapter 5

## GARCH Regression Models

### 5.1 Methodology & Implementation

In this section we will discuss the theoretical background of conditional variance models and their combination with a fundamental regression model in order to estimate electricity spot prices. Furthermore, we will reason the specific model choice and outline the applied estimation procedure.

#### 5.1.1 GARCH (Regression) Models

Generalized Autoregressive Conditional Heteroskedasticity (GARCH) models were first introduced by Bollerslev (1986) as a generalization of ARCH models pioneered by Engle (1982). Today they constitute a well-established approach to examine time series exhibiting patterns of non-constant and clustered volatility as for example observed in electricity spot prices.<sup>1</sup> Usually, GARCH models are applied as AR(1)-GARCH( $p, q$ ) processes with an autoregressive component for modelling the conditional mean which takes the form

$$y_t = \rho_1 y_{t-1} + \epsilon_t. \tag{5.1}$$

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<sup>1</sup>See Bystrom (2005), Mugele et al. (2005), or Knittel & Roberts (2005).

In 5.1 the disturbance term is defined as the product of the square root of the variance (volatility) and a standard normally distributed (white noise) variable:

$$\epsilon_t = \sqrt{\sigma_t^2} u_t, u_t \sim N \quad (5.2)$$

Variance  $\sigma_t^2$  is conditional on the information set at  $t$  and modelled by the following generalized process:

$$\sigma_t^2 = \omega + \sum_{i=1}^q \phi_i \epsilon_{t-i}^2 + \sum_{j=1}^p \psi_j \sigma_{t-j}^2 \quad (5.3)$$

where  $\phi$  is the coefficient for past realized squared error terms (included up to lag  $q$ ) and  $\psi$  is the coefficient for lagged variances (included up to lag  $p$ ). Thus,  $\phi$  can be interpreted as the sensitivity of the conditional variance towards market shocks while  $\psi$  can be viewed as a persistence measure for the prevailing variability.  $\omega$  is a constant and in combination with  $\phi$  and  $\psi$  constitutes the long-term average volatility  $\bar{\sigma}^2 = \omega / (1 - (\phi + \psi))$ . In order to ensure a stationary volatility process,  $\sum_{i=1}^q \phi_i + \sum_{j=1}^p \psi_j < 1$  has to apply. Furthermore,  $\omega > 0$  and  $\phi_i, \psi_i \geq 0$  have to hold.<sup>2</sup>

As discussed in chapter 3, the estimation of conventional AR(1)-GARCH( $p, q$ ) processes can lead to erroneous results due to present spike patterns in electricity spot prices.<sup>3</sup> Franses & Ghijssels (1999) discuss this issue for time series in general. As a remedy for experienced problems with spiky series, Karakatsani & Bunn (2010) propose to replace (or enrich) the AR process for the conditional mean by a matrix of explanatory variables  $X$  which results in a mean specification similar to a standard regression model:

$$y_t = c + x_t' \beta + \epsilon_t \quad (5.4)$$

where  $x_t$  denotes a time-dependent vector, which is part of matrix  $X$ , including all relevant exogenous variables at  $t$ . Equation 5.4 combined with a conditional variance specification constitutes the *GARCH regression model*.

### 5.1.2 Extended Models

So far, we have assumed the stochastic component  $u_t$  in equation 5.2 to be independently and identically normally distributed. Variance models built on that assumption

<sup>2</sup>See Bollerslev (1986).

<sup>3</sup>See Duffie et al. (1998) or Escribano et al. (2002).

are therefore labelled *normal* GARCH models. Bollerslev (1987) introduced an alternative formulation which assumes  $u_t$  to follow a Student-t distribution. The resulting *Student-t* GARCH model implies conditionally t-distributed market shocks which is a reasonable assumption for many financial time series. In a preliminary step we have estimated our GARCH regression models with both, a normal GARCH process and a Student-t GARCH process finding that the latter provides superior results for most hours.<sup>4</sup> In case of a Student-t GARCH variance, an additional parameter (degree of freedom) has to be estimated.

In equation 5.3 we assume symmetry in the volatility process which means that the variance reacts likewise to positive and negative shocks. In addition to the conventional GARCH representation, models which allow for asymmetric volatility effects have been developed. Examples are the threshold GARCH (GJR-GARCH)<sup>5</sup> or the exponential GARCH (E-GARCH)<sup>6</sup> model. Asymmetric GARCH models are often applied to equity and commodity prices, motivated by the fact that, in general, volatility increases more rapidly following negative price changes than following positive price changes in the case of equities and vice versa in the case of commodities.<sup>7</sup> In addition to the Student-t GARCH(1, 1) representation we have estimated our model with an E-GARCH(1, 1) methodology for the variance. However, the results proved to be inferior to what we obtained from symmetric GARCH models.<sup>8</sup> Also, we have not found the leverage factor<sup>9</sup> to be statistically significant for most hours. Hence, asymmetric volatility processes do not seem to be an appropriate modelling technique for our dataset which is why we will not continue considering them.

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<sup>4</sup>Respective measures of comparison are reported in table B.1 in the appendix.

<sup>5</sup>See Glosten et al. (1993).

<sup>6</sup>See Nelson (1991).

<sup>7</sup>See Alexander (2008).

<sup>8</sup>We have estimated E-GARCH regression models with both, normally and t-distributed disturbances. Our comparison was based on the information criteria AIC and BIC as well as on the value of the respective likelihood function. Detailed results are available upon request.

<sup>9</sup>The leverage factor is an integral component of the E-GARCH model. It defines the magnitude and direction of asymmetry effects in volatility. For a symmetric volatility model, this factor is zero.

### 5.1.3 Selected Model & Estimation Procedure

Given the aforementioned findings from preliminary analyses we formulate the following conditional mean model for day-ahead electricity spot prices:

$$P_t = c + x_t' \beta + \epsilon_t, \epsilon_t = \sqrt{\sigma_t^2} u_t, u_t \sim t \quad (5.5)$$

where  $x_t$  contains fundamental variables as outlined in the preceding chapter. In terms of the dependent variable (the day-ahead electricity spot price)  $t$  indicates the delivery day (i.e. the day after the auction). In terms of exogenous (fundamental) variables,  $t$  refers to the point in time when the information of the respective factor was updated for the last time before the auction takes place.<sup>10</sup> We model the conditional variance process via a GARCH(1, 1) representation:<sup>11</sup>

$$\sigma_t^2 = \omega + \phi \epsilon_{t-1}^2 + \psi \sigma_{t-1}^2 \quad (5.6)$$

To all our knowledge, Karakatsani & Bunn (2010) are the only ones having implemented a GARCH regression model for electricity prices before. They applied both, a symmetric and an asymmetric variance representation to prices from the British electricity market (in-sample only).

We estimate the GARCH regression models using a maximum likelihood approach. In general, maximum likelihood estimation (MLE) is used to estimate  $\theta$ , denoting a set of  $k$  parameters, in such a way that the probability of the estimates corresponding to the real parameters is maximized. For this purpose, the likelihood function (LLF)

$$L(\theta) = \prod_{t=1}^T f(y_t | \theta) \quad (5.7)$$

is to be maximized. Assuming independence, the LLF denotes the joint density function of  $y_t$  built by multiplying the conditional density functions for all observations  $y_t, t \in \{1, \dots, T\}$ . In order to facilitate maximization, the LLF is transformed by the natural logarithm which yields

$$\ln L(\theta) = \sum_{t=1}^T \ln f(y_t | \theta). \quad (5.8)$$

<sup>10</sup>Expected wind infeed is an exception as explained in the preceding chapter.

<sup>11</sup>We have found that adding additional lags to the variance representation does neither improve goodness of fit measures for the model itself nor price forecasts. Furthermore, statistical tests applied to residuals indicate that the chosen specification is appropriate.



The combination of parameters in  $\hat{\theta}_i, i \in \{1, \dots, k\}$  which maximizes the value of the LLF constitutes the optimal maximum likelihood estimators.

The log likelihood function for the Student-t GARCH(1,1) regression model is based on the Student-t density function

$$f_{\nu}(t) = [(\nu - 2)\pi]^{-0.5} \Gamma\left(\frac{\nu}{2}\right)^{-1} \Gamma\left(\frac{\nu + 1}{2}\right) \left(1 + \frac{t^2}{\nu - 2}\right)^{-\frac{\nu+1}{2}} \quad (5.9)$$

and can be formulated as follows:<sup>12</sup>

$$\begin{aligned} \ln L(\theta) = & - \sum_{t=1}^T \left\{ \ln(\sigma_t^2) + \left(\frac{\nu + 1}{2}\right) \ln \left[ 1 + (\nu - 2)^{-1} \left(\frac{\epsilon_t}{\sigma_t}\right)^2 \right] \right\} \\ & + T \times \ln \left\{ [(\nu - 2)\pi]^{-0.5} \Gamma\left(\frac{\nu}{2}\right)^{-1} \Gamma\left(\frac{\nu + 1}{2}\right) \right\} \end{aligned} \quad (5.10)$$

In 5.9 and 5.10  $\nu$  denotes degrees of freedom and  $\Gamma$  denotes the gamma function. In order to simultaneously estimate all required parameters from the conditional mean and the conditional variance equation  $\theta = (c, \beta, \omega, \phi, \psi, \nu)$ , 5.10 needs to be minimized (as we deal with a negative LLF).

#### 5.1.4 Correlations of Regressors

An essential precondition of linear regression analysis is that regressors are not highly correlated. A high (positive or negative) correlation between two or more explanatory variables in a regression model impedes a precise estimation of coefficients and can therefore result in misleading statistical inference. This problem, known as *multicollinearity*, is often observed in fundamental factor models. In the extreme case, if some regressors are exact linear combinations of each other, the covariance matrix  $\sum_{i=1}^N x_i x_i'$  is no longer invertible which would be necessary to obtain the least square estimator.<sup>13</sup> In the presence of multicollinearity it can often be observed that t-statistics of individual coefficients are very weak (driven by increased standard errors) while the overall explanatory power of the model (e.g. represented by the coefficient of determination) is rather strong.

In general, multicollinearity is less of a problem if the researcher's only intention is to forecast a variable. However, if he is interested in explaining a dependent variable by changes in regressors incorporated in the model, it becomes an issue.

<sup>12</sup>See Alexander (2008).

<sup>13</sup>See Verbeek (2008).

In the models to be estimated we will use a broad range of fundamental variables which are closely linked to each other from an economic point of view. It is conceivable that this could potentially give rise to multicollinearity issues. In order to check for this we examine correlations among the selected fundamental variables. Table 5.1 depicts the correlation matrix of regressors for hour 12 for illustrative purposes.

The average correlation coefficient across all hours is closely to zero. Most of the chosen variables prove to have very low correlations. As expected, there is high correlation among the fuel prices, i.e. coal, gas, and oil as well as between the lag spot price and the lag average spot price. The highest correlation can be observed between coal and gas and amounts to 0.80-0.90 for most hours. Overall, we are not concerned about serious multicollinearity problems. Moreover, in section 5.2.1 we will decide to eliminate the oil price as a variable which will lessen multicollinearity issues due to correlated fuel prices.

	S(-1)	Av.(-1)	Vol.	Coal	Gas	C/G	Oil	CO <sub>2</sub>	Wind	PPA	Exp.D.	D(-1)	Seas.
Spot(-1)	1.00	0.84	0.15	0.44	0.54	-0.37	0.27	0.20	-0.20	-0.28	-0.09	-0.05	-0.07
Av. Spot(-1)	0.84	1.00	0.07	0.63	0.68	-0.30	0.45	0.09	-0.15	-0.22	-0.06	-0.08	0.12
Spot Volatility	0.15	0.07	1.00	-0.12	-0.06	-0.09	-0.20	-0.02	0.04	0.03	0.14	0.18	0.16
Coal Price	0.44	0.63	-0.12	1.00	0.90	-0.07	0.83	-0.05	0.08	-0.37	-0.12	-0.26	-0.08
Gas Price	0.54	0.68	-0.06	0.90	1.00	-0.49	0.76	-0.11	0.06	-0.31	-0.07	-0.17	-0.06
Coal/Gas Ratio	-0.37	-0.30	-0.09	-0.07	-0.49	1.00	-0.05	0.07	0.04	0.00	-0.06	-0.12	0.04
Oil Price	0.27	0.45	-0.20	0.83	0.76	-0.05	1.00	-0.25	0.11	-0.40	-0.24	-0.34	-0.09
CO <sub>2</sub> Price	0.20	0.09	-0.02	-0.05	-0.11	0.07	-0.25	1.00	-0.23	-0.02	-0.24	-0.09	-0.25
Wind Forecast	-0.20	-0.15	0.04	0.08	0.06	0.04	0.11	-0.23	1.00	0.06	0.27	0.11	0.22
Power Plant Av.	-0.28	-0.22	0.03	-0.37	-0.31	0.00	-0.40	-0.02	0.06	1.00	0.57	0.56	0.71
Exp. Demand	-0.09	-0.06	0.14	-0.12	-0.07	-0.06	-0.24	-0.24	0.27	0.57	1.00	0.59	0.59
Demand(-1)	-0.05	-0.08	0.18	-0.26	-0.17	-0.12	-0.34	-0.09	0.11	0.56	0.59	1.00	0.55
Seasonality	-0.07	0.12	0.16	-0.08	-0.06	0.04	-0.09	-0.25	0.22	0.71	0.59	0.55	1.00

**Table 5.1:** Correlation matrix of regressors for hour 12. Correlations have been computed on the in-sample dataset, i.e. after correcting for weekends, holidays, bridge days, and outliers.

## 5.2 Empirical Results

In estimating the GARCH regression models and presenting the results we will proceed as follows. First, we will report which variables we exclude based on preliminary data analyses. Then, we will estimate the models in-sample for each single hour twice, with and without including the wind forecast variable. This shall allow us to draw conclusions regarding the explanatory power of expected wind infeed. After elaborating on the seasonal patterns, which we obtain from the results, we will rerun the estimation on an out-of-sample dataset. Whereas so far, all efforts serve to explain the conditional mean of day-ahead spot prices, we will conclude the empirical part by discussing obtained conditional volatility estimates.

We will estimate 24 GARCH regression models, i.e. one model for every single hour. This approach is motivated by various reasons. First, separating the modelling of different hours seems to us an efficient way to dispose of intraday seasonality which otherwise would have to be done by further extending the model with deterministic functions and/or dummy variables. Second, to derive hourly price forecasts for the entire next day, our approach requires estimating 24 one-step-ahead forecasts. This might produce less noise than deriving forecasts up to 24 steps ahead from one model. Third, it can be argued that due to very diverse constitutions of the demand and supply side at different hours, hourly electricity products qualify as separate commodities.<sup>14</sup>

### 5.2.1 Elimination of Variables

In order to check the appropriateness of our variable selection we run the GARCH regression models including all variables in a preliminary step (detailed results will not be shown). Looking at the t-statistics of the coefficients

$$t_{\hat{\beta}} = \frac{\hat{\beta} - \beta_0}{s.e.(\hat{\beta})} \quad (5.11)$$

across all 24 hours of the day, we decide to remove the following two variables.

The *oil price* coefficient is not significantly different from zero except for hours 4 to 6 and hours 22 to 24.<sup>15</sup> Moreover, the high correlation with other fuel prices

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<sup>14</sup>See for example Nan et al. (2010).

<sup>15</sup>A possible reason for statistical significance at the mentioned times may be that these are the hours when hard coal is the price setting technology. Normally, hard coal is imported and needs significant transportation efforts until the good is at the respective power plant. Related transportation costs are driven by the oil price to a significant extent.

as discussed in section 5.1.4 serves as a motivation to exclude this variable also for multicollinearity reasons.

The second variable we decide to exclude from our dataset is *lagged demand*. The fact that the corresponding coefficients are not significant at the 90% level for 20 hours suggests that all relevant information of past demand is already included appropriately in the spot price of the preceding day and/or in the demand forecast variable which we estimate using an autoregressive component (see chapter 4).

In order to obtain more insight into the role of the two omitted variables we perform F-tests on the entire models in addition to t-tests on individual coefficients. In doing so, we investigate whether a significant increase in  $R^2$  can be achieved by including one of the two variables although their t-statistics are not significant for most hours. In the case of multicollinearity, correlated variables often improve the overall model fit while the coefficients themselves are not significant. The appropriate F-test statistic is

$$F = \frac{(R_1^2 - R_0^2)/j}{(1 - R_1^2)/(n - k)} \quad (5.12)$$

where  $R_0^2$  denotes the coefficient of determination of the model excluding the variable at interest,  $R_1^2$  denotes the coefficient of determination of the model including the variable,  $j$  is 1 corresponding to the difference in the number of variables in the models to compare,  $n$  is the number of observations, and  $k$  is the number of parameters to be estimated.<sup>16</sup> We perform individual F-tests for all hours for models including the oil price as well as for models including the lagged demand variable. Test results are reported in tables B.2 and B.3 in the appendix. Overall, the results reveal that despite modest increases in  $R^2$  when adding one of the omitted variables, the model fit cannot be improved significantly for most hours. For some hours, a significant improvement can be achieved which, as mentioned above, is typical in the presence of multicollinearity. One of the main objectives of this thesis is to obtain a better understanding of the sensitivity of spot prices towards different fundamental variables. This makes significant and thus interpretable parameter coefficients essential which is the concluding rationale for omitting the two variables.

We keep all other variables which are not significant for only some hours, mainly based on economic reasoning.

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<sup>16</sup>For a more detailed introduction to F-tests see for example Verbeek (2008).

## 5.2.2 Conditional Mean Modelling

### 5.2.2.1 In-Sample Results

The observation period for the in-sample estimation starts on January 1, 2010 and ends on December 31, 2011.

Table 5.2 presents a summary of the estimation results of all 24 GARCH regression models without including wind forecast as an explanatory variable. The  $R^2$  spans over a range between 0.45 and 0.74 with an average of 0.60. This means that the fundamental models without wind forecast are able to explain 60 percent of the electricity spot price variability on average. The mean absolute percentage error (MAPE) is between 5.2% and 32.0% with an average of 9.5%. The results indicate a rather good fit for hours 18, 19, 20, and 21, for which  $R^2$  is at or above 0.70 and the MAPE amounts to 6.4-7.2%. There is a consistently worse fit for early morning hours.

To examine potential serial correlation in residuals we report Durbin-Watson test statistics:<sup>17</sup>

$$dw = \frac{\sum_{t=2}^T (\epsilon_t - \epsilon_{t-1})^2}{\sum_{t=1}^T \epsilon_t^2} \quad (5.13)$$

Under the null hypothesis of the Durbin-Watson test, error terms  $\epsilon_t$  are independently distributed. A value of  $dw$  close to 2 indicates zero first-order autocorrelation among the residuals, which would be desired. Looking at in-sample results, the Durbin-Watson test statistic shows values of close to 2 for almost all hours meaning that there is no need to be concerned about autocorrelated residuals that could distort standard errors of coefficient estimates. As for coefficient significance, we find t-statistics to be above the critical level required for a 95% significance for the majority of all coefficients across the day.<sup>18</sup>

In a next step we re-estimate the 24 models including the wind forecast variable. Table 5.3 provides a summary of the estimation results. The overview shows that for all hours, the coefficients of determination have clearly improved. For convenience, figure 5.1 compares the most important goodness of fit measures. In terms of  $R^2$ , improvements of 0.05 to 0.17 can be observed with an average increase of 0.09. In other words, the inclusion of expected wind infeed as a variable seems to be able to explain an additional 10 percent of price variability on average. A more intuitive

<sup>17</sup>See Durbin & Watson (1950).

<sup>18</sup>We do not display detailed estimation results for all 24 hourly models.

interpretation can be derived when looking at mean absolute errors<sup>19</sup>. Here we have reductions of between 0.65 and 0.17 with an average reduction of 0.45. In monetary terms this means that in the estimated models, adding wind forecast as a variable is worth 45 cents per MWh. From the comparative illustrations in figure 5.1 we see that the improvements are strongest for early morning hours and noon peak hours. When including wind forecast we end up at an average  $R^2$  of 0.69 with a minimum value of 0.58 (hour 7) and a maximum value of 0.80 (hour 18).

Durbin-Watson statistics for the models with wind are somewhat less powerful than for the models without wind, though still within an acceptable range overall.

In figures 5.2 we plot model forecasts and according realized spot prices for hours 3, 12, and 18. In addition, detailed estimation results for hours 12 and 18 are provided in tables 5.4 and 5.5 as examples.

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<sup>19</sup>The mean absolute error (MAE) is defined as  $\frac{1}{N} \sum_{n=1}^N |y_n - \hat{y}_n|$ .

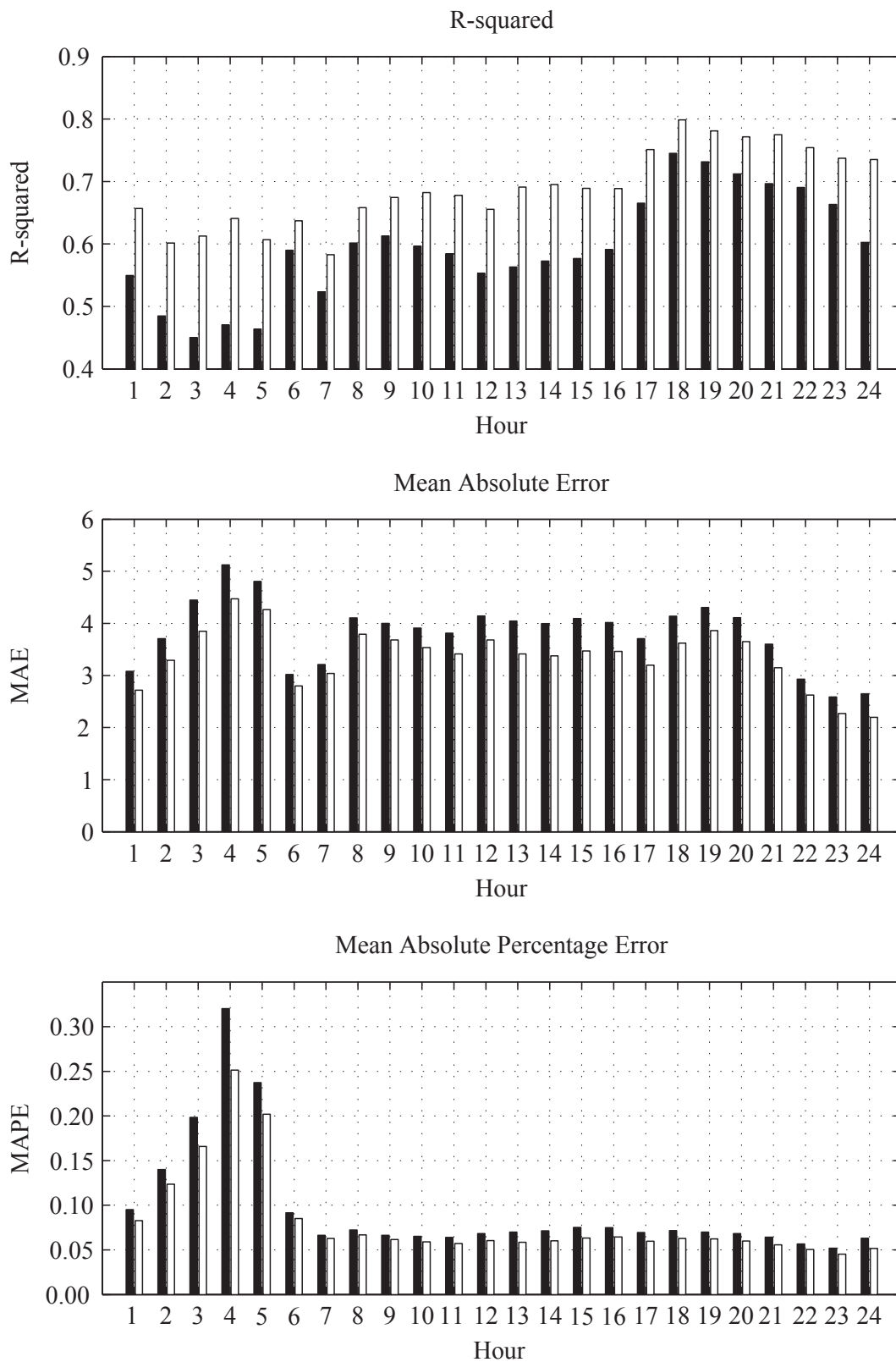
Hour	1	2	3	4	5	6	7	8
Obs.	486	487	486	491	489	484	489	492
$R^2$	0.55	0.48	0.45	0.47	0.46	0.59	0.52	0.60
$\bar{R}^2$	0.54	0.47	0.44	0.46	0.45	0.58	0.51	0.59
MAE	3.08	3.71	4.45	5.12	4.81	3.02	3.21	4.10
MAPE	0.095	0.140	0.198	0.320	0.237	0.092	0.066	0.072
$\sigma_\epsilon$	4.67	5.63	6.45	7.18	6.80	4.30	4.47	5.60
D/W	1.83	1.75	1.86	1.85	1.87	1.92	2.07	1.81
LLF	-1341	-1432	-1517	-1611	-1573	-1337	-1384	-1496
Hour	9	10	11	12	13	14	15	16
Obs.	491	491	490	490	492	492	492	491
$R^2$	0.61	0.60	0.58	0.55	0.56	0.57	0.58	0.59
$\bar{R}^2$	0.60	0.59	0.57	0.54	0.55	0.56	0.57	0.58
MAE	4.00	3.91	3.81	4.14	4.04	4.00	4.09	4.01
MAPE	0.066	0.065	0.064	0.068	0.070	0.071	0.075	0.075
$\sigma_\epsilon$	5.43	5.20	5.02	5.39	5.16	5.13	5.22	5.12
D/W	1.92	2.05	2.06	2.12	2.11	2.10	2.05	2.02
LLF	-1492	-1482	-1474	-1510	-1492	-1488	-1497	-1480
Hour	17	18	19	20	21	22	23	24
Obs.	487	486	484	489	492	491	488	490
$R^2$	0.67	0.74	0.73	0.71	0.70	0.69	0.66	0.60
$\bar{R}^2$	0.66	0.74	0.73	0.71	0.69	0.68	0.66	0.59
MAE	3.70	4.14	4.31	4.11	3.60	2.93	2.59	2.65
MAPE	0.069	0.072	0.070	0.068	0.064	0.057	0.052	0.063
$\sigma_\epsilon$	4.81	5.64	5.98	5.61	4.88	3.92	3.41	3.67
D/W	2.02	1.99	2.04	2.07	2.14	2.14	2.17	2.14
LLF	-1430	-1474	-1483	-1496	-1443	-1339	-1279	-1295

**Table 5.2:** Summary of in-sample results for GARCH regression models excluding expected wind infeed. The underlying dataset starts on January 1, 2010 and ends on December 31, 2011. LLF denotes the value of the log likelihood function used to estimate the models.

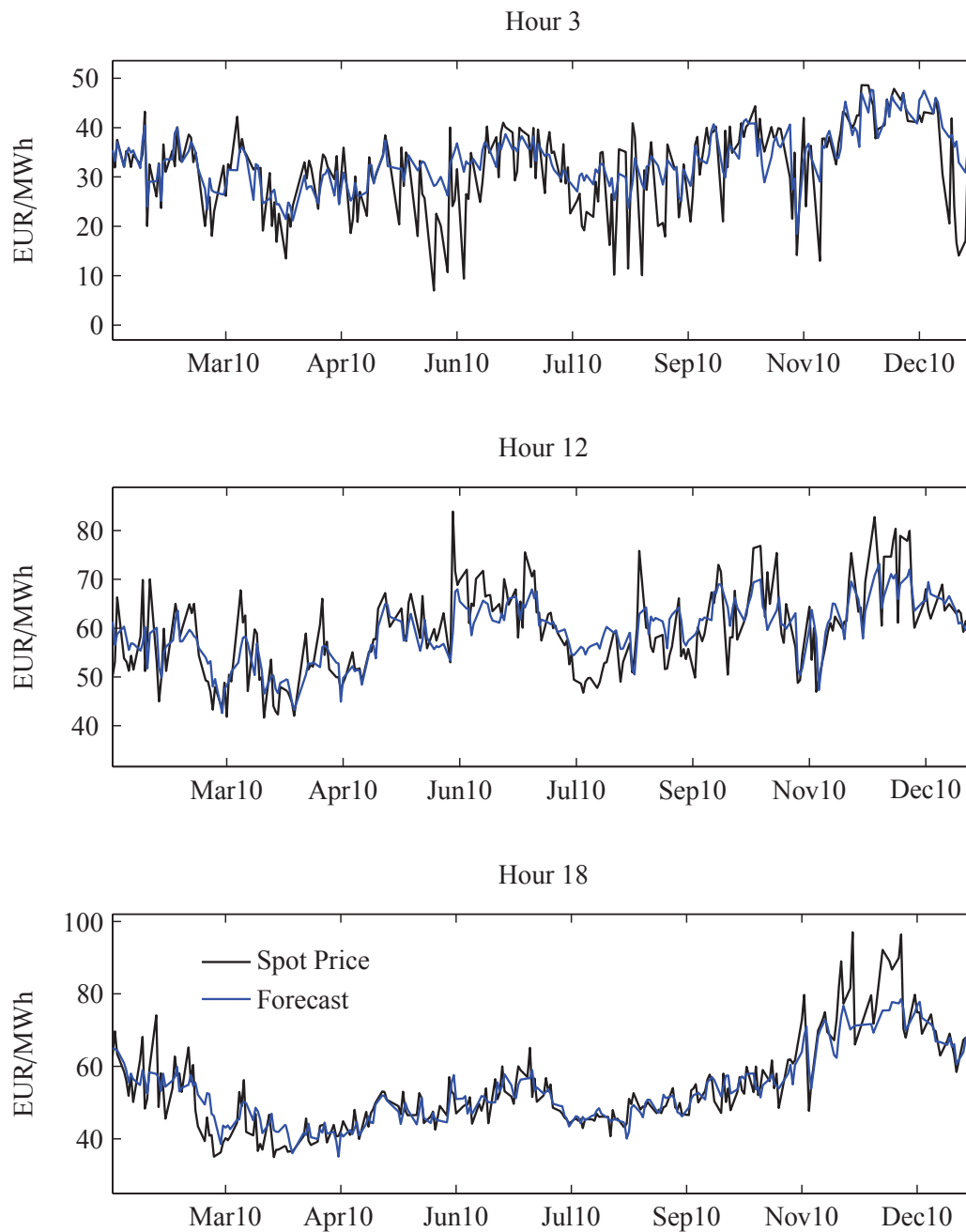


<b>Hour</b>	<b>1</b>	<b>2</b>	<b>3</b>	<b>4</b>	<b>5</b>	<b>6</b>	<b>7</b>	<b>8</b>
Obs.	486	487	486	491	489	484	489	492
$R^2$	0.66	0.60	0.61	0.64	0.61	0.64	0.58	0.66
$\bar{R}^2$	0.65	0.59	0.60	0.63	0.60	0.63	0.57	0.65
MAE	2.72	3.29	3.85	4.47	4.26	2.80	3.04	3.79
MAPE	0.083	0.124	0.166	0.251	0.202	0.085	0.063	0.067
$\sigma_\epsilon$	4.09	4.95	5.46	5.99	5.85	4.04	4.19	5.18
D/W	1.76	1.70	1.67	1.59	1.76	1.82	2.04	1.67
LLF	-1286	-1379	-1456	-1548	-1514	-1299	-1352	-1456
<b>Hour</b>	<b>9</b>	<b>10</b>	<b>11</b>	<b>12</b>	<b>13</b>	<b>14</b>	<b>15</b>	<b>16</b>
Obs.	491	491	490	490	492	492	492	491
$R^2$	0.67	0.68	0.68	0.66	0.69	0.70	0.69	0.69
$\bar{R}^2$	0.67	0.67	0.67	0.65	0.68	0.69	0.68	0.68
MAE	3.68	3.53	3.42	3.68	3.41	3.38	3.47	3.46
MAPE	0.062	0.059	0.057	0.060	0.059	0.060	0.063	0.064
$\sigma_\epsilon$	4.99	4.62	4.42	4.73	4.34	4.33	4.47	4.46
D/W	1.70	1.78	1.76	1.77	1.68	1.71	1.70	1.68
LLF	-1443	-1428	-1412	-1443	-1403	-1402	-1420	-1407
<b>Hour</b>	<b>17</b>	<b>18</b>	<b>19</b>	<b>20</b>	<b>21</b>	<b>22</b>	<b>23</b>	<b>24</b>
Obs.	487	486	484	489	492	491	488	490
$R^2$	0.75	0.80	0.78	0.77	0.77	0.75	0.74	0.74
$\bar{R}^2$	0.74	0.79	0.78	0.77	0.77	0.75	0.73	0.73
MAE	3.20	3.62	3.86	3.65	3.15	2.62	2.27	2.20
MAPE	0.060	0.063	0.062	0.060	0.056	0.050	0.045	0.052
$\sigma_\epsilon$	4.15	5.00	5.39	4.98	4.19	3.49	3.01	3.00
D/W	1.68	1.52	1.75	1.70	1.91	1.96	1.99	1.82
LLF	-1352	-1395	-1423	-1435	-1381	-1286	-1216	-1208

**Table 5.3:** Summary of in-sample results for GARCH regression models including expected wind infeed. The underlying dataset starts on January 1, 2010 and ends on December 31, 2011. LLF denotes the value of the log likelihood function used to estimate the models.



**Figure 5.1:** Improvement in goodness of fit for GARCH regression models if expected wind infeed is included as exogenous variable. Black bars denote goodness of fit measures for in-sample estimated models excluding wind, measures denoted by white bars include wind.



**Figure 5.2:** *In-sample fit of GARCH regression models for hours 3, 12, and 18. The observation period starts on January 1, 2010 and ends on December 31, 2011.*

Observations	490	$R^2$	0.66
AIC	2916	$\bar{R}^2$	0.65
BIC	2979	MAE	3.68
DoF	9.38	MAPE	0.060
D/W	1.77	$\sigma_\epsilon$	4.73
		<i>Coefficient</i>	<i>t-Statistic</i>
Constant		13.802	2.05
Spot(-1)		0.175	3.03
Spot Av.(-1)		0.169	2.19
Spot Price Volatility		0.087	0.85
Coal Price		-0.269	-9.48
Gas Price		0.020	14.10
Coal/Gas Price Ratio		278.971	685.03
CO <sub>2</sub> Price		0.820	7.80
Expected Wind Infeed		-0.0007	-11.86
Exp. Power Plant Availability		-0.0006	-7.68
Exp. Demand		0.0006	6.00
Seasonality		1.885	3.40
$\omega$		1.957	1.44
$\phi$		0.104	1.99
$\psi$		0.810	9.02

**Table 5.4:** *In-sample results of the GARCH regression model for hour 12. The underlying dataset starts on January 1, 2010 and ends on December 31, 2011.*

Observations	486	$R^2$	0.80
AIC	2820	$\bar{R}^2$	0.79
BIC	2883	MAE	3.62
DoF	12.40	MAPE	0.063
D/W	1.52	$\sigma_\epsilon$	5.00
		<i>Coefficient</i>	<i>t-Statistic</i>
Constant		-12.937	-1.81
Spot(-1)		0.253	5.07
Spot Av.(-1)		0.032	0.59
Spot Price Volatility		0.437	3.85
Coal Price		-0.014	-0.62
Gas Price		0.011	8.23
Coal/Gas Price Ratio		108.240	253.56
CO <sub>2</sub> Price		0.329	3.31
Expected Wind Infeed		-0.0006	-15.46
Exp. Power Plant Availability		-0.0004	-5.55
Exp. Demand		0.0010	8.33
Seasonality		1.525	2.45
$\omega$		1.210	2.34
$\phi$		0.279	3.60
$\psi$		0.694	10.50

**Table 5.5:** *In-sample results of the GARCH regression model for hour 18. The underlying dataset starts on January 1, 2010 and ends on December 31, 2011.*

### 5.2.2.2 Seasonal Patterns

In this section we discuss intraday seasonal patterns of price sensitivities towards different fundamental factors. Figures 5.3 and 5.4 plot all (significant and non-significant) coefficients of the different variables we incorporated into the models for every single hour of the day. The constant and the seasonality variable have been omitted.

The *lag-1 spot price* and the *lag-1 average spot price* display coefficients in the range between 0 and 0.4 with converse patterns over the day. In combination they explain approximately one quarter of the day-ahead forecast. This is not a lot but might be reasoned by the consideration of a rich set of fundamental variables which already includes most of the information represented in hourly spot prices.

Until mid-afternoon the coefficient of *spot price volatility* is not significantly different from zero. Then, this changes and its value increases gradually and reaches a top during evening peak hours. At this time we can observe coefficients which are significantly greater than zero indicating that the price uncertainty over the past week impacts market participants' bidding behaviour and thus day-ahead prices. After having peaked at hour 19, the coefficient drops back to close to zero.

For coal and gas prices we can report highly significant factor loads for most hours with distinct seasonal patterns throughout the day. The significance of *gas price* coefficients is particularly high during hours of high demand. This corresponds with the fact that gas is the price setting technology during high demand hours due to short ramp-up times. *Coal prices* prove to have a positive impact on prices for hours of low demand only, namely the hours after the peak-on/peak-off switch in the evening and for some early morning hours. However, the significance is not persistent across low demand hours, possibly arising from the fact that lignite is the price setting technology in periods of extremely low demand. There is no transparent market price for lignite available and, in addition, our chosen coal price variable may not be an adequate approximation (it was mainly chosen as a variable for the hard coal price). A fact to be highlighted are the peaks of coefficient values at hours 8 and 19. At these times, the peak-on/peak-off switch, which impacts the composition of available power generation technologies, takes place. Concretely, in the morning around 08:00 am gas fired plants are switched on to serve higher demand during peak hours and replace hard coal as the price setting technology. By the same token, gas is replaced by coal in the evening around 07:00 pm when gas fired plants are switched off. At the time when gas fired plants start or end their production, prices are highly sensitive. This behavior is confirmed by the coefficients of the *coal/gas price ratio*.

The coefficient pattern of the *CO<sub>2</sub> price index* peaks at hour 12 on the upside slightly above 0.8 and at hour 19 on the downside reaching a value close to zero

meaning that changes in CO<sub>2</sub> prices have little impact on spot prices then.

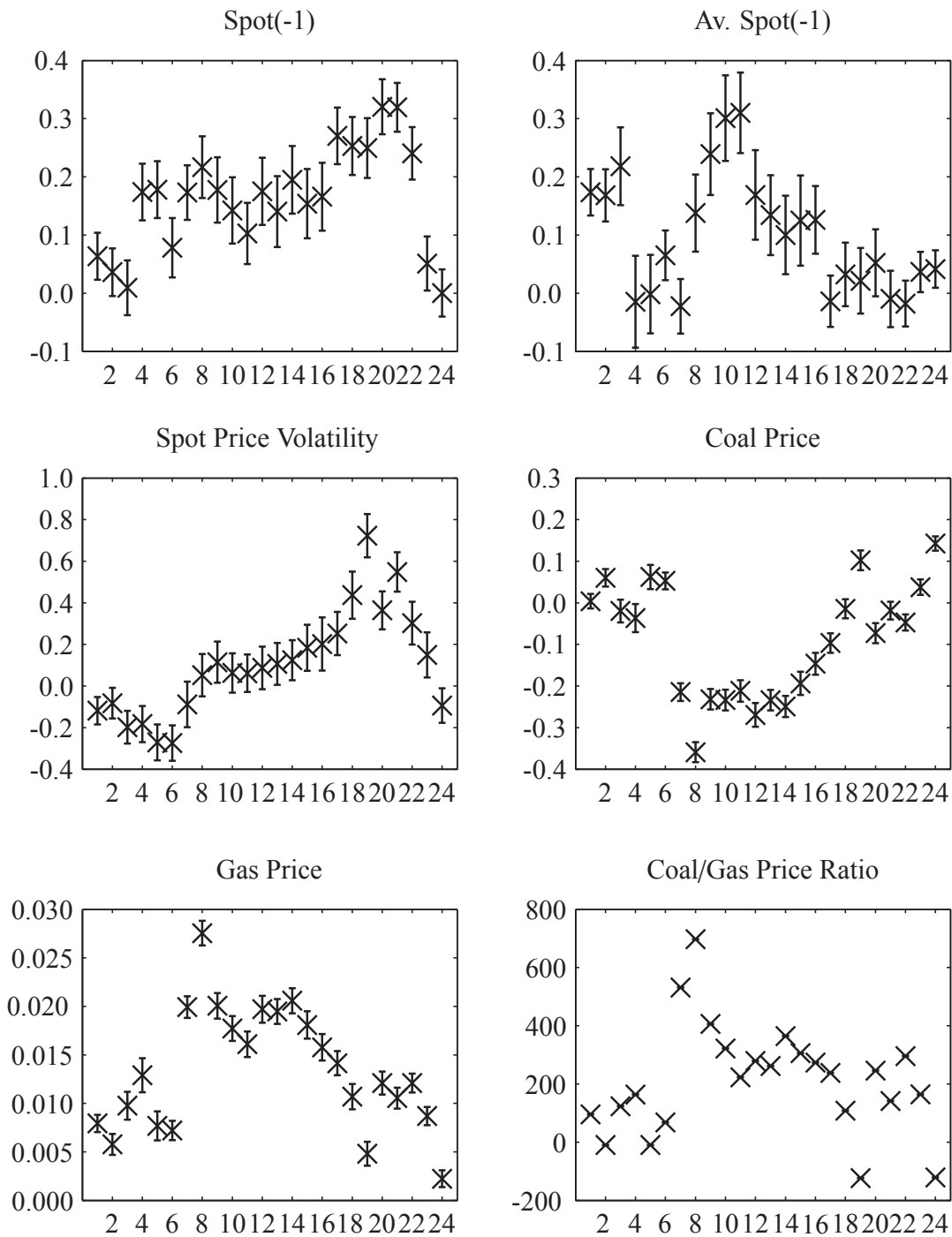
Spot prices display negative sensitivities towards the *expected wind infeed* for the entire day which is expected as higher wind infeed increases supply. This pattern is particularly pronounced during the first hours of the day when demand is at its low. Then, an increase in wind speed can quickly lead to excessive supply and thus even result in negative prices. These findings have been reported by other researchers before.<sup>20</sup> The absolute sensitivity generally lessens for peak hours, whereas for hours with highest demand, i.e. during the noon and the evening peak, it increases again. At these hours, the market situation is tense and any relevant change on the supply side of the market can easily lead to price spikes.

Factor loads of *expected power plant availability* are negative over the entire day which is expected as an increase in available production resources removes tension from the market and vice versa. We can observe that absolute factor loads tend to be higher for peak hours which is sensible since changes in available production resources are much more influential because most power plants run and since there is little unused capacity.

Logically, *expected demand* proves to have a positive impact on electricity spot price levels. We can observe intraday peaks in the morning as the ramp-up period of peak demand serving power plants starts, shortly after the evening peak when more expensive production sites are in the ramp-down phase, and - locally and less pronounced - at the noon peak.

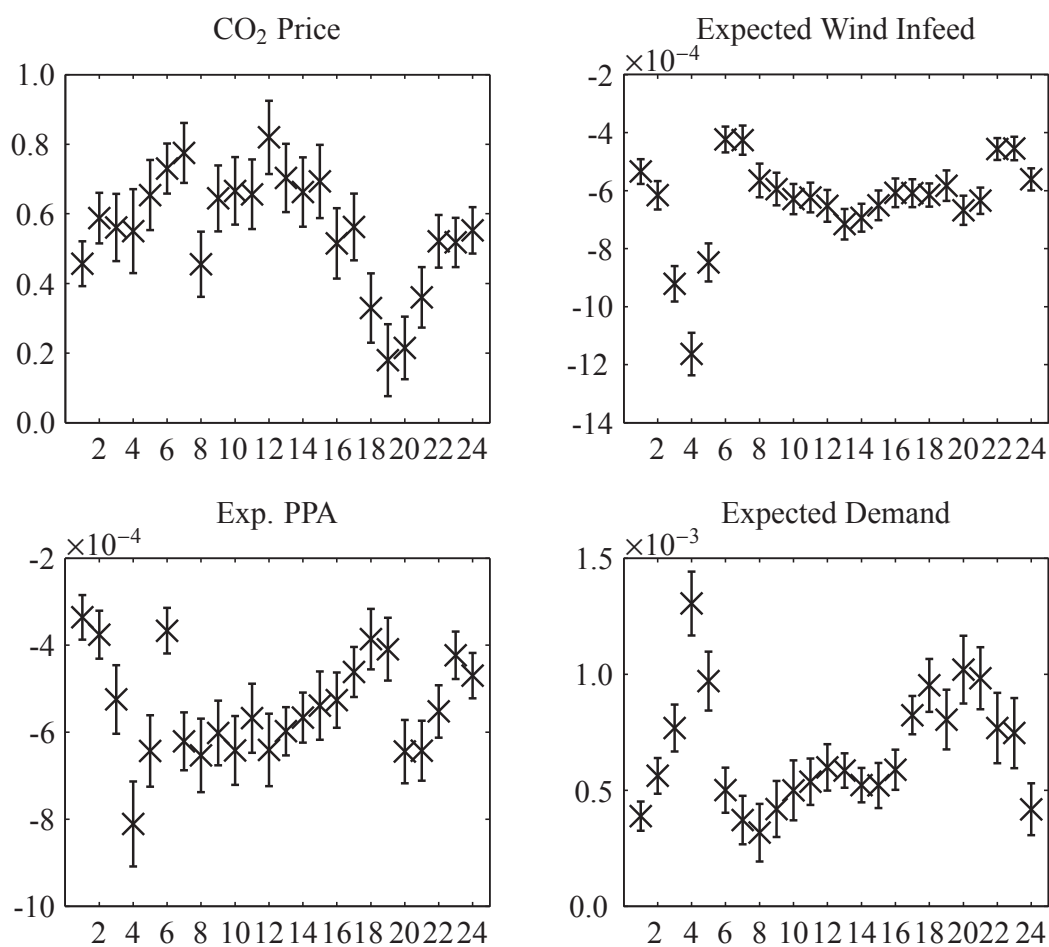
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<sup>20</sup>See for example Nicolosi (2010) or Fanone et al. (2012).



**Figure 5.3:** Coefficients of GARCH regression models for all hours across the day based on the in-sample model estimation. Error bars indicate the range of one standard error around estimated coefficient values.





**Figure 5.4:** Coefficients of GARCH regression models for all hours across the day based on the in-sample model estimation. Error bars indicate the range of one standard error around estimated coefficient values.

### 5.2.2.3 Out-of-Sample Results

Following the estimation of the 24 models in section 5.2.2.1 we are interested in how the models perform in predicting day-ahead electricity spot prices out-of-sample. For this purpose we apply all estimated models (with inclusion of wind forecast) to a period of 4 months following the in-sample estimation period. The out-of-sample observation period thus reaches from January 1, 2012 to April 30, 2012.

Tables 5.6 summarizes goodness of fit measures for the out-of-sample forecasting. The model works best for hours 8, 9, 18, 19, 22, and 23 where  $R^2$  measures between 0.58 and 0.66 can be reported.<sup>21</sup> For afternoon hours 13 to 16 we obtain negative coefficients of determination meaning that averaging the observed actual spot price would provide a more reliable prediction than the forecasts delivered by our model. Excluding hours 13 to 16 we can report an average  $R^2$  of 0.48, an average MAE of 5.86, and an average MAPE of 13.2%.

When excluding expected wind infeed from the dataset we obtain an average  $R^2$  of 0.23 for all 24 hours and an average  $R^2$  of 0.31 when disregarding hours 13 to 16 which indicates a rather poor fit. For the latter case, the average MAE is 6.28 and the average MAPE is 14.7%. We will not show detailed results.

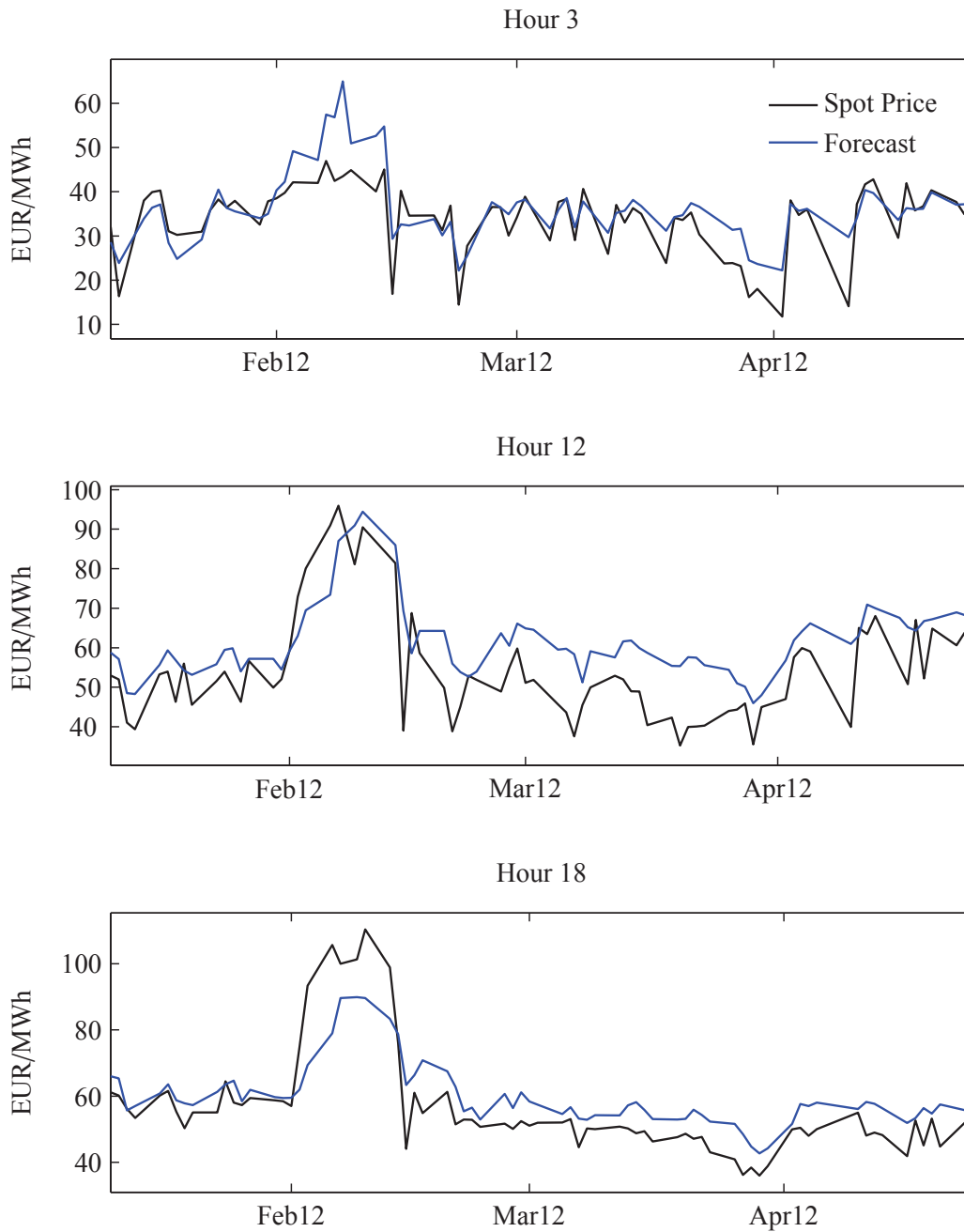
Figure 5.5 exemplarily illustrates the out-of-sample fit for hours 3, 12, and 18 (including wind infeed).

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<sup>21</sup>We do not report  $\bar{R}^2$  as the relatively low number of observations would distort results.

<b>Hour</b>	<b>1</b>	<b>2</b>	<b>3</b>	<b>4</b>	<b>5</b>	<b>6</b>	<b>7</b>	<b>8</b>
Obs.	76	75	76	77	77	76	77	77
$R^2$	0.41	0.47	0.43	0.47	0.55	0.42	0.44	0.61
MAE	4.29	4.13	4.26	4.31	3.78	3.34	4.43	7.65
MAPE	0.158	0.168	0.158	0.162	0.152	0.114	0.096	0.135
$\sigma_\epsilon$	5.28	5.20	5.46	4.78	4.68	4.77	5.82	9.16
<b>Hour</b>	<b>9</b>	<b>10</b>	<b>11</b>	<b>12</b>	<b>13</b>	<b>14</b>	<b>15</b>	<b>16</b>
Obs.	76	75	77	76	75	76	76	76
$R^2$	0.58	0.48	0.49	0.23	-0.26	-0.21	-0.43	-0.14
MAE	7.39	7.48	9.21	9.51	10.46	9.94	8.74	7.02
MAPE	0.115	0.132	0.178	0.200	0.241	0.227	0.203	0.162
$\sigma_\epsilon$	10.44	9.32	9.83	8.13	7.19	6.71	5.86	5.52
<b>Hour</b>	<b>17</b>	<b>18</b>	<b>19</b>	<b>20</b>	<b>21</b>	<b>22</b>	<b>23</b>	<b>24</b>
Obs.	76	77	75	77	76	77	77	76
$R^2$	0.27	0.66	0.62	0.49	0.30	0.59	0.58	0.55
MAE	5.99	7.22	9.23	8.81	5.88	3.46	3.11	3.67
MAPE	0.137	0.135	0.146	0.127	0.107	0.067	0.066	0.094
$\sigma_\epsilon$	5.38	8.10	11.56	13.32	6.83	4.61	4.03	4.39

**Table 5.6:** Summary of out-of-sample results for GARCH regression models including expected wind infeed as exogenous variable. The dataset starts on January 1, 2012 and ends on April 30, 2012.



**Figure 5.5:** Out-of-sample fit of GARCH regression models for hours 3, 12, and 18. The dataset starts on January 1, 2012 and ends on April 30, 2012.

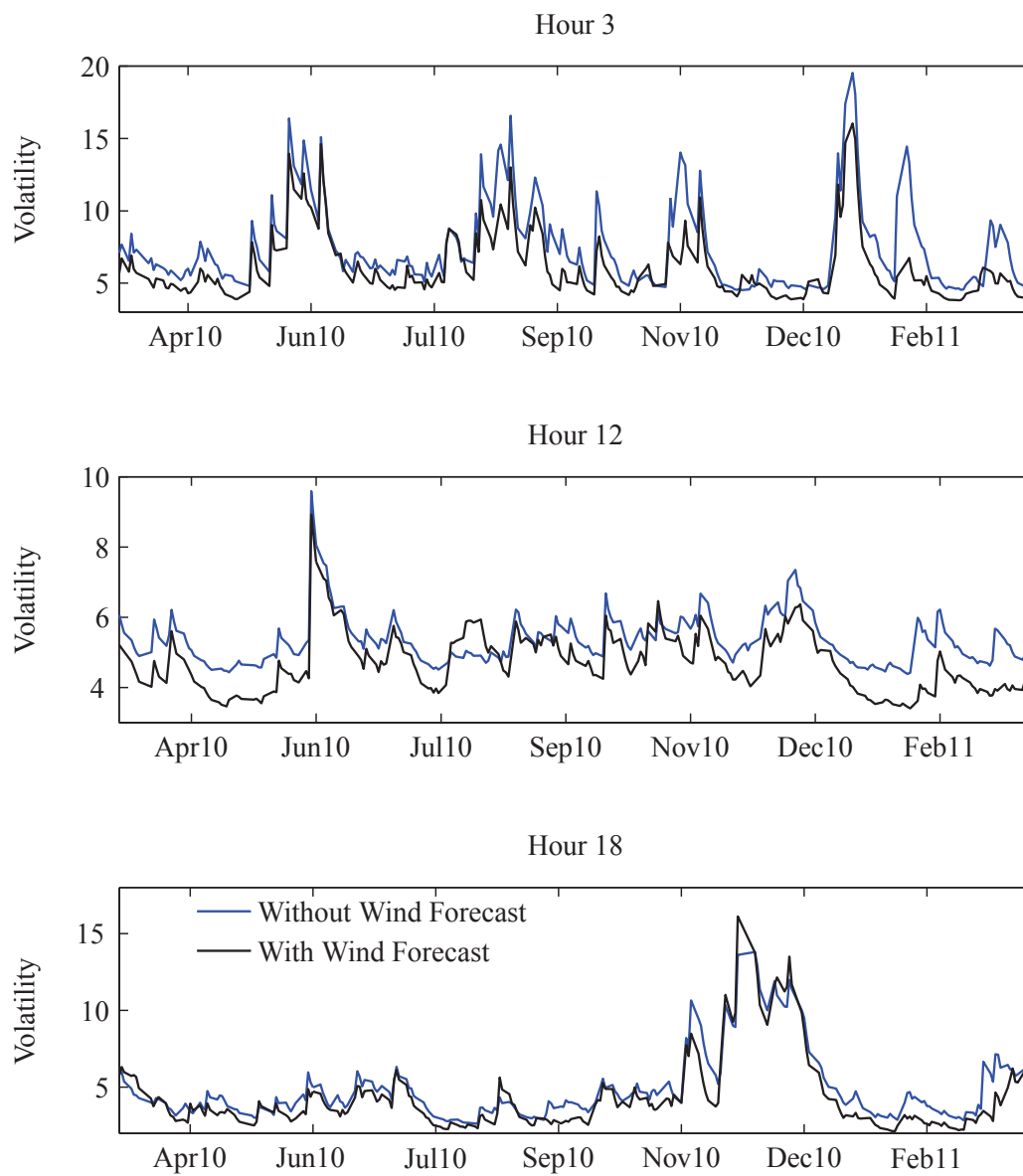
### 5.2.3 Conditional Volatility Modelling

Table 5.7 depicts the coefficients of the Student-t GARCH (1, 1) model for all hours. Looking at the lag error coefficients  $\phi$  we note that their values are higher (i.e. above 0.20) for early morning hours 1 to 5 as well as for hours 8, 9, 17, and 18. As the lag error parameter represents market shocks, high values indicate that for these hours, price volatility is more sensitive to market events. The lag variance coefficient  $\psi$  shows a relatively stable pattern over the day with a generally rather low value which only for five hours is higher than 0.80. A low coefficient for the last realized variance indicates that increased volatility in general fades away rather fast. Our empirical findings correspond to the nature of electricity spot prices which, after having spiked, often move back to previous levels very fast.

Figure 5.6 exemplarily depicts conditional volatilities for hours 3, 12, and 18 for both model specifications, with and without wind forecast. The mapped conditional volatility corresponds to the square root of  $\sigma_t^2$  as defined in equation 5.6. To give a better graphical overview we have selected 250 observation days only between March 2010 and March 2011. The persistence in conditional volatility, which can be repetitively observed in the plotted series, catches volatility clusters which constitute a distinctive characteristic of electricity spot prices. All three plots confirm that the remaining variance can overall be reduced by incorporating expected wind infeed as an explanatory variable into the conditional mean specification. The effect is most apparent for hour 3 (which is representative for morning hours in general) when wind infeed into the system often creates significant (negative) price spikes.

<b>Hour</b>	<b>1</b>	<b>2</b>	<b>3</b>	<b>4</b>	<b>5</b>	<b>6</b>	<b>7</b>	<b>8</b>
$\omega$	6.27	6.16	5.78	5.42	3.74	1.59	1.32	3.13
t-Stat	(1.59)	(1.68)	(2.25)	(2.13)	(2.17)	(1.72)	(1.67)	(2.32)
$\phi$	0.29	0.41	0.35	0.22	0.27	0.17	0.10	0.25
t-Stat	(1.56)	(1.69)	(2.37)	(2.63)	(2.52)	(2.07)	(2.14)	(3.14)
$\psi$	0.56	0.59	0.59	0.66	0.69	0.79	0.82	0.66
t-Stat	(3.36)	(5.62)	(5.52)	(5.72)	(8.38)	(10.72)	(11.48)	(7.90)
<b>Hour</b>	<b>9</b>	<b>10</b>	<b>11</b>	<b>12</b>	<b>13</b>	<b>14</b>	<b>15</b>	<b>16</b>
$\omega$	4.88	2.21	2.28	1.96	3.24	2.32	2.38	2.87
t-Stat	(2.27)	(1.68)	(1.54)	(1.44)	(2.04)	(1.92)	(1.70)	(2.30)
$\phi$	0.26	0.12	0.11	0.10	0.17	0.14	0.11	0.19
t-Stat	(2.85)	(2.19)	(1.92)	(1.99)	(2.71)	(2.62)	(2.22)	(2.84)
$\psi$	0.56	0.78	0.78	0.81	0.66	0.74	0.77	0.66
t-Stat	(4.28)	(8.27)	(7.29)	(9.02)	(5.41)	(7.80)	(7.71)	(6.46)
<b>Hour</b>	<b>17</b>	<b>18</b>	<b>19</b>	<b>20</b>	<b>21</b>	<b>22</b>	<b>23</b>	<b>24</b>
$\omega$	2.38	1.21	1.15	1.49	0.34	0.33	0.98	2.35
t-Stat	(2.59)	(2.34)	(2.04)	(1.93)	(1.29)	(1.36)	(1.08)	(324.03)
$\phi$	0.23	0.28	0.18	0.18	0.09	0.10	0.06	0.00
t-Stat	(3.35)	(3.60)	(3.34)	(3.09)	(2.73)	(2.71)	(1.65)	(0.00)
$\psi$	0.63	0.69	0.78	0.77	0.90	0.88	0.84	0.73
t-Stat	(6.82)	(10.50)	(15.22)	(11.92)	(24.65)	(21.63)	(6.99)	(17.66)

**Table 5.7:** Conditional variance parameters of the Student- $t$  GARCH(1,1) process embedded in GARCH regression models including expected wind infeed.  $\omega$  denotes the constant,  $\phi$  denotes the coefficient for past realized squared error terms, and  $\psi$  denotes the coefficient for lagged variances. The underlying in-sample observation window starts on January 1, 2010 and ends on December 31, 2011.



**Figure 5.6:** Conditional volatility obtained from Student- $t$  GARCH(1,1) specifications in GARCH regression models with and without wind forecast for hours 3, 12, and 18. The observation window includes 250 observations between March 25, 2010 and March 30, 2011.

### 5.3 Intermediate Summary

We have shown that market participants trading electricity respond to changes in fundamental variables. Including wind forecast into fundamental models with a linear conditional mean process can significantly improve in-sample results. In general, fundamental variables prove to be well suited to describe price dynamics in day-ahead electricity spot prices. However, when applying the calibrated models to out-of-sample data, we obtain rather imprecise forecasts for most hours.

We believe that the poor results from out-of-sample testing of fundamental GARCH regression models may be caused by the following reasons:

1. **Short out-of-sample period:** With less than 80 observations the underlying out-of-sample dataset is rather short. While our in-sample set spans over two entire annual cycles, the out-of-sample set only covers two winter and two spring months.
2. **Extreme price variability:** During February 2012 electricity spot prices showed unusually extreme moves. When looking for example at hour 12, prices in 2010 and 2011 have evolved within a band of 40-85 EUR/MWh. In contrast, prices in February 2012 spiked to nearly 100 EUR/MWh. The main drivers of extreme prices in February 2012 were very low temperatures and, to some extent consequently, a jump in gas prices. Between January 1, 2012 and April 30, 2012 the gas price evolved relatively stable within a band between EUR 2,000 and EUR 2,500 with the exception of the first half of February when prices used to be at much higher levels between EUR 3,000 and EUR 3,500.
3. **Imprecise Demand Forecast:** The expected electricity demand which we estimate applying our own model reveals an out-of-sample MAPE of 4.8% on average.<sup>22</sup> Using a flawed demand forecast as an exogenous variable creates multiplicative errors in predicting electricity-spot prices. It can be assumed that this effect would be mitigated if a more precise demand forecast model was available.
4. **Assumption of one linear relationship:** Our representation of the conditional mean assumes that there exists one single linear relationship between the spot price and the explanatory variables. Possibly, this is not enough to cope with

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<sup>22</sup>See section 4.1.



the complex dependencies of electricity prices on fundamental variables and allowing for more than one regime could improve results.

5. **Seasonality in sensitivities:** Based on the aforementioned paragraph, electricity spot prices may have sensitivities towards fundamental factors that vary significantly with certain seasonal patterns and/or adapt to changing market environments and regulatory amendments. Hence, the application of a model which assumes constant coefficients may not be appropriate.

In order to remedy deficiencies related to the assumption of one single constant linear relationship and in order to gain further insight into the explanatory power of expected wind infeed in fundamental forecast models we will introduce two additional model types in the following chapters. First, we will estimate so-called *threshold regression models* which allow for two regimes with different linear relationships. Second, we will estimate *time-varying parameter (TVP) regression models* which allow a more subtle modelling of seasonal patterns in price sensitivities.



## Chapter 6

# Threshold Regression Models

In this chapter we will estimate threshold regression models following a methodology introduced by Hansen (2000). The estimation of threshold regression models has the following two main purposes. First, it shall help to gain a deeper understanding of the spot price sensitivities towards expected wind infeed as well as the interactions of wind forecast with other fundamental variables. Second, we want to work out whether allowing for two regimes can help to significantly improve the out-of-sample forecast of electricity spot prices compared to beforehand estimated GARCH regression models.

### 6.1 Methodology

Subsequently, we will briefly discuss the estimation of threshold regression models as introduced by Hansen (2000). The underlying idea of the concept is a splitting of the sample into two subsamples where for each subsample different linear relations between the dependent and the explanatory variables apply. The original sample is split into two regimes according to the threshold variable which has to be preliminarily defined and which can be any of the explanatory variables in the model.

#### 6.1.1 Threshold Estimation

Formally, the standard regression model

$$y_t = \beta' x_t + \epsilon_t \tag{6.1}$$

is transferred into a threshold regression model of the form

$$y_t = \theta_1' x_t + \epsilon_t, q_t \leq \gamma \quad (6.2)$$

$$y_t = \theta_2' x_t + \epsilon_t, q_t > \gamma \quad (6.3)$$

where  $\theta_1$  and  $\theta_2$  denote the respective coefficient sets for the two linear relations.  $q_t$ , which can be an element of  $x_t$ , denotes the value of the threshold variable at  $t$  and  $\gamma$  denotes the threshold level. Hence, for all observations for which  $q_t \leq \gamma$  holds, coefficients of  $\theta_1$  apply whereas for observations where  $q_t > \gamma$  holds, coefficients of  $\theta_2$  apply. Hansen shows that the threshold  $\gamma$  can be estimated via least square. In a preliminary step 6.2 and 6.3 are combined into one single equation which reads as

$$y_t = \theta_2' x_t + \delta_n' x_t \mathbf{I}_{q_t \leq \gamma} + \epsilon_t \quad (6.4)$$

where  $\delta_n = \theta_1 - \theta_2$  is called *threshold effect* and  $\mathbf{I}_{q_t \leq \gamma}$  denotes the indicator variable which can take the values 1 (for  $q_t \leq \gamma$ ) or 0 (for  $q_t > \gamma$ ). In a next step,  $n \times 1$  vectors  $Y$  and  $e$  as well as  $n \times m$  matrices  $X$  and  $X_\gamma$  are formed by stacking all vectors  $y_t$ ,  $\epsilon_t$ ,  $x_t$ , and  $x_t \mathbf{I}_{q_t \leq \gamma}$  to obtain 6.4 in matrix notation:

$$Y = X\theta_2 + X_\gamma \delta_n + e \quad (6.5)$$

In order to receive estimators for  $\theta_2$ ,  $\delta_n$ , and  $\gamma$ , Hansen builds the sum of squared errors function

$$S_n(\theta_2, \delta_n, \gamma) = (Y - X\theta_2 - X_\gamma \delta_n)' (Y - X\theta_2 - X_\gamma \delta_n) \quad (6.6)$$

which as a concentrated sum reads as

$$\begin{aligned} S_n(\gamma) &= S_n(\hat{\theta}_2(\gamma), \hat{\delta}_n(\gamma), \gamma) \\ &= Y'Y - Y'X_\gamma^* (X_\gamma^{*'} X_\gamma^*)^{-1} X_\gamma^{*'} Y \end{aligned} \quad (6.7)$$

where  $X_\gamma^* = XX_\gamma$ .  $\hat{\theta}_2(\gamma)$  and  $\hat{\delta}_n(\gamma)$  are conditional OLS estimators. The estimator of the threshold level  $\hat{\gamma}$  minimizes 6.7 and is defined as

$$\hat{\gamma} = \arg \min_{\gamma \in \Gamma_n} S_n(\gamma). \quad (6.8)$$

$\Gamma = [\underline{\gamma}, \bar{\gamma}]$  defines the set to which  $\gamma$  is bounded and is approximated by a grid in case of a very high number of observations. For iid  $\mathcal{N} \sim (0, \sigma^2)$  distributed error terms  $\epsilon_t$ , the LS estimator  $\hat{\gamma}$  qualifies as the maximum likelihood estimator (MLE).

### 6.1.2 Confidence Interval & Threshold Significance

Assuming that  $\epsilon_t$  in 6.4 is independent and identically normally distributed with mean 0 and constant variance  $\sigma^2$ , the likelihood ratio statistic provided by Hansen is

$$LR_n(\gamma) = n \frac{S_n(\gamma) - S_n(\hat{\gamma})}{S_n(\hat{\gamma})}. \quad (6.9)$$

Critical values, which are used to define the confidence interval of the threshold for different significance levels, are derived via inversion of the distribution function of this likelihood ratio. If the assumption of iid  $\mathcal{N} \sim (0, \sigma^2)$  distributed error terms does not hold, the asymptotic distribution function of the likelihood ratio is perturbed by a nuisance parameter and a normalized likelihood ratio statistic has to be applied:

$$LR_n^*(\gamma) = \frac{LR_n(\gamma)}{\hat{\eta}^2} = \frac{S_n(\gamma) - S_n(\hat{\gamma})}{\hat{\sigma}^2 \hat{\eta}^2} \quad (6.10)$$

where  $\hat{\eta}^2$  denotes the nuisance parameter (reducing to unity in case of homoskedasticity) which can be consistently estimated via polynomial or kernel regression.

To test whether the estimated threshold is significant, the standard approach in case of homoskedasticity is to formulate the null hypothesis  $H_0 : \gamma = \gamma_0$  and to reject it for large enough values of the likelihood ratio statistic. The corresponding p-value reads as

$$p_n = 1 - \left(1 - e^{-0.5 \times LR_n(\gamma_0)^2}\right)^2. \quad (6.11)$$

In case of heteroskedasticity in  $\epsilon_t$ , the distribution function of the nuisance parameter is no longer available in closed form which would be a requirement to test the significance of the threshold. For heteroskedastic error terms, Hansen (1996) therefore introduces a Lagrange multiplier (LM) test with p-values that are estimated using a bootstrap method simulating the asymptotic null distribution. We will follow this testing approach given the heteroskedastic patterns observed in electricity spot prices and apply 1,000 bootstrap replications.

## 6.2 Empirical Results

In this section we will present results for threshold models with expected wind infeed as the threshold variable. In-sample results will be followed by out-of-sample results. The elimination procedure for outliers (i.e. excluding all observations three standard deviations below and above the mean observed spot price) is the same as in the preceding chapter and so are the observation windows for in-sample and out-of-sample

data. The estimation procedure is implemented as discussed in the preceding section with  $y_t$  being the vector of observed day-ahead electricity spot prices and  $x_t$  being the time-varying vector of exogenous variables in equations 6.2 and 6.3.

## 6.2.1 In-Sample Results

### 6.2.1.1 Threshold Estimates & Goodness of Fit

Table 6.1 summarizes the most important estimation results for all hours of the day. The results show that a highly significant threshold is found for most hours. Using the Lagrange Multiplier statistic and p-values obtained from the bootstrap algorithm by Hansen (1996), the null hypothesis of no threshold effect can be rejected at a 99% significance level for most hours. Hours 15, 17, and 24 are exceptions as we cannot reject the null hypothesis on a 95% significance level in these cases. For on/off-peak switching hours 7 and 19, a high number of observations fall into the 95% confidence interval of the threshold and thus cannot be classified into one of the two regimes. In addition, we can see that only a very low percentage of observations is located above the threshold for these hours (i.e. less than 30 observations) which increases the uncertainty about the level of the threshold. Given the poor results, we will exclude hours 7, 15, 17, 19, and 24 from the following discussion. For most other hours the confidence region is rather narrow meaning that the threshold can be located with a much higher certainty.

Looking at the threshold levels across the day we can report that threshold levels for peak hours are in general lower than for early morning hours when demand is at lower levels. It is our belief that this can be derived from intermittent infeed of electricity from wind replacing supply from sources which have much higher marginal costs in times of higher demand. In such situations, a relatively small additional amount of supply from wind power plants can have a disproportional impact on the price formation. Contrary, during low demand night hours, wind energy replaces electricity from comparatively inexpensive coal plants; hence the impact might be much weaker.

Figure 6.1 presents the normalized likelihood ratio  $LR_n^*$  as well as the 95% confidence interval for six selected hours. We have estimated the nuisance parameter  $\eta^2$ , which is required to obtain  $LR_n^*$ , using an Epanechnikov Kernel<sup>1</sup> with a plug-in bandwidth as proposed by Hansen.<sup>2</sup> The plots illustrate that results generally prove to be

<sup>1</sup>See Epanechnikov (1969).

<sup>2</sup>See Hansen (2000) and Durlauf & Johnson (1995) for details.

more clear-cut for hours in the first half of the day. When looking for example at hour 3, we can observe a very clear indication for a threshold at slightly above 5,000 MWh. In contrast to this, we have less distinct results for hours later in the day (e.g. hours 12 or 18).

For comparison, we additionally report  $R^2$  measures for a regression model without a threshold in table 6.1. A closer look at goodness of fit measures reveals that  $R^2$  can be improved between 0.01 and 0.05 when allowing for two regimes depending on the level of expected wind infeed. The increase is highest for hours in the first half of the day and lower for hours after 12:00 pm. This is in line with the aforementioned observation that a threshold can be evaluated more clearly for early morning and morning hours.

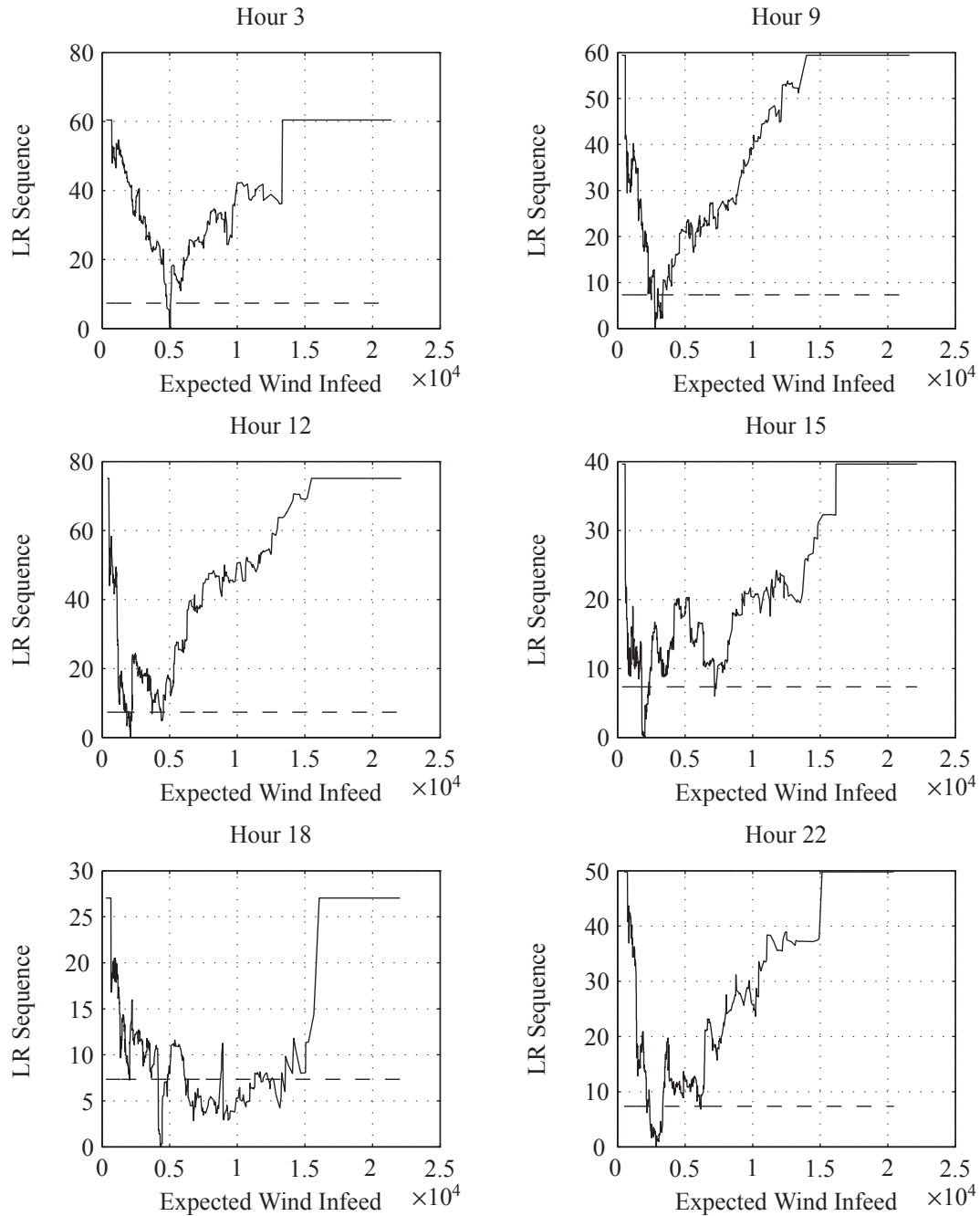
Compared to in-sample results of GARCH regression models we can report that  $R^2$  measures for threshold regression models are between 0.02 and 0.11 higher. This seems sensible as we allow for a second linear relationship.

Reported Durbin/Watson statistics are at reasonable levels and do not give rise to autocorrelation concerns.

Hour	1	2	3	4	5	6	7	8
Obs.	486	487	486	491	489	484	489	492
Threshold	4421	5075	5052	4604	6500	6454	11592	2564
Lower Confid. Int.	3922	4829	4778	4335	5654	5226	1683	2449
Upper Confid. Int.	5619	5336	5061	5793	10596	6610	11904	3817
Obs. below TH (pct.)	62.3	68.8	68.1	62.9	77.7	78.1	95.5	41.3
Obs. above TH (pct.)	37.7	31.2	31.9	37.1	22.3	21.9	4.5	58.7
LM-Statistic	42.05	46.04	41.11	33.68	39.17	37.19	24.80	32.35
p-Value	0.000	0.000	0.000	0.000	0.002	0.000	0.022	0.002
$R^2_{without\ Threshold}$	0.69	0.66	0.64	0.65	0.64	0.66	0.59	0.67
$R^2_{with\ Threshold}$	0.72	0.71	0.69	0.69	0.68	0.69	0.62	0.71
$R^2_{below\ TH}$	0.70	0.69	0.68	0.69	0.66	0.69	0.60	0.77
$R^2_{above\ TH}$	0.69	0.65	0.61	0.60	0.60	0.63	0.81	0.64
MAE	2.63	3.15	3.67	4.23	4.04	2.76	2.97	3.56
MAPE	0.075	0.106	0.142	0.214	0.170	0.080	0.062	0.062
$\sigma_e$	3.74	4.36	4.95	5.64	5.39	3.75	4.02	4.77
D/W	1.71	1.76	1.71	1.66	1.68	1.91	2.08	1.88
Hour	9	10	11	12	13	14	15	16
Obs.	491	491	490	490	492	492	492	491
Threshold	2766	3082	2234	2110	1800	1913	2020	7340
Lower Confid. Int.	2489	1901	1645	1781	1381	1585	1802	734
Upper Confid. Int.	3326	3435	3963	4542	2254	3784	7283	9031
Obs. below TH (pct.)	45.2	49.1	36.9	33.1	28.0	29.5	29.5	75.8
Obs. above TH (pct.)	54.8	50.9	63.1	66.9	72.0	70.5	70.5	24.2
LM-Statistic	40.08	51.46	46.16	42.27	34.94	29.88	21.12	24.77
p-Value	0.000	0.000	0.000	0.000	0.002	0.001	0.074	0.018
$R^2_{without\ Threshold}$	0.68	0.68	0.68	0.66	0.69	0.70	0.69	0.70
$R^2_{with\ Threshold}$	0.73	0.73	0.72	0.69	0.72	0.72	0.71	0.71
$R^2_{below\ TH}$	0.72	0.73	0.72	0.65	0.73	0.69	0.67	0.69
$R^2_{above\ TH}$	0.71	0.70	0.69	0.68	0.70	0.71	0.69	0.60
MAE	3.51	3.28	3.22	3.44	3.23	3.26	3.33	3.35
MAPE	0.058	0.054	0.054	0.056	0.055	0.058	0.061	0.062
$\sigma_e$	4.58	4.23	4.15	4.49	4.12	4.15	4.32	4.29
D/W	1.80	1.76	1.75	1.78	1.72	1.73	1.74	1.81
Hour	17	18	19	20	21	22	23	24
Obs.	487	486	484	489	492	491	488	490
Threshold	1050	4345	13262	10382	6742	2847	3147	14600
Lower Confid. Int.	779	2037	6514	9626	5885	2155	2532	3030
Upper Confid. Int.	7633	13531	13404	13696	6899	6161	3284	14600
Obs. below TH (pct.)	11.1	56.2	95.0	90.4	76.6	41.3	45.1	97.1
Obs. above TH (pct.)	88.9	43.8	5.0	9.6	23.4	58.7	54.9	2.9
LM-Statistic	20.21	24.89	25.54	25.00	33.88	28.87	24.60	21.72
p-Value	0.144	0.031	0.024	0.025	0.001	0.002	0.019	0.057
$R^2_{without\ Threshold}$	0.76	0.82	0.79	0.78	0.78	0.76	0.74	0.74
$R^2_{with\ Threshold}$	0.77	0.83	0.81	0.79	0.80	0.78	0.76	0.75
$R^2_{below\ TH}$	0.77	0.84	0.81	0.79	0.82	0.82	0.78	0.73
$R^2_{above\ TH}$	0.76	0.78	0.71	0.76	0.68	0.71	0.71	0.95
MAE	3.11	3.49	3.80	3.53	2.99	2.50	2.18	2.14
MAPE	0.059	0.061	0.062	0.059	0.053	0.049	0.044	0.049
$\sigma_e$	3.99	4.61	5.09	4.77	3.93	3.28	2.88	2.90
D/W	1.83	1.82	1.85	1.92	1.87	1.81	1.88	1.75

**Table 6.1:** Summary of in-sample results for hourly threshold regression models. The underlying dataset starts on January 1, 2010 and ends on December 31, 2011. LM-statistics and according p-values are based on the Lagrange Multiplier test as proposed by Hansen (1996). The p-value denotes the probability that the null hypothesis of no threshold is wrongly rejected.





**Figure 6.1:** Normalized likelihood ratio sequences  $LR_n^*(\gamma)$  plotted as functions of the threshold variable (expected wind infeed). The dashed line denotes the 95% confidence interval of the threshold.

### 6.2.1.2 Factor Loads

In order to investigate the role of wind forecast in price building dynamics, we take a closer look at coefficients of different fundamental variables below and above the threshold level. For some of the variables we see distinct patterns whereas our results do not reveal rich information for others. In tables 6.2 and 6.3 detailed estimation results for two exemplary hours (hour 3 representing off-peak hours and hour 12 representing peak hours) are presented.

The factor load of *expected wind infeed* itself proves to be relatively stable in the regimes above the estimated threshold levels for most hours. However, for times during which the expected level of wind infeed is below the threshold, our estimation yields higher absolute coefficient values (in the negative area) meaning that spot prices react much more sensitively to changes in expected wind infeed within these regimes. This is particularly the case for peak hours with high demand around noon. A possible explanation for this pattern could be derived from the fact that when demand is relatively high and wind infeed is low at the same time, the situation is tense. Accordingly, spot prices are particularly sensitive to intermittent supply from renewables. This rational makes especially sense given that thresholds for noon peak hours are generally low and the regime below the threshold represents a tense situation. The observed pattern is clearly revealed by the results for hour 12 which are presented in table 6.3. For hour 12, the factor load of wind forecast below the threshold is more than four times the factor load above the threshold (i.e. -0.0029 versus -0.0007).

In the coefficients referring to *prices of emission allowances* we can observe the most distinct pattern. In times of expected wind levels above the threshold, absolute coefficient values of CO<sub>2</sub> prices are higher than in times of expected wind levels below the threshold. This behavior might be reasoned by the fact that higher wind infeed into the grid shifts the merit order curve to the right. As a result, the spot price is no longer set by gas fired plants but rather by CO<sub>2</sub> intense hard coal fired plants (for peak hours) or even more CO<sub>2</sub> intense lignite fired plants (for off-peak hours) for which prices of emission allowance certificates are more relevant. The observed pattern is reflected in the results for hours 3 and 12 presented in tables 6.2 and 6.3. For these hours, factor loads below the threshold are 0.60 and 0.71 while they amount to 0.85 and 0.92 in the regime above the threshold.

Looking at the factor loads of *fuel prices*, we can generally note that the statistical significance of coefficients is rather poor, driven by high standard errors. This observation is contrary to the results from GARCH regression models where factor loads for fuel prices were more or less persistently highly significant.

Expected demand and expected power plant availability have highly significant

factor loads in both regimes for all hours. For *expected demand*, results indicate higher (positive) factor loads for the regime above the threshold. For *expected power plant availability*, absolute sensitivities (negative coefficient values) in the regime above the threshold are in general relatively higher for early morning hours with low demand levels and relatively lower for peak hours.

Observations	486	$R^2_{without TH}$	0.64		
Threshold	5052	$R^2_{with TH}$	0.69		
LM-Statistic	41.11	MAE	3.67		
p-Value	0.000	MAPE	0.142		
D/W	1.71	$\sigma_e$	4.95		
<b>Regime 1: Wind Forecast <math>\leq</math> Threshold</b>			<b>Regime 2: Wind Forecast <math>&gt;</math> Threshold</b>		
<i>Variable</i>	<i>Coefficient</i>	<i>t-Stat</i>	<i>Variable</i>	<i>Coefficient</i>	<i>t-Stat</i>
Constant	24.63	1.41	Constant	-46.28	-1.13
Spot(-1)	-0.04	-0.85	Spot(-1)	0.07	0.66
Spot Av.(-1)	0.22	2.69	Spot Av.(-1)	0.21	1.19
Spot Price Volatility	-0.21	-2.15	Spot Price Volatility	-0.38	-1.60
Coal Price	0.32	2.32	Coal Price	-0.90	-2.92
Gas Price	-0.01	-0.99	Gas Price	0.06	3.41
Coal/Gas Price Ratio	-407.58	-1.48	Coal/Gas Price Ratio	1464.15	2.47
CO <sub>2</sub> Price	0.60	5.61	CO <sub>2</sub> Price	0.85	3.33
Wind Forecast	-0.0013	-6.20	Wind Forecast	-0.0010	-5.12
Exp. Power Plant Av.	-0.0005	-4.62	Exp. Power Plant Av.	-0.0016	-5.97
Exp. Demand	0.0009	5.70	Exp. Demand	0.0018	5.66
Seasonality	2.80	4.41	Seasonality	6.49	3.37
Observations	331		Observations	155	
$R^2$	0.68		$R^2$	0.61	

**Table 6.2:** *Threshold regression in-sample results for hour 3 using expected wind infeed as threshold variable. The observation window starts on January 1, 2010 and ends on December 31, 2011. T-statistics are computed based on White-corrected error terms to account for heteroskedasticity (for details see White (1980)).*

Observations	490	$R^2_{without TH}$	0.66		
Threshold	2110	$R^2_{with TH}$	0.69		
LM-Statistic	42.27	MAE	3.44		
p-Value	0.000	MAPE	0.056		
D/W	1.78	$\sigma_e$	4.49		
<b>Regime 1: Wind Forecast <math>\leq</math> Threshold</b>			<b>Regime 2: Wind Forecast <math>&gt;</math> Threshold</b>		
<i>Variable</i>	<i>Coefficient</i>	<i>t-Stat</i>	<i>Variable</i>	<i>Coefficient</i>	<i>t-Stat</i>
Constant	26.06	0.84	Constant	0.89	0.04
Spot(-1)	0.43	4.70	Spot(-1)	0.08	1.17
Spot Av.(-1)	0.13	0.71	Spot Av.(-1)	0.16	1.82
Spot Price Volatility	-0.15	-0.98	Spot Price Volatility	0.15	1.41
Coal Price	-0.36	-1.74	Coal Price	-0.30	-1.74
Gas Price	0.02	1.68	Gas Price	0.02	2.49
Coal/Gas Price Ratio	405.72	1.01	Coal/Gas Price Ratio	396.59	1.12
CO <sub>2</sub> Price	0.71	3.80	CO <sub>2</sub> Price	0.92	7.58
Wind Forecast	-0.0029	-4.23	Wind Forecast	-0.0007	-12.26
Exp. Power Plant Av.	-0.0008	-4.31	Exp. Power Plant Av.	-0.0006	-6.20
Exp. Demand	0.0005	3.45	Exp. Demand	0.0007	5.04
Seasonality	4.16	3.70	Seasonality	1.37	1.97
Observations	162		Observations	328	
$R^2$	0.65		$R^2$	0.68	

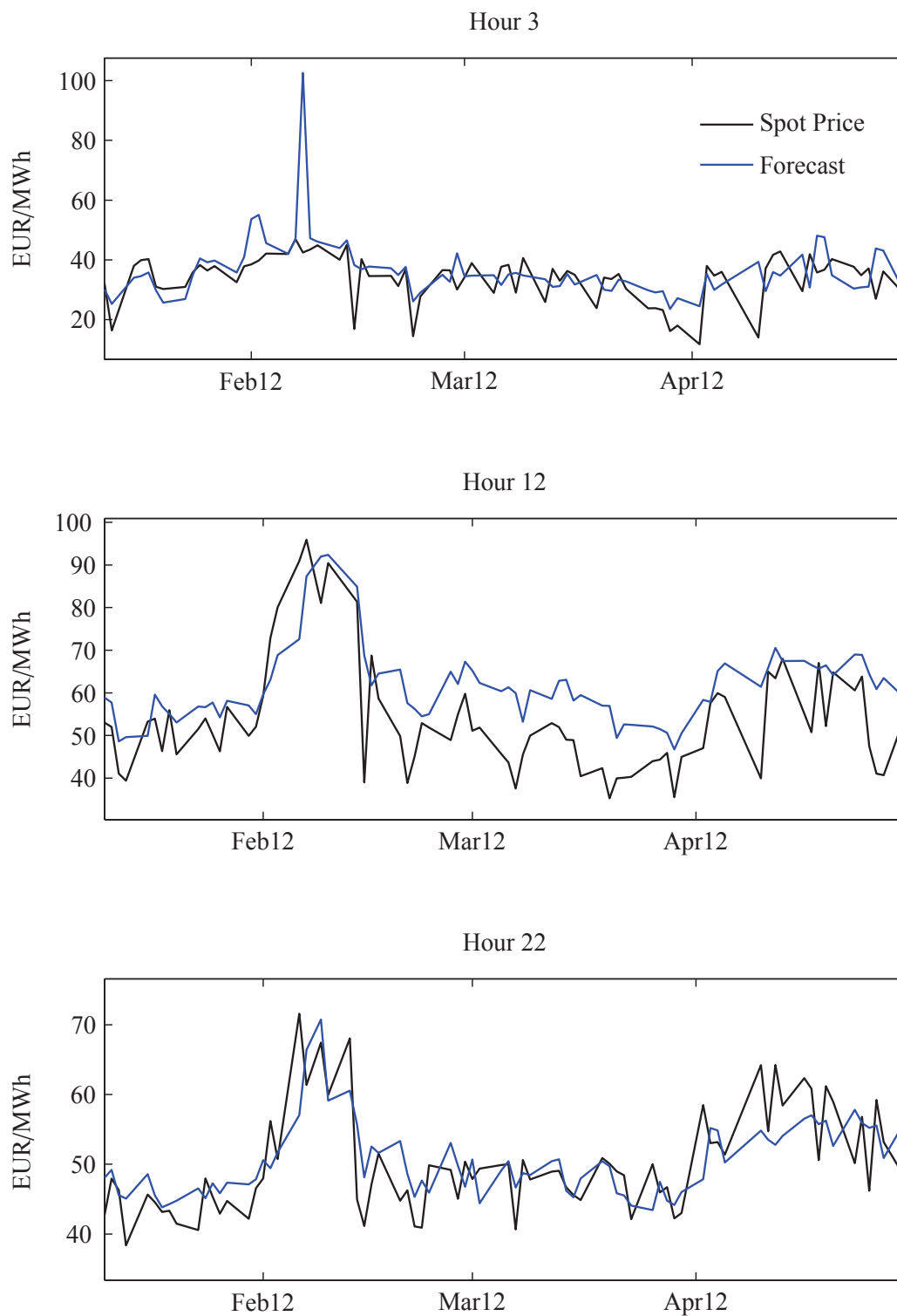
**Table 6.3:** *Threshold regression in-sample results for hour 12 using expected wind infeed as threshold variable. The observation window starts on January 1, 2010 and ends on December 31, 2011. T-statistics are computed based on White-corrected error terms to account for heteroskedasticity.*

### 6.2.2 Out-of-Sample Results

Table 6.4 depicts a summary of the out-of-sample results we obtained by applying the calibrated threshold regression models to data between January 1, 2012 and April 30, 2012. Already at first sight we realize that the fit is rather poor. While the in-sample fit was superior to the results from GARCH regression models, out-of-sample results of threshold regression models seem to be worse. We observe that especially for early morning and afternoon hours, results indicate a rather bad fit with  $R^2$  being negative and the MAPE ranging between 15% and 25%. When considering exclusively hours which we did not exclude from discussion due to lack of statistical significance in section 6.2.1, we note that the MAPE for out-of-sample threshold regression models is, on average, about 10 percentage points higher than for GARCH regression models. This is contrary to in-sample results where we were able to improve the fit by splitting the sample. Figure 6.2 displays out-of-sample fits for hours 3 and 12 where the fit proves to be rather bad as well as for hour 22 where we observe a better fit with an  $R^2$  of 0.54.

Hour	1	2	3	4	5	6	7	8
Obs.	76	75	76	77	77	76	77	77
$R^2$	0.30	-0.29	-0.70	-0.16	0.18	0.13	-0.95	0.61
MAE	4.48	5.62	6.49	5.46	4.77	3.68	6.36	7.32
MAPE	0.154	0.199	0.235	0.212	0.195	0.133	0.148	0.127
$\sigma_e$	6.17	8.94	9.87	7.70	6.37	5.89	10.69	9.13
Hour	9	10	11	12	13	14	15	16
Obs.	76	75	77	76	75	76	76	76
$R^2$	0.56	0.40	0.46	0.22	-0.21	-0.18	-0.26	-0.31
MAE	7.34	8.00	9.73	9.48	10.30	9.81	8.14	7.42
MAPE	0.112	0.142	0.191	0.200	0.238	0.224	0.189	0.169
$\sigma_e$	10.70	9.91	9.57	8.27	7.20	6.71	5.89	6.21
Hour	17	18	19	20	21	22	23	24
Obs.	76	77	75	77	76	77	77	76
$R^2$	0.24	0.68	0.51	0.46	0.21	0.54	0.50	-21.09
MAE	6.13	7.25	10.90	9.65	6.67	3.85	3.43	8.10
MAPE	0.140	0.139	0.181	0.142	0.123	0.077	0.071	0.224
$\sigma_e$	5.38	7.32	12.74	13.37	6.52	4.91	4.45	31.32

**Table 6.4:** Summary of out-of-sample results for hourly threshold regression models. The underlying out-of-sample dataset starts on January 1, 2012 and ends on April 30, 2012.



**Figure 6.2:** *Out-of-sample fit of threshold regression models for hours 3, 12, and 22. The underlying dataset starts on January 1, 2012 and ends on April 30, 2012.*

### 6.3 Intermediate Summary

In general, out-of-sample results of the threshold regression models prove to be poor. As possible explanations for the inferior out-of-sample fits we see the following. On the one hand, the location of the thresholds may be heavily driven by the occurrence of rather extreme and non-repetitive values in spot prices as well as in explanatory variables. Moreover, allowing for two regimes might still be too restrictive given the dynamic behavior of electricity spot markets. Complementary to GARCH regression results, threshold regression models confirm that static regression models perform rather well in catching price dynamics by fundamental variables in-sample. However, out-of-sample there is obviously a need to apply models which allow for more flexibility.

Given our findings from both, GARCH regression and threshold regression models, we will introduce a more dynamic approach in the following chapter. We will estimate a time-varying parameter regression model which allows continuous adaption of price sensitivities.





## Chapter 7

# Time-Varying Parameter Regression Models

When applying conventional regression models, researchers have to assume that factor sensitivities are constant over time. This assumption is often inappropriate either because it conflicts with empirical data or because the author is interested in the intertemporal changes of factor loads. Formulating a state space model which allows for changing regression coefficients and estimating it by means of a filter developed by Kalman (1960) is a viable approach to tackle the problem.

Subsequently, we will first introduce the Kalman filter algorithm in its general form. We will then explain how we shall apply it to estimate a fundamental regression model allowing for time-varying coefficients. This methodical part will be followed by the presentation and discussion of empirical results and a comparison of forecast accuracy with mainly GARCH regression but also threshold regression models.

## 7.1 The Kalman Filter

### 7.1.1 Main Algorithm

The approach pioneered by Kalman (1960) is based on the following filtering problem represented by a stochastic, linear discrete-time state space model:<sup>1</sup>

$$y_t = x_t' \beta_t + v_t \quad (7.1)$$

$$\beta_t = A\beta_{t-1} + Bu_{t-1} + Gw_t \quad (7.2)$$

Equation 7.1 is referred to as the *measurement equation* which relates a known quantity (vector of exogenous variables)  $x_t$  to an observed variable  $y_t$  while 7.2 describes the process governing the unobserved *state*  $\beta_t$  and is called *transition equation*. Moreover, the two equations are composed of structural matrices  $A$ ,  $B$ , and  $G$  and zero-mean error components  $v_t \sim \mathcal{N}(0, R)$  and  $w_t \sim \mathcal{N}(0, Q)$ . Measurement noise covariance  $R = E v_t v_t'$  and transition noise covariance  $Q = E w_t w_t'$  can either be constant or change with every time step. The control input  $u$  is often omitted in applications. In order to solve the problem of estimating the unknown quantity  $\beta_t$ , Kalman provides a powerful set of equations which works in a recursive mode and on which we will elaborate subsequently.

Throughout the estimation algorithm, as we run from  $t = 1$  to  $t = T$ , we distinguish between two possible states of knowledge, namely the *a priori* state when observations (i.e. elements of vector  $y$ ) up to  $t - 1$  are known and the *posterior* state when observations up to  $t$  are available. Following this separation we define

$$\hat{\beta}_t^- = E(\beta_t | y_{t-1}) \quad (7.3)$$

as the *a priori* state estimate conditional on  $y_{t-1}$  and

$$\hat{\beta}_t = E(\beta_t | y_t) \quad (7.4)$$

as the *posterior* state estimate conditional on  $y_t$ . The true state  $\beta_t$  is, as mentioned, unobserved and unknown. Consequently, any estimation procedure will result in estimation errors

$$e_t^- \equiv \beta_t - \hat{\beta}_t^- \quad (7.5)$$

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<sup>1</sup>When introducing the Kalman filter algorithm, we follow Welch & Bishop (2006).

$$\text{and } e_t \equiv \beta_t - \hat{\beta}_t \quad (7.6)$$

which are known as *a priori* and *posterior* estimation error, respectively. They have corresponding error covariances

$$P_t^- = E[e_t^- e_t^{-'}] \quad (7.7)$$

$$\text{and } P_t = E[e_t e_t']. \quad (7.8)$$

At any time step  $t$  the *a priori* state estimate conditional on  $y_{t-1}$  is governed by the *posterior* state estimate of the last time step, by an optional control input, and by the structural transition matrices  $A$  and  $B$ :

$$\hat{\beta}_t^- = A\hat{\beta}_{t-1} + Bu_{t-1} \quad (7.9)$$

The covariance of the corresponding estimation error  $e_t^-$  is projected by the following linear equation:

$$P_t^- = AP_{t-1}A' + Q \quad (7.10)$$

7.9 and 7.10 are called *prediction equations*. As  $y_t$  becomes observable we can derive the *measurement innovation* which is defined as the discrepancy of  $\hat{y}_t$  (estimated using the *a priori* state estimate  $\hat{\beta}_t^-$  and any exogenous variable available at time step  $t$ ) and the true  $y_t$ :

$$\begin{aligned} \xi_t &= y_t - \hat{y}_t \\ &= y_t - x_t' \hat{\beta}_t^- \end{aligned} \quad (7.11)$$

The goal is now to compute the *posterior* state estimate  $\hat{\beta}_t$  minimizing the covariance of its estimation error, namely  $P_t$ . The linear equation defining the *posterior* state estimate is defined as the sum of the *a priori* state estimate and the weighted *measurement innovation*

$$\hat{\beta}_t = \hat{\beta}_t^- + K_t \xi_t \quad (7.12)$$

with a corresponding *posterior* covariance

$$P_t = P_t^- - P_t^- K_t x_t. \quad (7.13)$$

7.12 and 7.13 are called *update equations*. The weighting factor  $K_t$  is called Kalman gain and chosen to minimize  $P_t$  and is defined as follows:

$$K_t = \frac{P_t^- x_t'}{x_t P_t^- x_t' + R} \quad (7.14)$$

After the *posterior* state estimate in 7.12 has been computed, it is used to project the next state  $\hat{\beta}_{t+1}^-$ . The entire procedure is repeated until all information in  $y = \{y_0, \dots, y_T\}$  has been used.<sup>2</sup>

Assuming normally distributed disturbances, the recursion is optimal in the sense that it minimizes the mean square error among all estimators. Another advantage of the method is that no information is lost. Instead, all information is transmitted from one stage to the next, mainly through the prediction equations. The updating equations care for a discounting of older information and over time, older observations become less important. The prediction errors are not weighted with a fixed or given factor. Instead, the weight is determined by the dynamic Kalman gain  $K_t$ .

As an additional (optional) step, a smoothing algorithm can be performed when the filter procedure has come to an end. The so-called *Kalman smoother* re-estimates state variables  $\beta_t$  given the full information set of  $y$  up to  $T$  applying a backwards working iteration. In finance the Kalman smoother is a popular method to estimate style exposures of hedge funds *ex post*.<sup>3</sup> We will not explain the smoother in more detail as this feature will not be used in the thesis at hand. An introduction to the smoothing algorithm can be found in Kim & Nelson (1999) or, in a more detailed format, in Chui & Chen (2009).

For our estimation of a time-varying-parameter (TVP) regression model, the measurement equation of the state space model is implemented as in 7.1 with  $y_t$  denoting the day-ahead electricity spot price and regressor matrix  $x_t$  incorporating all fundamental variables including a constant (we repeat it for convenience):

$$y_t = x_t' \beta_t + v_t \quad (7.15)$$

Furthermore, the evolution of the state  $\beta_t$  (i.e. the time-varying regression coefficient) is assumed to follow a random walk according to

$$\beta_t = \beta_{t-1} + w_t \quad (7.16)$$

meaning that we represent matrices  $A$  and  $G$  by the identity matrix. We do not use the optional control variable  $u_{t-1}$  in 7.2. The predicted day-ahead electricity spot price for time  $t$  is projected applying the *a priori* estimated regression coefficient  $\hat{\beta}_t^-$  of this stage to the observed exogenous variables.

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<sup>2</sup>For a more detailed derivation of the filter see Hamilton (1994a) or Hamilton (1994b).

<sup>3</sup>See for example Lhabitant (2004).

From a methodological perspective, this approach of modelling time-varying parameter regression models has for example been followed by Kim & Nelson (1989), Song & Wong (2003), or Karakatsani & Bunn (2010).

### 7.1.2 Initialization of Critical Parameters

Besides the state  $\beta_t$  there is a set of parameters  $(A, B, G, R, Q)$  which are also unobserved. While finding an appropriate specification for the structural matrices  $A$ ,  $B$ , and  $G$  is usually a rather easy task, initializing the covariances of the disturbances  $R$  and  $Q$  is less obvious. In some rare cases, the two are partly or even completely known from former empirical work. However, most of the time they have to be determined by the researcher. This is a critical task as ill defined matrices  $R$  and  $Q$  lead to a filter which is no longer optimal.<sup>4</sup> Mohamed & Schwartz (1999) elaborate on the consequences of suboptimal a priori statistics. For example, if  $R$  and/or  $Q$  on the one hand have too small values, the probability band around the true value decreases accordingly and the resulting state estimate may be biased. On the other hand, if they are chosen too large, the result can be a practical divergence of the filter.

Following the introduction of the filter in the sixties, several methods to mitigate the risk of initializing wrong covariances  $R$  and  $Q$  have been introduced. Odelson et al. (2005) group them into four main categories: Bayesian, maximum likelihood, covariance matching, and correlation techniques. Filter routines applying one of these methods to derive matrices  $R$  and  $Q$  are often summarized under the term *Adaptive Kalman Filters*. Adaptive Kalman filtering means that the overall estimation procedure is split into two related steps. Noise covariances are determined during data processing in a first step and applied to estimate the unobserved state via the Kalman filter algorithm in a second step. Bayesian and maximum likelihood approaches were largely pioneered and developed by Schweppe (1965), Hilborn & Lainiotis (1969), Kashyap (1970), Bohlin (1976), and Alspach (1974). In the thesis at hand we will apply the maximum likelihood approach as explained subsequently.

In chapter 5 we have introduced the generalized form of the log likelihood function which reads as

$$\ln L(\theta) = \sum_{t=1}^T \ln(f(y_t|\theta)). \quad (7.17)$$

Schweppe (1965) and also Harvey (1981) show that the log likelihood function to

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<sup>4</sup>See Oussalah & de Schutter (2000) or Odelson et al. (2005).

estimate noise covariances  $R$  and  $Q$  can be built on the *measurement innovations* as introduced in equation 7.11 and is formulated as follows:

$$\ln L(\theta) = -\frac{1}{2} \sum_{t=1}^T (\ln |C_t| + \xi_t' C_t^{-1} \xi_t) \quad (7.18)$$

where  $C_t$  denotes the contemporaneous variance covariance matrix of the measurement innovations  $\xi_t$  which corresponds to the denominator in equation 7.14,  $x_t P_t^- x_t' + R$ . Matrix  $C_t$  is dependent on  $R$  and additionally on  $Q$ , namely through the estimation error covariance  $P_t^-$  as defined in equation 7.10. Based on the assumption that innovations  $\xi_t$  are normally distributed we can rewrite the log likelihood function more detailed:

$$\begin{aligned} \ln L(\theta) &= -\frac{1}{2} \sum_{t=1}^T \left\{ \ln [2\pi \times (x_t P_t^- x_t' + R)] + \frac{(y_t - x_t \hat{\beta}_t^-)^2}{(x_t P_t^- x_t' + R)} \right\} \\ &= -\frac{1}{2} \sum_{t=1}^T \ln [2\pi \times (x_t (A P_{t-1}^- A' + Q) x_t' + R)] \\ &\quad - \frac{1}{2} \sum_{t=1}^T \left\{ \frac{(y_t - x_t \hat{\beta}_t^-)^2}{(x_t (A P_{t-1}^- A' + Q) x_t' + R)} \right\} \end{aligned} \quad (7.19)$$

The log likelihood function is constructed using a first limited number of observations (in our case 50 observations) and is then minimized in order to obtain estimates for covariance matrices  $R$  and  $Q$  which we assume to be constant over time. Afterwards, the pre-estimated covariances are passed to the second stage, where the Kalman filter is applied using the remaining observations in order to obtain the desired estimates for the state  $\beta_t$ .

## 7.2 Empirical Results

Subsequently, we will present empirical results for the in-sample and out-of-sample estimation of the hourly TVP regression models and compare them with the results from GARCH regression and threshold regression models. When discussing results for the in-sample dataset we will additionally investigate seasonal patterns in the regression coefficients of expected wind infeed. Furthermore, we will analyze the conditional volatility of measurement innovations which we extract from the Kalman filter algorithm.

## 7.2.1 Conditional Mean Modelling

### 7.2.1.1 In-Sample Results

In order to measure the explanatory power of expected wind infeed we first estimate the models excluding the expected wind infeed variable and then we account for it in a second step.

Table 7.1 depicts the in-sample results for the estimated models without wind forecast and table 7.2 shows the in-sample results of the estimated models including wind forecast. Without considering wind we can observe an  $R^2$  of between 0.46 and 0.75 with an average of 0.58 across the 24 hourly models while the MAPE is between 5.4% and 31.9% with an average of 9.5%. The MAE amounts to between 2.71 and 5.40 with an average of 3.86. Similar to the GARCH regression models, it is obvious that particularly for early morning hours the model fit proves to be at rather poor levels.

Including the wind variable we can improve the fit significantly and report an  $R^2$  of between 0.57 and 0.85 with an average of 0.71. At the same time, the MAPE for models including wind reduces to between 4.7% and 21.7% with an average of 7.5% and the MAE reduces to between 2.32 and 4.16 with an average of 3.19. Table 7.1 illustrates the improvements which are achieved by the inclusion of the wind variable.  $R^2$  is improved by 0.13 on average and the MAPE is reduced by 2.0% on average. The MAE is reduced by 0.67 on average which in economic terms means that including the wind variable has a value of 67 cents per MWh. When comparing TVP regression in-sample results with the results for the GARCH regression models, it is apparent that for the case where wind is excluded, the model fit of GARCH regression models is slightly better for most hours. Results for the TVP models are, however, clearly better than the GARCH regression estimates for most hours when including wind. Exceptions are hours 22 to 24 where GARCH regression models dominate TVP regression models for in-sample results. At first sight it may seem dubious that TVP regression models do not consistently provide superior results. Allowing for time-varying coefficients, they should be much better able to model price variability. However, one has to consider that estimations via TVP regression models only use information which is available before the respective spot price is observed whereas GARCH-regression models applied to in-sample data use all information provided by the entire dataset. It is therefore sensible that despite time-varying coefficients, the results for TVP regression models are not consistently superior compared to GARCH regression models for the in-sample period.



Hour	1	2	3	4	5	6	7	8
Obs.	436	437	436	441	439	434	439	442
$R^2$	0.54	0.50	0.46	0.49	0.48	0.59	0.50	0.61
$\bar{R}^2$	0.47	0.43	0.38	0.41	0.41	0.53	0.43	0.55
MAE	3.32	4.09	4.86	5.40	5.09	3.16	3.28	4.03
MAPE	0.097	0.142	0.201	0.319	0.235	0.092	0.068	0.070
$\sigma_\epsilon$	4.79	5.75	6.70	7.37	6.97	4.36	4.43	5.51
D/W	1.88	1.88	1.93	1.85	1.87	1.98	2.05	2.05
LLF	-1304	-1388	-1451	-1510	-1478	-1256	-1277	-1382
Hour	9	10	11	12	13	14	15	16
Obs.	441	441	440	440	442	442	442	441
$R^2$	0.60	0.59	0.56	0.54	0.54	0.54	0.53	0.55
$\bar{R}^2$	0.55	0.53	0.50	0.47	0.47	0.48	0.46	0.49
MAE	3.94	3.76	3.79	4.05	3.98	3.92	4.06	3.97
MAPE	0.065	0.062	0.063	0.066	0.068	0.069	0.073	0.073
$\sigma_\epsilon$	5.43	5.15	4.99	5.30	5.11	5.06	5.25	5.12
D/W	2.10	2.10	2.02	2.01	1.93	1.90	1.92	2.02
LLF	-1373	-1349	-1333	-1361	-1350	-1346	-1361	-1346
Hour	17	18	19	20	21	22	23	24
Obs.	437	436	434	439	442	441	438	440
$R^2$	0.65	0.74	0.74	0.75	0.69	0.61	0.58	0.51
$\bar{R}^2$	0.60	0.70	0.71	0.71	0.64	0.56	0.52	0.45
MAE	3.60	4.12	4.20	4.02	3.55	3.04	2.71	2.76
MAPE	0.066	0.071	0.069	0.067	0.062	0.058	0.054	0.064
$\sigma_\epsilon$	4.71	5.68	5.99	5.38	4.83	4.12	3.57	3.88
D/W	1.96	2.03	2.05	1.90	1.97	1.99	2.09	2.05
LLF	-1297	-1345	-1361	-1363	-1323	-1251	-1181	-1223

**Table 7.1:** Summary of in-sample results for TVP regression models excluding expected wind infeed as exogenous variable. The observation period starts on January 1, 2010 and ends on December 31, 2011. LLF denotes the value of the log likelihood function used to estimate  $R$  and  $Q$  in a preliminary step.

In order to test whether our models are appropriately specified we investigate potential autocorrelation in one step ahead forecast errors as suggested by Engle & Watson (1981).<sup>5</sup> For this purpose we apply the Ljung-Box Q-test as introduced in section 4.1.3 to the measurement innovations  $\xi_t$  (as defined in equation 7.11) for all 24 hours. We test autocorrelation at lags 1, 5, 10, and 20 and find that for most hours, we cannot

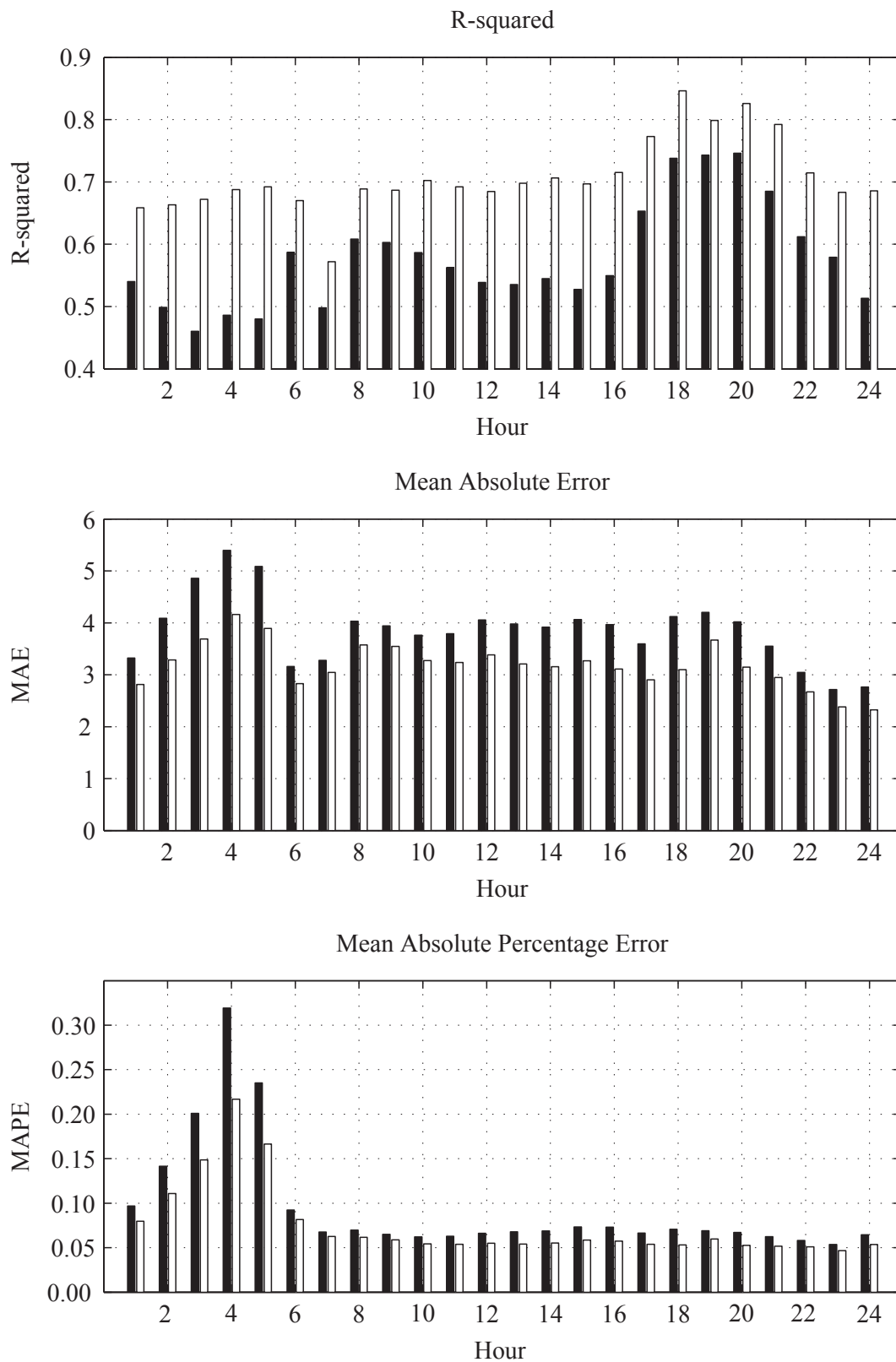
<sup>5</sup>We exclusively test forecast errors of the models including expected wind infeed.



Hour	1	2	3	4	5	6	7	8
Obs.	436	437	436	441	439	434	439	442
$R^2$	0.66	0.66	0.67	0.69	0.69	0.67	0.57	0.69
$\bar{R}^2$	0.61	0.62	0.62	0.64	0.65	0.62	0.51	0.64
MAE	2.81	3.29	3.69	4.16	3.89	2.83	3.05	3.58
MAPE	0.080	0.111	0.149	0.217	0.166	0.082	0.063	0.062
$\sigma_\epsilon$	4.13	4.71	5.22	5.75	5.36	3.89	4.09	4.92
D/W	1.85	1.92	1.95	1.97	1.95	1.94	2.05	2.03
LLF	-1220	-1279	-1323	-1371	-1336	-1199	-1237	-1331
Hour	9	10	11	12	13	14	15	16
Obs.	441	441	440	440	442	442	442	441
$R^2$	0.69	0.70	0.69	0.68	0.70	0.71	0.70	0.72
$\bar{R}^2$	0.64	0.66	0.65	0.64	0.65	0.66	0.65	0.67
MAE	3.54	3.27	3.23	3.38	3.21	3.16	3.27	3.11
MAPE	0.059	0.054	0.054	0.055	0.054	0.055	0.059	0.057
$\sigma_\epsilon$	4.82	4.37	4.19	4.39	4.12	4.06	4.20	4.07
D/W	2.12	2.08	2.04	2.01	1.97	2.01	2.01	2.06
LLF	-1320	-1275	-1255	-1276	-1250	-1244	-1258	-1238
Hour	17	18	19	20	21	22	23	24
Obs.	437	436	434	439	442	441	438	440
$R^2$	0.77	0.85	0.80	0.83	0.79	0.71	0.68	0.69
$\bar{R}^2$	0.74	0.82	0.77	0.80	0.76	0.67	0.64	0.64
MAE	2.90	3.10	3.67	3.15	2.95	2.67	2.38	2.32
MAPE	0.054	0.053	0.060	0.053	0.052	0.051	0.047	0.053
$\sigma_\epsilon$	3.81	4.36	5.30	4.46	3.92	3.53	3.10	3.12
D/W	2.02	2.17	2.21	2.01	2.04	2.06	2.05	1.92
LLF	-1195	-1226	-1277	-1273	-1217	-1175	-1119	-1116

**Table 7.2:** Summary of in-sample results for TVP regression models including expected wind infeed as exogenous variable. The observation period starts on January 1, 2010 and ends on December 31, 2011. LLF denotes the value of the log likelihood function used to estimate  $R$  and  $Q$  in a preliminary step.

reject the null hypothesis of no autocorrelation at a 99% significance level (the same applies in general for a significance level of 95%). Exceptions are hours 18, 19, and 22 where we can reject the null hypothesis at the 99% level (for higher lags). Detailed results are reported in table C.1 in the appendix. These results are confirmed by the Durbin/Watson statistics in table 7.2.

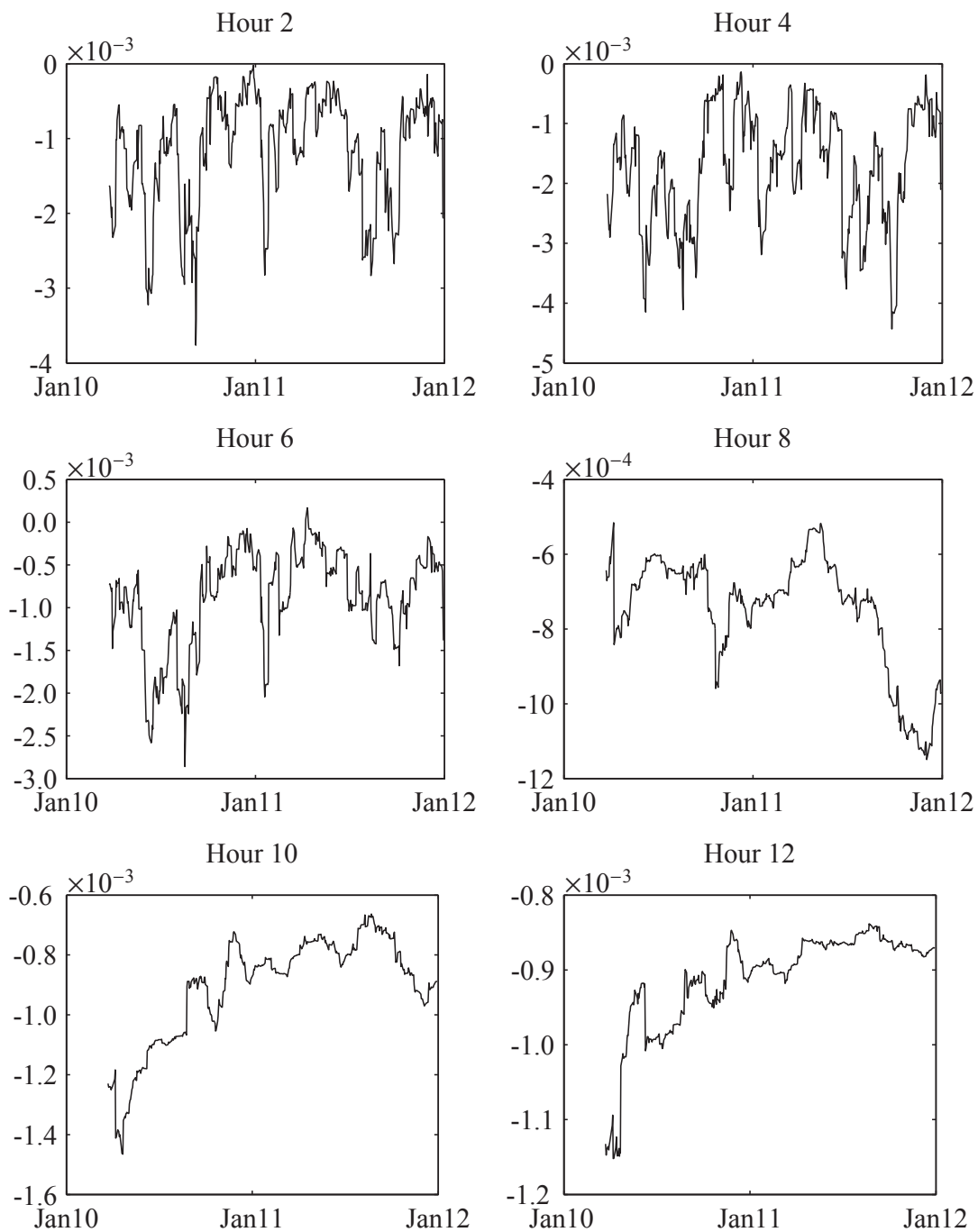


**Figure 7.1:** Improvement in goodness of fit for TVP regression models if expected wind infeed is included as exogenous variable. Black bars denote goodness of fit measures for in-sample estimated models excluding expected wind infeed, measures denoted by white bars include expected wind infeed.

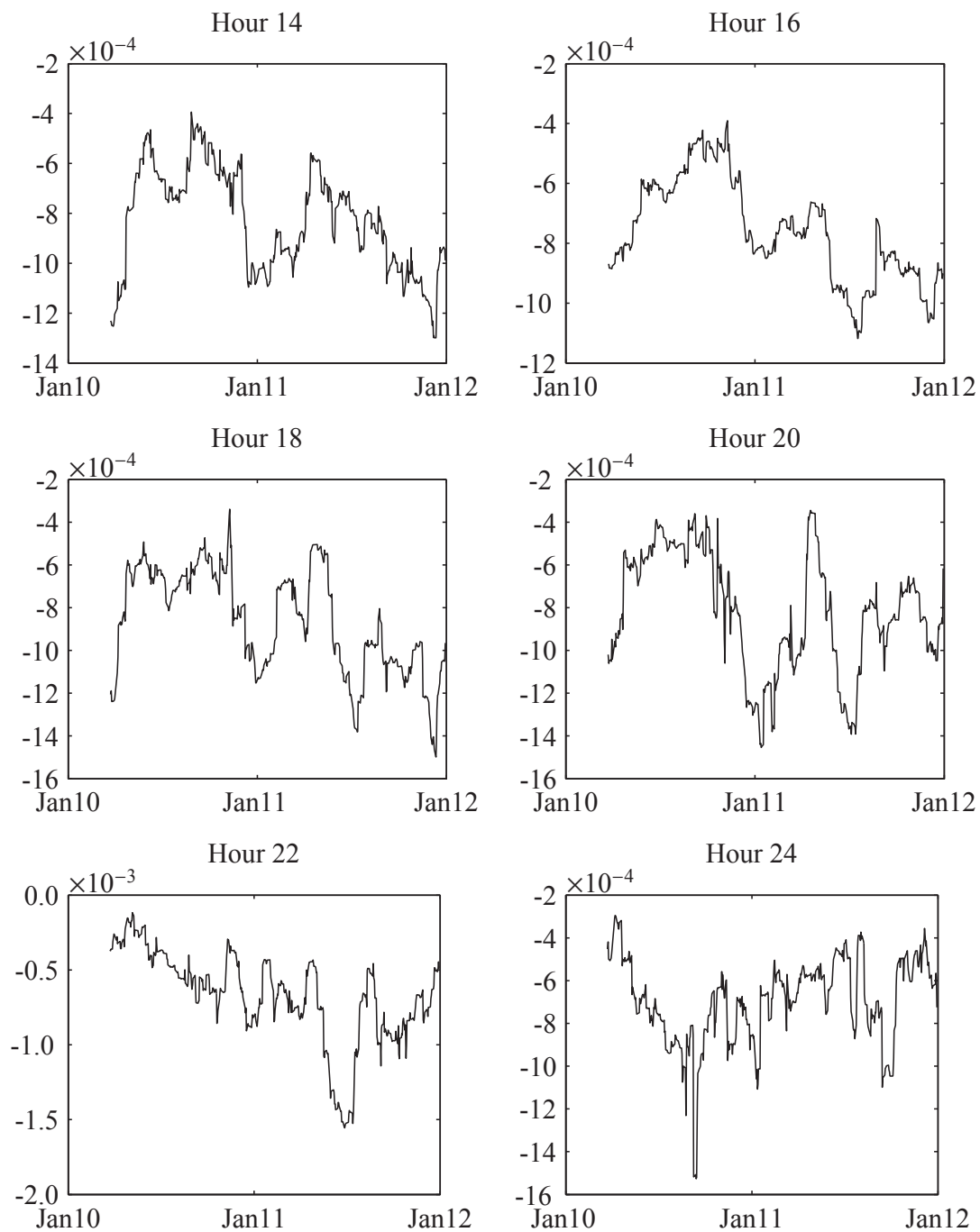
### 7.2.1.2 Seasonal Patterns

In this section we look at the evolution of time-varying regression coefficients for in-sample estimated models. As expected wind infeed is the focus variable of this thesis, we present corresponding regression coefficients for 12 selected hours in figures 7.2 and 7.3. As can be seen from the plots, the first 50 days in 2010 have no values because these days were used to estimate covariances  $R$  and  $Q$  using maximum likelihood in a preliminary step. Overall, the plots provide evidence that the price sensitivity towards expected wind infeed displays different patterns depending on the hour of the day. When looking at hours 2, 4, and 6 we see that for early morning hours the coefficient evolves highly dynamically with a tendency for higher absolute sensitivities during summer months and lower absolute sensitivities during winter months. For peak hours until noon we can observe more constant patterns which stabilize within a certain range once enough information has become available. As section 4.2.4 has revealed, expected wind infeed is at a low for these hours which comes along with a lower infeed volatility. This probably makes prices more robust with regard to the expected infeed. After noon, when higher wind levels are observed, we discern again a more volatile behavior with some evidence for seasonality noting higher absolute sensitivities during winter time and lower absolute sensitivities during summer time (see especially hour 14 in figure 7.3). Overall, the regression coefficient of expected wind infeed evolves highly dynamically for most hours which supports the application of TVP regression models.

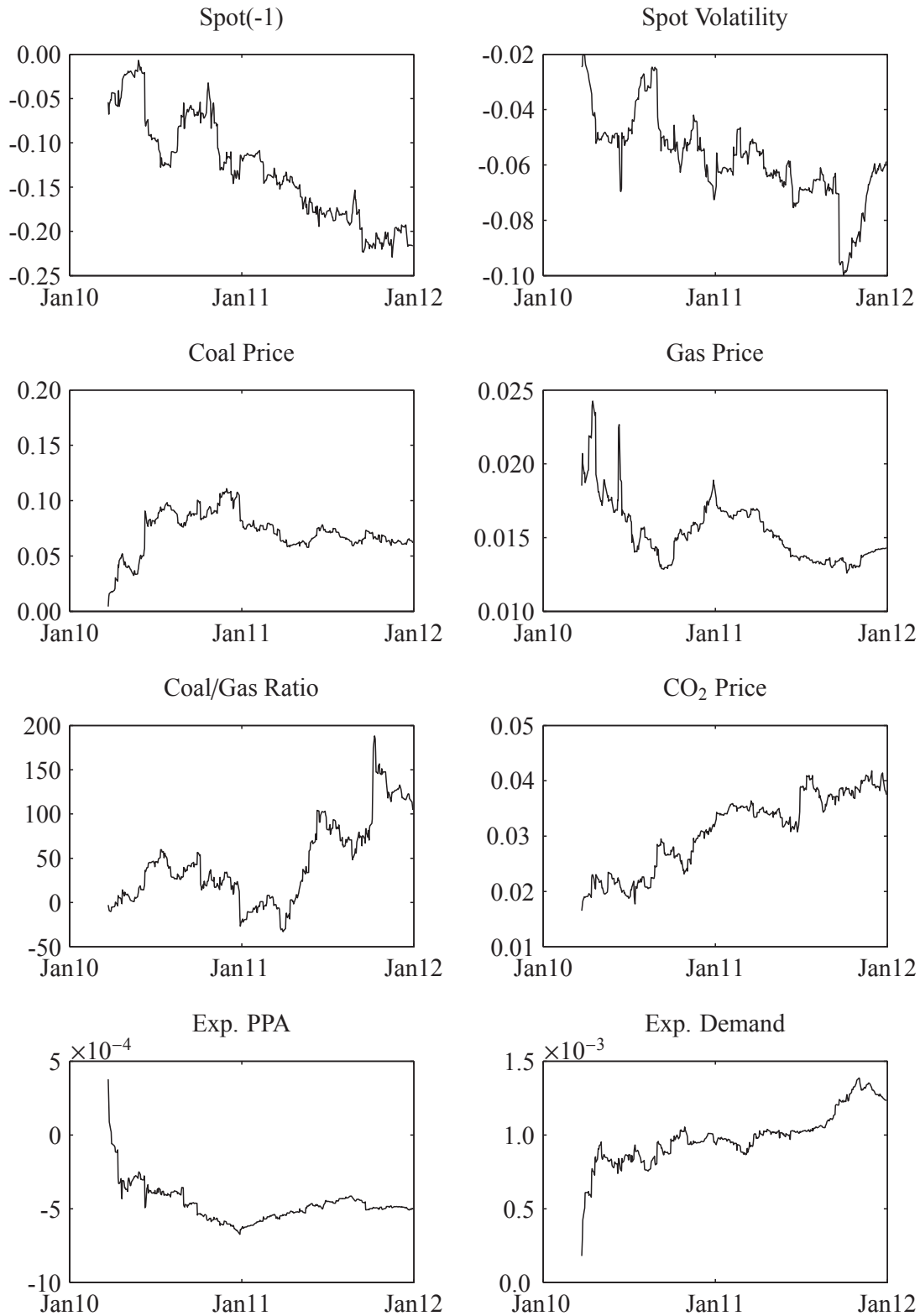
We will not discuss the evolution of other regression coefficients in detail as this is beyond the scope of this thesis whose main focus is the role of expected wind infeed. However, coefficients of selected variables for hours 12 and 18 are depicted in figures 7.4 and 7.5 for illustrative purposes. A comparison of the dynamics of various regression coefficients with patterns observed for expected wind infeed shows that the former are less volatile and behave more persistent in most cases. For example, we can study the coefficients of the expected power plant availability which stabilize once enough information is available. There are exceptions such as the evolution of the coefficient for CO<sub>2</sub> prices at hour 18 which displays a rather dynamic pattern.



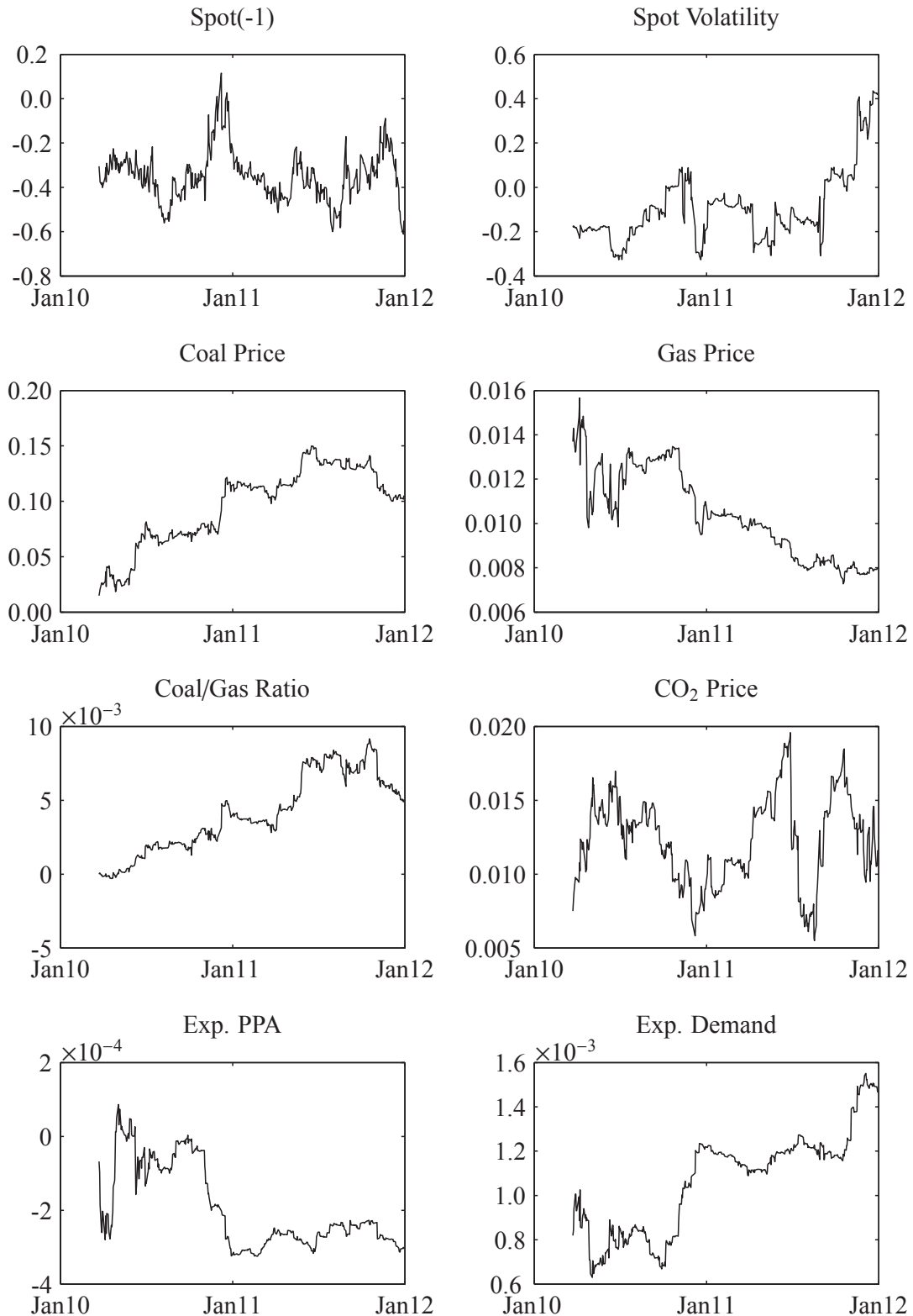
**Figure 7.2:** Evolution of expected wind infeed regression coefficients over the in-sample observation period between January 1, 2010 and December 31, 2011 for hours 2, 4, 6, 8, 10, and 12. The coefficients have been estimated applying the introduced Kalman filter routine. 50 initial observations are excluded as they were used for preliminary initialization of unknown covariances  $R$  and  $Q$ .



**Figure 7.3:** Evolution of expected wind infeed regression coefficients over the in-sample observation period between January 1, 2010 and December 31, 2011 for hours 14, 16, 18, 20, 22, and 24. The coefficients have been estimated applying the introduced Kalman filter routine. 50 initial observations are excluded as they were used for preliminary initialization of the unknown covariances.



**Figure 7.4:** Evolution of various regression coefficients over the in-sample observation period between January 1, 2010 and December 31, 2011 for hour 12. The coefficients have been estimated applying the introduced Kalman filter routine. 50 initial observations are excluded as they were used for preliminary initialization of unknown covariances  $R$  and  $Q$ .



**Figure 7.5:** Evolution of various regression coefficients over the in-sample observation period between January 1, 2010 and December 31, 2011 for hour 18. The coefficients have been estimated applying the introduced Kalman filter routine. 50 initial observations are excluded as they were used for preliminary initialization of unknown covariances  $R$  and  $Q$ .

### 7.2.1.3 Out-of-Sample Results

In order to test the hourly TVP regression models out-of-sample, we estimate them on the same dataset as GARCH regression and threshold regression models which starts on January 1, 2012 and ends on April 30, 2012. As input variables  $P_t$  and  $\hat{\beta}_t$  for  $t = 0$  we use the last updates of these variables from the in-sample estimation. Thus, all past information is preserved in the system which is a characteristic feature of the Kalman filter algorithm. Covariances  $R$  and  $Q$  remain the same as in the in-sample dataset. Throughout the out-of-sample dataset the predicted electricity spot price at time  $t$  is again computed using consecutively generated *a priori* estimated regression coefficients for this stage.

Summaries of out-of-sample results are depicted in table 7.3 (excluding wind forecast) and table 7.4 (including wind forecast). Looking at out-of-sample results from models including wind forecast we can report an  $R^2$  of 0.61 on average with a minimum value of 0.31 (hour 20) and a maximum value of 0.83 (hour 18). The best fit can generally be observed for early morning hours. From hours 1 to 5,  $R^2$  is at 0.73 or higher. This is insofar remarkable as these hours proved to have the least accurate fit when estimating in-sample. The worst out-of-sample results we report for hours 13, 20, and 21.

The results from models excluding expected wind infeed are clearly worse by 0.27 on average in terms of  $R^2$ . An analysis of MAE measures shows an improvement of 1.29 EUR/MWh on average when accounting for wind, which we consider a respectable result. Also the MAPE can be reduced by 4 percentage points on average to a MAPE of 10% on average. Overall, incorporating expected wind infeed in TVP regression models can significantly improve out-of-sample results.

Comparing TVP regression results with GARCH regression and threshold regression out-of-sample results (including wind) we note a significant improvement in fit. Compared to GARCH regression models, the  $R^2$  is higher by 0.25 on average. Compared to threshold regression models, where out-of-sample results are generally disappointing, we obtain an improvement of even 0.48 (excluding hour 24) on average. Figure 7.6 depicts the out-of-sample fit for hours 3, 12, and 18. The plots underpin the superior ability of TVP regression models to forecast day-ahead electricity spot prices compared to GARCH regression and threshold regression models.

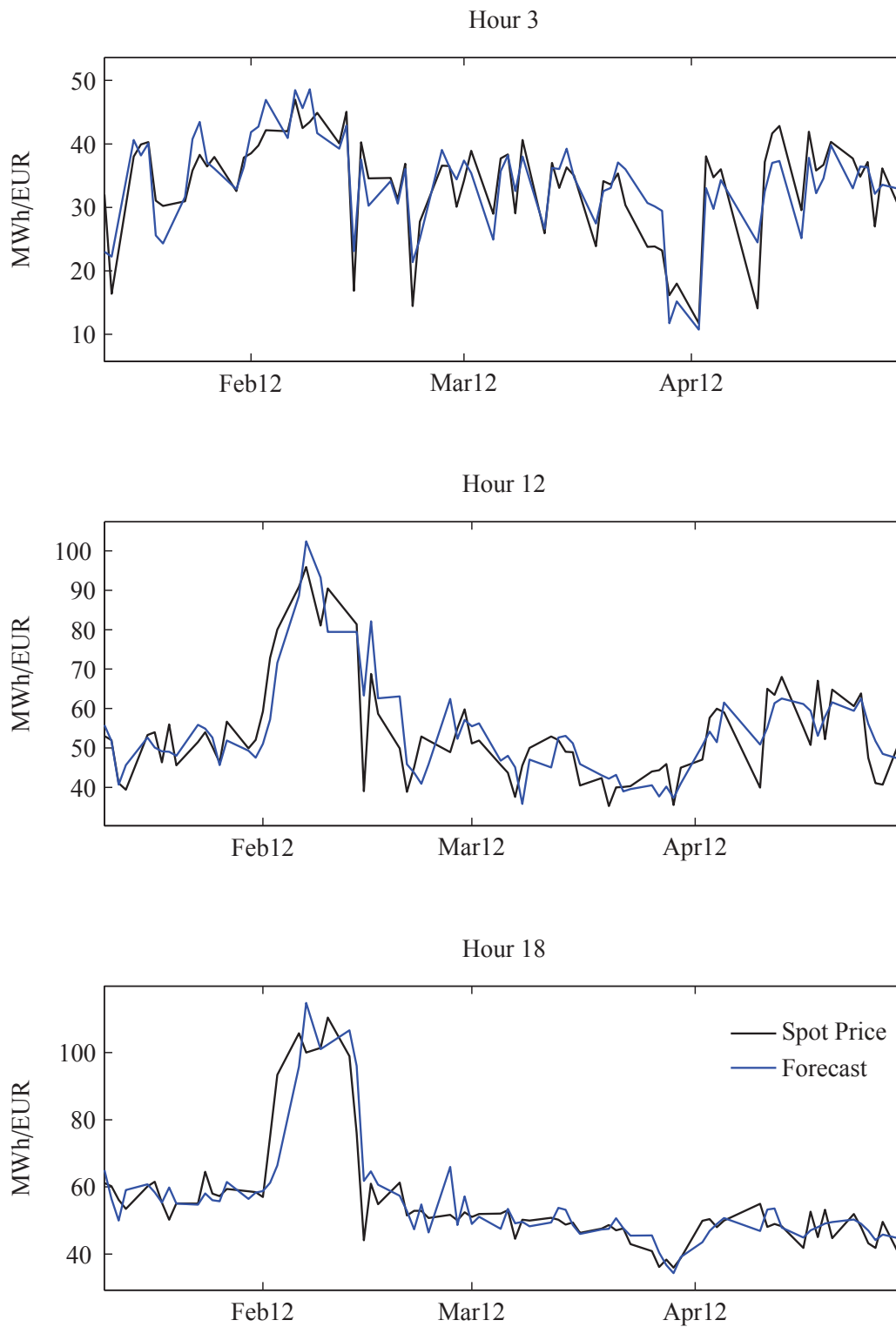


<b>Hour</b>	<b>1</b>	<b>2</b>	<b>3</b>	<b>4</b>	<b>5</b>	<b>6</b>	<b>7</b>	<b>8</b>
Obs.	76	75	76	77	77	76	77	77
$R^2$	0.26	0.20	0.11	0.14	0.13	0.14	0.24	0.50
MAE	5.04	5.59	5.67	5.33	5.15	4.26	5.04	7.38
MAPE	0.172	0.202	0.209	0.203	0.198	0.141	0.109	0.127
$\sigma_\epsilon$	6.91	7.32	7.40	7.14	7.03	5.98	6.80	11.13
<b>Hour</b>	<b>9</b>	<b>10</b>	<b>11</b>	<b>12</b>	<b>13</b>	<b>14</b>	<b>15</b>	<b>16</b>
Obs.	76	75	77	76	75	76	76	76
$R^2$	0.38	0.27	0.53	0.52	0.42	0.46	0.43	0.46
MAE	8.28	7.94	7.68	6.80	6.38	5.61	4.60	3.96
MAPE	0.129	0.134	0.139	0.134	0.140	0.123	0.103	0.089
$\sigma_\epsilon$	12.78	11.67	10.78	8.93	8.34	7.65	6.37	5.76
<b>Hour</b>	<b>17</b>	<b>18</b>	<b>19</b>	<b>20</b>	<b>21</b>	<b>22</b>	<b>23</b>	<b>24</b>
Obs.	76	77	75	77	76	77	77	76
$R^2$	0.48	0.59	0.27	0.20	0.28	0.47	0.39	0.24
MAE	4.11	5.82	9.64	10.87	5.08	3.75	3.71	4.57
MAPE	0.089	0.101	0.139	0.155	0.091	0.075	0.079	0.117
$\sigma_\epsilon$	6.26	9.88	16.96	16.59	7.68	5.24	4.90	5.85

**Table 7.3:** Summary of out-of-sample results for TVP regression models excluding expected wind infeed as exogenous variable. The dataset starts on January 1, 2012 and ends on April 30, 2012.

<b>Hour</b>	<b>1</b>	<b>2</b>	<b>3</b>	<b>4</b>	<b>5</b>	<b>6</b>	<b>7</b>	<b>8</b>
Obs.	76	75	76	77	77	76	77	77
$R^2$	0.74	0.78	0.75	0.77	0.73	0.66	0.47	0.69
MAE	2.88	2.99	3.22	3.00	3.01	2.89	4.33	5.92
MAPE	0.088	0.097	0.114	0.111	0.114	0.094	0.092	0.100
$\sigma_\epsilon$	4.11	3.85	3.91	3.71	3.88	3.78	5.70	8.77
<b>Hour</b>	<b>9</b>	<b>10</b>	<b>11</b>	<b>12</b>	<b>13</b>	<b>14</b>	<b>15</b>	<b>16</b>
Obs.	76	75	77	76	75	76	76	76
$R^2$	0.66	0.60	0.60	0.70	0.44	0.64	0.62	0.61
MAE	6.35	6.05	6.59	5.55	5.72	4.54	3.83	3.75
MAPE	0.096	0.100	0.118	0.107	0.121	0.097	0.084	0.083
$\sigma_\epsilon$	9.45	8.60	9.92	7.11	8.21	6.31	5.19	4.89
<b>Hour</b>	<b>17</b>	<b>18</b>	<b>19</b>	<b>20</b>	<b>21</b>	<b>22</b>	<b>23</b>	<b>24</b>
Obs.	76	77	75	77	76	77	77	76
$R^2$	0.63	0.83	0.34	0.31	0.37	0.56	0.57	0.64
MAE	3.66	4.30	8.55	9.71	4.74	3.43	3.16	3.17
MAPE	0.077	0.075	0.123	0.136	0.084	0.068	0.066	0.078
$\sigma_\epsilon$	5.31	6.46	16.01	15.37	7.19	4.77	4.13	4.01

**Table 7.4:** Summary of out-of-sample results for TVP regression models including expected wind infeed as exogenous variable. The dataset starts on January 1, 2012 and ends on April 30, 2012.



**Figure 7.6:** TVP regression out-of-sample fit for hours 3, 12, and 18. The dataset starts on January 1, 2012 and ends on April 30, 2012.

## 7.2.2 Conditional Volatility Modelling

As a side product, the Kalman Filter estimation procedure provides the variance of forecast errors  $\xi_t^-$  conditional on information up to  $t - 1$  which, at every time step  $t$ , is derived from the variance-covariance matrix of the *a priori* state estimates  $\hat{\beta}_t^-$  and  $R$ , which is the variance of the disturbance term of the measurement equation  $v_t$ :<sup>6</sup>

$$\Psi_t^- = x_t P_t^- x_t' + R \quad (7.20)$$

From equation 7.20 it follows that within the TVP regression framework, conditional volatility is not only derived from past error terms information (as in a GARCH setting) but directly incorporates information on current exogenous variables (included in  $x_t$ ). Therefore, uncertainty consists of two parts: First, uncertainty which is related to dynamically evolving regression coefficients (introduced by the error covariance  $P_t^-$ ) and, second, uncertainty which arises because of future disturbances (introduced by  $R$  and assumed constant in our case).<sup>7</sup>

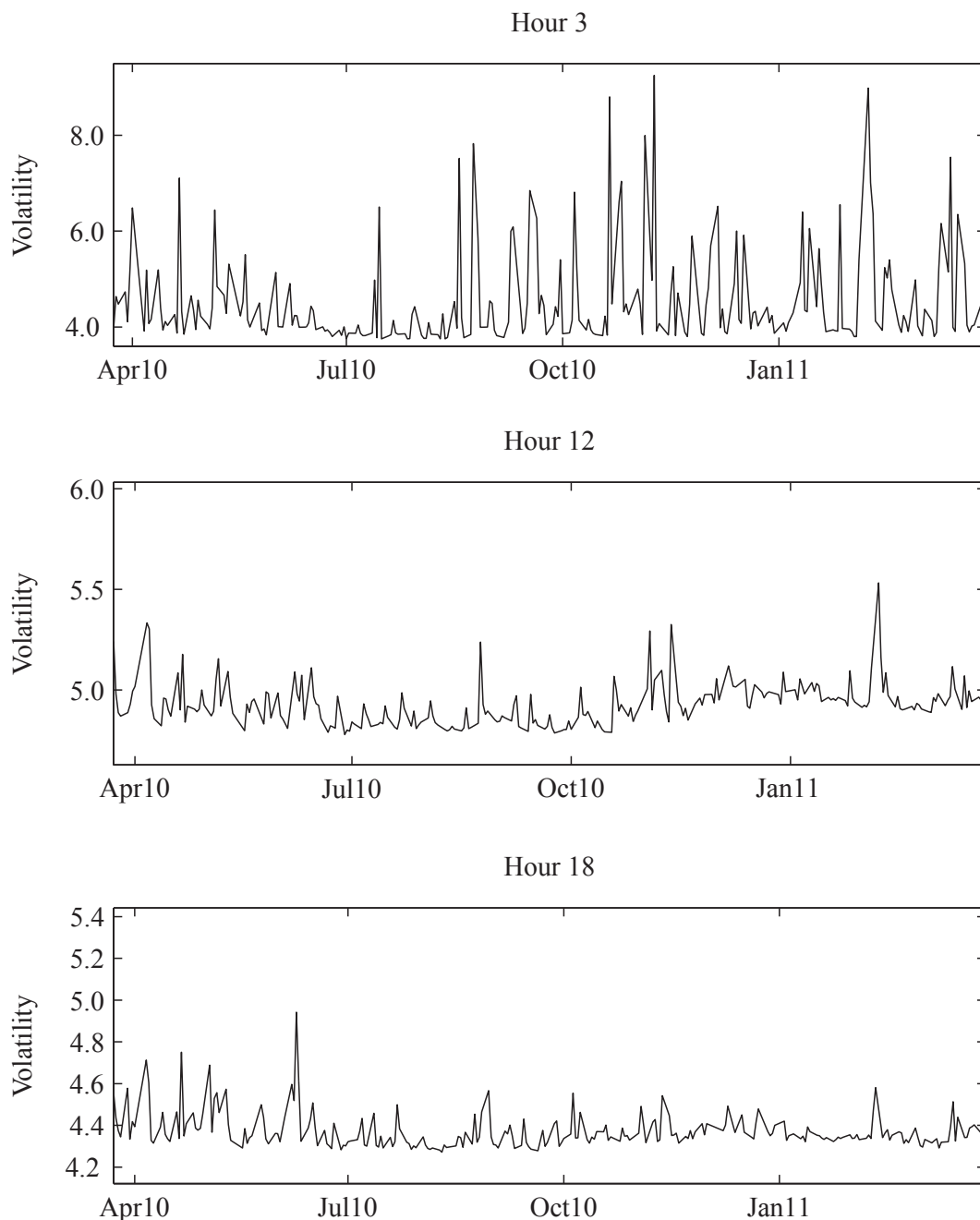
We can now compare the conditional volatility extracted from TVP regression models with the conditional volatility from GARCH regression estimations. Figure 7.7 displays the conditional volatility (i.e. the square root of the conditional variance as defined in equation 7.20) estimated via the Kalman Filter for hours 3, 12, and 18. Similar to chapter 5, we plot 250 trading days within the in-sample dataset. Comparing the conditional volatility plots of GARCH regression (figure 5.6) and TVP regression models, we can observe that the *a priori* volatility structure extracted from the TVP regression models is finer and at a consistently lower level. On the contrary, conditional volatility from the GARCH(1, 1) model specification is more persistent and overall higher. For hour 12 for example, the conditional volatility of the GARCH regression model evolves within a range between 4.0 and 7.0, whereas volatility from the TVP regression model stays within a range between 4.8 and 5.2 most of the time.

The results give evidence that by allowing for time-varying regression parameters uncertainty can be reduced significantly although GARCH volatility is by construction derived from all information available (before and after the observation date), unlike TVP volatility. Furthermore, a time-varying specification obviously removes the autoregressive structure in volatility as we can observe it in the plots of conditional volatility from GARCH regression models (figure 5.6).

<sup>6</sup>See Kim & Nelson (1999). The same time-dependent specification of the variance is incorporated in the log likelihood function in equation 7.19.

<sup>7</sup>See Kim & Nelson (1989).

When investigating spot prices of the British electricity market by TVP regression models, Karakatsani & Bunn (2010) observe the same phenomenon concluding that TVP regression models better reflect characteristics of electricity prices which are instantaneous and highly adaptive.



**Figure 7.7:** Conditional volatility of forecast errors obtained by the Kalman filter algorithm for hours 3, 12, and 18. The observation window includes 250 observations between March 23, 2010 and March 29, 2011.

### 7.3 Intermediate Summary

The estimation of time-varying parameter regression models reveals that results can be significantly improved when allowing for dynamic regression coefficients. We see that the sensitivity of prices towards various fundamental variables is governed by distinct patterns. Looking at the evolving coefficient for expected wind infeed - the focus variable of this thesis - we can observe a seasonal behavior which is more or less pronounced depending on the hour of the day.

Especially when comparing out-of-sample results with results from GARCH regression and threshold regression models we can report significant improvements. Using a TVP regression approach we can for example explain about 75% of the total spot price variation for early morning hours, whereas within the GARCH regression framework we can only explain between 40% and 55% of price variability for the same hours. These findings expand to the remaining hours of the day. There is, however, a handful of hours where the approach is not able to provide solid results. This applies particularly for hours 19 to 21, when the on/off-peak switch takes place. We report  $R^2$  measures below 0.40 which, for hours 19 and 20 only, is slightly inferior to the fit of the GARCH regression models.

Overall, significant improvements can be reported when comparing TVP regression models without and with consideration of the expected wind infeed variable. Within the TVP regression framework expected wind infeed proves to have a high in-sample as well as out-of-sample explanatory power for day-ahead electricity spot prices.

The analysis of the conditional volatility of forecast errors obtained by the Kalman filter algorithm additionally provides evidence that accounting for the adaptive nature of electricity spot prices reduces forecasting uncertainty considerably.

Autocorrelation tests applied to obtained measurement errors generally confirm that the hourly TVP regression models are appropriately specified.

# Chapter 8

## Forward Market Analysis

### 8.1 Economic & Regulatory Background

This thesis analyzes and models electricity prices by means of fundamental variables. Special attention is paid to the role of expected wind infeed. Over the last decade, wind energy has become an increasingly important market driver along with the development of laws and policies on renewable energies in Germany. As discussed in section 2.2, with the introduction of the equalization mechanism ordinance (AusglMechV) which became effective as of beginning 2010, the way of how electricity from renewable sources enters the system has changed significantly.

In the old regime, transmission system operators (TSOs) sold renewable energy through so-called monthly bands (economically comparable to forward contracts) to energy supply companies (ESCs). In order to eliminate the resulting market exposure it is conceivable that ESCs hedged their positions by entering a forward contract at the EEX which would have resulted in additional short positions in front-month EEX electricity futures.<sup>1</sup> Under the new ordinance, electricity from renewable sources is to be sold on the day-ahead spot market directly. This theoretically lowers supply on the forward market and increases supply on the day-ahead market, leading to an increasing spread (the risk premium) between the two. However, it is probable that before, ESCs also used other markets in addition to front-month futures (e.g. OTC or other futures contracts than the front-month) to hedge market exposures. It can be assumed that under the new regime, ESCs have to buy electricity somewhere in

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<sup>1</sup>Today, there is not enough transparency to investigate this exactly.

the market in order to close the gap which results from the abandoning of delivery through monthly bands. If the required volumes are bought on the day-ahead market, the excess supply caused by increased green electricity sales by TSOs is most likely eliminated. Unfortunately, getting sufficient information to draw a final conclusion on how the behavior of all involved parties has changed is not possible given today's level of transparency.

In its report published two years after the introduction of the ordinance, the German Federal Network Agency notes that volumes and thus liquidity on the day-ahead market have immediately and significantly increased as of January 2010. The authority reports a total trading volume of TSOs in the day-ahead market of 81 TWh in 2010 versus 20 TWh in 2009. They find no such structural change when looking at the price pattern considering the price level and volatility though. Only for the very first weeks of 2010 a higher price volatility was observed as a result of uncertainty introduced by the new rule.<sup>2</sup> These findings are confirmed by von Rintelen & Wragge (2010).

In addition to the observed, we intend to get a clearer idea on how market participants' behavior changed on the short-term forward markets following the introduction of the *AusglMechV*. We will therefore analyze risk premia on two sorts of electricity forward contracts. First, we will investigate risk premia on day-ahead contracts which are traded at the Energy Exchange Austria (EXAA) earlier in the day and which can be settled in the same geographical area. Second, we will look at premia on EEX front-month futures contracts which have the same time-to-maturity as the before existing monthly bands of green electricity delivery.

In the remainder of this chapter we will proceed as follows. First, we will give an introduction to forward pricing and risk premia in commodity markets in general as well as in electricity markets specifically. Afterwards, we will provide an overview on according research which has been done on electricity forward markets. We will then perform an empirical analysis on day-ahead prices traded in Vienna/Austria which, following other researchers, we define as the shortest possible forward contracts for EEX day-ahead prices. Our empirical analysis will focus on the development of risk premia as well as their dependence on expected wind infeed. In addition to risk premia we will estimate the market price of risk and see whether our findings are confirmed. Besides premia on EXAA contracts we will also investigate front-month EEX futures contracts and finally conclude the chapter with a summary of our findings.

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<sup>2</sup>See Federal Network Agency (2012).



## 8.2 Forward Pricing in Electricity Markets

### 8.2.1 Commodity Forward Pricing Concepts

Keynes (1930) was one of the first to put forward a fundamental theory to explain price differences between commodity spot and forward prices. Known as the *Expectations Theory* it claims forward prices of a commodity to be equal to the expected future spot price:

$$F(t, T) \approx \mathbb{E}[S(T)|\mathcal{F}_t] \quad (8.1)$$

where  $F(t, T)$  denotes the forward price at time  $t$  with delivery at time  $T$  (could be a period instead) and  $\mathbb{E}[S(T)|\mathcal{F}_t]$  denotes the expected future spot price adapted to the filtration  $\mathcal{F}_t$ . The difference between the forward and the (expected) later on realized spot price is referred to as risk premium:

$$\pi(t, T) = F(t, T) - \mathbb{E}[S(T)|\mathcal{F}_t] \quad (8.2)$$

Following the formula, a positive risk premium is a premium paid by the buyer to the seller of the futures contract and vice versa.

According to Keynes, commodity markets typically exhibit negative risk premia meaning that forward prices trade below spot prices. He characterizes this situation a *Normal Backwardation* formation which he deems the result of a market situation that is made up of two main sorts of participants. On the one hand, commodity producers ('hedgers') enter short positions in futures contracts in order to hedge against possible future price decreases. On the other hand, their counterparties ('speculators'), who assume the long positions, are paid a risk premium compensating them for bearing the future price uncertainty. In Keynes' Normal Backwardation theory hedgers always act as insurance buyers and thus exert downside pressure on forward prices. This strict assumption is considered one of the main limitations of the approach. Working (1942) for instance mentions that market participants are often prepared to pay a premium for carrying the price uncertainties in futures contracts on speculation purposes.

As an alternative to Keynes' approach, a concept known as the *Theory of Storage* was developed by Kaldor (1939) and advanced by Working (1942 & 1949), Brennan (1958), Telser (1958), Weymar (1966), and Williams (1986). It concentrates less on the hedging and speculation intentions of market participants but links the intertemporal pricing relationship between spot and forward markets to the cost of storing a commodity and the stream of benefits resulting from holding it. The latter is known

as the convenience yield. The Theory of Storage can be summarized by the following equation:

$$F(t, T) = S(t)e^{(r-\delta+s)(T-t)} \quad (8.3)$$

where  $r$  denotes the continuously compounded risk-free rate,  $\delta$  denotes the cost of storage, and  $s$  denotes the convenience yield earned by the holder of the commodity.<sup>3</sup>

The concept of the absence of arbitrage in a perfect market implies that any deviation of (8.3) from its equilibrium can be exploited by market participants pushing the relationship between spot and forward prices back to comply with the stated equation. One of the main prerequisites for the arbitrage condition to hold is the storability of the underlying commodity. Whereas for most commodities such as precious metals, base metals, or oil, storability seems a reasonable assumption, it fails for others. Electricity is a case in point for which the storability requirement is hardly fulfilled and the commodity has to be consumed immediately once purchased. We use the term *hardly* intentionally as one could argue that the presence of hydroelectric power plants (HPPs) offers potential storage capacities in the way that during times of (too) low spot prices, electricity could be bought and used to pump water up into storage lakes and then retransformed into electricity at a later stage when spot prices are at higher levels.<sup>4</sup> Switzerland is a prominent example where such a behavior of market participants can be observed, particularly in summertime when mountain lakes are not frozen. However, as the focus of this thesis is the German electricity market where storage opportunities as described do not exist to a relevant extent, we will assume non-storability of electricity. Non-storability makes the application of the storage or convenience yield related approach to price derivatives in general and forward contracts specifically impossible.<sup>5</sup> Instead, a focus on hedging needs of producers and consumers, and thus risk premia, seems more reasonable when analyzing differences between quoted futures and spot electricity prices.

### 8.2.2 Risk Premia

As reasoned in the preceding paragraph, due to the characteristics of the underlying commodity, risk premia seem an appropriate instrument to analyze and discuss the

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<sup>3</sup>Some researchers argue that the convenience yield in commodity pricing is comparable to the dividend yield used in standard (equity) option pricing theory (see e.g. Burger et al. (2007)).

<sup>4</sup>Benth et al. (2008) refer to this as *indirect storability*.

<sup>5</sup>See Eydeland & Geman (1999).

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dynamics of electricity forward prices. Following the liberalization of electricity markets, research on the existence and dynamics of risk premia in exchange traded futures contracts has increasingly been performed over the last decade. There are basically two types of risk premia discussed.

The *ex ante risk premium* is evaluated at time  $t$  and defined as the difference between the current forward market price and the expected future spot price:

$$\pi(t, T) = F(t, T) - \mathbb{E}_t[S(T)] \quad (8.4)$$

The *ex post risk premium* is only computed at time  $T$  once the realized spot price is known and is defined as follows:

$$\pi(T) = F(t, T) - S(T) \quad (8.5)$$

Obviously, the identification of the *ex ante risk premium* is much more critical. To define  $\mathbb{E}_t[S(T)]$  in (8.4) an appropriate estimation model needs to be in place which requires numerous assumptions regarding the underlying spot price dynamics. This is probably the reason why most researchers base their analyses on the *ex post premium*. Both, the *ex ante risk premium* as well as the *ex post risk premium* can be expressed either in absolute terms or in relative terms as a percentage of the forward price.

### 8.2.3 Literature Review

Bessembinder & Lemmon (2002) were among the first to deal with risk premia in electricity markets. Based on hedging arguments they formulated a moment-based equilibrium model finding that risk premia are negatively related to the variance and positively related to the skewness of the expected spot price:

$$\pi(t, T) = \alpha + \beta Var(\mathbb{E}_t[S(T)]) + \gamma Skew(\mathbb{E}_t[S(T)]) \quad (8.6)$$

They allow for negative as well as positive risk premia which is in contrast to Keynes (1930) who exclusively assumes the existence of negative and thus one-sided risk premia.

Shortly after, Longstaff & Wang (2004) investigated hourly spot and futures prices from a two year dataset in the Western US (PJM market) finding strong support for the Bessembinder-Lemmon (B/L) model. They also identify significant risk premia in forward prices which are positive on average and exhibit systematic behavior patterns driven by economic factors like spot prices, volatility, or intraday changes in demand. Douglas & Popova (2008) derive an empirical model for the PJM market confirming the results of Longstaff & Wang (2004). In addition they analyze how risk premia are affected by natural gas storage finding a measurable negative effect of gas inventories on premia in times of high demand for electricity and low expected future demand for gas.

With liberalization developments in Europe, which were to follow the efforts in the US, risk premia in the newly formed electricity markets started to arouse researchers' interest. The Scandinavian Nordpool electricity market, which together with the UK market initiated the liberalization process in Europe, was analyzed by Botterud et al. (2002) who find positive risk premia for futures contracts with a maturity of up to one year. With a longer dataset of 11 years at hand, they confirm their findings and in addition identify a convenience yield which is negative on average.<sup>6</sup> By considering the convenience yield they refer to the indirect storability of electricity which seems a reasonable assumption for the Nordpool market due to the extensive existence of hydro reservoirs and related pump facilities. Lucia & Torro (2008) investigate the four nearest week futures at the Nordpool from 1998 to 2007 confirming the existence of positive risk premia. On top of that, their results are consistent with the B/L model. Redl et al. (2009) compare the relationship between spot and forward prices in the Nordic and the German markets between 2003 and 2008 mentioning similarities in the futures price formation. Especially, they provide evidence that market participants' behavior is heavily influenced by current spot prices rather than fundamental models when trading in the forward market. Overall, their findings agree with earlier analyses demonstrating positive risk premia (especially for short-term contracts).

Given the later start of exchange-based electricity trading, most literature on the German market has come forward over the last couple of years. Bierbrauer et al. (2007) introduce and estimate different spot price models for data between 2000 and 2003. Applying the model results to compute risk premia, their results underpin earlier findings from other electricity markets that risk premia for short-term futures contracts are significantly positive. Pietz (2009) performs a comprehensive analysis on a longer dataset, reaching from 2002 to 2008, looking at monthly, quarterly, and yearly contracts. His results indicate that electricity consumers mainly use short-term futures to hedge their risks whereas producers mainly trade in long-term contracts. Besides, he discerns a term structure as well as seasonal patterns for risk premia in German electricity futures. For contracts with delivery during winter months, risk premia prove to be positive whereas for contracts with delivery during summertime, there is evidence for an opposite sign. These results are in accordance with earlier analyses by Benth et al. (2008) who investigate data from 2002 to 2006. Relating the market power of producers to risk premia they find high positive risk premia for short maturities and significant discounts for longer maturities. Additional results confirming the existence of significant risk premia in German electricity futures contracts were

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<sup>6</sup>See Botterud et al. (2010).

published by Diko et al. (2006), Daskalakis & Markellos (2009), and Spitzen (2010).

While the so far mentioned literature mainly focuses on monthly futures and, to some extent, on quarterly and yearly contracts, a smaller group of researchers has investigated risk premia on contracts which are auctioned only a very short time before the German day-ahead spot contracts. For the German market Kolos & Ronn (2008), Ronn & Wimschulte (2009), and Viehmann (2011) use prices of day-ahead EXAA spot contracts traded in Vienna as the 'nearest' futures contracts for EEX day-ahead contracts. This seems an acceptable approach for mainly two reasons. First, EXAA contracts facilitate delivery in exactly the same market area (Germany, Austria) as EEX contracts. Second, the auction of EXAA day-ahead contracts takes place about two hours before EEX day-ahead contracts are auctioned. Hence, at the time when market participants submit their bids for the EEX auction, they already possess full information on the results of the EXAA auction. Having said this, EXAA contracts are the last possibility for market participants at the EEX to hedge their exposures. Viehmann (2011) reports that the B/L equilibrium model is, to a large extent, valid for EXAA risk premia confirming the hypothesis of energy traders behaving like risk averse agents. He also finds seasonal patterns in risk premia for different hours of the day as well as for summer and winter months. The observed period reaches from 2005 to 2008. Kolos & Ronn (2008) and Ronn & Wimschulte (2009) consider datasets between 2004 and 2007.<sup>7</sup> Unlike the research discussed so far, they do not analyze risk premia the conventional way. Instead, they estimate the market price of risk by applying both, parametric and non-parametric approaches. When looking at EXAA and EEX data they arrive at the conclusion that market participants are willing to pay a positive risk premium for contracts which trade earlier in the day, identifying 'intra-day' forward prices (EXAA contracts) as upward-biased predictors of expected spot prices (EEX contracts).

The existing literature is in wide agreement on the statistically significant existence of risk premia in electricity markets. However, especially with regards to futures contracts with longer maturities, findings about the sign of the premia differ.

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<sup>7</sup>Whereas Kolos & Ronn (2008) consider US and foreign electricity and other energy markets, Ronn & Wimschulte (2009) solely investigate the German/Austrian electricity market.



## 8.3 EXAA Risk Premia

### 8.3.1 Methodology

In our analyses, we will focus on absolute risk premia. We have verified that, despite varying price levels over the analyzed periods, investigating relative instead of absolute risk premia would neither change our findings and interpretations nor the statistical significance of them.

The absolute ex post risk premium on EXAA contracts for hour  $t$  is defined as

$$RP_t = \frac{1}{N} \sum_{n=1}^N (DA_{EXAA,n,t} - DA_{EEX,n,t}) \quad (8.7)$$

where  $DA$  denotes day-ahead spot prices and  $n$  denotes the observation out of the set of relevant delivery dates  $\{n = 1, \dots, N\}$ . To investigate the existence of significant EXAA risk premia in 2009, 2010, and 2011, we apply a straight-forward t-test with test statistic

$$t = \frac{\bar{x} - \mu}{s / \sqrt{n}} \quad (8.8)$$

where  $\bar{x}$  is the arithmetic mean of all  $n$  risk premia observed in the relevant period and  $s$  denotes their standard deviation. To investigate whether risk premia are different from zero, we set  $\mu = 0$ . We perform the test as both, a one-sided and a two-sided test. The two-sided test shall provide evidence whether risk premia are significantly different from zero whereas the one-sided test shall give further insight into whether - in case of significance - they are positive or negative.

In order to check whether EXAA risk premia grouped according to the expected level of wind infeed originate from different statistical distributions, we apply a distance-based Kolmogorov-Smirnov test with test statistic

$$\max[|F_1(x) - F_2(x)|] \quad (8.9)$$

to all 24 hours.<sup>8</sup>  $F_1$  and  $F_2$  denote the empirical distribution functions of two groups of risk premia, composed according to the expected level of wind infeed. Critical values of the test statistic for  $n > 40$  are approximately formula-based according to  $\sqrt{\ln(2/\alpha)} / \sqrt{2n}$  where  $\alpha$  denotes the level of significance.<sup>9</sup>

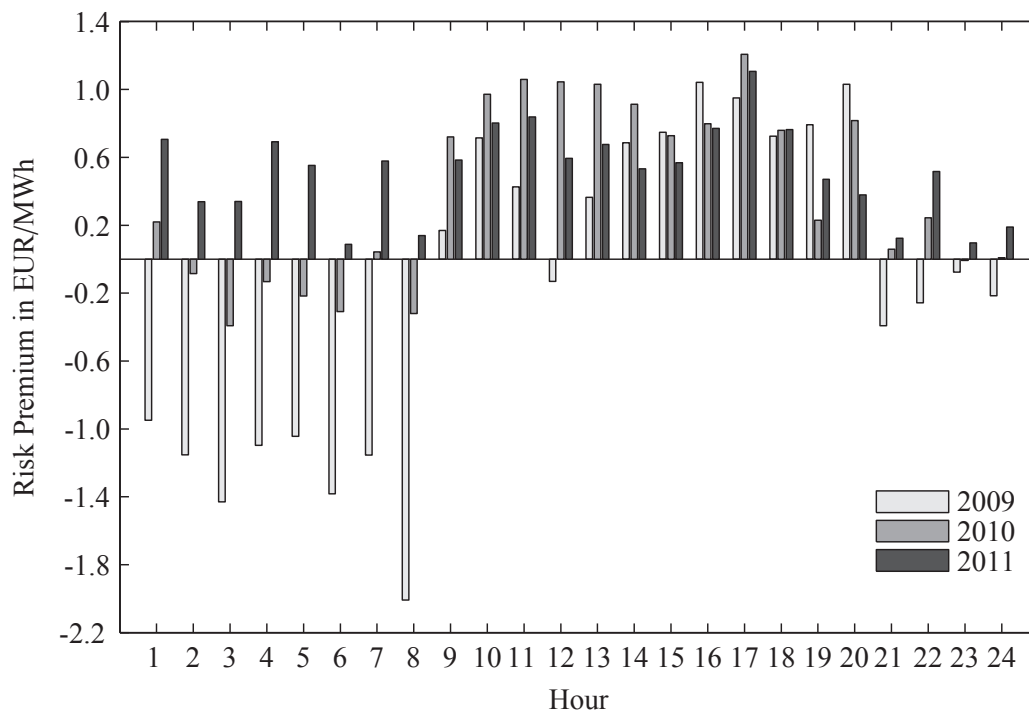
<sup>8</sup>For a detailed introduction see Massey (1951), Miller (1956), Stephens (1970), or Marsaglia et al. (2003).

<sup>9</sup>See Massey (1951).

### 8.3.2 Development of EXAA Risk Premia

We investigate only working day data and exclude all weekends, holidays, and bridge days. Because we do not want to have our results distorted by the influence of outliers which may occur due to technical breakdowns or other non-repetitive situations, we drop all risk premia three standard deviations below and above the mean risk premium in all subsequent analyses.

Figure 8.1 displays absolute risk premia on EXAA day-ahead contracts for years 2009 to 2011. Additionally, t-test results for all hours in 2009, 2010, and 2011 (the years before and after the introduction of the equalization mechanism ordinance) are shown in tables D.1, D.2, and D.3, respectively, in the appendix.



**Figure 8.1:** Absolute ex post risk premia on EXAA contracts for years 2009, 2010, and 2011. Hourly risk premia are computed as arithmetic averages of the risk premia of all relevant delivery days in the respective year, excluding weekends, holidays, and bridge days. Risk premia 3 standard deviations below and above the mean risk premium are excluded.

We first look at risk premia for *off-peak hours* 1 to 8 and 21 to 24. During this period we observe strongly negative premia for 2009 which have reduced in 2010 and turned positive in 2011. This observation is confirmed by the t-test results. For



2009 the  $H_0$  of a risk premium equal to zero as well as the  $H_0$  of a positive premium can both be rejected with high significance levels of 99% for hours 1 to 8, hence significant negative risk premia for this period are confirmed. When looking at the results for 2010, we observe that the  $H_0$  of a zero risk premium can no longer be rejected for these hours. Finally, for 2011 we can reject the null hypothesis of no risk premium as well as the null hypothesis of a negative risk premium for hours 1, 4, 5, and 7 at the 95% significance level. To summarize, we have overall evidence that risk premia for early morning hours until hour 8 have increased. The observed development is similar for off-peak hours in the evening, though not as pronounced as for early morning hours.

Looking at *peak hours* 9 to 20, figure 8.1 reveals positive premia for all hours except hour 12 in 2009 with highest values at hours 16 to 20. During 2010 and 2011 changes turn out to be mixed. For hours 9 to 13 and 17 to 18, i.e. the hours when demand is highest, we also observe an increase in risk premia in 2010 followed by some reduction in 2011, however, not back to the levels of 2009. The inference from graphical illustration is confirmed by the t-tests. Whereas for noon hours in 2009, risk premia were not significantly different from zero, they were in 2011 where we can even reject the  $H_0$  of negative risk premia at the 95% significance level. Also, afternoon and evening hours until hour 19 prove to have significant positive risk premia in 2011. Only for hour 20 we cannot reject the null hypothesis of a zero risk premium in 2011. Overall, we can report that risk premia have increased for peak hours 9 to 12 and 17 and 18, and somewhat reduced for other peak hours.

The overall increase in risk premia on EXAA day-ahead contracts implies that, with the introduction of the equalization mechanism ordinance, the willingness of market participants to pay more (in relative terms) for contracts traded earlier in the day has increased. It is conceivable that this behavior is the result of a market situation for EEX contracts that has become more uncertain with the obligation of the transmission system operators to market all electricity from renewable sources in the day-ahead auction. The observed changes are most pronounced for early morning hours (when intermittent wind infeed increases uncertainty) and peak demand hours (when a tense market situation caused by particular high demand increases uncertainty).

Our findings of increased risk premia for early morning hours are not only valid for years 2010 and 2011 compared to 2009. Viehmann (2011) investigated EXAA premia between October 2005 and September 2008. When comparing risk premia for 2010 and especially 2011 with his results, our conclusion of an increase for these hours is confirmed.

What seems unclear is why market participants are willing to accept a higher price (in absolute or relative terms) for EXAA contracts during early morning hours when

during this period, downward price spikes at the EEX are observed with a higher probability than upward spikes. Germany and Austria form a single market area meaning that electricity to be consumed at any point of the grids in the two countries can originate from either EXAA or EEX contracts. Accordingly, one would expect buyers of electricity to depart from the EXAA auction and take on the chance of getting a lower price at the EEX. As a result, demand for EXAA contracts would decrease and prices would fall on a relative basis, letting risk premia move downwards rather than increase. We have to assume that traders who are buying electricity at the EEX either face some restrictions to freely move between the two exchanges or that risk premia are not high enough to justify a switch and thus, EXAA prices deviate from EEX prices in early morning hours. In other words, what we observe may primarily be pressure on EEX prices through higher supply initiated by renewable energy laws and rigidity in traders' action rather than a risk-based behavior of market agents. For premia on EXAA contracts during peak hours the rationale seems more straightforward. During these hours, downward spikes in spot prices are less likely than upward spikes due to the tense market situation caused by very high demand levels. Electricity stemming from renewable sources, which around noon is not only wind but also solar energy to a large extent, influences the risk that we do or do not observe a price spike. In view of this, the attempt of electricity buyers to get a firm price at the earlier auction in Vienna seems reasonable.

### 8.3.3 Conditional Distributions of EXAA Risk Premia

In the previous section we have noted that price differences between EEX and EXAA traded contracts have changed over the last years. In order to get a deeper understanding whether the observed price differences are driven by renewable energies and related regulatory directives, we analyze the distribution of risk premia conditional on the expected wind infeed relevant for the German market area. We perform our analysis on the observation period starting on January 1, 2010 and ending on December 31, 2011. We only include working days and remove risk premia outside the 3-sigma band from our dataset.

Our results seem rather unambiguous for early morning hours. For illustration we show statistics of absolute risk premia for hour 4 in table 8.1 and the corresponding frequency distributions in figure 8.2. Being representative for all morning off-peak hours, the statistics indicate an increase in risk premia when the expected wind infeed into the German grid is higher. Whereas for hour 4, market participants pay a premium of 1.40 EUR/MWh when expected wind speed is high, they get EXAA contracts at a discount of 0.95 EUR/MWh when the wind speed is at lower levels. As for volatility,

the standard deviation of paid premia increases as expected wind infeed increases which implies a higher price uncertainty at high wind levels. Although as for skewness the picture is less clear-cut, there is some evidence that risk premia are negatively skewed at low wind levels and positively at high wind levels. Looking at the kurtosis, distributions of risk premia are more leptokurtic at low wind levels and less peaked at higher wind levels.

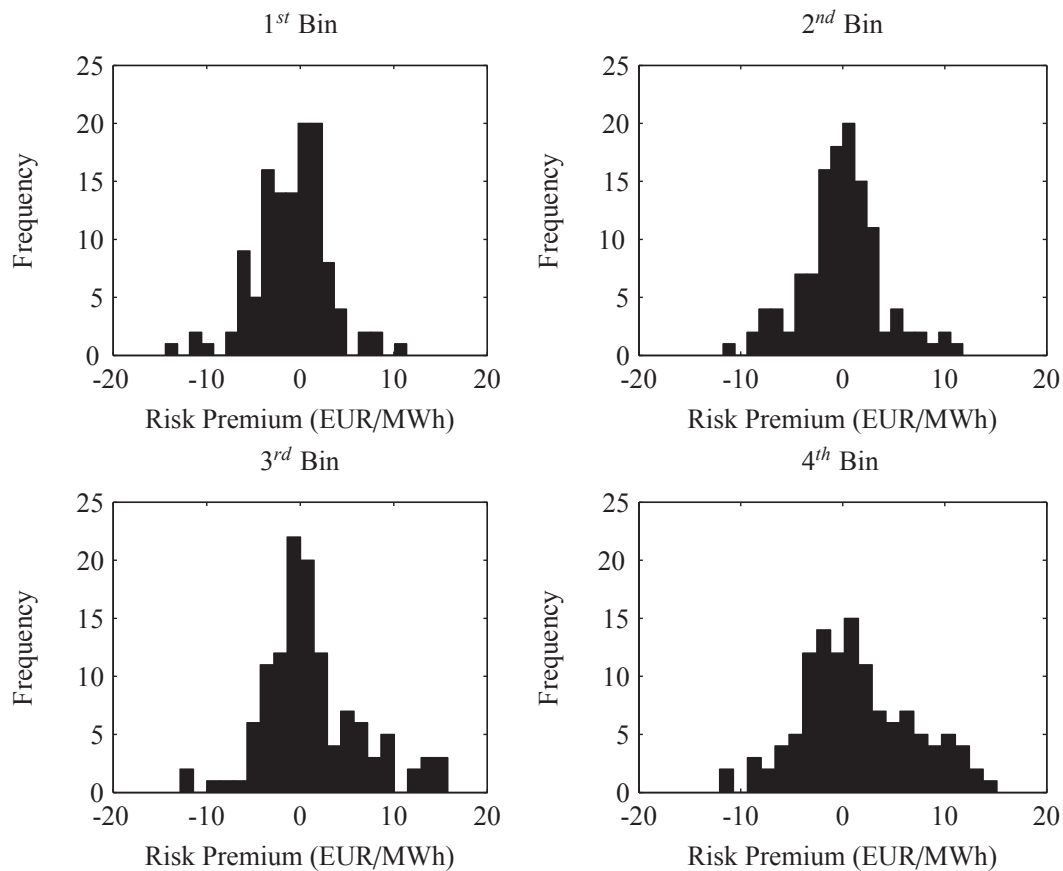
To test whether risk premia at different wind levels originate from different probability distributions, we apply a distance-based two-sample Kolmogorov-Smirnov test. Results for hour 4 are depicted in table 8.2. As can be seen from the table, for bin pairs 1/3, 1/4, and 2/4 we can reject the  $H_0$  that risk premia originate from the same sample. For adjacent bins, the null hypothesis cannot be rejected at a significance level of 95%.

Overall, the results are another evidence that differences between EEX and EXAA prices during early morning hours are heavily driven by renewables and that variation in EEX prices due to the new ordinance does not fully transmit to EXAA prices.

For peak hours the findings are different. As an illustrative example, table 8.3 shows risk premia statistics for different wind levels and table 8.4 depicts results of the Kolmogorov-Smirnov test for hour 12. Apparently, we cannot infer that differences between EEX and EXAA prices during peak hours are mainly driven by expected wind infeed. Hence, we have to conclude that for these hours, there must be other drivers which reason risk premia on EXAA contracts. It is conceivable that electricity from photovoltaics, which also has to be sold preferentially on the spot market, plays a major role during these hours. However, to test this, consistent data is not yet available for a sufficiently long time period. Hour 12 can be considered a representative example for most peak hours.

	1 <sup>st</sup> Bin	2 <sup>nd</sup> Bin	3 <sup>rd</sup> Bin	4 <sup>th</sup> Bin
Mean Wind Forecast	1172	2540	4612	9525
Mean	-0.95	-0.13	1.21	1.40
Standard Deviation	3.87	3.99	5.30	5.55
Skewness	-0.23	0.09	0.57	0.24
Kurtosis	4.41	3.95	3.87	2.76

**Table 8.1:** *Descriptive statistics of ex post absolute risk premia conditional on the expected wind infeed for hour 4. The observation period starts on January 1, 2010 and ends on December 31, 2011.*



**Figure 8.2:** Distributions of ex post absolute risk premia conditional on the expected wind infeed for hour 4. The observation period starts on January 1, 2010 and ends on December 31, 2011.

1 <sup>st</sup> Bin	2 <sup>nd</sup> Bin	$H_0$	test-Stat	p-Value
1	2	-	0.12	0.372
1	3	reject	0.20	0.015
1	4	reject	0.24	0.002
2	3	-	0.15	0.125
3	4	-	0.09	0.679
2	4	reject	0.21	0.010

**Table 8.2:** Results of two-sample Kolmogorov-Smirnov test on pairs of empirical distribution functions of absolute ex post risk premia for hour 4. The null hypothesis claims that the two compared data samples originate from the same continuous distribution and is tested applying a 95% significance level. The observation period starts on January 1, 2010 and ends on December 31, 2011.

	1 <sup>st</sup> Bin	2 <sup>nd</sup> Bin	3 <sup>rd</sup> Bin	4 <sup>th</sup> Bin
Mean Wind Forecast	957	2395	4760	11087
Mean	0.41	1.12	0.95	0.84
Standard Deviation	3.96	3.98	3.58	4.15
Skewness	-0.01	0.02	0.38	0.10
Kurtosis	3.46	3.22	2.75	3.55

**Table 8.3:** Descriptive statistics of ex post absolute risk premia conditional on the expected wind infeed for hour 12. The observation period starts on January 1, 2010 and ends on December 31, 2011.

1 <sup>st</sup> Bin	2 <sup>nd</sup> Bin	$H_0$	test-Stat	p-Value
1	2	-	0.12	0.291
1	3	-	0.09	0.679
1	4	-	0.08	0.786
2	3	-	0.07	0.879
3	4	-	0.06	0.985
2	4	-	0.08	0.786

**Table 8.4:** Results of two-sample Kolmogorov-Smirnov test on pairs of empirical distribution functions of absolute ex post risk premia for hour 12. The null hypothesis claims that the two compared data samples originate from the same continuous distribution and is tested applying a 95% significance level. The observation period starts on January 1, 2010 and ends on December 31, 2011.

## 8.4 EXAA Market Price of Risk

In order to further investigate the differences between EEX and EXAA day-ahead prices before and after the introduction of the equalization mechanism ordinance, we will estimate the market price of risk. The reason why we introduce this measure in addition to the risk premia analysis is its consideration of volatility. It thus allows us to perform a risk-adjusted analysis of changes in price differences between the two exchanges.

### 8.4.1 Methodology

Kolos & Ronn (2008) demonstrate a way of estimating the market price of risk for energy commodities which is summarized subsequently. In general, the market price of risk is defined as the return per unit standard deviation,  $\lambda \equiv \frac{\mu}{\sigma}$ .<sup>10</sup> Assuming that the market price of risk is constant, the evolution of the forward price  $F$  of a commodity can be expressed by the following stochastic differential equation:

$$\begin{aligned} dF &= \mu_t F dt + \sigma_t F dz \\ &= \lambda \sigma_t F dt + \sigma_t F dz \end{aligned} \quad (8.10)$$

In a discretized form, the SDE can be restated as follows:

$$\begin{aligned} \Delta \ln F_t &\equiv \ln \frac{F_{t+\Delta t}}{F_t} \\ &= \left( \lambda - \frac{\sigma_t}{2} \right) \sigma_t \Delta t + \sigma_t \sqrt{\Delta t} \epsilon_t \end{aligned} \quad (8.11)$$

with  $\sigma_t$  being dependent on the time to maturity.  $F_t$  denotes the price of the EXAA day-ahead contract and  $F_{t+\Delta t}$  denotes the price of the EEX day-ahead contract (which is actually considered the spot contract).

The authors argue that in the case of futures contracts with a very short time to maturity, issues related to the term structure of volatility can be neglected which facilitates the estimation of the market price of risk significantly. Essentially, they show that the maximum likelihood estimator of the market price of risk can easily be derived

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<sup>10</sup>In the case of equities the market price of risk is usually denoted as  $\frac{\mu-r}{\sigma}$  accounting for the riskless rate of return. Kolos and Ronn argue that unlike equity investments, commodity forward contracts are costless to enter which let them have a zero drift under the risk-neutral measure.

by reformulating (8.11) as the following process:

$$\ln \frac{F_{t+\Delta t}}{F_t} - \left( \lambda \sigma - \frac{\sigma^2}{2} \right) \Delta t = \sigma \sqrt{\Delta t} \epsilon_t \quad (8.12)$$

with the following first and second moment:

$$\sum_{t=1}^T \ln \frac{F_{t+\Delta t}}{F_t} - n \left( \lambda \sigma - \frac{\sigma^2}{2} \right) \Delta t = \sigma \sqrt{\Delta t} \sum_{t=1}^T \epsilon_t \quad (8.13)$$

$$\sum_{t=1}^T \left[ \ln \frac{F_{t+\Delta t}}{F_t} - \left( \lambda \sigma - \frac{\sigma^2}{2} \right) \Delta t \right]^2 = \sigma^2 \Delta t \sum_{t=1}^T \epsilon_t^2 \quad (8.14)$$

After rearranging they report the following estimator for the market price of risk:

$$\hat{\lambda} = \frac{\overline{\ln \frac{F_{t+\Delta t}}{F_t}}}{\hat{\sigma} \Delta t} + \frac{\hat{\sigma}}{2} \quad (8.15)$$

with

$$\hat{\sigma} = \frac{1}{\sqrt{\Delta t}} \sqrt{\frac{n}{n-1} \text{Var} \left( \ln \frac{F_{t+\Delta t}}{F_t} \right)} \quad (8.16)$$

Interpreting the EXAA day-ahead prices as the nearest futures contracts for EEX day-ahead prices qualifies for the use of such an estimator for the market price of risk.

This approach was applied to EXAA/EEX contracts by Kolos & Ronn (2008) and Ronn & Wimschulte (2009). However, their datasets consist of older observations dating back to the period between 2002 and 2007.

### 8.4.2 Empirical Results

Table 8.5 and figure 8.3 depict the market price of risk (MPR) for all hours in 2009, 2010, and 2011. Looking at morning hours we see a significant reduction in the market price of risk after 2009. Whereas in 2009, the MPR was positive for hours 1 to 8, it is zero to negative in 2011. This observation corresponds with our conclusion from the analysis of risk premia. The same applies to noon peak hours, evening peak hours, and late evening hours.

As for afternoon hours 14 to 16, the analysis of the MPR leads to slightly different results. The analysis of risk premia showed positive premia which decreased from 2009 to 2011. Accordingly, we would expect negative market prices of risk which decreased in absolute values for the same period. However, as can be seen from figure



8.3, the MPR for these hours increased in absolute values within the negative territory. This can be attributed to the decrease of volatility from 2009 to 2011 ( $\hat{\sigma}_{16,2009} = 1.72$  vs.  $\hat{\sigma}_{16,2011} = 1.00$ ) which lead to an absolute increase in the negative MPR.

Overall, the analysis of the MPR gives evidence that on a risk-adjusted basis, the price market participants pay for EXAA contracts compared to EEX contracts increased from 2009 to 2011.

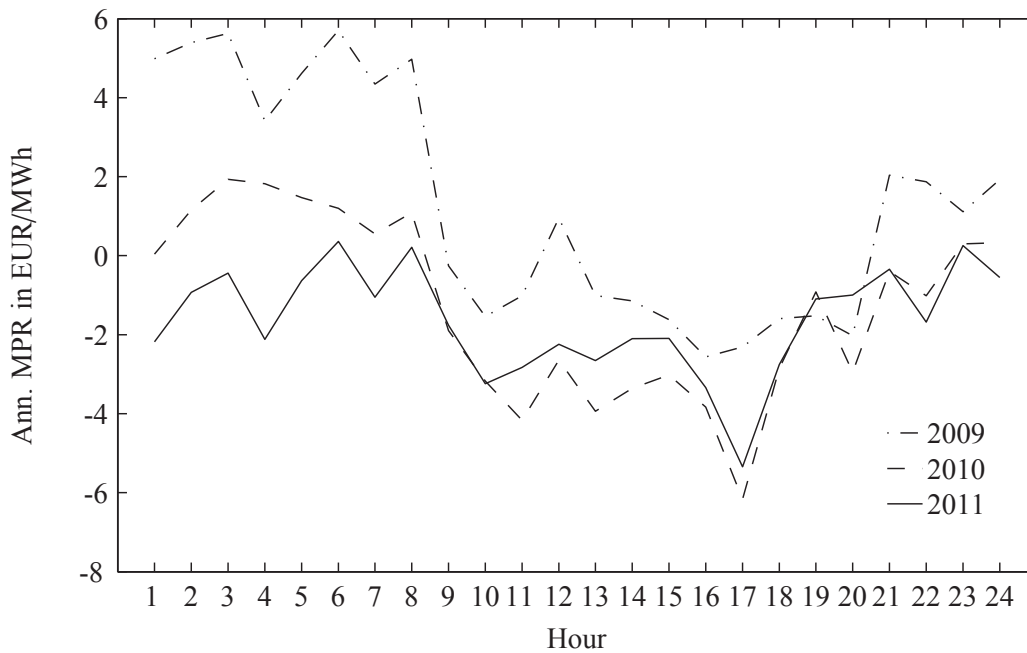
As we investigated the link between expected wind infeed and risk premia, we also want to analyze the relationship between wind and the market price of risk. Our aim is to find out whether patterns change if we account for volatility in the differences between EXAA and EEX prices. For our analysis, we sort EXAA and EEX data for the years 2010 and 2011 according to the expected wind infeed for the same hour. We then split them into bins 1 to 4 where bin 1 contains hours with the lowest wind levels and bin 4 contains hours with the highest wind levels. For every single bin we compute the market prices of risk.

Table 8.6 depicts all market prices of risk and figure 8.4 illustrates the data graphically. For early morning hours we discern a pattern which confirms our findings from the analysis of risk premia. As the expected wind infeed in the German market area increases, the market price of risk gets under pressure and for high wind levels becomes negative. For peak hours the figure indicates the existence of two regimes. For wind levels in the lowest bin, we observe a market price of risk which is around zero. On the contrary, clearly negative market prices of risk for bins 2 to 4 are hardly distinguishable. This gives evidence that if wind is not in the lowest quartile, it does obviously not directly impact the market price of risk. Rather, other factors might drive the MPR then.

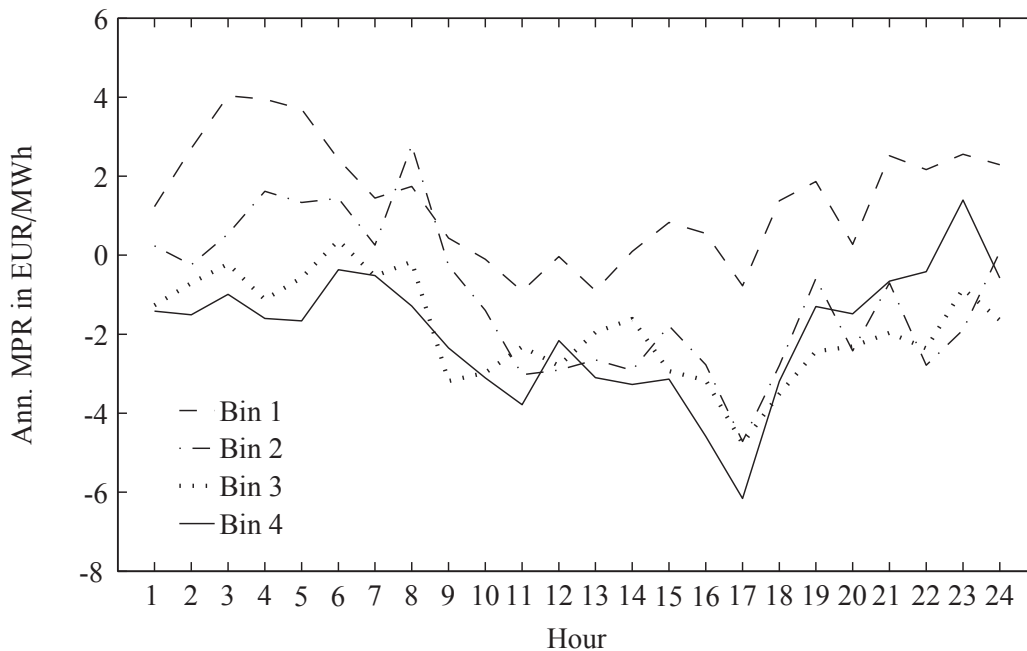


Hour	1	2	3	4	5	6	7	8
$\hat{\lambda}_{2009}$	4.99	5.39	5.63	3.41	4.61	5.70	4.34	4.97
$\hat{\lambda}_{2010}$	0.04	1.15	1.93	1.82	1.47	1.20	0.55	1.09
$\hat{\lambda}_{2011}$	-2.18	-0.93	-0.44	-2.12	-0.63	0.36	-1.05	0.21
$\hat{\sigma}_{2009}$	2.54	4.99	7.43	7.88	8.18	4.41	2.40	2.39
$\hat{\sigma}_{2010}$	2.33	2.94	3.29	3.72	3.63	2.68	1.41	1.40
$\hat{\sigma}_{2011}$	1.55	1.82	2.08	2.52	2.20	1.50	1.37	1.04
Hour	9	10	11	12	13	14	15	16
$\hat{\lambda}_{2009}$	-0.26	-1.54	-1.02	0.95	-1.01	-1.15	-1.62	-2.56
$\hat{\lambda}_{2010}$	-1.90	-3.17	-4.17	-2.65	-3.94	-3.36	-3.02	-3.83
$\hat{\lambda}_{2011}$	-1.76	-3.24	-2.83	-2.24	-2.65	-2.10	-2.10	-3.34
$\hat{\sigma}_{2009}$	1.77	1.53	1.53	1.71	1.56	1.56	1.69	1.72
$\hat{\sigma}_{2010}$	1.27	1.16	1.06	1.21	1.08	1.10	1.02	0.94
$\hat{\sigma}_{2011}$	0.92	0.90	0.84	0.89	0.94	0.89	0.99	1.00
Hour	17	18	19	20	21	22	23	24
$\hat{\lambda}_{2009}$	-2.31	-1.59	-1.52	-2.02	2.04	1.87	1.12	1.93
$\hat{\lambda}_{2010}$	-6.17	-2.85	-0.92	-2.95	-0.40	-1.01	0.30	0.33
$\hat{\lambda}_{2011}$	-5.35	-2.76	-1.10	-1.00	-0.35	-1.68	0.26	-0.55
$\hat{\sigma}_{2009}$	1.55	1.56	1.63	1.63	1.76	1.47	1.55	1.73
$\hat{\sigma}_{2010}$	0.92	1.14	1.13	1.16	0.99	0.94	0.97	1.17
$\hat{\sigma}_{2011}$	0.90	0.91	0.95	0.89	1.08	1.14	1.12	1.03

**Table 8.5:** Annualized market price of risk and volatility of EXAA day-ahead contracts for years 2009, 2010, and 2011 after correcting for weekends, holidays, bridge days, and outliers.



**Figure 8.3:** Annualized market price of risk of EXAA day-ahead contracts for years 2009, 2010, and 2011 after correcting for weekends, holidays, bridge days, and outliers.



**Figure 8.4:** Annualized market price of risk of EXAA day-ahead contracts conditional on the level of expected wind infeed. Bin 1 contains observations with lowest expected wind infeed and bin 4 contains observations with highest expected wind infeed. The observation period starts on January 1, 2010 and ends on December 31, 2011. Weekends, holidays, bridge days, and outliers are excluded.

Hour	1	2	3	4	5	6	7	8
$\hat{\lambda}_{Bin1}$	1.23	2.70	4.04	3.95	3.69	2.42	1.44	1.74
$\hat{\lambda}_{Bin2}$	0.24	-0.24	0.54	1.62	1.33	1.44	0.26	2.78
$\hat{\lambda}_{Bin3}$	-1.27	-0.71	-0.22	-1.12	-0.57	0.37	-0.53	-0.16
$\hat{\lambda}_{Bin4}$	-1.42	-1.51	-0.99	-1.60	-1.66	-0.37	-0.51	-1.29
$\hat{\sigma}_{Bin1}$	0.95	1.13	1.35	1.31	1.28	1.02	0.88	0.76
$\hat{\sigma}_{Bin2}$	0.90	1.00	1.19	1.45	1.38	0.77	0.97	0.76
$\hat{\sigma}_{Bin3}$	1.17	1.53	1.84	2.87	2.72	1.34	1.12	0.98
$\hat{\sigma}_{Bin4}$	3.13	2.68	3.51	2.81	2.80	2.55	1.06	0.96
Hour	9	10	11	12	13	14	15	16
$\hat{\lambda}_{Bin1}$	0.44	-0.10	-0.91	-0.04	-0.90	0.09	0.83	0.55
$\hat{\lambda}_{Bin2}$	-0.26	-1.40	-3.02	-2.91	-2.65	-2.91	-1.79	-2.77
$\hat{\lambda}_{Bin3}$	-3.19	-2.97	-2.31	-2.80	-1.95	-1.61	-2.93	-3.17
$\hat{\lambda}_{Bin4}$	-2.35	-3.11	-3.78	-2.16	-3.10	-3.27	-3.14	-4.59
$\hat{\sigma}_{Bin1}$	0.78	0.68	0.64	0.72	0.61	0.62	0.64	0.63
$\hat{\sigma}_{Bin2}$	0.78	0.67	0.64	0.67	0.72	0.62	0.61	0.68
$\hat{\sigma}_{Bin3}$	0.76	0.79	0.67	0.68	0.68	0.69	0.70	0.69
$\hat{\sigma}_{Bin4}$	0.82	0.78	0.81	0.80	0.78	0.82	0.82	0.75
Hour	17	18	19	20	21	22	23	24
$\hat{\lambda}_{Bin1}$	-0.77	1.38	1.87	0.27	2.52	2.17	2.55	2.29
$\hat{\lambda}_{Bin2}$	-4.72	-2.79	-0.59	-2.42	-0.70	-2.79	-1.89	0.08
$\hat{\lambda}_{Bin3}$	-4.77	-3.52	-2.43	-2.31	-1.96	-2.36	-0.85	-1.65
$\hat{\lambda}_{Bin4}$	-6.16	-3.19	-1.30	-1.48	-0.66	-0.42	1.40	-0.57
$\hat{\sigma}_{Bin1}$	0.64	0.68	0.59	0.62	0.58	0.57	0.57	0.63
$\hat{\sigma}_{Bin2}$	0.65	0.72	0.73	0.66	0.57	0.66	0.67	0.72
$\hat{\sigma}_{Bin3}$	0.66	0.74	0.81	0.74	0.76	0.80	0.75	0.79
$\hat{\sigma}_{Bin4}$	0.62	0.75	0.83	0.97	0.95	0.87	0.96	1.18

**Table 8.6:** Annualized market price of risk and volatility of EXAA day-ahead contracts conditional on the level of expected wind infeed. Bin 1 contains observations with lowest expected wind infeed and bin 4 contains observations with highest expected wind infeed. The observation period starts on January 1, 2010 and ends on December 31, 2011. Weekends, holidays, bridge days, and outliers are excluded.

## 8.5 Analysis of EEX Month-Futures Risk Premia

In a next step we investigate whether the introduction of the equalization mechanism ordinance has caused changes on the EEX forward market. It is conceivable that after the elimination of selling renewables in Germany via monthly bands, supply has disappeared from the front-month forward market. Therefore, we analyze risk premia on EEX front-month futures contracts.

In figures 8.5 and 8.6 we plot the evolution of absolute ex post risk premia on the base and peak forward contracts with delivery during the next month. Risk premia are computed according to equation 8.5 which we repeat for convenience:

$$\pi(T) = F(t, T) - S(T) \quad (8.17)$$

where  $F(t, T)$  is the price of the futures contract with delivery at (or during)  $T$  (front month) on trading day  $t$  and  $S(T)$  is calculated as the average spot price of day-ahead peak contracts for all relevant dates in the delivery month. A vertical line is plotted on December 1, 2009 when the front-month contract with delivery in January 2010 started trading. The underlying time window of this contract started as the new ordinance became effective.

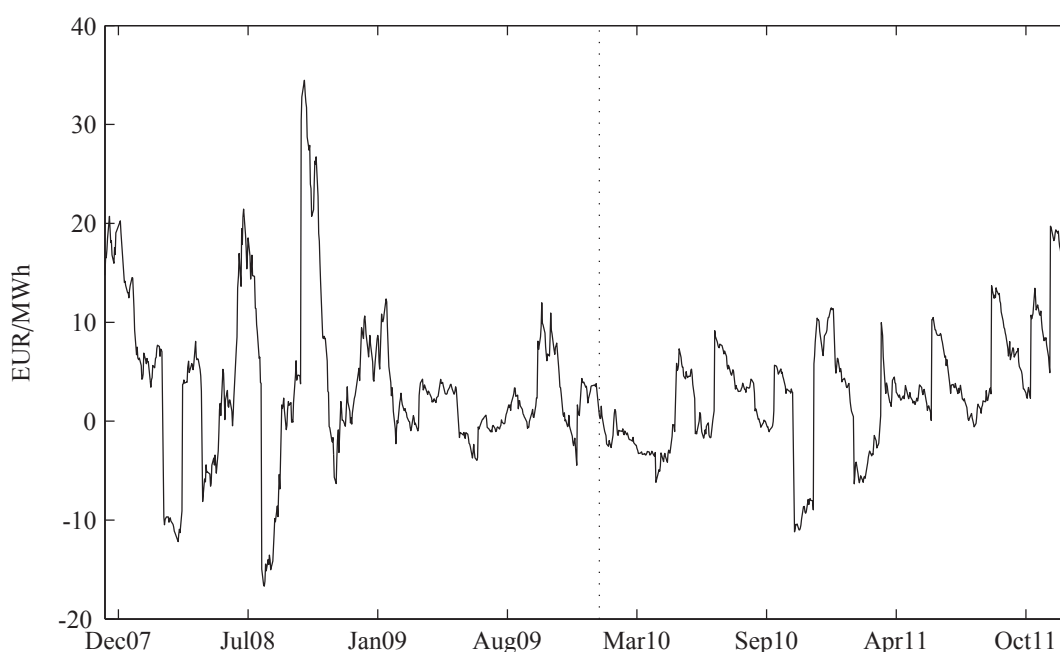
We can see that after the beginning of 2010, the volatility in risk premia seems to reduce somewhat, particularly compared to the year 2008. This could theoretically be reasoned by highly volatile electricity from renewable sources which as of 2010 was no longer offered through monthly bands. However, looking at risk premia on contracts with delivery starting 2, 3, and 5 months ahead (results are not reported), we can observe the same pattern. It is thus unlikely that the reduced volatility was caused by the introduction of the AusglMechV. The same applies to the level of risk premia. We looked at moving averages of risk premia which imply an increase in premia starting in 2010 (results are not reported). However, also this pattern is similar for contracts with a longer time-to-maturity.

The reason why we investigate front-month contracts is the implicit assumption that before 2010, ESCs hedged their obligations resulting from the creation of monthly bands via the corresponding forward market. With these monthly bands ceasing to exist, a reduction in supply on the front-month market should have led to higher forward prices (on a relative basis). However, as we see from figures 8.5 and 8.6, there is not even enough evidence for statistical testing procedures to make sense.

To investigate further we look at changes in risk premia for time-to-maturities of 1 to 5 months in different years. This means that we compute average risk premia for month futures based on trading prices in the fifth, the fourth, the third, the second, and

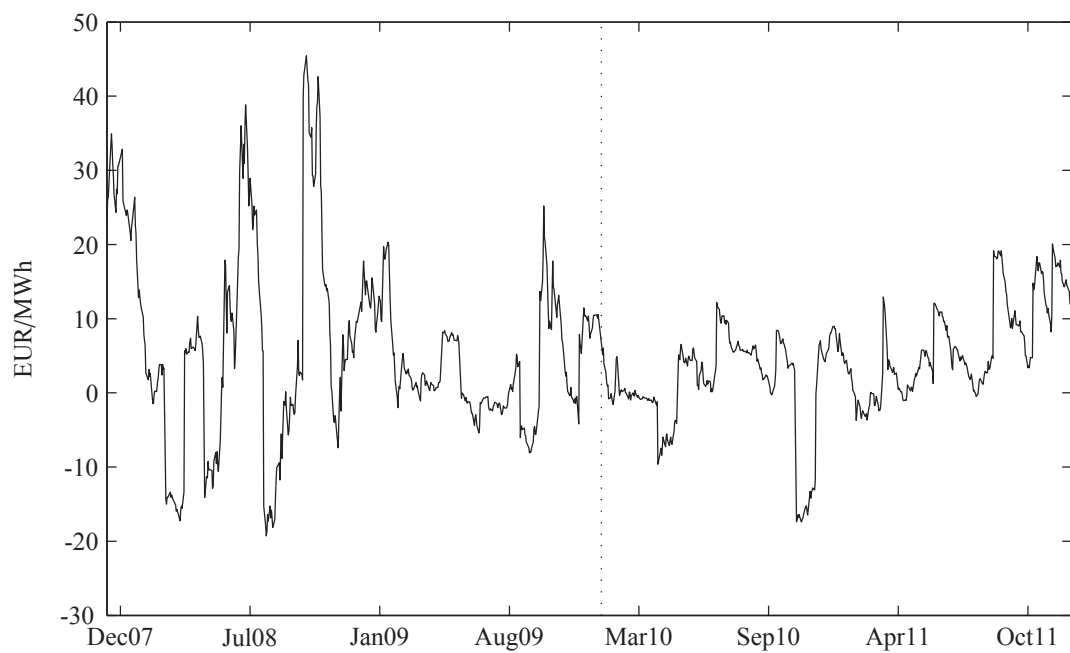
the first month before delivery starts.<sup>11</sup> Results for base delivery contracts are depicted in figure 8.7 and results for peak delivery contracts are depicted in figure 8.8. While we see much steeper curves for 2009, the curves for 2011 are comparable to the ones for 2008. Consequently, this analysis does not provide any reasonable indication that there has been a significant change in pattern as of January 2010 either.

To sum up, based on the data investigated we are not able to find evidence that the introduction of the equalization mechanism ordinance had an observable impact on the risk premia on front-month EEX forward contracts.

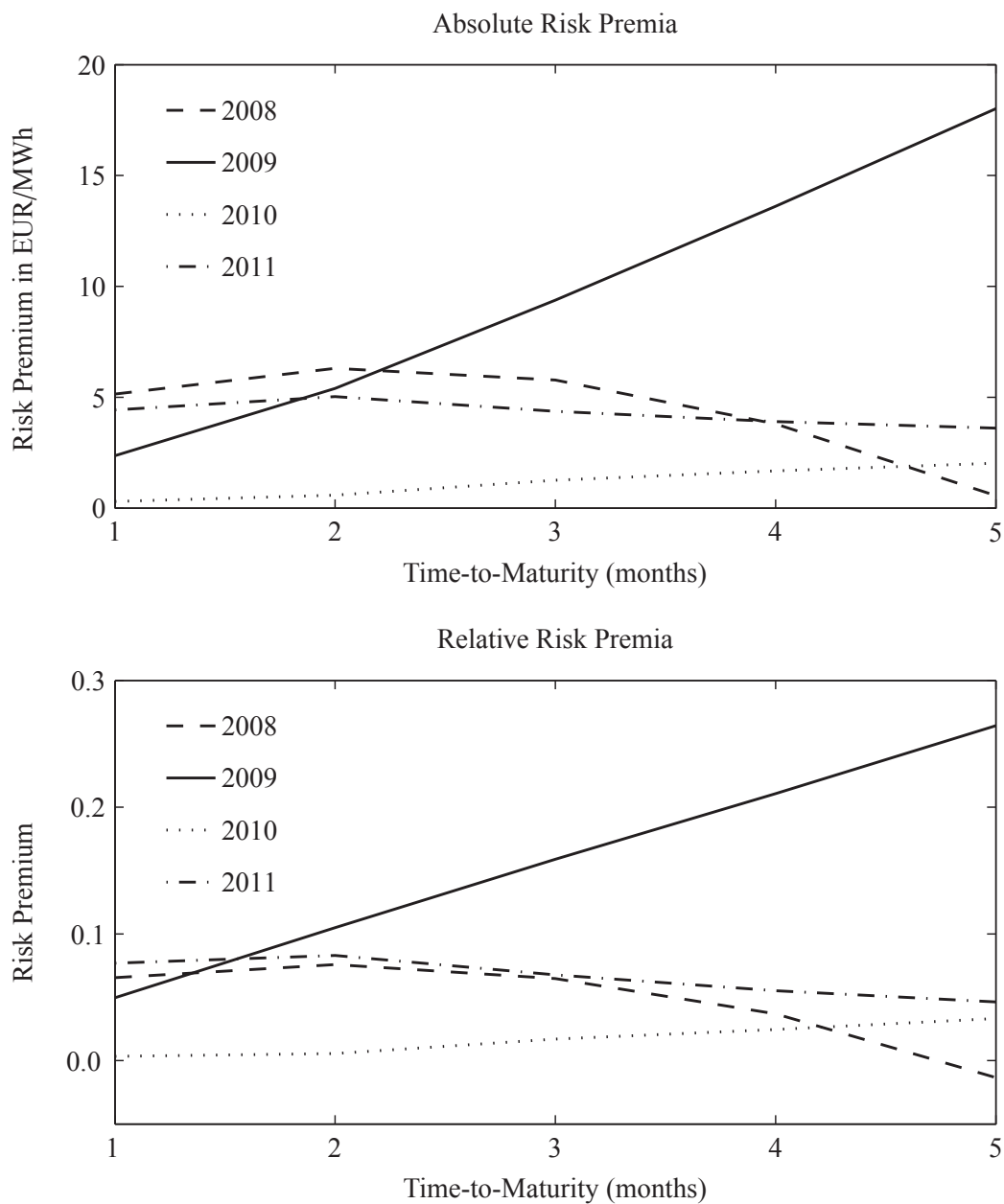


**Figure 8.5:** Absolute ex post risk premia on EEX front-month futures with base delivery. The vertical dashed line indicates the date (December 1, 2009) when trading of the contract with delivery in January 2010 started.

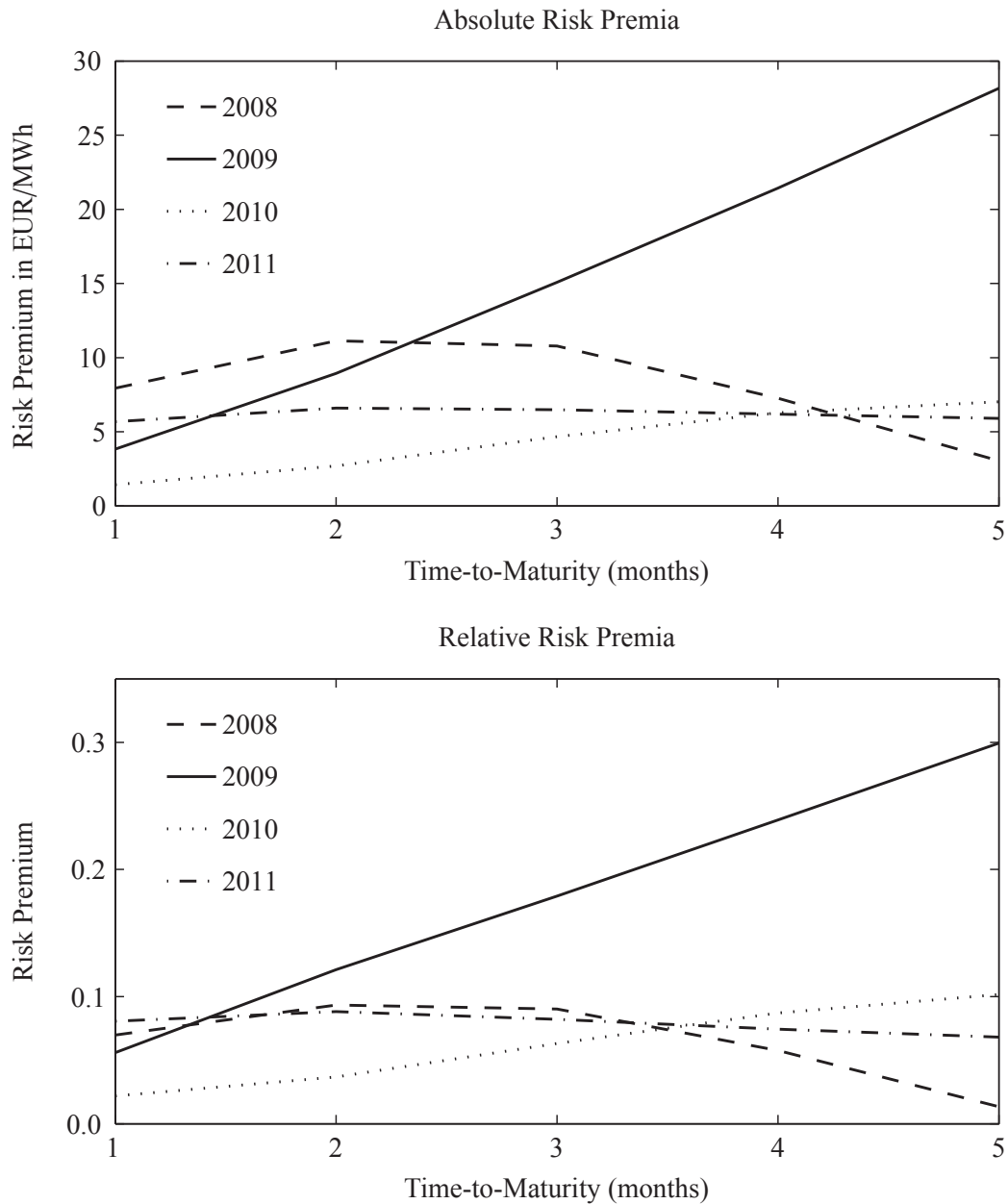
<sup>11</sup>Example: The risk premium for a two months time-to-maturity is computed using average effective spot prices for all individual months January to December of the respective year as well as average trading prices of the respective futures contracts in the period starting two months before the delivery starts and ending one month before the delivery starts (by that time the two-month contract becomes the front-month contract). This results in 12 risk premia which are again averaged to receive one final figure for the respective year.



**Figure 8.6:** *Absolute ex post risk premia on EEX front-month futures with peak delivery. The vertical dashed line indicates the date (December 1, 2009) when trading of the contract with delivery in January 2010 started.*



**Figure 8.7:** Absolute and relative ex post risk premia on EEX futures with base delivery for time-to-maturities of 1 to 5 months for years 2008 to 2011. Only trading dates within five months before the first delivery date have been considered.



**Figure 8.8:** Absolute and relative ex post risk premia on EEX futures with peak delivery for time-to-maturities of 1 to 5 months for years 2008 to 2011. Only trading dates within five months before the first delivery date have been considered.



## 8.6 Intermediate Summary

Our analyses on the differences between EXAA and EEX prices, defined as risk premia, reveal that EXAA prices have increased relative to EEX prices from 2009 to 2011. Investigating conditional risk premia distributions we obtain strong indications that the increased risk premia on EXAA prices are significantly driven by expected wind infeed in Germany during off-peak hours. At the same time we find no evidence that the increase in risk premia during peak hours is driven by expected wind infeed. However, we assume that during these hours, other renewables, especially photovoltaics, may be important driving factors. This would have to be investigated in further research as respective data become available for a sufficiently long time horizon.

When accounting for volatility by analyzing the market price of risk on EXAA contracts, we find our conclusion of increased EXAA prices relative to EEX prices confirmed. The analysis of market prices of risk depending on the expected wind infeed shows that during peak hours, unless expected wind infeed is at very low levels, it does not directly impact the magnitude of the MPR.

When analyzing front-month EEX contracts for peak and base delivery we were not able to discern any noteworthy change in pattern after the introduction of the equalization mechanism ordinance. As a potential explanation we consider the possibility that before 2010, exposures from monthly bands were not passed on through the one-month forward markets mainly, but other means were used, in particular OTC markets which constitutes a significant part of all electricity traded in Germany.



# Chapter 9

## Conclusion

Political efforts to promote electricity from renewable sources have changed the process of electricity price formation. In Germany, especially electricity from wind power plants is taking a more and more important role, not least because such electricity supply is highly volatile and practically unpredictable except in the very short run. At the same time, improving transparency in electricity markets makes fundamental modelling increasingly attractive to researchers. The application of fundamental forecast methods provides an alternative to established purely stochastic models which still constitute the most common approach in electricity spot markets. Considering these developments, it is the objective of this thesis to contribute to the existing research of electricity spot price modelling and, particularly, to investigate the explanatory power of expected wind (electricity) infeed in fundamental forecast models.

### 9.1 Summary of Main Results

We estimate three main classes of fundamental models on prices of day-ahead electricity contracts for the German market. We calibrate hourly models on an in-sample dataset consisting of data from January 1, 2010 to December 31, 2011. To test their forecasting power we apply the models to an out-of-sample dataset starting on January 1, 2012 and ending on April 30, 2012. Although the results of the three model classes are not directly comparable due to different assumptions underlying the methodologies, we can report distinct differences in their price forecasting abilities.

To start we estimate hourly Student-t GARCH(1,1) regression models which combine a multiple linear regression with a conditional variance specification. The in-sample fit of the models including expected wind infeed proves to be rather good with

an average  $R^2$  of 0.69 meaning that the models can explain nearly 70% of the spot price variability ex post. When comparing results with and without accounting for expected wind infeed as an exogenous variable, we can note an improvement in  $R^2$  of 0.09 on average. Investigating conditional volatility confirms that including expected wind infeed substantially reduces uncertainty in the models. Our analysis on the intra-day behavior of regression coefficients reveals that sensitivities of spot prices towards different fundamental variables possess distinct seasonal patterns. The patterns are mainly driven by the merit order curve and by the seasonal nature of electricity demand. Looking at out-of-sample results, GARCH regression models show a rather poor performance with an average  $R^2$  of less than 0.50.

In order to allow for two regimes depending on the level of expected wind infeed, we estimate threshold regression models on the spot prices. By statistical testing we find a significant threshold for most hours. Analyzing the factor loads of different hours for the two regimes reveals two main facts. First, absolute coefficient values of expected wind infeed are higher in the regimes below the threshold meaning that spot prices react much more sensitively to changes in expected wind infeed. This is particularly the case for peak hours with high demand around noon. We assign this behavior to a tense situation which is the result of high demand during these hours. Accordingly, spot prices are particularly sensitive to intermittent supply from renewables. Second, we observe that coefficients for CO<sub>2</sub> prices are much higher in times of expected wind levels above the threshold than below. We reason this behavior by the fact that higher wind infeed into the grid shifts the merit order curve to the right. As a result, the spot price is no longer set by gas fired plants but rather by CO<sub>2</sub> intense hard coal fired plants (for peak hours) or even more CO<sub>2</sub> intense lignite fired plants (for off-peak hours) for which prices of emission allowance certificates are more relevant. Although in-sample fits of threshold models are satisfying with an improvement in  $R^2$  of between 0.02 and 0.11 when allowing for a second regime, out-of-sample fits prove to be poor. Obviously, modelling approaches which assume static linear dependencies in electricity spot prices are rather well suited for ex post modelling but not capable of providing accurate out-of-sample forecasts.

By estimating time-varying parameter regression models via a Kalman filter algorithm we obtain results which are much more satisfying for the majority of all hours. This methodology, which assumes price sensitivities to evolve dynamically and thus allows for seasonalities in dependency structures, provides out-of-sample results with an  $R^2$  of 0.61 on average. Especially during early morning hours, when prices are very sensitive with regards to intermittent supply from wind power plants, we can report  $R^2$  measures of above 0.70. The best fit we find for hour 18 with an  $R^2$  of 0.83. On the contrary, the fit for hours 19 to 21, when the on/off-peak switch takes place, proves to

be clearly inferior with  $R^2$  measures below 0.40. Testing our models with and without including expected wind infeed we note an average improvement in  $R^2$  of 0.27. Looking at the mean absolute error we note an improvement of 1.29 EUR/MWh. We see this as a clear evidence that accounting for expected wind infeed can substantially improve forecasting accuracy. Furthermore, time-varying parameter regression models seem a reasonable method to incorporate information on intermittent supply from renewable sources when modelling electricity spot prices in the German market. The analysis of the behavior of the dynamic coefficients of expected wind infeed in the in-sample estimation reveals seasonal patterns depending on the hour of the day. In other words, the price formation process reacts differently to changes in electricity supply from wind power plants depending on the hour of the day and different seasons.

Although time-varying parameter regression models can significantly improve price forecasts, a considerable part of the price variability remains unexplained. We ascribe this to two main reasons. First, market power in the German electricity market is still rather concentrated. This means that market efficiency has not yet reached the level of more mature markets such as equities. Second, we believe that strategic and speculative behavior of market participants has a substantial impact on market prices. Such behavior is difficult to capture by quantitative models, especially given the current level of transparency in the market.

In a forward market analysis we investigate the impact of one of the latest regulatory amendments which became effective as of January 1, 2010 and which has changed the way of how electricity from renewable sources is marketed in Germany. We perform our analysis by defining day-ahead contracts traded at the EXAA in Austria as the nearest forward contracts for EEX day-ahead contracts. Looking at risk premia on EXAA contracts we find that they have increased, particularly for off-peak hours when demand is low and the intermittent supply of electricity from wind power plants causes large price movements. Whereas before January 1, 2010 negative risk premia can be observed during these hours, they have turned into positive premia by 2011. We can report an increase in risk premia for peak hours as well. However, during these times, we do not find evidence that expected wind infeed is the main driver which is the case for off-peak hours. Our findings are generally confirmed by an analysis of the development of the market price of risk on the same contracts between 2009 and 2011.

In addition to EXAA contracts we investigate risk premia on short-term EEX futures contracts. Following the latest amendment, electricity from renewable sources is no longer sold through monthly bands (economically comparable to monthly forward contracts). Considering this, the aim is to find out whether this has had any impact on the development of risk premia on short-term monthly futures contracts. However,

our analysis does not indicate any structural changes. Our findings let us assume that before 2010, exposures from monthly bands were to a large extent hedged by other means than monthly futures contracts.

## 9.2 Future Research

Modelling day-ahead electricity prices using fundamental variables is still rather undeveloped. One of the main reasons for this might be the fact that a lot of information has not been published for a long time, making the market quite intransparent.

We expect that day-ahead spot price forecasts from time-varying parameter regression models can be improved by the inclusion of expected electricity infeed from photovoltaic power plants. Over the last years electricity supply from this source has significantly increased. Especially during summertime and over noon electricity from photovoltaics represents an important part of today's energy mix. Similar to wind, expected infeed from photovoltaics is published by transmission system operators the day preceding the delivery. However, these publications started much later than publications of wind-related figures which is why there was no sufficiently long and consistent dataset available for our analysis. It is conceivable that out-of-sample,  $R^2$  measures for hours 9 to 17 for which, except hour 12, we obtained an  $R^2$  of below 0.70 in the TVP regression models, can be significantly improved when accounting for photovoltaics.

It would also be interesting to investigate to what extent differences between EXAA and EEX prices during peak hours can be attributed to intermittent infeed from photovoltaics. As discussed, we were not able to explain observed changes in risk premia for these hours by electricity from wind power plants.

We consider the derivation of a more accurate demand forecast model for the German market as another possible field of future research following this work. As demonstrated in the overview on existing literature, research in this area is still rather undeveloped. With a more accurate model at hand, day-ahead spot price forecasts could certainly be further improved.

## **Appendix A**

### **Additions Chapter 4**

<b>Hour</b>	<b>1</b>	<b>2</b>	<b>3</b>	<b>4</b>	<b>5</b>	<b>6</b>	<b>7</b>	<b>8</b>
Obs.	499	499	499	499	499	499	499	499
Min	25738	21144	20848	21904	23855	25876	29297	31999
Max	48092	45487	43845	43157	43265	45115	50846	56785
Mean	36111	34339	33407	33630	34718	37330	43052	47846
St.Dev.	3864	3767	3714	3597	3491	3221	3288	3650
Skewness	0.27	0.21	0.13	0.03	-0.10	-0.27	-0.73	-0.69
Kurtosis	2.93	3.17	3.05	2.97	2.89	3.17	4.22	4.54
<b>Hour</b>	<b>9</b>	<b>10</b>	<b>11</b>	<b>12</b>	<b>13</b>	<b>14</b>	<b>15</b>	<b>16</b>
Obs.	499	499	499	499	499	499	499	499
Min	33960	32551	31084	31793	31608	31155	30188	29843
Max	57944	58348	58176	58350	58151	58754	58492	58813
Mean	49131	48555	48418	48750	47779	47179	46591	46660
St.Dev.	3584	3695	3927	3997	4258	4395	4445	4439
Skewness	-0.57	-0.41	-0.29	-0.22	-0.07	0.01	0.05	0.07
Kurtosis	4.47	4.02	3.49	3.21	2.85	2.73	2.73	2.75
<b>Hour</b>	<b>17</b>	<b>18</b>	<b>19</b>	<b>20</b>	<b>21</b>	<b>22</b>	<b>23</b>	<b>24</b>
Obs.	499	499	499	499	499	499	499	499
Min	29170	29705	29924	30141	29593	28968	26781	23624
Max	60320	62015	61092	59483	57205	55450	54182	50704
Mean	46838	48376	49543	49697	47882	46006	43840	39896
St.Dev.	4641	5017	4820	4257	3533	3034	3338	3469
Skewness	0.24	0.35	0.01	-0.30	-0.76	-0.65	-0.21	0.09
Kurtosis	2.86	2.62	2.74	3.04	4.88	5.58	4.58	3.93

**Table A.1:** *Descriptive statistics of total electricity demand. Total demand is defined according to equation 4.1 in chapter 4. The observation window is between January 1, 2010 and December 31, 2011 and excludes weekends, holidays, and bridge days.*



Lag	1	5	10	15	20
<b>Critical Values</b>					
0.99-Significance	6.63	15.09	23.21	30.58	37.57
0.95-Significance	3.84	11.07	18.31	25.00	31.41
<b>Test Statistics</b>					
Hour 01	192.74	930.92	1658.68	2273.46	2720.14
Hour 02	193.38	927.86	1635.77	2223.05	2649.22
Hour 03	201.52	975.72	1740.81	2364.54	2829.12
Hour 04	199.71	962.81	1722.46	2327.30	2783.86
Hour 05	207.46	994.25	1786.05	2414.88	2898.12
Hour 06	219.69	986.27	1721.99	2277.58	2713.59
Hour 07	224.37	873.46	1330.29	1627.22	1835.42
Hour 08	259.05	911.58	1402.70	1707.76	1907.37
Hour 09	261.70	862.48	1318.05	1650.96	1859.12
Hour 10	247.36	812.22	1312.07	1738.12	2000.36
Hour 11	227.87	704.03	1128.44	1498.56	1729.06
Hour 12	219.67	692.52	1090.89	1461.91	1704.33
Hour 13	227.71	768.18	1238.64	1658.25	1965.37
Hour 14	245.89	838.98	1360.52	1813.04	2125.00
Hour 15	253.85	884.40	1438.88	1921.41	2273.62
Hour 16	266.89	1003.40	1677.34	2252.23	2686.66
Hour 17	300.67	1238.46	2152.44	2924.92	3517.63
Hour 18	345.20	1576.32	2874.89	3968.47	4815.67
Hour 19	328.52	1534.73	2819.69	3918.05	4782.77
Hour 20	284.45	1294.16	2304.12	3163.85	3833.48
Hour 21	200.86	875.83	1498.73	2027.93	2406.54
Hour 22	175.68	718.78	1212.00	1648.17	1967.16
Hour 23	260.82	1083.71	1872.85	2599.90	3126.78
Hour 24	319.84	1343.78	2338.51	3234.71	3881.42

**Table A.2:** Results of Ljung-Box  $Q$ -test performed on demand  $D_t$  in advance to the demand model estimation. Test statistics with values higher than the respective critical values reject the null hypothesis of no serial correlation. The observation window starts on January 1, 2010 and ends on December 31, 2011. Weekends, holidays, and bridge days are excluded.

<b>Lag</b>	<b>1</b>	<b>5</b>	<b>10</b>	<b>15</b>	<b>20</b>
<b>Critical Values</b>					
0.99-Significance	6.63	15.09	23.21	30.58	37.57
0.95-Significance	3.84	11.07	18.31	25.00	31.41
<b>Test Statistics</b>					
Hour 01	214.67	307.60	321.02	318.41	314.99
Hour 02	218.61	304.34	316.19	313.46	309.68
Hour 03	225.31	312.40	321.95	318.64	314.61
Hour 04	220.93	311.78	320.92	318.85	316.22
Hour 05	225.32	311.78	319.24	317.88	316.25
Hour 06	235.44	305.69	310.66	309.54	310.31
Hour 07	245.16	284.47	284.24	281.88	285.61
Hour 08	282.72	305.37	301.83	297.41	296.22
Hour 09	283.08	301.71	299.28	295.20	290.92
Hour 10	263.57	281.74	279.98	277.02	269.96
Hour 11	243.44	259.65	257.14	253.63	244.64
Hour 12	235.18	252.22	249.68	247.31	238.71
Hour 13	244.21	263.97	260.32	257.79	248.97
Hour 14	260.87	279.20	274.81	272.73	263.99
Hour 15	269.76	289.19	284.80	282.16	274.94
Hour 16	283.26	306.55	302.07	299.35	294.36
Hour 17	317.84	341.21	336.46	333.28	329.33
Hour 18	361.46	385.80	380.94	375.90	372.27
Hour 19	350.65	382.51	378.39	373.85	370.17
Hour 20	306.31	349.48	350.12	346.12	342.48
Hour 21	225.01	281.35	282.73	280.37	276.84
Hour 22	198.53	252.02	258.94	255.76	253.07
Hour 23	287.37	318.98	316.62	312.09	307.15
Hour 24	345.51	362.27	359.60	354.25	347.43

**Table A.3:** Results of Engle's ARCH test performed on demand  $D_t$  in advance to the demand model estimation. Test statistics with values higher than the respective critical values reject the null hypothesis of homoscedasticity. The observation window starts on January 1, 2010 and ends on December 31, 2011. Weekends, holidays, and bridge days are excluded.

Lag	1	5	10	15	20
<b>Critical Values</b>					
0.99-Significance	6.63	15.09	23.21	30.58	37.57
0.95-Significance	3.84	11.07	18.31	25.00	31.41
<b>Test Statistics</b>					
Hour 01	0.44	5.44	6.79	9.32	21.22
Hour 02	0.03	3.78	8.96	10.80	20.69
Hour 03	1.19	3.53	11.99	15.07	21.59
Hour 04	1.09	5.05	14.24	17.77	24.72
Hour 05	1.12	5.43	12.36	16.11	25.45
Hour 06	4.22	15.04	19.36	24.50	36.03
Hour 07	6.44	21.41	28.64	29.90	35.55
Hour 08	3.80	12.04	19.39	21.12	27.37
Hour 09	1.63	12.30	21.63	23.64	25.46
Hour 10	7.91	32.92	49.44	60.36	64.58
Hour 11	1.50	16.50	24.87	32.36	34.75
Hour 12	0.18	18.08	26.60	40.08	44.01
Hour 13	0.59	24.59	37.30	54.62	64.88
Hour 14	0.74	26.75	39.99	61.53	70.95
Hour 15	0.31	12.28	16.69	27.93	34.14
Hour 16	4.88	24.90	30.41	37.92	45.63
Hour 17	0.26	6.29	10.43	13.68	15.53
Hour 18	0.03	12.95	17.95	21.83	24.67
Hour 19	0.27	15.92	20.32	26.81	32.62
Hour 20	0.03	1.92	5.96	8.63	13.47
Hour 21	0.22	17.10	18.04	21.20	29.10
Hour 22	7.11	22.52	26.77	27.91	31.95
Hour 23	20.42	49.33	57.83	62.99	65.26
Hour 24	7.29	16.85	20.96	23.97	28.47

**Table A.4:** Results of Ljung-Box  $Q$ -test performed on standardized residuals after the estimation of demand models. Test statistics with values higher than the respective critical values reject the null hypothesis of no serial correlation. The observation window starts on January 1, 2010 and ends on December 31, 2011. Weekends, holidays, and bridge days are excluded.

<b>Lag</b>	<b>1</b>	<b>5</b>	<b>10</b>	<b>15</b>	<b>20</b>
<b>Critical Values</b>					
0.99-Significance	6.63	15.09	23.21	30.58	37.57
0.95-Significance	3.84	11.07	18.31	25.00	31.41
<b>Test Statistics</b>					
Hour 01	0.05	0.67	1.38	2.27	40.67
Hour 02	0.10	1.22	9.63	11.51	25.89
Hour 03	0.05	1.59	26.55	30.68	38.60
Hour 04	0.05	1.77	25.45	29.35	37.71
Hour 05	0.05	1.64	13.08	15.08	25.46
Hour 06	0.09	1.60	2.54	3.92	14.75
Hour 07	0.00	2.03	6.26	8.25	23.95
Hour 08	0.14	2.36	10.12	10.49	15.95
Hour 09	0.14	0.79	2.92	3.32	4.03
Hour 10	0.00	1.68	7.57	8.18	9.45
Hour 11	0.02	3.09	9.22	10.72	13.39
Hour 12	0.30	4.27	6.17	7.48	9.09
Hour 13	0.20	5.37	6.46	9.13	9.19
Hour 14	0.13	3.05	4.57	6.51	7.21
Hour 15	0.03	0.77	1.65	2.92	3.64
Hour 16	0.01	1.12	1.93	2.68	3.07
Hour 17	1.80	2.05	2.32	3.20	3.79
Hour 18	2.10	2.42	2.91	3.65	4.30
Hour 19	2.99	3.73	4.03	5.02	6.00
Hour 20	3.04	4.07	4.57	5.70	6.92
Hour 21	2.04	2.45	2.84	3.82	5.11
Hour 22	0.00	1.68	3.11	3.96	5.85
Hour 23	0.09	0.94	3.76	4.56	5.91
Hour 24	0.02	1.01	1.46	1.85	2.54

**Table A.5:** Results of Engle's ARCH test performed on standardized residuals after the estimation of demand models. Test statistics with values higher than the respective critical values reject the null hypothesis of homoscedasticity. The observation window starts on January 1, 2010 and ends on December 31, 2011. Weekends, holidays, and bridge days are excluded.

	Test Stat.	p-Value		Test Stat.	p-Value
Hour 01	-7.02	0.001	Hour 13	-7.12	0.001
Hour 02	-6.90	0.001	Hour 14	-6.89	0.001
Hour 03	-6.67	0.001	Hour 15	-6.68	0.001
Hour 04	-6.81	0.001	Hour 16	-6.22	0.001
Hour 05	-6.57	0.001	Hour 17	-5.50	0.001
Hour 06	-6.42	0.001	Hour 18	-4.31	0.001
Hour 07	-6.39	0.001	Hour 19	-4.32	0.001
Hour 08	-6.11	0.001	Hour 20	-5.02	0.001
Hour 09	-6.51	0.001	Hour 21	-6.33	0.001
Hour 10	-7.08	0.001	Hour 22	-6.97	0.001
Hour 11	-7.37	0.001	Hour 23	-5.68	0.001
Hour 12	-7.38	0.001	Hour 24	-4.90	0.001

**Table A.6:** Results of augmented Dickey-Fuller tests (ADF tests) applied to total demand. The null hypothesis claims that the tested variable follows a zero drift unit root process. The number of lags is 1 which corresponds to the later applied first order autoregressive component to forecast demand. The critical value for a 95% significance level is -2.87 (see Fuller (1976)).  $n$  is 499 for all hours.



## **Appendix B**

### **Additions Chapter 5**

	Normal GARCH			Student-t GARCH			LR-Test
	LLF	AIC	BIC	LLF	AIC	BIC	Test-Stat.
Hour 01	-1343	2716	2779	-1286	2603	2665	113.22
Hour 02	-1429	2887	2950	-1379	2788	2851	99.16
Hour 03	-1484	2997	3060	-1456	2942	3005	54.85
Hour 04	-1556	3142	3205	-1548	3126	3189	15.91
Hour 05	-1534	3097	3160	-1514	3059	3121	38.91
Hour 06	-1333	2696	2758	-1299	2629	2692	66.85
Hour 07	-1375	2781	2844	-1352	2734	2797	46.39
Hour 08	-1464	2959	3022	-1456	2942	3005	17.18
Hour 09	-1459	2947	3010	-1443	2916	2978	31.86
Hour 10	-1436	2901	2964	-1428	2887	2950	14.30
Hour 11	-1417	2864	2926	-1412	2853	2916	10.35
Hour 12	-1451	2931	2994	-1443	2916	2979	15.05
Hour 13	-1403	2836	2899	-1403	2836	2899	-0.08
Hour 14	-1401	2833	2896	-1402	2833	2896	-0.13
Hour 15	-1421	2871	2934	-1420	2870	2933	1.49
Hour 16	-1406	2843	2906	-1407	2844	2907	-0.77
Hour 17	-1352	2734	2797	-1352	2734	2797	0.00
Hour 18	-1397	2824	2887	-1395	2820	2883	4.37
Hour 19	-1433	2896	2958	-1423	2877	2940	18.70
Hour 20	-1446	2922	2985	-1435	2900	2963	22.36
Hour 21	-1394	2818	2881	-1381	2791	2854	27.03
Hour 22	-1294	2618	2681	-1286	2602	2665	16.04
Hour 23	-1224	2477	2540	-1216	2462	2525	15.47
Hour 24	-1231	2491	2554	-1208	2445	2508	45.94

**Table B.1:** Comparison of normal and Student-t GARCH models including expected wind infeed. AIC denotes the Akaike Information Criterion, BIC denotes the Bayesian Information Criterion (for both, a lower value implies superiority of the model), and LLF denotes the value of the log likelihood function (a lower absolute value implies superiority of the model). The test statistic for the likelihood ratio test is computed using the log likelihood function values of both models and one degree of freedom. For test statistics ( $\chi^2$ -distributed) with values above 3.84 the null hypothesis that the normal GARCH model is superior can be rejected at the 95% significance level (the critical value for the 90% level is 2.71). An introduction to likelihood ratio tests can for example be found in Hamilton (1994b).



	Model 1: $R^2$	Model 2: $R^2$	F-Stat.	p-Value
Hour 01	0.5497	0.5526	3.115	0.078
Hour 02	0.4847	0.4897	4.593	0.033
Hour 03	0.4502	0.4511	0.780	0.377
Hour 04	0.4704	0.4769	5.890	0.016
Hour 05	0.4637	0.4706	6.119	0.014
Hour 06	0.5898	0.5957	6.863	0.009
Hour 07	0.5234	0.5297	6.385	0.012
Hour 08	0.6014	0.5987		
Hour 09	0.6125	0.6127	0.198	0.657
Hour 10	0.5964	0.5968	0.467	0.495
Hour 11	0.5841	0.5857	1.905	0.168
Hour 12	0.5528	0.5557	3.075	0.080
Hour 13	0.5633	0.5664	3.479	0.063
Hour 14	0.5728	0.5751	2.623	0.106
Hour 15	0.5765	0.5781	1.713	0.191
Hour 16	0.5908	0.5952	5.167	0.023
Hour 17	0.6654	0.6676	3.044	0.082
Hour 18	0.7449	0.7449		
Hour 19	0.7315	0.7321	0.899	0.344
Hour 20	0.7121	0.7133	1.954	0.163
Hour 21	0.6964	0.6969	0.804	0.370
Hour 22	0.6905	0.6902		
Hour 23	0.6632	0.6692	8.563	0.004
Hour 24	0.6026	0.6057	3.838	0.051

**Table B.2:** Results of  $F$ -test to investigate whether including the oil price variable improves the model fit significantly. Model 1 excludes the oil price variable and model 2 includes it. Rejecting the null hypothesis implies that the increase in  $R^2$  is statistically significant. Critical values (the difference for separate hours is marginal) are 6.69 and 3.86 for a significance level of 99% and 95%, respectively. No test is performed for hours where  $R^2$  for model 2 is lower than for model 1.  $n$  is between 484 and 492. Models 1 and 2 exclude expected wind infeed as an explanatory variable. However, we have verified that including wind would not change the results of the analysis.

	Model 1: $R^2$	Model 2: $R^2$	F-Stat.	p-Value
Hour 01	0.5497	0.5501	0.433	0.511
Hour 02	0.4847	0.4865	1.688	0.194
Hour 03	0.4502	0.4498		
Hour 04	0.4704	0.4734	2.662	0.103
Hour 05	0.4637	0.4606		
Hour 06	0.5898	0.5898	0.043	0.835
Hour 07	0.5234	0.5280	4.655	0.031
Hour 08	0.6014	0.6056	5.028	0.025
Hour 09	0.6125	0.6157	3.954	0.047
Hour 10	0.5964	0.5976	1.339	0.248
Hour 11	0.5841	0.5844	0.373	0.542
Hour 12	0.5528	0.5529	0.097	0.755
Hour 13	0.5633	0.5634	0.162	0.688
Hour 14	0.5728	0.5730	0.285	0.594
Hour 15	0.5765	0.5766	0.042	0.838
Hour 16	0.5908	0.5914	0.733	0.392
Hour 17	0.6654	0.6676	3.027	0.083
Hour 18	0.7449	0.7465	3.074	0.080
Hour 19	0.7315	0.7316	0.006	0.936
Hour 20	0.7121	0.7130	1.593	0.208
Hour 21	0.6964	0.6966	0.430	0.512
Hour 22	0.6905	0.6908	0.386	0.535
Hour 23	0.6632	0.6634	0.295	0.587
Hour 24	0.6026	0.6021		

**Table B.3:** Results of F-test to investigate whether including the lagged demand variable improves the model fit significantly. Model 1 excludes the lagged demand variable and model 2 includes it. Rejecting the null hypothesis implies that the increase in  $R^2$  is statistically significant. Critical values (the difference for separate hours is marginal) are 6.69 and 3.86 for a significance level of 99% and 95%, respectively. No test is performed for hours where  $R^2$  for model 2 is lower than for model 1.  $n$  is between 484 and 492. Models 1 and 2 exclude expected wind infeed as an explanatory variable. However, we have verified that including wind would not change the results of the analysis.

## **Appendix C**

### **Additions Chapter 7**

<b>Lag</b>	<b>1</b>	<b>5</b>	<b>10</b>	<b>20</b>
<b>Critical Values</b>				
0.99-Significance	6.63	15.09	23.21	37.57
0.95-Significance	3.84	11.07	18.31	31.41
<b>Test Statistics</b>				
Hour 01	2.46	5.52	8.11	22.80
Hour 02	0.61	2.95	12.34	24.67
Hour 03	0.25	1.46	4.09	19.97
Hour 04	0.12	1.80	6.10	13.50
Hour 05	0.27	0.54	4.70	10.28
Hour 06	0.32	4.84	10.39	24.51
Hour 07	0.38	1.34	10.57	21.02
Hour 08	0.12	3.98	19.16	26.95
Hour 09	1.77	11.85	19.44	29.18
Hour 10	0.92	12.49	15.43	22.50
Hour 11	0.30	10.61	14.20	20.59
Hour 12	0.02	8.68	13.03	21.72
Hour 13	0.08	12.45	13.90	25.90
Hour 14	0.04	9.51	13.14	30.83
Hour 15	0.04	10.75	13.62	29.38
Hour 16	0.38	5.25	6.64	32.02
Hour 17	0.05	12.33	21.94	34.00
Hour 18	3.38	24.97	30.34	36.59
Hour 19	5.12	23.47	35.79	43.30
Hour 20	0.01	3.26	8.22	23.84
Hour 21	0.23	4.05	14.59	23.88
Hour 22	0.45	17.69	25.14	31.79
Hour 23	0.38	3.40	10.38	14.91
Hour 24	0.67	10.75	14.58	24.51

**Table C.1:** Results of Ljung-Box  $Q$ -test performed to estimation errors after the in-sample estimation of TVP regression models. Test statistics with values higher than the respective critical values reject the null hypothesis of no serial correlation. The observation window starts 50 trading days (pre-estimation period for parameter initialization) after January 1, 2010 and ends on December 31, 2011. Weekends, holidays, and bridge days are excluded.

## **Appendix D**

### **Additions Chapter 8**

Hour	Mean	DoF	$H_0 : RP = 0$		$H_0 : RP > 0$		$H_0 : RP < 0$	
			t-Stat	p-Value	t-Stat	p-Value	t-Stat	p-Value
1	-0.948	246	-3.59	0.000	-3.59	0.000	-3.59	1.000
2	-1.153	249	-3.37	0.001	-3.37	0.000	-3.37	1.000
3	-1.430	244	-4.23	0.000	-4.23	0.000	-4.23	1.000
4	-1.098	247	-2.95	0.003	-2.95	0.002	-2.95	0.998
5	-1.043	248	-2.71	0.007	-2.71	0.004	-2.71	0.996
6	-1.382	246	-4.31	0.000	-4.31	0.000	-4.31	1.000
7	-1.154	247	-3.25	0.001	-3.25	0.001	-3.25	0.999
8	-2.008	246	-4.07	0.000	-4.07	0.000	-4.07	1.000
9	0.169	247	0.43	0.666	0.43	0.667	0.43	0.333
10	0.714	244	2.13	0.034	2.13	0.983	2.13	0.017
11	0.426	245	1.16	0.249	1.16	0.875	1.16	0.125
12	-0.131	247	-0.31	0.760	-0.31	0.380	-0.31	0.620
13	0.364	245	1.10	0.271	1.10	0.865	1.10	0.135
14	0.685	245	2.14	0.033	2.14	0.984	2.14	0.016
15	0.748	244	2.32	0.021	2.32	0.989	2.32	0.011
16	1.042	246	3.29	0.001	3.29	0.999	3.29	0.001
17	0.949	247	3.17	0.002	3.17	0.999	3.17	0.001
18	0.725	248	2.05	0.042	2.05	0.979	2.05	0.021
19	0.792	245	1.96	0.051	1.96	0.974	1.96	0.026
20	1.030	247	2.76	0.006	2.76	0.997	2.76	0.003
21	-0.392	248	-1.19	0.235	-1.19	0.117	-1.19	0.883
22	-0.258	248	-1.04	0.302	-1.04	0.151	-1.04	0.849
23	-0.076	247	-0.31	0.760	-0.31	0.380	-0.31	0.620
24	-0.216	245	-0.93	0.353	-0.93	0.176	-0.93	0.824

**Table D.1:** Results of *t*-test applied to absolute risk premia on EXAA day-ahead contracts in 2009. The observation window is between January 1, 2009 and December 31, 2009. Weekends, holidays, and bridge days are excluded as well as all observation dates with risk premia 3 standard deviations above or below the mean risk premium of the respective time period.

Hour	Mean	DoF	$H_0 : RP = 0$		$H_0 : RP > 0$		$H_0 : RP < 0$	
			t-Stat	p-Value	t-Stat	p-Value	t-Stat	p-Value
1	0.220	244	0.80	0.422	0.80	0.789	0.80	0.211
2	-0.085	246	-0.28	0.781	-0.28	0.391	-0.28	0.609
3	-0.393	247	-1.17	0.243	-1.17	0.122	-1.17	0.878
4	-0.132	248	-0.39	0.699	-0.39	0.349	-0.39	0.651
5	-0.217	247	-0.66	0.511	-0.66	0.255	-0.66	0.745
6	-0.308	244	-1.12	0.264	-1.12	0.132	-1.12	0.868
7	0.043	245	0.17	0.865	0.17	0.567	0.17	0.433
8	-0.321	245	-1.05	0.296	-1.05	0.148	-1.05	0.852
9	0.720	246	2.42	0.016	2.42	0.992	2.42	0.008
10	0.972	246	3.61	0.000	3.61	1.000	3.61	0.000
11	1.059	244	4.05	0.000	4.05	1.000	4.05	0.000
12	1.044	244	3.69	0.000	3.69	1.000	3.69	0.000
13	1.030	246	4.11	0.000	4.11	1.000	4.11	0.000
14	0.912	246	3.90	0.000	3.90	1.000	3.90	0.000
15	0.727	246	3.46	0.001	3.46	1.000	3.46	0.000
16	0.798	246	4.14	0.000	4.14	1.000	4.14	0.000
17	1.206	247	6.17	0.000	6.17	1.000	6.17	0.000
18	0.760	244	2.91	0.004	2.91	0.998	2.91	0.002
19	0.229	244	0.79	0.428	0.79	0.786	0.79	0.214
20	0.816	244	3.15	0.002	3.15	0.999	3.15	0.001
21	0.058	245	0.28	0.779	0.28	0.610	0.28	0.390
22	0.244	248	1.37	0.171	1.37	0.914	1.37	0.086
23	-0.008	247	-0.04	0.966	-0.04	0.483	-0.04	0.517
24	0.009	246	0.04	0.966	0.04	0.517	0.04	0.483

**Table D.2:** Results of *t*-test applied to absolute risk premia on EXAA day-ahead contracts in 2010. The observation window is between January 1, 2010 and December 31, 2010. Weekends, holidays, and bridge days are excluded as well as all observation dates with risk premia 3 standard deviations above or below the mean risk premium of the respective time period.

Hour	Mean	DoF	$H_0 : RP = 0$		$H_0 : RP > 0$		$H_0 : RP < 0$	
			t-Stat	p-Value	t-Stat	p-Value	t-Stat	p-Value
1	0.706	243	3.16	0.002	3.16	0.999	3.16	0.001
2	0.339	241	1.49	0.137	1.49	0.931	1.49	0.069
3	0.341	240	1.44	0.150	1.44	0.925	1.44	0.075
4	0.692	245	2.48	0.014	2.48	0.993	2.48	0.007
5	0.553	243	2.22	0.028	2.22	0.986	2.22	0.014
6	0.088	243	0.41	0.685	0.41	0.657	0.41	0.343
7	0.578	245	1.99	0.048	1.99	0.976	1.99	0.024
8	0.139	247	0.52	0.603	0.52	0.698	0.52	0.302
9	0.584	246	2.36	0.019	2.36	0.990	2.36	0.010
10	0.802	247	3.41	0.001	3.41	1.000	3.41	0.000
11	0.837	247	3.81	0.000	3.81	1.000	3.81	0.000
12	0.593	246	2.63	0.009	2.63	0.995	2.63	0.005
13	0.676	245	3.01	0.003	3.01	0.999	3.01	0.001
14	0.532	246	2.45	0.015	2.45	0.992	2.45	0.008
15	0.568	247	2.44	0.016	2.44	0.992	2.44	0.008
16	0.771	247	3.38	0.001	3.38	1.000	3.38	0.000
17	1.107	244	5.37	0.000	5.37	1.000	5.37	0.000
18	0.763	245	3.34	0.001	3.34	1.000	3.34	0.000
19	0.471	244	1.82	0.069	1.82	0.965	1.82	0.035
20	0.379	245	1.52	0.129	1.52	0.936	1.52	0.064
21	0.124	245	0.49	0.627	0.49	0.687	0.49	0.313
22	0.516	245	2.14	0.033	2.14	0.983	2.14	0.017
23	0.097	245	0.41	0.680	0.41	0.660	0.41	0.340
24	0.189	245	0.91	0.362	0.91	0.819	0.91	0.181

**Table D.3:** Results of *t*-test applied to absolute risk premia on EXAA day-ahead contracts in 2011. The observation window is between January 1, 2011 and December 31, 2011. Weekends, holidays, and bridge days are excluded as well as all observation dates with risk premia 3 standard deviations above or below the mean risk premium of the respective time period.



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# Curriculum Vitae

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## Education

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| <b>2010 - 2012</b> | <b>University of St. Gallen (HSG)</b><br>Doctoral Studies in Finance                            |
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## Other Qualifications

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| <b>2011</b> | Chartered Financial Analyst (CFA)               |
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## Professional Experience

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