





# Essays in Electricity Price Modeling

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The President:

Prof. Dr. Thomas Bieger

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# Summary

The immediacy and directness of how physical properties of a good feed through to its economic price dynamics have probably never been more apparent than for electricity. In view of the current state of technology still at a loss to provide economically efficient storage solutions for electricity, the market-clearing (wholesale) price will always adjust for the flow of electricity produced in every instance of time to exactly equal the amount consumed by end customers, if system stability is to be maintained and unexpected blackouts to be avoided.

From a microeconomic point of view, the general non-storability of electricity causes the intertemporal linkages between economic agents' decisions today and in the future to break down: utility maximization, by allocating consumption and production over time, effectively reduces to a myopic one-period decision problem whereby the absence of storage possibilities combined with the empirically well-validated assumption of virtually completely inelastic demand set the stage. Generally, this can be seen as the ultimate cause for many of the unusual properties evident in the price trajectories of traded spot electricity contracts, such as excessive levels of volatility, seasonality, upward and downward price spikes, or the occurrence of negative prices – *stylized facts* that pose a significant challenge to market participants when it comes to pricing and hedging these contracts.

Whereas previous research on electricity price modeling has primarily been concerned with classic reduced-form approaches that have established as standard tools to model other (storable) commodities, interest rates, or currencies, this thesis focuses on structural pricing approaches instead. This relatively new class of models addresses said complexities in electricity price modeling by exploiting the comparatively transparent price formation mechanism in electricity markets and thus goes one step beyond standard reduced-form models: whereas the latter class mainly aims at reproducing the stochastic properties of the electricity price time series itself, structural approaches focus on the underlying fundamental factors (such as electricity demand, available generation capacity, and the prices for generating fuels), and derive prices based on an exogenous structural specification between these drivers.

In this thesis, we present and discuss fundamental frameworks for electricity pricing

and show how these can offer new solutions to recent modeling challenges for which reduced-form approaches are not well-suited and deliver suboptimal results. In the first part, we propose a new structural pricing model that not only incorporates demand, capacity, and fuel price as fundamental drivers, but additionally also makes use of demand and capacity *forecasts*. In an extensive empirical study, the value of using forward-looking information is clearly confirmed and thus highly recommended for electricity derivatives pricing.

In the second and third part of this thesis, we extend our model into a multi-market setting to investigate the effects of the increasing interconnectivity between electricity markets on price dynamics, thereby taking the continued roll-out of *market coupling* mechanisms as an example. As our findings show, changes in market design, such as the respective cross-border trading scheme prevailing, decisively impact the pricing and hedging of electricity contracts and could thus far not be reflected adequately in standard modeling approaches. Based on our structural setting, we offer new solutions to these challenges and provide a coherent modeling framework for spot, forward, and also spread-based derivative contracts when electricity markets are interconnected.

# Zusammenfassung

Die Unmittelbarkeit, mit welcher sich die physikalischen Eigenschaften eines Guts auf dessen ökonomische Preisdynamik auswirkt, tritt bei Elektrizität besonders deutlich zutage: Da der gegenwärtige Stand der Technik heute (noch) keine wirtschaftlichen Speicherlösungen für Elektrizität vorweisen kann, ist es stets der markträumende (Großhandels-) Preis, welcher letztendlich für die in jedem Zeitpunkt erforderliche Balance zwischen produziertem und abgenommenem Strom sorgen muss – nicht zuletzt zur Vermeidung von Stromausfällen oder sonstigen Beeinträchtigungen der Systemstabilität.

Aus mikro-ökonomischer Sicht hat die allgemeine Nicht-Speicherbarkeit von Strom zunächst zur Folge, dass rationale Agenten die Auswirkungen ihrer ökonomischen Entscheidungen von heute auf Folgeperioden vernachlässigen können: Denn anders als bei klassischen Problemen der Nutzenoptimierung stellt sich – mangels Speichermöglichkeiten sowie in Verbindung mit der empirisch gut belegten Annahme nahezu vollständig inelastischer Strom-Nachfrage – die Frage der Allokation von Konsum und wirtschaftlichem Angebot in jeder Periode von Neuem. Letztendlich ist hierin auch die Ursache für viele der Besonderheiten zu sehen, welche die Verlaufsmuster der Zeitreihen von Spot-Strompreisen kennzeichnen, wie z.B. äußerst hohe Volatilitäten, ausgeprägte Saisonalität, kurzfristige Preissprünge aufwärts und abwärts sowie das Auftreten negativer Preise. Diese *stylized facts* stellen Marktteilnehmer bekanntlich vor besondere Herausforderungen bei der Bewertung und Absicherung von Stromkontrakten.

Während sich die bisherige Forschung zur Strompreis-Modellierung vorwiegend mit dem Reduced-Form Ansatz beschäftigt hat, welcher sich als Standard zur Modellierung anderer (lagerbarer) Rohstoffe, Zinsen und Währungen etabliert hat, werden in der vorliegenden Dissertation stattdessen strukturelle Bewertungsansätze thematisiert. Diese relativ neue Modellklasse bezieht den auf Strommärkten vergleichsweise transparenten Preisbildungs-Mechanismus in den Modellrahmen mit ein und bietet auf diese Weise eine interessante Alternative, um die bekannten Schwierigkeiten bei der Strompreis-Modellierung zu adressieren. Im Vergleich zu den Modellen vom Reduced-Form Typ, welche primär die stochastischen Zeitreihen-Eigenschaften von Preisprozessen *direkt* abbilden, weisen strukturelle Ansätze eine tiefere ökonomische Struktur auf, und leiten Preisdynamiken *indirekt* aus dem Zusammenspiel von Fundamentalfaktoren (Strom-

nachfrage, verfügbare Erzeugerkapazität und Brennstoffpreise) und Modell-exogener Gleichgewichtsannahme ab.

Die vorliegende Dissertation stellt den Fundamental-Ansatz zur Strompreis-Modellierung vor und zeigt auf, wie sich mit dieser Modellklasse Lösungen für insbesondere solche neue Fragen und Herausforderungen finden lassen, die die klassischen Reduced-Form Ansätze vor (teilweise unüberwindbare) Probleme stellen. Im ersten Teil der Arbeit entwickeln wir ein neues Strukturmodell, welches auf den Fundamentalfaktoren Brennstoffpreis, Stromnachfrage und Erzeugerkapazität basiert, zusätzlich jedoch auch noch Marktvorhersagen für die letzteren beiden Faktoren verarbeiten kann. In der sich anschließenden empirischen Studie wird der Nutzen zukunftsgerichteter Informationen (in Form von Vorhersagen über Angebot und Nachfrage) für die Bewertung von Stromderivaten eindeutig bestätigt.

Im zweiten und dritten Teil dieser Arbeit überführen wir unser Strukturmodell in einen Multi-Markt-Ansatz, um die Auswirkungen der stetig zunehmenden Strommarkt-Integration auf Preisdynamiken zu untersuchen, wobei insbesondere die in Europa nun vorherrschenden Mechanismen zur *Marktkopplung* thematisiert werden. Hierbei wird deutlich, dass Änderungen im Marktdesign – wie etwa die Regelung und Organisation grenzüberschreitender Stromflüsse – die Bewertung und Absicherung von Stromkontrakten entscheidend beeinflussen können, was im Rahmen der klassischen Reduced-Form Modelle bisher jedoch nicht abgebildet werden kann. Unser infolgedessen erweitertes Strukturmodell stellt jedoch einen neuen Ansatz zur Lösung dieser Schwierigkeiten dar und bietet einen Rahmen, in dem Spot-, Forward-, aber auch Spread-basierte Stromkontrakte konsistent und unter Berücksichtigung zunehmender Marktintegration bewertet werden können.

# Chapter 1

## Introduction

### 1.1 Electricity Markets: On the Verge of Becoming Truly Financial Markets?

Compared to other mature commodity markets, electricity markets have come a long way to finally be recognized as financial markets in a broader sense at best. Following decades of “cost-plus” pricing and local quasi-monopolies, early deregulation efforts in the 1980s and early 1990s began to unfold with Chile and the UK among the first to lay out their plans for a comprehensive liberalization and reconstruction of their electricity markets.<sup>1</sup> Thus, and further promoted by the ongoing unbundling of vertically integrated utilities, several electricity markets across the US, Europe, and Australia had eventually laid the grounds for a level playing field amongst their market participants such as generators, suppliers, traders, and speculators.

To the extent that emerging power exchanges and a suitable institutional market design have helped to establish electricity price indices as reliable and transparent *price signals* against which financial contracts can be settled, liquid physical markets were complemented by active markets for financial futures and, to some extent, also options. Consequently, with trades being financial rather than physical, “outside” speculators may be more inclined to participate in transactions and, hence, more willing to take over idiosyncratic risks in order to diversify their broader portfolios. As is well-known, the

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<sup>1</sup>See, e.g., Pollitt (2004) for further information on electricity market reforms in Chile. However, the notion of Chile as a pioneer in electricity market liberalization is sometimes contested since the respective reforms did not immediately aim at promoting independent wholesale markets and breaking up vertically integrated monopolies (Joskow, 2008). For further details on the UK market reforms, see, e.g., Green and Newbery (1992) or Newbery and Pollitt (1997).

risk-bearing service by speculators, in turn, helps to promote market integration between electricity and other financial markets, thus finally reducing risk premia in the former markets (Bessembinder and Lemmon, 2002). In fact, for the (still hypothetical) case of a very high degree of integration between electricity markets and the broader financial markets for other asset classes, Bessembinder and Lemmon even conjecture forward price risk premia to vanish, due to the low correlation between electricity prices and aggregate market returns.<sup>2</sup>

However, the question of how liquid and integrated *financial* electricity markets will be – or whether they may even become similarly exposed to some form of *financialization* that other commodity markets have recently witnessed<sup>3</sup> – does not depend only on the above aspects of market design and post-liberalization policy framework. More importantly, it is the spiky and highly volatile nature of electricity prices that makes pricing and hedging in these markets very challenging in the first place. Price spikes, or a generally pronounced right-skewness in electricity prices, is inherited from the usually strongly convex supply/merit-order curves in electricity markets, combined with the inability to economically store electricity in most markets. As an immediate consequence, sudden upward price spikes can cause short positions of electricity forwards and futures to incur substantial losses<sup>4</sup> – and which may, ultimately, also affect financial investor participation.<sup>5</sup>

Adding to these complexities, certain types of derivative contracts, such as options on spot or forward electricity, may not be traded liquidly, so that market participants essentially face a double risk at least: holding a derivative contract on a highly volatile, spiky, and non-storable underlying on the one hand, while facing illiquidity of stale

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<sup>2</sup>See Bessembinder and Lemmon, 2002; p. 1355: “*If the outside speculators are risk-averse but hold diversified portfolios, then a CAPM-style result will be obtained, with the bias in the forward price as a predictor of the spot price dependent on the covariance between power prices and overall market returns. If (as might be expected) power prices are not significantly correlated with aggregate market returns, then frictionless models with unlimited numbers of either risk-averse or risk-neutral speculators will imply a zero risk premium for power contracts.*”

<sup>3</sup>See, e.g., Singleton (2012) or Tang and Xiong (2012).

<sup>4</sup>For instance, on a single day in 1998, US utility Illinova incurred trading losses approximately equal to its entire total earnings in that year. See Pirrong and Jermakyan (2008). As another example, Bessembinder and Lemmon (2002) mention two Californian electricity retailers, Southern California Edison and Pacific Gas and Electric, that had to default on debt servicing and other scheduled payments in January 2001 due to unexpectedly high electricity prices in the spot markets that, for regulatory reasons, could not be passed on to retail customers.

<sup>5</sup>For example, Bellmann et al. (2011) give proof of particularly high and volatile margining costs in the case of electricity derivatives, thus raising capital requirements to sometimes prohibitively high levels, especially for smaller financial players. This is confirmed empirically by, e.g., OFGEM (2009) and NordREG (2010).

positions and related risk premia on the other. For a financial investor, this is an obvious dilemma since, as confirmed by Eydeland and Geman (1999), the safest way to hedge the risks inherent to trading electricity (options) may ultimately only be to own a power plant.<sup>6</sup>

Against the above background, the current state of electricity markets today, taking Germany amidst the *Energiewende* and other European markets as examples, seems to contribute little to restoring investor confidence in electricity markets. Following a continued, strong shift towards generation from renewable energy resources, accompanied by nuclear phase-out policies in several countries, previously known “laws of the marketplace” and other empirical realities have started to break down, such as evidenced by: a significant increase in the number of negative price spikes and also first occurrences of days with negative *baseload* prices; a breakdown of the peakload/baseload price differential with peak prices more frequently falling below baseload prices whenever renewables feed-in is strong; or a change in the merit-order of supposedly “green” markets, leading to a revival of, for instance, coal-based generation, which in turn impacts overall price dynamics and hedging strategies in the respective market.

At the same time, however, a number of developments, projects, and initiatives are trying to address and mitigate the above challenges, thereby essentially contributing to creating electricity markets with more transparency and fewer frictions, and thus benefitting all market participants – regardless of whether with a financial or physical angle.

First, the recent years have seen a strong increase in the amounts of data being made available to market participants. Importantly, this not only relates to *outturn* data on electricity market fundamentals, such as historic levels of available generation or consumption, but even more so to *forecasts* about these factors. On the one hand, the shift towards “big data” in electricity markets is certainly driven by the strong proliferation of renewable generation and their inherent intermittency, which makes it increasingly important to focus on (regional) weather forecasts that not only include temperature levels, but also wind speed and sunshine. In a recent article, Carr (2013), for example, mentions that weather forecasting companies have lately seen double-digit year-on-year growth of their revenues brought in through energy companies. However,

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<sup>6</sup>See Eydeland and Geman (1999), p. 42.

while the relationship between weather (especially temperature levels) and electricity demand is reasonably well understood for most markets,<sup>7</sup> the link between weather data and available generation capacity from renewables is comparably weaker, thus confirming that more information does not always need to yield better trading decisions.<sup>8</sup> On the other hand, new regulatory requirements and transparency initiatives are also contributing to this development, and are forcing transmission system operators (TSOs) to release even more detailed and granular forecasts especially about (vertical) load and schedules of planned outages of major generation facilities. Forecasts of these “hard fundamentals” can more easily be assessed as regards their impact on electricity prices and are, hence, very welcome to traders and other market participants.

Second, market integration between European electricity wholesale markets is helping to finally realize the idea of one common electricity market across national borders. Again, although not a new development, it is the strong shift towards generation from renewable sources that has helped to accelerate the “unification” of the formerly fragmented European electricity markets. For especially in the case of “green” electricity, the location of generation sites might be far away from the actual centers of demand, thus putting electricity *transmission* under the spotlight: for instance, with rising numbers of on-shore and off-shore wind farms in the North Sea, new ways must be found in order to efficiently direct these electricity flows to high-demand areas in southern Germany, France, and the Benelux states. Hence, increasing interconnectivity – i.e., transmission capacity – between European electricity wholesale markets is a crucial prerequisite for a successful integration of renewable generation across Europe. Currently, however, interconnectors that link electricity markets are often congested and, therefore, require an efficient auction design of how to provide access to scarce transmission capacity for cross-border trading. In Europe, these capacity rights for interconnector use are currently made available either via (i) explicit ex-ante auctioning or (ii) implicit auctioning through market coupling. Coupling-based mechanisms have been particularly successful in further promoting price convergence amongst European electricity markets, thus factually creating one common market zone across countries. As is key to this development, the spreading of coupling-based mechanisms has thoroughly altered electricity price dynamics

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<sup>7</sup>See, e.g., Pardo et al. (2002) for further information.

<sup>8</sup>Carr (2013), for example, cites industry professionals who confirm that “*there are inaccuracies of 5GW and greater in these models, particularly when there are thunderstorms or [there is] snow coverage on the panels.*”

in the related markets, with both volatility and price spikes clearly reduced through more economic use of interconnection lines.

Finally, we observe an increased tendency to adjust the overall electricity market design to allow for more financial trades. To some extent, this may already be considered a consequence arising from the roll-out of market coupling, as can be observed for the case of transmission rights: a small change in the regulation of how to treat non-exercised rights has made possible both the retention of the primarily physical setting for cross-border trade and, at the same time, the opening of this market to financial investors that henceforth no longer need to worry about taking on the risk of physical delivery. From a risk management perspective, a quasi-financial setting for transmission rights also helps to increase the hedge-effectiveness of futures contracts in those markets that are still suffering from reduced liquidity and market-making activities.

As will be seen throughout this thesis, these developments are by far not limited to European markets or to just those markets that are characterized by particularly high levels of generation from renewables; instead, the above outlined structural developments can likewise also be anticipated for other electricity markets, such as in the US or Australia, since they are – directly or indirectly – related to the interplay of those fundamental factors that determine the electricity price formation mechanism in *every* market. However, if changing electricity market paradigms put more focus on underlying fundamental factors, model risk for market participants will inevitably increase: for previous approaches to pricing and hedging may become unreliable to the very extent that extrapolating future prices from historic time series may suffer from previously unknown regime changes which are triggered, e.g., by innovations such as market coupling. Obviously, while the developments outlined above are expected to cause financial investor participation in electricity markets to further increase, so will the problems related to pricing models that cannot cope with the implications of the changing marketplace.

In the remainder of this thesis, we show that an alternative pricing approach that gives more weight to underlying price drivers can actually master these challenges well: the class of structural electricity pricing models is presented in three essays. In each, we take the above developments in electricity markets as our main motivation to develop, analyze, and implement this new class of models – thereby, more generally, demonstrating to the reader our fundamental *credo* that especially for electricity markets, market structure and

pricing strategy are intricately intertwined and should always be considered a dynamic ensemble driven by a permanent state of flux.

In the next section, we give a brief introductory overview of the class of structural models for electricity pricing. Thereafter, the organization of this thesis is outlined and a brief summary of each of the three essays to follow is given.

## 1.2 The Structural Approach to Electricity Price Modeling

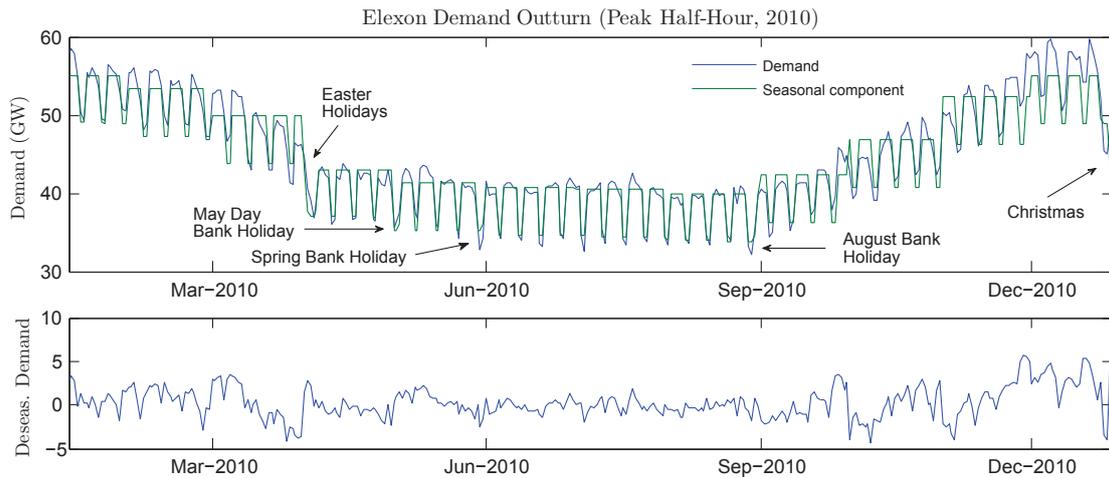
As opposed to standard reduced-form settings that aim at reproducing the key *stylized facts* of electricity prices (as observed from past price time series) by *directly* modeling price dynamics, structural approaches to electricity price modeling go beyond the historic price trajectories and instead follow a more detailed approach.<sup>9</sup> By modeling the impact of fundamental factors on electricity prices rather than modeling the price process itself, structural settings *endogenize* key stylized facts of electricity prices, such as upward/downward spikes, but also more intricate market developments such as a longer-lasting change in the merit-order for a certain generation park. At the same time, by imposing an *exogenously* given relationship between fundamental factors to yield electricity prices, the complexities of a (potentially dynamic) equilibrium setting can be avoided, which helps to retain tractability especially for practitioners.

Therefore, in a first step, the key building block for structural pricing models usually involves a supply/demand argument given that the interaction of these two fundamental factors is well-understood and easier to observe than for other markets.<sup>10</sup> In fact, whereas available generation capacity (i.e., supply) may occasionally be excluded from the modeling framework if assumed constant for simplicity, all of the structural models below explicitly include electricity demand as primary fundamental factor and cost driver next to fuel prices.

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<sup>9</sup>In order to avoid ambiguities, this class of models is sometimes also referred to in literature as *hybrid* or *fundamental* class of electricity pricing models. Throughout the remainder of this thesis, these classifications will be used interchangeably. However, in order to avoid confusion, and as will become obvious in this section, we do not subsume under the class of structural pricing models those bottom-up frameworks that are based on agent-based optimization and related endogenous determination of one- or multi-period equilibria. See, e.g., Bessembinder and Lemmon (2002) or Buehler and Mueller-Mehrbach (2007, 2009) for prominent examples of models of this kind.

<sup>10</sup>See, e.g., Carmona and Coulon (2012) on this argument.



**Figure 1.1: Daily Peak Electricity Demand for Great Britain in 2010**

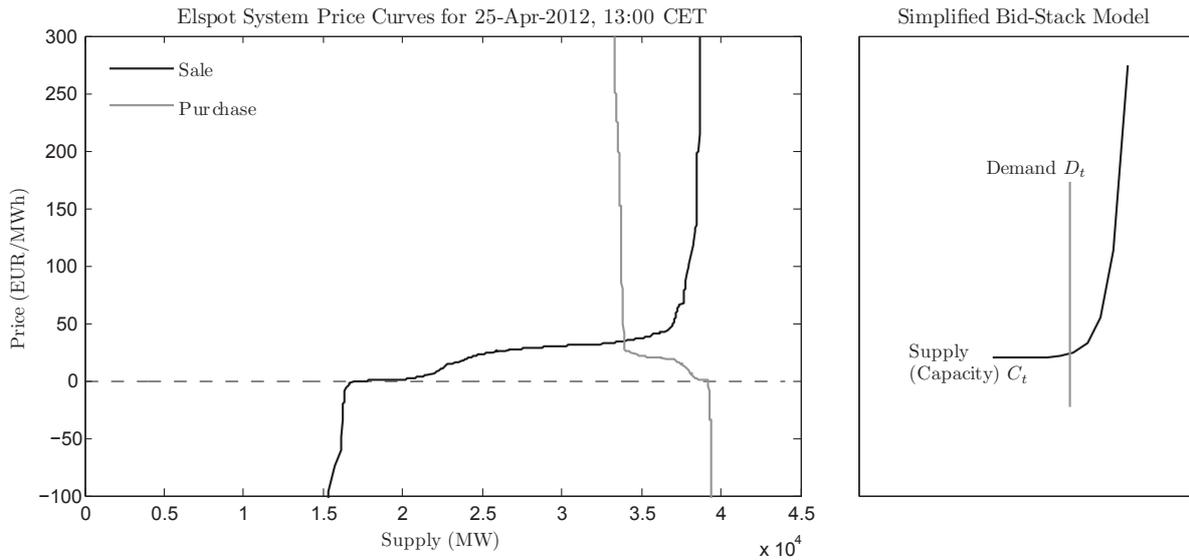
*The top graph illustrates the development of electricity demand in Great Britain during the daily peak half hour in 2010 along with a fitted deterministic component to capture seasonality effects.*

*The bottom graph shows the corresponding deseasonalized demand process.*

From a technical point of view, the time-series properties of electricity demand (and also available generation capacity) are characterized by distinct seasonalities, primarily induced by the close relationship between electricity consumption and underlying weather dynamics, with changes in temperature often accounting for more than 90% of overall daily demand variation.<sup>11</sup> In this context, the mean-reversion property of the temperature dynamics immediately feeds through into the demand process, which therefore is usually modeled as an Ornstein-Uhlenbeck (OU) process, as further illustrated in Figure 1.1.

Deferring the discussion regarding the inclusion of additional state variables, especially fuel prices, to Chapter 2, the next step for modelers now is to find an adequate specification for the functional relationship between the selected fundamental factors and electricity (spot) prices: in contrast to other equilibrium-based settings (e.g., Bessembinder and Lemmon, 2002), the class of models examined in this thesis derives electricity prices based on an exogenously-specified equilibrium assumption, thus mimicking the interplay of the involved fundamental factors within the price-setting mechanism, such as in a merit-order framework. More precisely, this functional relationship can be disentangled into an assumption on the structure of the (inverse) supply curve or bid-stack that is combined with the requirement that demand  $D_t$  always be equal to supplied capacity  $C_t$ , as is essential for electricity markets clearing. In addition, it is thanks to the empirically well-validated assumption of completely inelastic electricity

<sup>11</sup>See, e.g., Pardo et al. (2002) for further information.



**Figure 1.2: System Price Curves and Simplified Representation**

The LHS graph illustrates the price formation mechanism in electricity markets, taking bid/offer curves from the Nordic market (Nord Pool) as an example. In the RHS graph, the supply-demand equilibrium is represented in a simplified model with inelastic demand and exponential supply curve.

demand that said equilibrium can easily be solved to yield spot electricity prices.<sup>12</sup> While the assumption of inelastic demand is a common “ingredient” to all of the below presented models, the key differentiating factor is the functional form of the merit-order curve: most importantly, a good approximation of its characteristic slope and, especially, steepness for high levels of demand – depending on the respective markets under study – is required for an adequate reflection of the characteristic spikiness in electricity prices.

Importantly, for this class of models, spikes in electricity prices are no longer modeled exogenously using jump processes. Instead, they are generated endogenously by invoking those economic conditions that generally lead to the occurrence of spikes, such as low levels of available generation facing increased electricity demand. Figure 1.2 illustrates this aspect as well as the general idea of reflecting the price formation mechanism (i.e., balancing electricity demand with supply) with a simplified functional representation.<sup>13</sup>

As can be inferred from this stylized setting, even an exponential function may not

<sup>12</sup>Note that this assumption may not be valid for markets in which, e.g., “make-or-buy” decisions of generators can cause electricity demand to be less inelastic for certain price ranges, depending on the generators’ costs, and typically resulting in a “kinked” demand curve as shown in Figure 1.2. However, as shown by Coulon et al. (2014), even in these cases, the structural modeling set-up can be maintained by interpreting the sum of supply and demand “clusters” as a “slide” stack, which will then also allow to keep the assumption of inelastic electricity demand. For more information on the impact of market design (power pool vs. power exchange) on demand elasticity, also see Weron (2006).

<sup>13</sup>Regarding the integration of dynamics for one (or several) generating fuels into such a setting, we refer to Chapter 2.

always adequately match the steepness of the merit-order curve, which, on the one hand, might lead to suboptimal pricing results, especially when primarily focusing on *spot* rather than *derivatives* pricing. On the other hand, however, an exponential merit-order curve allows for analytic derivatives pricing formulae: a key benefit that not only pays off for practitioners upon calibration and for real-time pricing purposes; but that also provides deeper insights into the complex dependency structures between electricity prices and underlying fundamental factors, thus altogether accounting for much of the attention and popularity that the structural pricing approaches have recently seen. Note, however, that maintaining an exponential merit-order framework – as will be the case throughout the remainder of this thesis – still provides benefits also for modeling *spot* prices, e.g., in a multi-market framework, as will be seen in Chapter 3.<sup>14</sup>

In the remainder of this section, we briefly summarize some of the key elements to the field of structural electricity price modeling, thereby also giving an impression of how this class of models has evolved over time, and how the inclusion of additional state variables has added to their overall degree of technical sophistication.

Barlow (2002) sets up a simple framework in which electricity prices are determined by equating inelastic demand with constant, deterministic supply. Specifically, demand  $D_t$  is modeled as an Ornstein-Uhlenbeck process (using conventional notation):

$$dD_t = \kappa(\mu - D_t)dt + \sigma dW_t. \quad (1.1)$$

The supply curve (quantity as function of price) is defined as  $C(x) = \bar{C} - b_0x^\alpha$  with maximum system capacity  $\bar{C}$  and  $\alpha < 0$  determining the slope of the curve. Based on the inverse supply curve, the spot electricity price  $P_t$  is then defined as:

$$P_t = \begin{cases} \left( \frac{\bar{C} - D_t}{b_0} \right)^{1/\alpha}, & D_t < \bar{C} - \epsilon_0 b_0 \\ \bar{P}, & D_t \geq \bar{C} - \epsilon_0 b_0 \end{cases} \quad (1.2)$$

for some small  $\epsilon_0$  and  $\bar{P} = \epsilon_0^{1/\alpha}$  as the maximum system price.<sup>15</sup>

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<sup>14</sup>Also see the structural model by Coulon et al. (2013) presented further below who propose to combine their exponential framework with a regime-switching setting as an additional way to further improve the spot pricing performance of their model.

<sup>15</sup>Re-defining the first case in Equation (1.2) to  $P_t = f_\alpha(\tilde{D}_t) = (1 + \alpha\tilde{D}_t)^{1/\alpha}$  and applying the inverse Box-Cox transformation, this framework can be further generalized to allow for non-negative  $\alpha$ . In view of  $f_\alpha$ , the model is also often referred to as a non-linear OU-process.

Kanamura and Ohashi (2007), however, argue that the curvature of the inverse supply curve - as implied by the set-up used in Barlow (2002) - is not steep enough to adequately reflect price spikes and, therefore, propose an alternative specification based on a “hockey stick” shape. In a stylized way, they argue for a standard merit-order curve to be better represented by two linear functions - i.e., the flat baseload part and the steep part where plants with very high marginal costs are dispatched - and a quadratic curve to link them, resulting in the following piecewise defined inverse supply function  $P_t = f(C_t)$ :

$$P_t = \begin{cases} a_1 + \beta_1 C_t + \epsilon_t, & C_t < z - c \\ a_2 + \beta_2 C_t + \gamma_2 C_t^2 + \epsilon_t, & z - c \leq C_t < z + c \\ a_3 + \beta_3 C_t + \epsilon_t, & C_t \geq z + c \end{cases} \quad (1.3)$$

where  $C_t$  represents supplied generation capacity and with parameters specified such that the slope of the quadratic part at the connecting points  $z - c$  and  $z + c$  is equal to  $\beta_1$  and  $\beta_3$ , respectively.

Based on spot price data for the PJM<sup>16</sup> market, Kanamura and Ohashi (2007) show that the “hockey stick” model indeed captures the price spikiness more realistically than other benchmarks such as the Barlow (2002) model or the jump diffusion-model proposed by Clewlow and Strickland (2000); however, for the “hockey stick” model, as is also the case for the non-linear OU-model by Barlow, the availability (and tractability) of analytic pricing formulae for futures/forwards or other derivatives contracts is clearly affected by the functional form of the inverse supply curve. A simpler specification of the inverse supply curve based on an exponential function, as Kanamura and Ohashi (2007) argue, may come at the cost of reducing the steepness of the curve required for generating price spikes at times of high demand,<sup>17</sup> but at the same time shares the benefit of allowing for lognormally distributed spot prices and, hence, for closed-form solutions of electricity derivatives in some cases.

Skantze et al. (2000) propose a model in which the (log-) price is a function of the state variable demand,  $D_t$ , and a (residual) variable,  $b_t$ , that is meant to relate to supply conditions; given that (at the time the model was developed) data on actually dispatchable capacity was either very limited or non-observable,  $b_t$  is extracted indirectly from spot

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<sup>16</sup>Pennsylvania-New Jersey-Maryland

<sup>17</sup>Strictly speaking, the occurrence of price spikes should be attributed to a low reserve margin rather than a high level of demand only; see, e.g., Burger et al. (2004), Boogert and Dupont (2008), or Anderson and Davison (2008).

price data, implying that any movement in prices that is not attributable to  $D_t$  will be captured by  $b_t$ :

$$P_t = e^{aD_t + b_t}. \quad (1.4)$$

A similar framework is put forward by Villaplana (2004) and Cartea and Villaplana (2008). These authors explicitly incorporate capacity data into their analyses and model it as a distinct process, thus replacing the above residual state variable  $b_t$  with available generation capacity  $C_t$  and proposing the following alternative functional forms for the electricity spot price  $P_t$ :

$$P_t = C_t^\gamma \cdot \beta \cdot e^{\alpha D_t}, \text{ or} \quad (1.5)$$

$$P_t = \beta \cdot e^{\alpha D_t + \gamma C_t}, \quad (1.6)$$

where  $C_t$  can include jump components<sup>18</sup> as in Villaplana (2004), a potential stochastic long-run mean, or a deterministic seasonality component as in Cartea and Villaplana (2008). However, these authors solely focus on applying their model to extract risk premia from quoted futures/forwards prices for the PJM, England & Wales, and Nord Pool electricity markets, but do not provide any results that would allow to gauge the goodness-of-fit for their chosen specifications in Equations (1.5) and (1.6). Also, and from a technical point of view, it should be borne in mind that for these models, the capacity process  $C_t$  implicitly assumes that – by parallel shifting of the merit-order curve – capacity going on- or offline always relates to plants that will be operating, i.e. whose marginal costs are below the market-clearing price.

An alternative approach might instead be to consider the ratio of demand and capacity,  $D_t/C_t$ , implying that changes in available capacity affect all parts of the curve evenly; this idea has been applied to the "SMaPS" model – the model for spot market price simulation that has been proposed by Burger et al. (2004) and has been implemented at German utility EnBW. Burger et al. (2004) argue that spot electricity prices  $P_t$  should not be determined based on a merit-order framework only, but instead should also be influenced by non-technical determinants such as "*psychological aspects of the behavior*

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<sup>18</sup>Based on the work of Duffie et al. (2000), such setting of an affine jump-diffusion process still allows for analytic pricing formulae. Villaplana (2004) assumes jumps are negative and are based on an exponential distribution.

of speculators and other influence.”<sup>19</sup> The fundamental equation of their model can be written as:

$$P_t = \exp(f(t, D_t/c(t)) + X_t + Y_t), \quad (1.7)$$

where  $f(t, D_t/c(t))$  is a deterministic, non-parametric price-load (or inverse supply) curve,  $c(t)$  is a deterministic function specifying the expected availability of power plants, and  $X_t$  and  $Y_t$  are two processes that reflect the above mentioned additional factors. Specifically,  $X_t$  describes residual market fluctuations in the short-term,<sup>20</sup> whereas  $Y_t$  is modeled as a random walk that is separately estimated from traded long-term futures contracts and meant to reflect any longer-term variation in prices.<sup>21</sup> In order to derive forward price dynamics for this model, the authors, however, assume a non-zero market price of risk for  $Y_t$  only, leaving aside the non-hedgeable processes  $X_t$  and  $D_t$ .

The following class of fundamental/hybrid models for electricity spot prices enriches the supply-demand framework by accounting for the price dynamics of the underlying generating fuel(s) as additional state variable(s). In a series of articles, Pirrong and Jermakyan (1999, 2005, and 2008) propose the following multiplicative specification for the spot price of electricity  $P_t$ :

$$P_t = g_t^\gamma \cdot \phi(D_t), \quad (1.8)$$

where  $g_t$  is the spot price of natural gas, representing the marginal fuel in the market,<sup>22</sup> and  $\gamma \geq 0$  is the elasticity of the power price with respect to the fuel price; for  $\gamma = 1$ ,  $\phi(D_t)$  can also be interpreted as the “market heat rate” function that indicates the ratio between electricity and the respective underlying (marginal) fuel necessary to generate it. In order to further determine Equation (1.8), the authors propose three different ways: (i) modeling marginal costs of generation as a function of demand and fuel prices,<sup>23</sup> (ii) using econometric techniques and specifying some functional form for  $\phi(D_t)$ , or (iii)

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<sup>19</sup>Burger et al. (2004), p. 113.

<sup>20</sup>In a discretized setting, the authors model  $X_t$  using a SARIMA model.

<sup>21</sup>As Burger et al. (2004) argue, this long-term variation cannot be estimated from historic spot prices because, e.g., the EEX had only existed for two years at the time their study was performed.

<sup>22</sup>More generally, this could, of course, also refer to the spot price of any other marginal fuel, such as coal, for example.

<sup>23</sup>Yet this comes at the cost of assuming a perfectly competitive electricity market and neglecting potential effects of strategic bidding; see, e.g., Hortacsu and Puller (2008).

estimating a non-parametric heat-rate function based on generators' historic bid data that are used to construct "bid stack" (merit-order) curves.<sup>24</sup> Interestingly, with respect to the aforementioned approach (ii) to determine  $\phi(D_t)$ , and in order to increase the steepness of their model-implied merit-order curve, Pirrong and Jermakyan (1999) propose the following exponential function for the relationship between spot prices and state variables:

$$P_t = g_t^\gamma \cdot e^{\alpha D_t^2 + c(t)}, \quad (1.9)$$

where  $c(t)$  is a (non-specified) function meant to shift the curve over time. In contrast to the exponential settings put forward in Equations (1.5) and (1.6) (and irrespective of the inclusion of marginal fuel prices or not), Pirrong and Jermakyan (1999) argue that the convexity of  $\phi(D_t)$  should hence be increased by defining the first part of the exponent to be a convex function of demand itself.

As an alternative, Coulon et al. (2013) present a structural spot pricing model that is also cast in an exponential framework, yet addresses the (ultimately, still market-specific) problem of insufficient steepness of the implied merit-order curve by defining a fundamentals-based regime-switching setting, thus yielding more realistic spot trajectories and price spikes.<sup>25</sup> Defining two different regimes, a "normal" and a "spike regime," spot prices in either case are given by:

$$P_t = g_t \cdot \exp(\alpha_{m_k} + \beta_{m_k} D_t + \gamma_{m_k} C_t) \text{ for } t_k \leq t < t_{k+1}, k \in \mathbb{N}, \quad (1.10)$$

where  $g_t$ ,  $D_t$ , and  $C_t$  are defined as above and  $m_k \in \{1, 2\}$  determines which regime is prevailing: for  $m_k = 2$ , i.e., the "spike regime," the set of parameters  $(\alpha_2, \beta_2, \gamma_2)$  imply a steeper merit-order curve than for the "normal" regime with  $m_k = 1$ . The probabilities of reaching these regimes, in turn, are dependent on electricity demand:

$$m_k = \begin{cases} 1 & \text{with probability } 1 - p_s \Phi\left(\frac{D_{t_k} - \mu_s}{\sigma_s}\right) \\ 2 & \text{with probability } p_s \Phi\left(\frac{D_{t_k} - \mu_s}{\sigma_s}\right) \end{cases} \quad (1.11)$$

where  $p_s$  is the maximum probability for a spike to occur (i.e., for  $D_t \rightarrow \infty$ ),  $\mu_s$  and  $\sigma_s$

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<sup>24</sup>This approach is implemented in Pirrong and Jermakyan (2008) but generally faces the drawback that historic bid data – if publicly available at all – are often released with a lag of 3-6 months only.

<sup>25</sup>It should be noted that when specifically focusing on *spot* pricing, the models presented in this thesis can generally all be overlaid with a corresponding regime-switching setting. In order to avoid unnecessary complexity, however, we refrain from doing so in the following chapters.

are the mean and standard deviation of the stationary distribution of electricity demand, and  $\Phi$  is the standard normal cumulative distribution function.<sup>26</sup>

Finally, the models developed by Aïd et al. (2009), Aïd et al. (2011a), Coulon and Howison (2009), and Carmona et al. (2013) extend the above considerations into a multi-fuel framework. Generally, these models are very richly parameterized and will only briefly be outlined here.

Including  $n$  generating fuels in their setting, Aïd et al. (2009) and Aïd et al. (2011a) model electricity (spot and futures) prices as a weighted basket of fuel (spot and futures) contracts  $F_t$ , each individually scaled up by a factor  $h_i$  to reflect the respective heat rates. The weights, in turn, need to reflect the probability that fuel  $i$  ( $1 \leq i \leq n$ ) will be the marginal fuel, and hence are to be determined by comparing the outturn stochastic demand process with the cumulated capacity intervals  $I_t^i$  at any point in time:<sup>27</sup>

$$P_t = g(C_t^{max} - D_t) \sum_{i=1}^n h_i F_t^i \cdot \mathbb{I}_{\{D_t \in I_t^i\}}, \quad (1.12)$$

where  $\mathbb{I}$  denotes the indicator function and  $g(\cdot)$  is a “scarcity” function that reflects decreasing efficiencies within the portfolio of available power plants.

Coulon and Howison (2009) propose a different approach by dis-aggregating the empirically observed bid-stack (merit-order) curve into a histogram of bids by adding up the amount of capacity bid into the market within a certain price bin.<sup>28</sup> Coulon and Howison (2009) then fit density functions to this histogram and transform the mix of distributions<sup>29</sup> into a single mixture distribution, weighted by the share of the respective fuel in the overall generation mix for the market under study. Deriving its quantile-function  $B_t$ , the electricity spot price  $P_t = B_t(D_t/C_t)$  can generally be determined as the  $D_t/C_t$ -quantile of the mixture distribution.

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<sup>26</sup>Note that since spikes in electricity prices can only be observed on an hourly or half-hourly basis (depending on the respective market), the sequence of random draws of  $m_k$  is only defined for those times  $t_k \in \mathcal{T} = \{t_1, t_2, \dots\}$ , i.e., the set containing the start of each (half-) hourly delivery period.

<sup>27</sup> $I_t^i$  includes the aggregate capacities of all power plants (ordered by efficiency) up to, and including, those plants with their generation technology based on fuel  $i$ .

<sup>28</sup>In such a histogram, must-run bids, e.g., by nuclear generators, will add up to a separate bar in a zero-cost bin, whereas aggregate capacities bid by coal-, gas-, or oil-based generators will be more widely distributed across price bins, reflecting the different efficiencies of plants.

<sup>29</sup>Abstracting from “neighboring” fuel types within the merit-order, whose marginal costs might be overlapping, there will generally be a jump in marginal costs when the marginal fuel changes from, e.g., coal to natural gas. Hence, the bars in the bid histogram can roughly be grouped into bars that represent bids pertaining to the same fuel type. For each of these groups, a separate density function will be fitted.

While the framework above is very flexible and intuitive, it is a clear disadvantage that for the multi-fuel case, there are no explicit expressions even for spot, nor for forward contracts. Instead, Carmona et al. (2013) assume that for each fuel type, the different heat rates (i.e., levels of efficiency) of the corresponding plants in the respective market are distributed such that they can be approximated by exponential “sub-bid stacks.” For instance, in a market with coal- and gas-based generators, we define the following “sub-bid stacks”  $b_i$ :

$$b_c(x) = f_t e^{k_c + m_c x}, \text{ for } 0 \leq x \leq \bar{C}^c \text{ (coal-based generation),} \quad (1.13)$$

$$b_g(x) = g_t e^{k_g + m_g x}, \text{ for } 0 \leq x \leq \bar{C}^g \text{ (gas-based generation),} \quad (1.14)$$

where  $f_t$  is the spot price for coal,  $g_t$  is the price for natural gas,  $\bar{C}^c$  ( $\bar{C}^g$ ) is the total capacity of all coal-based (gas-based) generators in the market, and  $k_c$  and  $m_c$  are constants. The aggregated “market bid stack,” in turn, is a piecewise exponential function and is given by:

$$B(x) = (b_c^{-1} + b_g^{-1})^{-1}(x), \text{ for } 0 \leq x \leq \bar{C} = \bar{C}^c + \bar{C}^g \quad (1.15)$$

The spot price  $P_t$ , however, depends on the ordering of the respective start and end points of the underlying “sub-bid stacks”  $b_i$ . Assuming, for instance, that in the merit-order of the respective market, coal-based generation is cheaper than gas-based generation (yet including some area of overlapping bids), we yield the following piecewise defined spot price:

$$P_t(D_t, f_t, g_t) = \begin{cases} b_c(D_t) & \text{for } 0 \leq D_t \leq D_1 \\ f_t^\alpha g_t^{1-\alpha} e^{\gamma + \delta D_t} & \text{for } D_1 \leq D_t \leq D_2 \\ b_g(D_t - \bar{C}^c) & \text{for } D_2 \leq D_t \leq \bar{C} \end{cases} \quad (1.16)$$

where  $\alpha$ ,  $\beta$ ,  $\gamma$ , and  $\delta$  are constants that are defined by  $m_i$  and  $k_i$  for  $i = \{c, g\}$ ; the breakpoint  $D_1$  is defined as  $D_1 = (1/m_c)(\log(g_t/f_t) + k_g - k_c)$ , with  $D_2$  defined analogously. The resulting framework thus is especially useful for markets in which the marginal fuel type changes very frequently, with the key benefit of still allowing for closed-form formulae for forwards, options, and even dark or spark spread options. However, it should be noted that including more fuels will clearly lead to too much complexity given the increasing number of permutations within the bid stack, reflecting the multiple different orderings

of the fuel types. Nevertheless, the analytic expressions are still reasonably handable for the two-fuel case, which, at the same time, is the most relevant case in practice when implementing the model for a specific market.

### 1.3 Contribution and Organization

The main contribution of this thesis is to offer a comprehensive and rigorous treatment of structural approaches to electricity price modeling, thereby especially focusing on how the versatility of this class of models can be exploited to provide new solutions to current pricing challenges in electricity markets. Since reduced-form approaches for electricity pricing have traditionally been analyzed and used more widely by both practitioners and scholars, extant research in this field is abundant; by contrast, this work extends the smaller, yet growing strand of literature on electricity price modeling based on structural approaches.<sup>30</sup> However, in view of a number of recent major contributions,<sup>31</sup> it seems justified to say that although the theoretical groundwork has been laid, “the jury is still out” both on *(i)* how reliably structural models perform in practice and *(ii)* where they can actually offer advantages over the well-tested and widely used reduced-form approaches. It is in this context that the following chapters provide new insights and findings, thereby specifically investigating how the additional flexibility inherited from using a structural approach can be used to address those pricing challenges where classic modeling approaches start to see limitations.

This thesis has been organized in three self-contained chapters that each contain a more detailed overview of the respective contributions to extant literature, an individual introduction to the topics covered, and an overview of previous research and findings on the subject. In the following, we provide a short summary of each chapter and briefly outline our main approaches and results.

In **Chapter 2**, we analyze how forecasts of electricity demand and available generation capacity can be taken advantage of for derivatives pricing purposes.<sup>32</sup> Thanks

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<sup>30</sup>For a general and comprehensive overview on electricity price modeling, we refer to Chapter 2.

<sup>31</sup>See, e.g., Cartea and Villaplana (2008), Aïd et al. (2009), Carmona and Coulon (2012), Aïd et al. (2013), Carmona et al. (2013), and Coulon et al. (2013).

<sup>32</sup>This chapter is based on the corresponding working paper entitled “*Electricity Derivatives Pricing with Forward-Looking Information*” co-authored with Roland Füss and Marcel Prokopczuk. Cf. Füss et al. (2013a).

to both legal developments and voluntary initiatives by transmission system operators (TSOs), forward-looking information is publicly available and mainly relates to forecasts of expected (domestic) electricity demand and available generation capacity. Reliable outturn and forecast data released by TSOs is clearly appreciated by market participants since the transparency allows for more informed trading decisions. Nevertheless, most of the widely-used reduced-form approaches to electricity price modeling are clearly unsuited to incorporate forecasts of fundamental factors into their model dynamics. By contrast, the structural modeling approach presented in this chapter lends itself well to integrating forward-looking information, while retaining a focus on ease of implementation and tractability to allow for analytic derivatives pricing formulae. In an extensive futures pricing study, the pricing performance of our model is shown to further improve based on the inclusion of electricity demand and capacity forecasts, thus confirming the importance of forward-looking information for electricity derivatives pricing.

In **Chapter 3**, we focus on the increasing interconnectivity between electricity wholesale markets, as can be observed for European but also major US markets.<sup>33</sup> Given that existing interconnection lines between adjacent markets tend to be congested, an efficient allocation scheme is required in order to provide access to scarce cross-border transmission capacities.

In both the US and Europe, existing schemes have primarily induced economically inefficient interconnector use given that flows have to be nominated prior to spot market clearing. By contrast, the market coupling mechanisms recently rolled out in parts of Europe avoid these inefficiencies by implicitly allocating cross-border transmission capacity upon spot market clearance. In this chapter, we show that these institutional aspects of market design clearly manifest in the empirical dynamics of both electricity spot and derivatives prices, and hence, do have important implications for pricing and hedging in these markets. Since traditional reduced-form models fail to reproduce such effects of market microstructure, we extend our fundamental framework developed in Chapter 2 into a multi-market setting for electricity pricing in order to analyze how the different allocation schemes affect the key *stylized facts* of electricity prices. Under market coupling, for instance, a more economically efficient allocation of interconnector

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<sup>33</sup>This chapter is based on the corresponding working paper entitled “*Electricity Spot and Derivatives Pricing When Markets Are Interconnected*” co-authored with Roland Füss and Marcel Prokopczuk. Cf. Füss et al. (2013b).

capacities clearly mitigates spot price spikes in the participating markets and furthermore allows for an important volatility reduction effect. The latter effect, in turn, has strong implications for derivatives pricing: using a variety of comparative-static analyses and sensitivities, we show how changes in market design can significantly impact the term structure of electricity futures prices, e.g., by reversing the entire curve from contango into backwardation (and vice versa).<sup>34</sup>

In **Chapter 4**, we re-visit and examine the close relationship between market design and price dynamics with special focus being laid on transmission rights valuation.<sup>35</sup> In Europe, where transmission rights are mainly related to cross-border transactions between adjacent markets, said dependency between aspects of market (micro-) structure and electricity pricing has even further strengthened since the roll-out of market coupling mechanisms across the Central Western European (CWE) electricity markets: as a result of CWE spot prices henceforth coinciding more frequently, modeling the dynamics of price spreads between adjacent markets has become more intricate and complex, which cannot generally be achieved with classic reduced-form approaches commonly used for transmission rights valuation. In this chapter, we demonstrate how these challenges can instead be addressed with a structural modeling approach, based on the multi-market setting presented in the previous chapter. In this context, we analyze in detail how transmission rights can be valued as spread options on the spot prices derived from this framework – so that the aforementioned intricacies of spread dynamics under market coupling are adequately captured and reproduced. As is important especially from a practitioner’s point of view, we yield an analytic option pricing formula that is well-suited to price transmission rights, and illustrate how related pricing implications compare against the standard Margrabe (1978) benchmark.

**Chapter 5** contains concluding remarks.

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<sup>34</sup>The impact of reduced volatility on other derivatives, such as spread options, is treated separately in Chapter 4.

<sup>35</sup>This chapter is based on the corresponding working paper entitled “*Transmission Rights Valuation for Coupled Electricity Markets: An Option-based Approach*”. Cf. Mahringer (2013).

# Chapter 2

## Electricity Derivatives Pricing with Forward-Looking Information<sup>\*</sup>

### 2.1 Introduction

Following the liberalization of electricity markets in many countries, utility companies and other market participants have been facing an increasing need for new pricing models in order to accurately and efficiently evaluate spot and derivative electricity contracts. In addition, the end of cost-based pricing and the transition towards a *deregulated* market environment also gave rise to new financial risks, threatening to impose substantial losses especially for sellers of electricity forward contracts. As such, the necessity to now optimize against the market for both standard electricity products as well as tailored contingent claims additionally required effective and integrated risk management strategies to be developed.

These developments have to be seen in the context of the unique behavior of electricity (spot) prices, which is primarily induced by the non-storability of this commodity: apart from hydropower with limited storage capabilities, an exact matching of flow supply

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<sup>\*</sup>This chapter is based on the corresponding working paper entitled “*Electricity Derivatives Pricing with Forward-Looking Information*” co-authored with Roland Füss and Marcel Prokopczuk. Cf. Füss et al. (2013a). The paper has been presented at the Conference on Energy Finance 2012, Trondheim, the Conference of the Swiss Society for Financial Market Research 2013, Zurich, the Eastern Finance Association Meeting 2013, St. Pete Beach, the Financial Management Association Conference 2013, Luxembourg, and at the ICMA Centre 2013 Research Seminar, Reading. Status: revise and resubmit to the *Journal of Economic Dynamics and Control*.

and flow demand for electricity is required at every point in time. The resulting price dynamics with their well-known *stylized facts* such as spikiness, mean-reversion, and seasonality, have extensively been analyzed in the literature,<sup>1</sup> yet still pose a challenge to both practitioners and researchers in terms of adequately modeling and forecasting their trajectories.

However, the non-storability of electricity has further implications for the price formation mechanism. First, unlike in a classic storage economy, it is the instantaneous nature of electricity that causes the intertemporal linkages between economic agents' decisions today and tomorrow to break down. In fact, this forms the basis for electricity markets usually being characterized as very transparent with respect to their underlying economic factors, including electricity demand, available levels of generation capacity, as well as the costs for generating fuels and emissions allowances. Against this background, structural approaches taking this information explicitly into account appear especially appealing to electricity price modeling (see, e.g., Pirrong, 2012). Second, and as the above implies, the classic assumption that the evolution of all relevant pricing information, i.e., the information filtration, is fully determined by the price process of the commodity itself, does not hold for non-storable assets such as electricity. In other words, today's electricity prices do not necessarily reflect forward-looking information that is publicly available to all market participants.<sup>2</sup> At the same time, legal requirements and voluntary initiatives to increase data transparency have had power transmission system operators (TSOs) publish an increasing amount of data regarding the condition of their network, including, e.g., forecasts about expected electricity demand or updated schedules of planned short-term outages.<sup>3</sup> Pricing electricity spot and derivatives contracts based on models that make use of historical information only, may hence result in substantial errors since the model

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<sup>1</sup>See, e.g., Johnson and Barz (1999), Burger et al. (2004) or Fanone et al. (2013).

<sup>2</sup>Benth and Meyer-Brandis (2009) provide several examples in support of this argument, such as the case of planned maintenance for a major generating unit, which is likely to be public information available to all market participants. Assuming a stylized setting, this outage will necessarily affect electricity spot prices expected to prevail during the time the unit is offline. Likewise, the outage will also affect today's prices of derivative contracts such as forward and futures contracts if their delivery periods overlap with the period of scheduled maintenance. However, in the absence of any means to economically store electricity bought at (cheaper) spot prices today and to sell it at higher prices during the time of the outage, there is no opportunity for arbitrage in such situation. This consequently implies that today's electricity *spot* prices will remain virtually unaffected by the announcement of the outage.

<sup>3</sup>In Europe, Regulations (EC) No. 1228/2003, its follow-up No. 714/2009, and annexed "Congestion Management Guidelines" (CMG) may serve as the most prominent example, requiring, e.g., that "*the TSO shall publish the relevant information on forecast demand and on generation (...)*" (CMG, article 5.7). In the US, similar standards are in place, e.g., as issued by the Federal Energy Regulatory Commission (FERC).

leaves aside important, forward-looking information, although it is publicly available and likely to play a key role for individual trading decisions.

In this chapter, we hence focus on the prominent role of forward-looking information in electricity markets and empirically investigate its impact on pricing performance. As such, we contribute to the literature in the following ways:

First, we propose a new fundamental model for electricity pricing including fuel, demand, and capacity dynamics that successfully captures the stylized facts of this commodity and provides analytic derivatives pricing formulae. Second, most studies that propose new fundamental electricity pricing models do not calibrate the models to market data. If so, however, they either focus on time series fitting or provide pricing results for selected individual forward contracts for illustration purpose only. In contrast, we test our model in an extensive empirical study, using a comprehensive data set of forward contracts traded in the British electricity market. This also allows us to address several important implementation challenges that arise during the calibration procedure. Third, and to the best of our knowledge, we are the first to empirically investigate the pricing of derivative contracts in electricity markets by explicitly making use of forward-looking information. By means of an enlargement-of-filtration approach, we show how to properly integrate forecasts of electricity demand and available capacity into our setting, and thus account for the apparent asymmetry between the historical filtration and the (enlarged) market filtration in electricity markets.

In general, existing literature on electricity *spot* price modeling can be grouped into two categories: on the one hand, often allowing for analytic derivatives pricing formulae, considerable attention has been devoted to reduced-form models that either directly specify dynamics for the electricity spot price process itself or, alternatively model the term structure of forward contracts, where spot dynamics are derived from a forward contract with immediate delivery (see, e.g., Clewlow and Strickland, 2000; Koekebakker and Ollmar, 2001; or Benth and Koekebakker, 2008). Starting with traditional commodity modeling approaches via mean-reverting one- or two-factor models (Lucia and Schwartz, 2002), a more adequate reflection of the stylized facts of electricity spot price dynamics demands for more elaborate settings including affine jump diffusion processes and/or regime-switching approaches (see, e.g., Bierbrauer et al., 2007; Weron, 2009; or Janczura

and Weron, 2010, for a comprehensive overview). However, this may still not be sufficient to reliably differentiate between spike- and non-spike regimes as observed in reality, or to adequately capture the absolute spikiness of electricity prices. As a solution, additional enhancements have been proposed, such as considering non-constant deterministic or stochastic jump intensities (see, e.g., Seifert and Uhrig-Homburg, 2007) and their impact on possibly different speeds of mean-reversion of the underlying Ornstein-Uhlenbeck (OU) process, which, in turn, negatively affects analytic tractability. The same applies when trying to mitigate other common drawbacks such as models precluding successive upward jumps or leaving jump intensities unaffected by previous jumps. Extensions like Barone-Adesi and Gigli (2002) try to address these problems but must resort to non-Markovian models, which, however, restricts the applicability for contingent claim valuation. Finally, and as a point of structural criticism, reduced-form models obviously fail to analyze the dependence structure between prices and the electricity markets' underlying drivers, which not only leaves unexplained important features such as the occurrence of price spikes, but also affects their applicability for fields such as cross-commodity option valuation (unless, e.g., a co-integration setting is employed such as in Emery and Liu, 2002; de Jong and Schneider, 2009; or Paschke and Prokopczuk, 2009). In this context, and given the above mentioned increase in publicly available fundamental data released by TSOs, it must be seen as a drawback of classic reduced-form models that they obviously fail to take direct benefit from this increasing transparency.<sup>4</sup>

On the other hand, the class of structural/fundamental electricity price models subsumes a wide spectrum of more diverse modeling approaches; starting with equilibrium-based models (Bessembinder and Lemmon, 2002; Buehler and Mueller-Mehrbach, 2007; Aïd et al., 2011b) or even more richly parameterized full production cost models (Eydeland and Wolyniec, 2002) on the one end, but also including, on the other end, econometric approaches such as regression-based settings (Karakatsani and Bunn, 2008) or time-series models whose efficiency is enhanced by including exogenous fundamental variables (Weron, 2006; or Misiorek et al., 2006).

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<sup>4</sup>We note that it is still possible to integrate information about the dynamics of fundamental state variables (such as demand or, e.g., also temperature) into reduced-form models by means of correlated processes. For an example, see Benth and Meyer-Brandis (2009). However, even though such models may bridge the gap between classic reduced-form and fundamental approaches, it is still questionable whether a single correlation parameter may be sufficient to reflect the rich dependence structures between electricity prices and a fundamental state variable – all the more if the dynamics of several underlying variables are to be taken into account at the same time.

Often referred to as hybrid approach, the class of models focused on in this study may be seen in the middle of this spectrum.<sup>5</sup> In its most general form, fundamental settings of this kind comprise of a selection of separately modeled underlying factors, such as electricity demand, available generation capacity, and fuels. Along with a specification of the functional relationship between these factors and electricity spot prices, this setting can hence be interpreted as merit-order framework.<sup>6</sup> The main challenge in this context is to be seen in an adequate reflection of the characteristic slope and curvature of the merit-order curve that is usually characterized by significant convexity.<sup>7</sup> As a matter of simplification, many studies (see, e.g., Skantze et al., 2000; Cartea and Villaplana, 2008; or Lyle and Elliott, 2009) propose to approximate the merit-order curve with an exponential function. While there may be other functional specifications yielding a better fit, such as a piecewise defined “hockey stick” function (Kanamura and Ohashi, 2007) or power laws (Aïd et al., 2013), the exponential setting offers the key advantage of yielding log-normal electricity spot prices, allowing for analytic derivatives pricing formulae.

In order to provide timely pricing information to market participants by retaining tractability, our model also adopts an exponential setting for representing the merit-order curve. As regards the inclusion of generating fuels, we follow Pirrong and Jermakyan (2008) by modeling a stylized one-fuel market, leaving aside more flexible multi-fuel approaches (see Aïd et al., 2009, 2013; Coulon and Howison, 2009; Carmona et al., 2013). While our one-fuel setting avoids a model-endogenous determination both of the merit-order and the marginal fuel in place, it remains to be discussed how this reduction in flexibility affects pricing results, and for which markets such a simplification may be feasible at all.

Regarding the question of how to account for forward-looking information in this

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<sup>5</sup>In order to avoid ambiguities, when we refer to *fundamental* electricity price models throughout the rest of this chapter (and within the entire thesis), we shall actually mean the hybrid class of models *within* this category.

<sup>6</sup>Alternatively, this functional relationship can also be seen as inverse supply curve or bid-stack, if we abstract from generators submitting bids exceeding marginal costs. Also, our setting implicitly assumes electricity demand being completely inelastic, which is a basic assumption for models of this kind. See Carmona and Coulon (2012) for further reference as well as for a general and comprehensive review of the fundamental modeling approach.

<sup>7</sup>This is a non-trivial issue given that the curvature is determined by both the individual composition of generating units for each marketplace as well as their marginal cost structure which, in turn, depends stochastically on other factors such as underlying fuel prices, weather conditions, (un-)planned outages, and daily patterns of consumption. Additional factors may relate to market participants exercising market power by submitting strategic bids, but also regulatory regimes awarding, e.g., preferential feed-in tariffs to renewable energy producers.

context, many of the above presented models could in fact be modified to accommodate short-, mid- or long-term forecasts about future levels of electricity demand or available capacity. However, previous literature mainly focuses on the benefits of using day-ahead demand/capacity forecasts in order to improve day-ahead electricity pricing performance (see, e.g., Karakatsani and Bunn, 2008; Bordignon et al., 2013). A different approach regarding the integration of forecasts into a pricing model is proposed by Cartea et al. (2009). In their study, a regime-switching setting is invoked where the ratio of expected demand to expected available capacity is used to determine an exogenous switching component that governs the changes between “spiky” and “normal” spot price regimes. In this way, the modeling of spikes present in spot prices can be improved, although the model only resorts to very few forecast points per week and available forecasts are not explicitly part of the price formation mechanism. Burger et al. (2004) also present a model that requires as input normalized electricity demand, i.e. demand scaled by available capacity. For the latter, the usage of forecasts of future capacity levels is suggested, but not focused on in more detail.

Finally, the application of the enlargement-of-filtration approach to electricity markets was initially proposed by Benth and Meyer-Brandis (2009). Focusing on risk premia rather than on forward pricing, Benth et al. (2013) use this concept in order to analyze the impact of forward-looking information on the behavior of risk premia in the German electricity market. The authors develop a statistical test for the existence of an *information premium*<sup>8</sup> and show that a significant part of the oftentimes supposedly irregular behavior of risk premia can be attributed to it.<sup>9</sup>

The remainder of this chapter is structured as follows: in the next section, we develop our underlying electricity pricing model. Section 2.3 introduces the concept of the enlargement-of-filtration approach in the context of fundamental electricity price modeling. Section 2.4 starts with the empirical part by describing the data, the estimation methodology, as well as the general structure of the pricing study. Section 2.5 discusses the pricing results. Section 2.6 concludes.

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<sup>8</sup>The information premium is defined as the difference between forward prices, depending on whether or not forward-looking information is entering the price formation mechanism.

<sup>9</sup>On a more general note, the idea to resort to forward-looking information, of course, extends to numerous other fields of academic research. Another “natural” candidate is, by way of example, the pricing of weather derivatives. For studies that resort to temperature forecasts in order to price temperature futures, see, e.g., Jewson and Caballero (2003), Dorfleitner and Wimmer (2010), and Ritter et al. (2011).

## 2.2 A Fundamental Electricity Pricing Model

### 2.2.1 Electricity Demand

Electricity demand is modeled on a daily basis with its functional specification chosen to reflect typical characteristics of electricity demand such as mean-reverting behavior, distinct seasonalities as well as intra-week patterns. On a filtered probability space  $(\Omega, \mathcal{F}^D, \mathbb{F}^D = (\mathcal{F}^D)_{t \in [0, T^*]}, \mathbb{P})$  with natural filtration  $\mathbb{F}^q = (\mathcal{F}^q)_{t \in [0, T^*]}$  (for  $\mathcal{F}_t^D = \mathcal{F}_0 \vee \mathcal{F}_t^q$ ), demand  $D_t$  is assumed to be governed by the following dynamics:

$$D_t = q_t + s^D(t), \quad (2.1)$$

$$dq_t = -\kappa^D q_t dt + \sigma^D e^{\varphi(t)} dB_t^D, \quad (2.2)$$

$$s^D(t) = a^D + b^D t + \sum_{i=2}^{12} c_i^D M_i(t) + c_{WE}^D WE(t) + \sum_{j=1}^4 c_{PH_j}^D PH_j(t), \quad (2.3)$$

$$\varphi(t) = \theta \sin(2\pi(kt + \zeta)), \quad (2.4)$$

where  $q_t$  is an OU-process with mean-reversion parameter  $\kappa^D$  and a standard Brownian motion  $B_t^D$ . Since volatility of electricity demand has often been found to exhibit seasonal levels of variation (see, e.g., Cartea and Villaplana, 2008),<sup>10</sup> we apply a time-varying volatility function as proposed by Geman and Nguyen (2005) or Back et al. (2013), with  $\theta \geq 0$ , a scaling parameter  $k = \frac{1}{365}$ , and  $\zeta \in [-0.5; 0.5]$  to ensure uniqueness of parameters.<sup>11</sup> In order to additionally reflect absolute-level demand-side seasonality, the deterministic component  $s^D(t)$  contains monthly dummy variables  $M_i(t)$  as well as indicators for weekends  $WE(t)$  and public holidays.<sup>12</sup> A linear trend is also included in  $s^D(t)$  in order to capture the effect of structural developments in the respective market that may lead to an increase or decrease of electricity demand in the long term.

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<sup>10</sup>As our estimation results will show, volatility of electricity demand in the British market is higher during winter months than during summer months. However, this effect may be less pronounced or even reversed for other markets where, e.g., the need for air conditioning during summer months drives electricity demand to higher and more volatile levels than during winter months.

<sup>11</sup>This volatility specification allows for continuous differentiability, which is a technical necessity in the context of the enlargement-of-filtration approach. See the technical appendix to this chapter for further information.

<sup>12</sup>Since the extent of a demand reduction induced by a public holiday strongly depends on the respective season prevailing, three different groups of public holidays shall be distinguished: those occurring in winter ( $PH_2$ ), the Easter holidays ( $PH_3$ ), and others ( $PH_4$ ). Additionally, the days with reduced electricity demand between Christmas and New Year are treated as *quasi*-public holidays ( $PH_1$ ). This may appear overly detailed; however, almost all coefficients turn out to be highly significant. See Buehler and Mueller-Mehrbach (2009) for an even more detailed approach.

## 2.2.2 Available Capacity

Available capacity  $C_t$  is modeled in a similar manner as electricity demand. Hence, on a filtered probability space  $(\Omega, \mathcal{F}^C, \mathbb{F}^C = (\mathcal{F}^C)_{t \in [0, T^*]}, \mathbb{P})$  with the natural filtration defined as  $\mathbb{F}^m = (\mathcal{F}^m)_{t \in [0, T^*]}$  (for  $\mathcal{F}_t^C = \mathcal{F}_0 \vee \mathcal{F}_t^m$ ), we specify the following dynamics:

$$C_t = m_t + s^C(t), \quad (2.5)$$

$$dm_t = -\kappa^C m_t dt + \sigma^C dB_t^C, \quad (2.6)$$

$$s^C(t) = a^C + b^C t + \sum_{i=2}^{12} c_i^C M_i(t) + c_{WE}^C WE(t) + \sum_{j=1}^4 c_{PH_j}^C PH_j(t) + c_R^C R(t), \quad (2.7)$$

where  $m_t$  is again an OU-process with mean-reversion parameter  $\kappa^C$  and constant volatility  $\sigma^C$ .<sup>13</sup>  $B_t^C$  is a standard Brownian motion and  $s^C(t)$  is defined equivalently to  $s^D(t)$ . Finally, another dummy variable  $R(t)$  is included in order to reflect the fact that, unlike for the electricity demand data used in this study, capacity data relating to generation units from Scotland is available only after April 2005.<sup>14</sup>

## 2.2.3 Marginal Fuel

In addition to the processes for electricity demand and available capacity, we introduce the dynamics for our third state variable, i.e., the marginal fuel used for generation. For the sake of simplicity, we assume that the marginal fuel for the respective electricity market under study does not change. While this certainly is a restrictive assumption, it may still seem justified for markets that are strongly relying on one generating fuel only so that during baseload/peakload hours, spot markets are primarily cleared by plants that use the same fuel for generation. Reflecting the dominant role of natural gas as marginal generating fuel in the British market – and, more generally, in several other major electricity markets – we choose it as the single fuel to be included into our overall pricing model.

Although for modeling natural gas, a variety of multi-factor approaches with varying degree of sophistication have been proposed by recent literature (see, e.g., Cartea and

<sup>13</sup>In contrast to demand  $D_t$ , available capacity  $C_t$  is generally not found to exhibit seasonality in volatility levels.

<sup>14</sup>The introduction of the British Electricity Trading and Transmission Agreements (BETTA) as per April 2005 is generally referred to as the starting point of a UK-wide electricity market. Prior to that, and although linked via interconnectors, the electricity markets of England/Wales and Scotland were operating independently.

Williams, 2008, for an overview), we seek to limit both complexity and (the already high) parametrization of the model and, therefore, apply the mean-reverting one-factor model initially proposed by Schwartz (1997). On a given filtered probability space  $(\Omega, \mathcal{F}^g, \mathbb{F}^g = (\mathcal{F}^g)_{t \in [0, T^*]}, \mathbb{P})$ , the log gas price,  $\ln g_t$ , is assumed to be governed by the following dynamics:

$$\ln g_t = X_t + s^g(t), \quad (2.8)$$

$$dX_t = -\kappa^g X_t dt + \sigma^g dB_t^g, \quad (2.9)$$

$$s^g(t) = a^g + b^g t + \sum_{i=2}^{12} c_i^g M_i(t), \quad (2.10)$$

where  $X_t$  is the logarithm of the de-seasonalized price dynamics and  $s^g(t)$  reflects the strong seasonality component that is inherent in natural gas prices. Note that the overall structure of our power price model as well as the availability of closed-form solutions will be retained when introducing refinements such as a multi-factor log-normal model for natural gas.<sup>15</sup>

## 2.2.4 Pricing Model

In order to link the three state variables – marginal fuel  $g_t$ , electricity demand  $D_t$ , and capacity  $C_t$  – with electricity spot prices  $P_t$ , we employ an exponential setting, thus reflecting the convex relationship between prices and load/capacity as induced by the merit-order curve. At the same time, we assume power prices to be multiplicative in the marginal fuel. Both assumptions can be considered common practice and yield the following structural relationship between spot prices and state variables:<sup>16</sup>

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<sup>15</sup>Applying a one-factor model for natural gas prices may be seen as simplistic since the structure of this model implies that all natural gas forward/futures contracts are perfectly correlated across maturities. However, note that we primarily focus on pricing short-term electricity forward contracts for which only the short end of the curve may be relevant. In contrast, when pricing longer-term electricity contracts, we suggest employing a two-factor natural gas price model instead.

<sup>16</sup>As mentioned in the overview of literature, note that – as is characteristic for this class of models – we thus derive electricity *spot* prices based on an exogenously given relationship between fuels, supply, and inelastic demand (see, e.g., Skantze et al., 2000, Cartea and Villaplana, 2008, Pirrong and Jermakyan, 2008, Lyle and Elliott, 2009 or, more generally, Carmona and Coulon, 2012). Taking the dynamics for the spot as given, forward pricing formulae are then derived based on a no-arbitrage argument. Although a (possibly dynamic) equilibrium setting that explicitly models the optimization behavior of all participants in both spot and forward markets might provide additional insights, such as on the determinants of endogenously derived forward risk premia, we refrain from doing so here. Given that we primarily focus on the *pricing* impact of using forward-looking information, dynamic equilibrium settings might be unsuited for that purpose, e.g. due to a number of additionally required assumptions and/or unobserved variables, leading to calibration challenges, and implying reduced flexibility in general.

$$P_t = \alpha g_t^\delta e^{\beta D_t + \gamma C_t}, \quad (2.11)$$

or, in log-form:

$$\ln P_t = \ln \alpha + \delta \ln g_t + \beta D_t + \gamma C_t, \quad (2.12)$$

where  $\delta$  can be interpreted as the elasticity of the electricity spot price with respect to changes in the natural gas price. Setting  $\delta = 1$  would thus allow to interpret  $e^{\beta D_t + \gamma C_t}$  as heat rate function.<sup>17</sup> However, given that we primarily investigate baseload power prices in the empirical part of this chapter, we acknowledge that the elasticity of baseload power prices with respect to natural gas may be varying and, hence, do not impose the restriction  $\delta = 1$ .

Also, and as will be seen later, there is a subtle form of dependence between the parameters  $\alpha$  and  $\delta$ . In order to give an intuition for the role of  $\alpha$ , and at the same time providing an abstract link to structural *multi*-fuel power price models, Equation (2.11) can be re-written as follows (also cf. Equation (1.16) in Chapter 1):

$$P_t = \underbrace{f_t^{(1-\delta)}}_{\alpha} g_t^\delta e^{\beta D_t + \gamma C_t}. \quad (2.13)$$

In Equation (2.13),  $\alpha$  can hence be interpreted as reflecting the dynamics of another generating fuel  $f_t$  (such as coal) which, however, will be held constant for simplicity.

Following classic theory, futures prices equal the expectation of the spot price at maturity under a suitably chosen risk-neutral measure  $\mathbb{Q}$  (Cox and Ross, 1976, and Harrison and Kreps, 1979). However, the non-storability of electricity creates non-hedgeable risks, leading to an incomplete market setting. Therefore, the risk-neutral measure  $\mathbb{Q}$  cannot be determined uniquely, but will instead be inferred from market prices of traded forward contracts, as will be shown in Section 2.4. In order to govern the change of measure, and following Girsanov's theorem, we introduce separate market prices of risk  $\lambda^D$ ,  $\lambda^C$ , and  $\lambda^g$  for the different sources of uncertainty in our model. These market prices of demand, capacity, and fuel price risk are assumed constant. Given that  $P_t$  is log-normal in the state variables, the log futures price,  $\ln F_t(T)$ , at time  $t$  with delivery date  $T$  is

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<sup>17</sup>The *heat rate* indicates how many units of natural gas (or, more generally, of any other generating fuel) are required to produce one unit of electricity. In our case, the “market” heat rate would refer to the price-setting plant that generates the marginal unit of electricity.

given as follows:

$$\ln F_t(T) = \mathbb{E}^{\mathbb{Q}}[\ln P_T \mid \mathcal{F}_t] + \frac{1}{2}\mathbb{V}^{\mathbb{Q}}[\ln P_T \mid \mathcal{F}_t] \quad (2.14)$$

$$\begin{aligned} &= \ln \alpha + \delta \mathbb{E}^{\mathbb{Q}}[\ln g_T \mid \mathcal{F}_t] + \beta \mathbb{E}^{\mathbb{Q}}[D_T \mid \mathcal{F}_t] + \gamma \mathbb{E}^{\mathbb{Q}}[C_T \mid \mathcal{F}_t] \\ &\quad + \frac{1}{2}\delta^2 \mathbb{V}^{\mathbb{Q}}[\ln g_T \mid \mathcal{F}_t] + \frac{1}{2}\beta^2 \mathbb{V}^{\mathbb{Q}}[D_T \mid \mathcal{F}_t] + \frac{1}{2}\gamma^2 \mathbb{V}^{\mathbb{Q}}[C_T \mid \mathcal{F}_t], \end{aligned} \quad (2.15)$$

where  $\mathbb{E}^{\mathbb{Q}}[\cdot \mid \mathcal{F}_t]$  and  $\mathbb{V}^{\mathbb{Q}}[\cdot \mid \mathcal{F}_t]$  indicate expectation and variance under  $\mathbb{Q}$  and conditional on  $\mathcal{F}_t$  which is defined as  $\mathcal{F}_t := \mathcal{F}_t^D \vee \mathcal{F}_t^C \vee \mathcal{F}_t^g$ .<sup>18</sup> As further outlined in Section 2.3, when pricing forward contracts by making use of forecasts of electricity demand and capacity, forward prices will be computed as risk-neutral expectations of the spot price during the delivery period, conditional on  $\mathcal{G}_t$  rather than  $\mathcal{F}_t$ . Consequently, Equation (2.14) will need to be replaced by  $\ln F_t(T) = \mathbb{E}^{\mathbb{Q}}[\ln P_T \mid \mathcal{G}_t] + \frac{1}{2}\mathbb{V}^{\mathbb{Q}}[\ln P_T \mid \mathcal{G}_t]$ , where  $\mathcal{G}_t := \mathcal{G}_t^D \vee \mathcal{G}_t^C \vee \mathcal{F}_t^g$  and  $(\mathcal{G}_t)_{t \in [0, T^*]}$  (or, more precisely,  $(\mathcal{G}_t^D)_{t \in [0, T^*]}$  and  $(\mathcal{G}_t^C)_{t \in [0, T^*]}$ ) is the enlarged market filtration containing forecasts of expected demand and capacity levels, respectively.

Also note that Equation (2.14) refers to a contract with delivery of electricity at some future date  $T$ , whereas standard electricity forward contracts specify the delivery of electricity throughout a delivery period  $[\underline{T}, \bar{T}]$  (with  $\underline{T} < \bar{T}$ ), e.g., one week or one month. Following Lucia and Schwartz (2002), we compute the price of a forward contract with delivery period  $[\underline{T}, \bar{T}]$ , containing  $n = \bar{T} - \underline{T}$  delivery days, as the arithmetic average of a portfolio of  $n$  single-day-delivery forward contracts with their maturities spanning the entire delivery period, i.e.:

$$F_t(\underline{T}, \bar{T}) = \frac{1}{\bar{T} - \underline{T}} \sum_{i=1}^n F_t(\tau_i). \quad (2.16)$$

Finally, calculating electricity forward prices based on Equation (2.16) also requires us to have available the corresponding fuel forward prices with single-day maturities, i.e., one also needs to compute  $\mathbb{E}^{\mathbb{Q}}[\ln g_{\tau_i} \mid \mathcal{F}_t^g]$  (as well as the conditional variance) for every day  $\tau_i$  within the delivery period  $[\underline{T}, \bar{T}]$ . For that purpose, we take the log-spot price implied by the natural gas forward curve at time  $t$  as a starting point to compute for every day  $\tau_i$  within the delivery period the price of a hypothetical natural gas forward contract that matures on the same day. Hence, we calibrate to the gas curve for every pricing

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<sup>18</sup>Note that the second part of Equation (2.14) reflects our implicit assumption of all state variables being independent of each other.

day before subsequently fitting the power forward curve. In this context, as a simplified approach, only one average value for  $\mathbb{E}^{\mathbb{Q}}[\ln g_{\tau_i} \mid \mathcal{F}_t^g]$  during the entire delivery period could be used (e.g., based on the current value of the month-ahead natural gas forward, when pricing month-ahead electricity forwards). However, this may pose problems for non-standard delivery periods and would require identically defined delivery periods for gas and power.<sup>19</sup>

## 2.3 The Enlargement-of-Filtration Approach

Non-storability of a given asset  $Z$  implies that forward-looking information can neither be inferred from, nor is reflected in the historical evolution of its price trajectory  $Z_t$  (Benth and Meyer-Brandis, 2009). Mathematically speaking, given a finite horizon  $T^*$  and letting  $(\Omega, \mathcal{F}, \mathbb{F} = (\mathcal{F})_{t \in [0, T^*]}, \mathbb{P})$  be a filtered probability space, the natural filtration  $\mathbb{F}^Z = (\mathcal{F}^Z)_{t \in [0, T^*]}$  (with  $\mathcal{F}_t = \mathcal{F}_0 \vee \mathcal{F}_t^Z$ ) may not reflect all forward-looking information available to market participants.

Assume that agents have access to some (non-perfect) forecast of the price of  $Z$  at some future point in time  $t^* \in [0, T^*]$ . Then, there exists a sigma-algebra  $\mathcal{G}_\tau$  with  $\mathcal{F}_\tau \subset \mathcal{G}_\tau$  for all  $\tau < t^*$ , where  $\mathcal{G}_\tau$  reflects all available information including the forecast, whereas  $\mathcal{F}_\tau$  does not. For  $\tau \geq t^*$ , i.e. for times beyond the forecast horizon, we however have  $\mathcal{F}_\tau = \mathcal{G}_\tau$ , since no further forward-looking information is assumed to be available.

Next, note that whereas electricity clearly serves as most prominent example for non-storable underlyings, the above outlined incompleteness of natural filtrations with respect to forward-looking information can generally be extended to any kind of non-storable underlying. Therefore, and strictly speaking, we do not enlarge the filtration of the electricity spot price in order to incorporate forecasts, like Benth and Meyer-Brandis (2009) do in their reduced-form setting. Instead, we focus on electricity demand  $D_t$  in Equation (2.1) and available capacity  $C_t$  in Equation (2.5) which are, of course, non-storables as well, and hence do not reflect forward-looking information either. Therefore, and more precisely, it is the filtrations relating to the demand and capacity processes,

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<sup>19</sup>Note, however, that in the UK, electricity forward contracts (still) trade according to the EFA (electricity forward agreement) calendar, grouping every calendar year into four quarters with three delivery months with lengths of 4/4/5 calendar weeks, respectively. Consequently, delivery months of electricity forward contracts may not exactly overlap with corresponding delivery months of traded natural gas futures contracts.

respectively, that need to be enlarged in order to integrate forecasts provided by the system operator.

In the following, all formulae derived in this section relate to *available capacity* and forecasts thereof. Additional theoretical background as well as how to derive respective formulae for the more general case of deterministic, but non-constant volatility (as for *electricity demand*  $D_t$ ) is provided in the technical appendix 2.7.1. For notational convenience, we work with de-seasonalized forecasts that relate to  $m_t$  instead of  $C_t$ ;  $\mathcal{F}_t^C$  and  $\mathcal{G}_t^C$  are defined as further above.<sup>20</sup>

In this setting, the (de-seasonalized) forecast of generation capacity available at time  $t$  with forecast horizon  $T$  is interpreted as  $\mathcal{G}_t$ -conditional expectation and can be expressed as:

$$\mathbb{E}^{\mathbb{P}}[m_T \mid \mathcal{G}_t^C] = m_t e^{-\kappa^C(T-t)} + \sigma^C \mathbb{E}^{\mathbb{P}} \left[ \int_t^T e^{-\kappa^C(T-u)} dB_u^C \mid \mathcal{G}_t^C \right]. \quad (2.17)$$

This raises the question of how to treat expectations like  $\mathbb{E}^{\mathbb{P}}[\int_t^T e^{-\kappa^C(T-u)} dB_u^C \mid \mathcal{G}_t^C]$  that are conditional on  $\mathcal{G}_t^C$  (i.e., the sigma-algebra including forecasts) when  $B_t^C$ , however, is an  $\mathcal{F}_t^C$ -adapted Brownian motion. Consequently,  $B_t^C$  may no longer be a standard Brownian motion with respect to  $(\mathcal{G}_t^C)_{t \in [0, T^*]}$ . Even more importantly, and following the “average approach” in Equation (2.16), the pricing of, e.g., a forward contract with delivery period of one month will require us to ideally have capacity forecasts for every day within the delivery period. Yet, as is outlined in Section 2.4, detailed forecasts on a daily basis, as released by National Grid for the British market, only cover a window of the next 14 days. For longer-term prognoses, such as expected available capacity in 21 days, only forecasts of weekly granularity are published. Consequently, we may at best cover a certain first part of the delivery period with daily forecasts, whereas for the rest of the period, only a few weekly forecast points will be available, thus leaving several delivery days “uncovered” by forecasts. Hence, another key question is how to consistently determine  $\mathbb{E}^{\mathbb{P}}[m_T \mid \mathcal{G}_t^C]$  when forecasts for capacity on delivery day  $T$  are not available, but only for times  $T_1$  and

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<sup>20</sup>One could argue that there exist, of course, numerous other forecasts about expected available capacity that market participants might also have access to. E.g., capacity forecasts released by the system operator that relate to intermittent energy sources, such as wind or solar power, might be adjusted based on a utility’s proprietary model involving different meteorological assumptions, e.g., more windy conditions or fewer sunshine hours. Likewise, the same is true for demand forecasts if market participants expect, e.g., higher temperatures than implied by the forecast of the system operator. Therefore, if we speak of  $\mathcal{G}$  as the sigma-algebra “including forecasts”, we assume away the existence of other forecasts and only mean to refer to those forecasts released by the TSO.

$T_2$  with  $T_1 \leq T \leq T_2$ . This leads to the following proposition:

**Proposition 2.3.1.** *Suppose that market participants are provided with forecasts of available capacity at future points in time  $T_1$  and  $T_2$ , i.e.,  $\mathbb{E}^{\mathbb{P}}[m_{T_1} \mid \mathcal{G}_t^C]$  and  $\mathbb{E}^{\mathbb{P}}[m_{T_2} \mid \mathcal{G}_t^C]$ . Then, for  $t \leq T_1 \leq T \leq T_2$ , capacity expected to be available at time  $T$  is given as:*

$$\begin{aligned} \mathbb{E}^{\mathbb{P}}[m_T \mid \mathcal{G}_t^C] &= \mathbb{E}^{\mathbb{P}}[m_{T_1} \mid \mathcal{G}_t^C] e^{-\kappa^C(T-T_1)} \\ &+ \mathbb{E}^{\mathbb{P}} \left[ \int_{T_1}^{T_2} e^{\kappa^C u} dB_u^C \mid \mathcal{G}_t^C \right] \sigma^C \frac{e^{\kappa^C T} (1 - e^{-2\kappa^C(T-T_1)})}{e^{2\kappa^C T_2} - e^{2\kappa^C T_1}}. \end{aligned} \quad (2.18)$$

The first part of the second term on the RHS of Equation (2.18) is given as follows:

$$\mathbb{E}^{\mathbb{P}} \left[ \int_{T_1}^{T_2} e^{\kappa^C u} dB_u^C \mid \mathcal{G}_t^C \right] = \frac{1}{\sigma^C} \left( \mathbb{E}^{\mathbb{P}}[m_{T_2} \mid \mathcal{G}_t^C] e^{\kappa^C T_2} - \mathbb{E}^{\mathbb{P}}[m_{T_1} \mid \mathcal{G}_t^C] e^{\kappa^C T_1} \right). \quad (2.19)$$

*Proof.* This directly follows from Propositions 3.5 and 3.6 in Benth and Meyer-Brandis (2009). Detailed derivations for the more general case of non-constant deterministic volatility are provided in the technical appendix 2.7.1.  $\square$

Note that we do not impose any specific structure on the nature of the enlarged filtration  $(\mathcal{G}_t^C)_{t \in [0, T^*]}$  apart from the facts that (i) the forecasts released by the TSO are interpreted as  $\mathcal{G}_t$ -conditional expectations and (ii) the  $\mathcal{F}_t$ -adapted process  $B_t^C$  (likewise  $B_t^D$ ) is a semi-martingale under the enlarged filtration. The latter is a common and well-studied approach in the enlargement-of-filtration theory, although more recent studies (Biagini and Oksendal, 2005; Di Nunno et al., 2006) have shown that such assumption could in fact be relaxed. As is shown in the appendix to this chapter in more detail, the general idea in this case is that  $B_t^C$  under the enlarged filtration  $(\mathcal{G}_t^C)_{t \in [0, T^*]}$  decomposes into a standard Brownian motion  $\hat{B}_t^C$  and a drift term  $A(t) = \int^t \vartheta(s) ds$  which is usually referred to as the information drift. Hence, the additional information is essentially incorporated in the drift term  $\vartheta(t)$ , so that the dynamics for  $m_t$  in Equation (2.6) can be re-written as follows:

$$dm_t = -\kappa^C \left( m_t - \frac{\sigma^C}{\kappa^C} \vartheta(t) \right) dt + \sigma^C d\hat{B}_t^C. \quad (2.20)$$

Based on Equation (2.20) – or, equivalently, on Proposition 2.3.1 – we can now compute  $\mathcal{G}_t$ -conditional expectations which relate to those points in time where no TSO forecasts are

available, but which are still consistent with the modified stochastic dynamics as imposed by the *available* forecast points. Although a related concept, the change of the drift for the above capacity process has not been obtained through a change of the probability measure, i.e.,  $\hat{B}_t^C$  is a  $\mathcal{G}_t$ -adapted Brownian motion under the statistical measure  $\mathbb{P}$ . Therefore, when it comes to derivatives pricing under a risk-neutral measure  $\mathbb{Q}$  in Section 2.5, we consequently look for a  $\mathcal{G}_t$ -adapted standard  $\mathbb{Q}$ -Brownian motion  $\check{B}_t^C = \hat{B}_t^C - \Lambda_{\mathcal{G}}^C(t)$ , where  $\Lambda_{\mathcal{G}}^C(t)$  is a finite variation process representing the market price of risk that will be inferred from prices of electricity derivative contracts.

Finally, we briefly discuss why we propose to use this specific approach of integrating demand and capacity forecasts here. In fact, one may think of alternative ways of how the incorporation of forward-looking information could be dealt with.

Assuming the forecast data to be reasonably reliable, one approach would be to interpret the forecasts as being released under *perfect foresight* and, hence, treating  $D_t$  and  $C_t$  as deterministic processes. In such case, demand and capacity forecasts, ultimately represented by expected values in Equation (2.14), would be replaced by constants, so that the corresponding variance terms vanish. Although appealing by its simplicity, this approach raises several issues: first, when pricing, e.g., a forward contract with monthly delivery period, it is often the case that detailed forecast data on a daily basis is not available for all days of the delivery month. Especially for mid- to long-term forecasts, granularity of forecast points tends to be rather low, i.e., only expected maximum weekly, monthly, or seasonal demand (capacity) levels may be indicated. Irrespective of the question of whether long-term forecasts are still sufficiently accurate at all in order to justifiably treat them as deterministic, the necessary interpolation of missing long-term forecasts will induce some kind of arbitrariness. Given a variety of different interpolation methods to choose from, pricing results would consequently be quite sensitive to the specific technique selected. Second, and as is analyzed further below, future capacity levels are generally known to be hard to predict, in particular for the British market (see, e.g., Karakatsani and Bunn, 2008). This results in slightly less reliable forecasts, hence invalidating the assumption of deterministic forecasts in the first place, and leading to increased modeling risk otherwise.

A related approach has been presented by Ritter et al. (2011) and Haerdle et al. (2012) in the context of weather derivatives pricing. In case of missing daily forecast points for

periods beyond the horizon of the daily forecasts, they propose to proceed as follows: the respective stochastic process is estimated based on a time series of historical data that has been extended to also include a given set of available daily forecasts, treating the latter as if they were actually observed. Missing forecasts are then replaced with expectations derived from the estimated process. Generally speaking, estimating parameters based on historic and forecast data at the same time may come close to the general idea of enlarging the information filtration. However, implementing this approach again comes at the cost of having to consider daily forecasts as deterministic whenever available. In addition, it is not a priori clear how to implement the “combined” estimation strategy when estimating a process on a daily basis and forecasts are given, e.g., on a weekly basis only, yet shall nevertheless be included in the estimation procedure. Importantly, parameter estimates, such as the speed of mean-reversion, may be critically affected, especially when during the estimation procedure, more weight is given to the forecast data relative to the realized data. Note that this issue is avoided by modifying demand and capacity dynamics as proposed in Equation (2.20), while retaining the basic  $\mathcal{F}$ -implied stochastic properties of the respective processes at the same time.

Finally, as a third approach that might be appealing to practitioners, techniques similar to yield curve calibration in fixed income could be used: while the naïve approach of incorporating demand forecasts directly into the seasonal function  $s^D(T)$  is wrong,<sup>21</sup> it is possible to re-fit the mean-reversion level and let  $s^D(T)$  adjust so that expectations will correctly match the forecast points. For forecast data with high granularity (e.g., daily), results will be similar to the enlargement-of-filtration approach, but additional assumptions for interpolation will be necessary for the case of more widely spaced forecast points, as well as on which functional representation to use for  $s^D(T)$  when expectations beyond the forecast horizon are to be computed.

Nevertheless, both the enlargement-of-filtration approach as well as the alternatives discussed above share the common drawback of not adequately reflecting the relationship between forecast horizon and process variance. As confirmed empirically later (see Section 2.5 and Figure 2.4), especially demand forecasts are more accurate in the short-term, i.e., their reliability helps to reduce process variance. Importantly, such variance in the

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<sup>21</sup>Note that this will distort the relationship between the *speed* of mean-reversion of the OU-component and the actual *level* of mean-reversion, as imposed by the parameter estimates determined from historical data.

difference between forecast and realized demand tends to be lower than what is implied by a standard OU-process until approximately a horizon of  $t+8$ . Thus, relative to *realized* demand, the above techniques either imply that there is not enough variance in *forecast* demand (e.g., no variance at all as in the case of perfect foresight), or they leave variance levels unchanged altogether, and hence, too high.

## 2.4 Data and Estimation Approach

### 2.4.1 Fundamental Data

The data set used in this study for the fundamental variables demand and capacity comprises of ten years of historical data for the British electricity market, covering the period from 2002 up to 2011. These contain both historical realized as well as historical forecast data, and were obtained from National Grid, the British TSO,<sup>22</sup> and Elexon, the operator of the balancing and settlement activities in the British market.<sup>23</sup> Figure 2.1 shows the development of the realized demand and capacity data during the period from 01-Jan-2007 to 31-Dec-2011, i.e., the period covered by our pricing study, whereas the prior five years are used as estimation period.

With respect to electricity demand, *realized* data is based on the outturn average megawatt (MW) value of electricity demand in England, Wales, and Scotland during the peak half-hour of the day, as indicated by operational metering.<sup>24</sup> Specifically, we use the demand metric *IO14 DEM* which includes transmission losses and station transformer load, but excludes pump storage demand and net demand from interconnector imports/exports.<sup>25</sup>

*Forecasts* of expected electricity demand can be classified into two categories: first, National Grid releases daily updated demand forecasts covering the next 2 weeks ahead with daily granularity. These are forecasts of electricity demand expected to prevail during

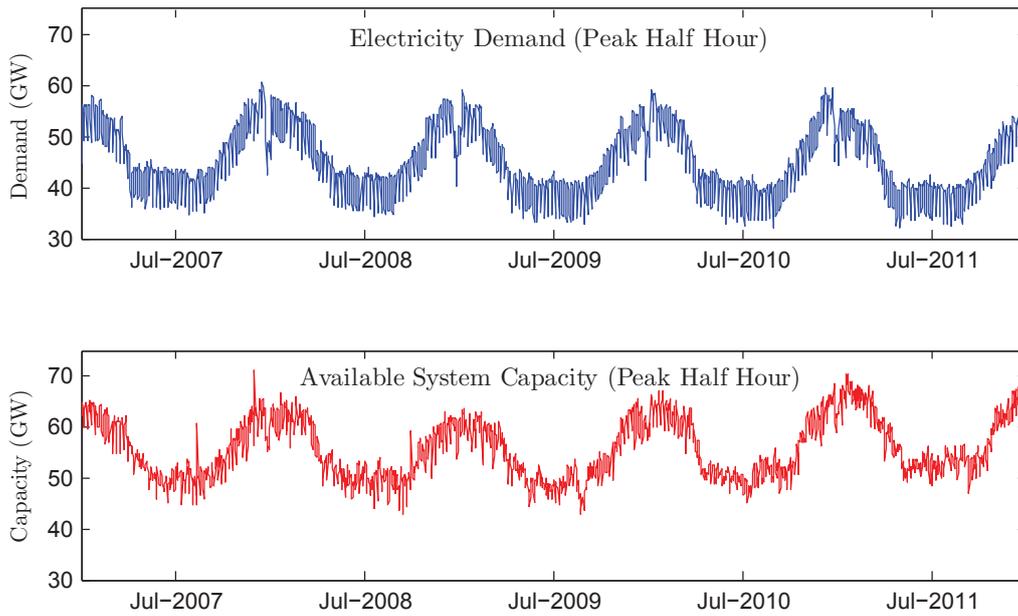
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<sup>22</sup>National Grid both owns and operates the systems in England and Wales. Since the start of BETTA in April 2005, it has also been operating the high-voltage networks in Scotland owned by Scottish and Southern Energy as well as Scottish Power.

<sup>23</sup>The following websites were accessed: <http://www.nationalgrid.com>, <http://www.bmreports.com>, and <http://www.elexonportal.co.uk>.

<sup>24</sup>In contrast to most other markets, electricity in Great Britain is traded on a half-hourly basis, corresponding to 48 settlement periods per day.

<sup>25</sup>The British electricity market is connected to neighbouring markets via interconnectors such as to/from France (IFA), the Netherlands (BritNed), or the Moyle Interconnector (connection to Northern Ireland).



**Figure 2.1: Daily Electricity Demand and Available System Capacity**

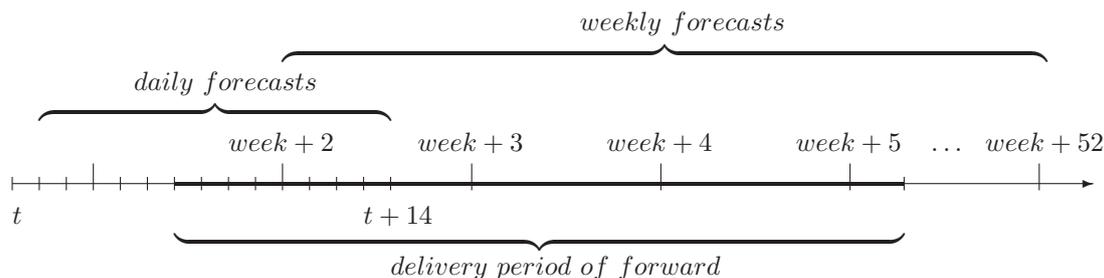
*This figure shows the time series of realized daily electricity demand and available system capacity in the British market during the period from 01-Jan-2007 to 31-Dec-2011. Displayed demand and capacity data both relate to the same daily peak (demand) half hour. All data shown were obtained from National Grid and Elexon.*

the peak half-hour of the respective day, which is the reason why we use peak demand instead of average baseload demand throughout this study. Second, longer-term forecasts of expected demand are released once a week, covering the next 2–52 weeks ahead with weekly granularity. These forecasts relate to expected demand during the peak half-hour of the respective week. Figure 2.2 provides a schematic overview of the different forecast horizons in the context of pricing a forward contract with monthly delivery period. Finally, note that special attention was paid to the realized and forecast data employed in our study being defined consistently.

In terms of *realized* capacity available, National Grid records maximum export limits (MEL) for each of the units that are part of the overall balancing mechanism (BM).<sup>26</sup> These limits quantify the maximum power export level of a certain BM unit at a certain time and are indicated by generators to the TSO prior to gate closure for each settlement period.<sup>27</sup> In case of an (un-)expected outage for some generation unit, generators will accordingly submit a MEL of zero during the time of the outage for this unit. Moreover,

<sup>26</sup>These are approximately 300 units in Great Britain, with one plant comprising several units.

<sup>27</sup>In the British market, gate closure is set at one hour before each half-hourly trading period. It refers to that point in time by when all market participants have to give notice about their intended physical positions so that the TSO can take action to balance the market.



**Figure 2.2: Schematic Overview of Forecast Horizons for the GB Market**

Daily forecasts are available on a 2- to 14-days-ahead basis; additionally, forecasts of expected maximum demand (capacity) per week are released for weeks 2–52. In this example, the first nine delivery days of some given forward contract are covered by daily forecasts, expected demand and capacity for each of the remaining days must be derived based on Proposition 2.3.1.

since MEL do include volatile interconnector flows as well as anticipated generation from intermittent/renewable sources, they can be seen as a good real-time proxy of available generation capacity that either is in use for production, or could additionally be dispatched into the transmission system immediately.

Capacity forecasts are released by National Grid, too, but primarily relate to the expected “market surplus” *SPLD*. This variable gives an indication on expected excess capacity beyond the levels required to satisfy expected demand and reserve requirements, but is structurally different from the MEL-approach that we follow for the realized capacity data. Amongst other reasons, this is due to *SPLD* including a statistically derived reserve-allowance which is based on average loss levels and forecast errors, rather than actual reserve levels held in operational timescales which are probably less pessimistic as well. As such, and in order to consistently define realized and forecast capacity levels, we instead use forecasts of expected total generation availability (which are also released by National Grid) and adjust them for few additional items.<sup>28</sup> Both timescale and updating structure of these forecasts are similar to the demand case.

When feeding the forecast data into our model, note that the weekly updated demand forecasts with a forecast horizon ranging from 2–52 weeks ahead are specified

<sup>28</sup>Even when using generation availability instead of *SPLD*, and unlike for the case of demand data, capacity forecasts still slightly differ in definition from the capacity metric on which the realized data is based (i.e., MEL). There are several reasons for this: *Inter alia*, volatile interconnector flows are hard to predict and, hence, are set at float throughout all forecast horizons. Also, a small number of generating units submit a MEL which, however, is not included into the forecast of generation capacity. We roughly adjust for these items to still arrive at consistently defined metrics, e.g., by carrying over latest observed values/forecast deviations into the future. At the same time, special focus is laid on our adjustments to remain simple, easily reproducible, and hence likely to be used by market participants. Further details are available from the authors upon request.

in correspondence to the expected peak half-hour within the respective week, i.e., it is not tied to a specific business day. Weekly capacity forecasts then relate to this very same half hour of expected peak demand, but do not specify an exact date either, which, however, is required in order to apply Proposition 2.3.1. Based on historic data, the peak half-hour of demand during a given week was most often found to occur between Tuesday and Thursday. For the sake of simplicity, we hence assume that weekly demand and capacity forecasts always refer to the Wednesday of the respective week.<sup>29</sup>

Finally, an important *caveat* applies: while forward-looking information may presumably be beneficial for derivatives pricing, *outdated* forward-looking information may certainly lead to the opposite. In fact, depending on both maturity and length of the delivery period for the respective contract to be priced, it may be the case that  $\mathbb{E}^{\mathbb{P}}[D_{\tau} \mid \mathcal{G}_t^D]$  and  $\mathbb{E}^{\mathbb{P}}[C_{\tau} \mid \mathcal{G}_t^C]$  for  $\tau = \underline{T} \dots \bar{T}$  are exclusively determined based on longer-term forecast points which are only updated weekly, as opposed to the *daily* updated 2–14 day-ahead forecasts. Focusing specifically on capacity forecasts, it may, however, happen that a major unplanned outage occurs just after the most recent weekly forecasts have been released. Even worse, for few periods in our data sample, forecast updates are missing altogether, leaving gaps of up to several weeks between successive forecast updates. Feeding such outdated forecasts into our (or any other) model and not updating for significant outages (whenever indicated) that move the market, hence unduly punishes the forecast-based model. Therefore, in case of missing updates or major unplanned outages not reflected in the most recent set of capacity forecasts, we have adjusted for such events by combining the forecast data with information provided on Bloomberg’s “UK VOLTOUT” page as well as in news reports from ICIS Heren. Note that this information was available to market participants at the time of trading.<sup>30</sup>

## 2.4.2 Electricity Spot and Forward Data

Following the historic development of electricity market regulation and especially since the inception of the New Electricity Trading Arrangements (NETA) regime in 2001, wholesale

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<sup>29</sup>Pricing errors have proven to be rather insensitive to this assumption, i.e., fixing the weekly forecasts to relate to each Tuesday or Thursday of a given week (or even alternating, based on the business day for which the weekly peak-hour during the preceding week was observed) did not visibly change results.

<sup>30</sup>Prominent examples, amongst others, relate to several of the unplanned trippings of nuclear generation units during 2007/08, which along with increased retrofitting activities of coal-based plants at that time led to extremely tight levels of available capacity in the British market.

trading in the British market is predominantly characterized by OTC forward transactions with physical settlement. The forward market, defined as covering maturities from day-ahead up to several years ahead of delivery, makes up for about 90% of overall electricity volume traded in the UK (Wilson et al., 2011). Compared to other major European electricity markets such as Germany or the Nordic market, financially-settled trades are less common and mainly concentrate on limited exchange-based trading activity such as at the Intercontinental Exchange or at the APX UK exchange. More recently, the N2EX platform, operated by Nord Pool Spot and Nasdaq OMX Commodities and established in order to re-strengthen exchange-based trading, has also started to list cash-settled power futures contracts for the British market. Despite these developments, exchange-based derivatives trading activity still seems to be rather limited, with member participation in futures trading increasing at slow pace only (see OFGEM, 2011).

In view of this dispersed market structure with the vast majority of trades still being bilateral or broker-based, our electricity price data is exclusively based on OTC contracts and was obtained from two sources: first, Bloomberg provides historical forward prices which are defined as composite quotes from a panel of OTC brokers. Second, we obtained a comprehensive data set from Marex Spectron, a leading independent energy broker that operates one of Europe's largest and most established marketplaces for electricity. This second data set is entirely based on trade data (including time stamp of trade, executed through platform or voice brokers) and contains a variety of additional types of electricity contracts, out of which a second OTC sample was formed. These two samples, for which pricing errors are analyzed separately in Section 2.5, contain the following types of contracts:

*“Bloomberg Data Set”:*

- 1-month ahead forward contracts

*“Marex Spectron Data Set”:*

- 1-month ahead forward contracts
- 2-months ahead forward contracts

All selected forward contracts are baseload contracts. Moreover, electricity spot (i.e., day-

ahead)<sup>31</sup> price data is additionally used for model calibration purposes, but is not analyzed further in the main study. We deliberately focus on pricing the above types of baseload contracts, leaving aside other instruments with quarterly, seasonal, or yearly delivery periods. This is due to the following reasons: first, we are primarily interested in the pricing impact when considering demand and capacity forecasts, compared to a situation when disregarding such forecasts. Since these forecasts are more accurate for short-term horizons,<sup>32</sup> our study focuses on contracts with short maturities and delivery periods. Second, trading activity generally concentrates on front months with liquidity at the longer end of the curve rapidly decreasing (OFGEM, 2011). Finally, and again primarily for liquidity reasons, we have chosen to analyze baseload contracts instead of peakload contracts. The fact that we are pricing *baseload* contracts, although using demand and capacity during *peak* half-hours as inputs, may seem inconsistent, but is ultimately due to the forecast data being available in this format only. It might be possible to convert the peakload demand and capacity forecasts into corresponding baseload predictions, e.g., by applying scaling factors that are based on historical averages. However, this is already indirectly accounted for by the estimation procedure outlined in the following subsection.

An overview of the two data samples is provided in Table 2.1 where descriptive statistics as well as further contractual characteristics for the day-ahead and forward contracts are summarized. As can be seen, the data exhibits well-known characteristics of electricity prices, such as substantial levels of volatility and excess kurtosis. While generally these effects are more pronounced for spot than for forward contracts, we also note the obvious difference in skewness of log-returns between both types of contracts.

### 2.4.3 Estimation Approach and Estimation Results

The individual processes for the state variables demand ( $D_t$ ) and available capacity ( $C_t$ ) are estimated by discretizing Equations (2.2) and (2.6) and using maximum likelihood. Based on annually rolling windows of five years of time series data, parameters are re-estimated annually, but held constant throughout every subsequent year when used for

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<sup>31</sup>We follow the classic assumption in literature according to which the day-ahead electricity price is interpreted as (quasi-) spot price. Although it would be possible to use *within-day* rather than *day-ahead* data for calibration purposes, we refrain from doing so given that within-day markets are more technical in nature, implying that short-term balancing needs may strongly overlay with our supply/demand dynamics.

<sup>32</sup>Longer-term forecasts rely on statistical averages and, thus, should convey no significant additional information as compared to the “no-forecast” case that is characterized by filtration  $(\mathcal{F}_t)_{t \in [0, T^*]}$ .

Table 2.1: **Samples of Baseload Spot and Forward Contracts**

This table reports summary statistics for the samples of electricity spot (day-ahead) and forward prices covering the period from January 2, 2007 until December 30, 2011.  $[\underline{T}, \bar{T}]$  denotes the average delivery period (in days) and  $\underline{T} - t$  the average maturity (in days) as measured until the start of the delivery period. All contracts from both the Bloomberg and Marex Spectron samples are baseload contracts. Displayed log-returns for 1- and 2-month(s) ahead forward contracts are adjusted to account for roll-over of contracts as well as for missing quotes.

	Mean	Median	Std. Dev.	Skewness	Kurtosis	$[\underline{T}, \bar{T}]$	$\underline{T} - t$	
<b>Bloomberg Data</b>	<b>1-Day Ahead</b>							
	$\ln P_t$	3.7543	3.7600	0.3891	0.0818	-0.1121	1.0	1.0
	$\ln P_t - \ln P_{t-1}$	-0.0019	-0.0019	0.0717	1.2812	12.8443		
	<b>1-Month Ahead</b>							
	$\ln F_t$	3.7781	3.7899	0.3737	0.2196	0.2131	30.4	15.9
	$\ln F_t - \ln F_{t-1}$	-0.0009	-0.0003	0.0219	-0.2365	5.4731		
<b>Marex Spectron Data</b>	<b>1-Day Ahead</b>							
	$\ln P_t$	3.7500	3.7612	0.3829	0.1225	0.0516	1.0	1.0
	$\ln P_t - \ln P_{t-1}$	-0.0027	-0.0021	0.0721	1.2138	10.9920		
	<b>1-Month Ahead</b>							
	$\ln F_t$	3.7856	3.7956	0.3604	0.2359	0.4641	30.4	16.1
	$\ln F_t - \ln F_{t-1}$	-0.0010	-0.0014	0.0205	-0.1641	4.8328		
<b>2-Months Ahead</b>								
$\ln F_t$	3.8003	3.7975	0.3532	0.3031	0.4328	30.8	44.8	
$\ln F_t - \ln F_{t-1}$	-0.0009	-0.0005	0.0177	-0.3552	4.5943			

pricing purposes. Estimation results and robust standard errors are presented in Tables 2.6 and 2.7 in the appendix to this chapter. The reported significance levels underline the distinct seasonalities for both demand and capacity, with our chosen specifications capturing well the most prominent characteristics.

Given the already very high number of parameters to be estimated, we have chosen a rather simple one-factor approach to model the dynamics of the marginal fuel used for generation, i.e., natural gas in our case. Since the spot component  $X_t$  in Equation (2.8) cannot be observed directly, estimation of all parameters for the natural gas model is instead performed based on futures data, by using the Kalman filter and maximum likelihood. Reformulating the model into state-space representation with corresponding

transition and measurement equations is a standard exercise (see, e.g., Schwartz, 1997). Since our study primarily focuses on the pricing of short-term electricity forward contracts, we refer to the short end of the natural gas curve and, hence, seek to infer the log-spot natural gas dynamics from corresponding short-term futures contracts with maturities ranging from one to four months. Relevant data is sourced from Bloomberg and relates to natural gas futures contracts traded at the Intercontinental Exchange (ICE) with physical delivery at the National Balancing Point (NBP), the virtual trading hub for natural gas in Great Britain. Parameter estimates for the dynamics of natural gas are summarized in Table 2.8. Again, the estimates are statistically highly significant and clearly reflect the strong seasonal component that is inherent in natural gas prices.

Having estimated the parameters that govern the dynamics of the respective underlying variables  $D_t$ ,  $C_t$ , and  $g_t$ , the parameters  $\alpha$ ,  $\beta$ ,  $\gamma$ , and  $\delta$  that link the three fundamental factors yet remain to be determined. Generally, two approaches appear suitable:

1. Based on Equation (2.12), historic log electricity spot prices,  $\ln P_t$ , are regressed on corresponding time series data of  $D_t$ ,  $C_t$ , and  $\ln g_t$ . This approach is proposed by Cartea and Villaplana (2008) for a structurally similar model that, however, does not include marginal fuel dynamics or forward-looking information.
2. Implicitly (re-)estimating the parameters over time, based on a cross-section of electricity spot and forward prices.

Given evidence that  $\alpha$ ,  $\beta$ ,  $\gamma$ , and  $\delta$  may not be constant over time, we favor the second approach. For instance, Karakatsani and Bunn (2008) also apply fundamentals-based models in their study on electricity spot price forecasting in the British market. They conclude that the models with the best pricing performance are those that allow for time-varying coefficients to link the fundamental factors. Moreover, the changing level of dependence of electricity spot prices on each fuel price due to mixing of bids and merit order changes as proposed in the model by Carmona et al. (2013) may be seen in the same spirit. Therefore, and although treated as constants in our model, the time-varying nature of the parameters  $\alpha$ ,  $\beta$ ,  $\gamma$ , and  $\delta$  is captured by implicitly extracting and re-estimating them weekly from the cross-section of quoted power prices. Likewise, the parameters  $\lambda^D$

and  $\lambda^C$  governing the change of measure from  $\mathbb{P}$  to  $\mathbb{Q}$  are inferred in the same way.<sup>33</sup>

In order to implicitly estimate these parameters, the following objective function is minimized:<sup>34</sup>

$$\begin{aligned}\Phi_W^* &= \arg_{\Phi_W} \min RMSPE(\Phi_W) \\ &= \arg_{\Phi_W} \min \left[ \sqrt{\frac{1}{N_W^P} \sum_{i=1}^{N_W^P} \left( \frac{\hat{P}_{W,i}(\Phi_W^{\mathbb{P}}) - P_{W,i}}{P_{W,i}} \right)^2} + \sqrt{\frac{1}{N_W^F} \sum_{i=1}^{N_W^F} \left( \frac{\hat{F}_{W,i}(\Phi_W^{\mathbb{Q}}) - F_{W,i}}{F_{W,i}} \right)^2} \right],\end{aligned}$$

where  $\Phi_W \equiv \{\alpha, \beta, \gamma, \delta, \lambda^D, \lambda^C\}$  with the two subsets  $\Phi_W^{\mathbb{Q}}$  and  $\Phi_W^{\mathbb{P}}$  defined as  $\Phi_W \equiv \Phi_W^{\mathbb{Q}}$  and  $\Phi_W^{\mathbb{P}} \equiv \Phi_W^{\mathbb{Q}} \setminus \{\lambda^D, \lambda^C\}$ . To minimize the root mean squared percentage error (RMSPE) over the in-sample period  $W$ , we assemble all available day-ahead prices  $P_{W,i}$  (totaling  $N_W^P$  quotes) as well as all available forward prices  $F_{W,i}$  ( $N_W^F$  quotes) and compare against prices  $\hat{P}_{W,i}$  and  $\hat{F}_{W,i}$  as predicted by our model based on Equations (2.12) and (2.14).<sup>35</sup> For in-sample estimation windows  $W$ , we use a length of eight weeks, e.g.,  $w_1 - w_8$ , for the Bloomberg sample. Out-of-sample testing of the model is performed during the subsequent week (i.e.,  $w_9$ ), employing the parameters estimated over  $W$  – thus only using information available up to the respective pricing day. Finally, the in-sample period is shifted by one week (i.e., new window:  $w_2 - w_9$ ) and parameters are re-estimated. For the Marex Spectron sample, we shorten the length of the in-sample estimation windows to six weeks since more price observations per week are available, thus allowing for a robust estimation with a shorter window. Furthermore, these changes in the in-sample set-up may provide additional robustness to our findings examined in Section 2.5, so as to ensure that pricing improvements from using forecasts do not rely on a specific mix of contracts or length of in-sample estimation windows.

Different sets of implied parameter estimates  $\Phi_W^*$  are obtained for the Bloomberg and

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<sup>33</sup>Note that for pricing power derivatives in our structural framework, risk-neutral dynamics are also required for the natural gas component. The corresponding market price of risk  $\lambda^g$  which is assumed constant, however, has already been determined by the Kalman filter estimation (see Table 2.8). We hence assume that the “look-through” risk premium of natural gas indirectly inherent to power derivatives is equal to the one for (outright) traded natural gas futures contracts. While  $\lambda^g$  could easily be re-estimated by including it into the set of implicitly determined parameters  $\Phi$ , we refrain from doing so and instead prefer to reduce the number of free parameters here.

<sup>34</sup>Unlike price time series in financial markets, such as stock prices, the time series of electricity prices in our sample are stationary, so that we refrain from differencing the data when calibrating the model.

<sup>35</sup>Note that our sample of day-ahead quotes does not contain any observations of negative prices. However, if we were to model hourly electricity prices (where the occurrence of negative prices is more likely than on a day-ahead level), and depending on the regulation for the respective market under study, our model could be extended to also produce negative prices.

Marex Spectron samples (which are priced separately), as well as depending on whether or not forecasts of demand and/or available capacity are used during the estimation procedure. As an example, Table 2.2 summarizes implied estimates for the Bloomberg sample, when using both demand and capacity forecasts.<sup>36</sup> Although the table only provides an aggregate view on the estimates, their corresponding means and standard errors indicate significant weekly variation among the parameters which our model could not capture when holding constant the “fundamental” parameters  $\alpha$ ,  $\beta$ ,  $\gamma$ , and  $\delta$  in  $\Phi_W^*$  otherwise.

Examining the development of the parameter estimates over time, we observe that  $\beta$  and  $\gamma$ , the parameters governing the sensitivity of the power price with respect to changes in demand and capacity, respectively, culminate in 2008 and gradually decline thereafter. As is further outlined in the next section, this can be well explained by the fact that in terms of excess capacity, the British power market was especially tight in 2008, as is clearly reflected in the behavior of day-ahead and month-ahead forward prices displayed in Figure 2.3. The following years are marked by a massive increase in installed generation capacity by more than 10 gigawatts (GW), leading to oversupply especially of thermal generation and, consequently, to tightening spreads (especially spark spreads) for generators. As a consequence of these abundant capacity levels, changes in demand and capacity are of less importance for power price dynamics at that time, as evidenced by rather small absolute values for the estimates of  $\beta$  and  $\gamma$  in the years 2009–2011.<sup>37</sup> As will be seen, this strongly affects the relative advantage of using forecasts of demand and capacity.

Recalling that  $\delta$  can be interpreted as the elasticity of the power price with respect to changes in the fuel price, we observe that between 2009 and 2011, the estimate for  $\delta$  more than doubles. This increase in the power-gas sensitivity may come as a surprise given that at the same time, spark spreads have continued to decline. However, it is the heavily gas-based structure of the British generation park that causes especially the short end of the power price curve to track the NBP gas curve very closely. Hence, the link between gas and power markets may have become even stronger recently, owing to the

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<sup>36</sup>Estimation results for the other sets of parameters are available from the authors upon request.

<sup>37</sup>However, the variation of  $\beta$  and  $\gamma$  and especially the increase in absolute values for 2008 could, at least to some extent, also be due to insufficient convexity of our functional representation of the merit order curve. The curve is likely to be steeper during times of low system margin than the corresponding levels implied by our exponential-form representation.

Table 2.2: Implied Parameter Estimates for  $\Phi_W^*$  (Bloomberg Sample / Forecasts for  $D_t$  and  $C_t$ )

This table reports yearly average values and corresponding standard errors [in brackets] of the implied estimates for the “fundamental” and risk-neutral parameters  $\Phi_W^* = \{\alpha, \beta, \gamma, \delta, \lambda_G^D, \lambda_G^C\}$ . Parameters are obtained by minimizing the root mean squared percentage errors (RMSPE) between observed market prices and theoretical model prices for both 1-day ahead and 1-month ahead forward contracts from the Bloomberg (BBG) data sample. Throughout the estimation procedure, forecasts of both electricity demand as well as available capacity are used. In-sample estimation is performed based on a time window  $W$  of eight weeks with weekly shifting. Then, for every parameter in  $\Phi_W^*$ , the below displayed average values are computed based on the set of estimates implied from all in-sample windows in the respective year.

	2007	2008	2009	2010	2011	07-11
$\alpha$	1.5203 [0.1386]	3.0155 [0.1956]	2.7241 [0.0981]	2.0338 [0.1025]	1.8217 [0.1276]	2.2220 [0.0697]
$\beta$	0.0279 [0.0020]	0.0286 [0.0027]	0.0164 [0.0009]	0.0135 [0.0008]	0.0106 [0.0003]	0.0193 [0.0008]
$\gamma$	-0.0175 [0.0015]	-0.0344 [0.0035]	-0.0150 [0.0006]	-0.0116 [0.0004]	-0.0117 [0.0004]	-0.0180 [0.0009]
$\delta$	0.4317 [0.0334]	0.4369 [0.0388]	0.2713 [0.0269]	0.4674 [0.0253]	0.5578 [0.0317]	0.4324 [0.0151]
$\lambda_G^D$	-16.3475 [10.2246]	3.9719 [14.4244]	-45.1915 [12.5963]	-30.9245 [7.5293]	-58.2689 [8.8900]	-29.5412 [5.0647]
$\lambda_G^C$	20.2954 [15.0584]	11.3382 [18.2663]	52.4444 [14.5850]	36.0655 [7.6381]	64.8668 [9.5236]	37.1602 [6.1353]
$\beta\lambda_G^D + \gamma\lambda_G^C$	-0.0817 [0.4304]	-0.8503 [0.8125]	-1.5816 [0.4360]	-0.6957 [0.1589]	-1.2935 [0.1913]	-0.9034 [0.2087]

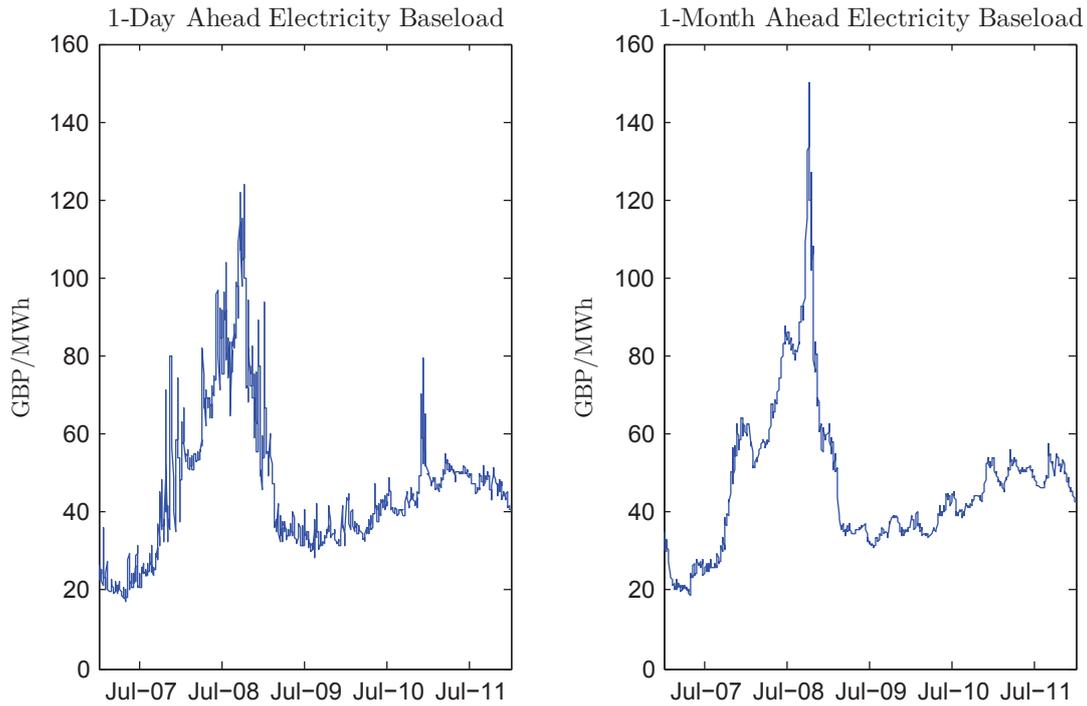


Figure 2.3: **1-Month Ahead Forward and 1-Day Ahead Electricity Prices**

*This figure shows the time series of daily forward prices for 1-month ahead and 1-day ahead electricity contracts (baseload) during the period from 01-Jan-2007 to 31-Dec-2011. All data shown were obtained from Bloomberg; for dates with missing quotes/prices, the last observed historic price was carried over.*

fact that (i) the LCPD<sup>38</sup> has started to reduce availability levels of coal plants and that (ii) new generation coming online has primarily been of CCGT-type.<sup>39</sup> We also note that the increase in value for  $\delta$  during 2009–2011 goes in line with a corresponding decrease in value for  $\alpha$ , which appears reasonable when recalling the interpretation of  $\alpha = f_t^{1-\delta}$  in Equation (2.13).

Finally, in view of rather large estimates for the market prices of demand and capacity risk,  $\lambda^D$  and  $\lambda^C$ , it is important to mention that since these two parameters are estimated simultaneously, they interact with each other during the estimation procedure and cannot be determined uniquely. It might hence be more convenient to think of a “combined” market price of (reserve) margin risk  $\beta\lambda^D + \gamma\lambda^C$  which is also shown in Table 2.2.

<sup>38</sup>The UK Large Combustion Plant Directive (LCPD) limits the amount of Sulphur Dioxide, Nitrous Oxides, and dust that coal- and oil-fired power stations are allowed to emit. As an alternative to complying with the tighter emissions regulations, power stations that were “opted-out” either face restrictions of operational hours and/or have to close by 2015.

<sup>39</sup>Combined cycle gas turbine (CCGT) plants are natural gas fired generation plants which, due to their enhanced technology, achieve high levels of thermal efficiency and offer sufficient flexibility in generation to meet sudden fluctuations in electricity demand.

## 2.5 Pricing Results

In order to examine the pricing impact of using forward-looking information in more detail, we distinguish between three cases: using (i) no forecasts, (ii) demand forecasts only, and (iii) forecasts of both demand and available capacity. Results are reported for each of the five years covered by our study as well as on an aggregate basis for 2007–2011. Table 2.3 summarizes pricing results for 1-month ahead forward contracts from the Bloomberg data set. As can be seen, employing demand and capacity forecasts clearly improves pricing performance on an overall basis, reducing pricing errors by up to 50%: aggregate RMSPE over the entire sample period reduces to less than 6% as compared to an RMSPE of about 10% when no forecasts are used; corresponding absolute-level RMSE even halves and decreases by some £4.00/MWh, which also underlines the economic significance of the pricing improvements achieved by incorporating forecasts into our model – especially in view of average contract volumes of several thousands of megawatt hours (MWh).

In order for the analysis of pricing errors to be consistent with our estimation procedure, we mainly focus on root mean squared-based error measures, given that this objective function has also been used for estimation. However, we also note that the relative improvement in pricing performance when employing forecasts is generally smaller when looking at the absolute percentage error (MAPE) as opposed to RMSPE, which underlines that incorporating forecasts seems to pay off mainly in situations of unusually high demand or low capacity. Hence, before analyzing the breakdown of pricing errors on a yearly basis, it is important to recall that primarily during the first 2–2.5 years of our study, the British power market has suffered from exceptionally poor expected levels of power plant availability, with reserve margins clearly falling below long-term averages, especially in 2008. Consequently, the model excluding forecasts fares clearly worse than during any other period of our study. By contrast, the model including both demand and capacity forecasts gives strong evidence of its capabilities, reducing pricing errors even in times of extreme fluctuations in day-ahead and forward price levels, i.e., during times demanding utmost flexibility from any type of model. Reconsidering Figure 2.3, the extreme spike in month-ahead forward prices during September/October 2008 was clearly driven by ever-increasing supply fears,<sup>40</sup> and it is obvious that such a trajectory

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<sup>40</sup>This is supported by our analysis of market commentary covering the respective trading days. Importantly, in these days, prices of month-ahead natural gas were approximately flat.

Table 2.3: **Out-of-Sample Pricing Results: 1-Month Ahead Forward Contracts (BBG)**

*This table reports yearly (and aggregate) pricing errors for 1-month ahead electricity forward contracts from the Bloomberg (BBG) data sample; implemented models either do not rely on forecasts (“No FC”) or incorporate demand and/or capacity forecasts (“FC for  $D_t$ ” and “FC for  $D_t$  &  $C_t$ ”, respectively). We use weekly subperiods for out-of-sample pricing, with in-sample fitting of the model being performed based on the cross-section of 1-day ahead and 1-month ahead forward quotes collected during the preceding eight weeks. Error measures shown are mean percentage error (MPE), mean absolute percentage error (MAPE), root mean squared error (RMSE), and root mean squared percentage error (RMSPE).*

	MPE	MAPE	RMSE	RMSPE	MPE	MAPE	RMSE	RMSPE
	<b>2007</b>				<b>2008</b>			
No FC	-3.33%	5.19%	£2.90	7.17%	0.98%	9.83%	£17.22	20.57%
FC for $D_t$	-4.54%	5.86%	£3.78	7.76%	0.71%	7.73%	£10.29	12.04%
FC for $D_t$ & $C_t$	-4.30%	5.53%	£3.47	7.49%	-1.19%	6.65%	£7.73	8.99%
	<b>2009</b>				<b>2010</b>			
No FC	-0.79%	3.18%	£2.06	5.99%	-1.02%	2.00%	£1.31	2.96%
FC for $D_t$	-0.11%	2.68%	£1.36	3.90%	-0.36%	1.94%	£1.16	2.69%
FC for $D_t$ & $C_t$	-0.28%	2.83%	£1.46	4.16%	-0.31%	2.17%	£1.32	3.03%
	<b>2011</b>				<b>2007-2011</b>			
No FC	1.15%	2.59%	£1.73	3.63%	-0.62%	4.56%	£7.90	10.31%
FC for $D_t$	1.06%	2.18%	£1.37	2.81%	-0.67%	4.09%	£5.00	6.87%
FC for $D_t$ & $C_t$	1.47%	2.41%	£1.57	3.23%	-0.94%	3.93%	£3.95	5.91%

can only be captured (albeit not perfectly) by a model that includes forward-looking information about the capacity levels that are expected to prevail during the respective delivery months.<sup>41</sup>

The pricing performance of the models during the year 2007 provides another opportunity to further discuss what kind of forward-looking information we actually consider to be contained in the enlarged filtration  $(\mathcal{G}_t^C)_{t \in [0, T^*]}$  – and what is not contained therein. Based on a detailed analysis of single-day pricing errors, the model including both forecast types yields very satisfactory pricing results throughout this year, except for a period of rather poor pricing performance during November and December 2007, for which forward prices are clearly underestimated. Although market commentary may generally be criticized for over-emphasizing alleged causal relationships between specific events and strong market movements, several reports released at that time stress, amongst other reasons, the then very high continental power prices that are said to have impacted British power prices as well. In fact, French power prices had reached record levels in November 2007, fueled by strikes in the energy sector that led to temporary production cuts of about 8,000 MW. This, in turn, raised concerns about French electricity supplies for the rest of the year, which ultimately could have resulted in Britain becoming a net exporter of power to France via its interconnector, putting an additional drain on the already tight British system.<sup>42</sup> However, although market commentary indicates that (British) market participants do seem to have “priced in” such a scenario, and although pricing errors for the forecast-based variant of our model would have clearly been reduced, we have decided not to incorporate this belief (i.e., a longer-lasting strike in France having interconnectors switch from imports to exports) into our capacity forecasts:  $(\mathcal{G}_t^C)_{t \in [0, T^*]}$  is only based on forecasts released by the TSO and supplemented with updates of major unplanned

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<sup>41</sup>The benefit of using forecasts during times of high demand and/or tight reserve margins can also be confirmed by regressing the related reduction in pricing errors on a measure that quantifies by how much the forecasts deviate from corresponding long-term seasonal levels. More precisely, on every pricing day, we feed into our model estimates of both demand and available capacity that are expected to prevail on every day within the delivery period of the respective contract (see Equation (2.16)). Thus, for every *delivery* day, we compute the percentage deviation between these sets of demand (capacity) expectations when based on forward-looking information and when excluding it. For our regression, we define the regressors as the maximum of these demand (capacity) deviations, i.e., where on every pricing day, the maximum is taken over all delivery days. Especially for the years 2007 and 2008, regressing the reduction in RMSPE on these regressors yields highly statistically significant coefficients at the 1%-level.

<sup>42</sup>The interconnector that links British and French electricity markets has a capacity of approx. 2,000 MW; Britain has “traditionally” been an importer of French electricity – which especially during peak hours tends to be cheaper, also in view of the higher share of nuclear baseload generation capacity. Yet at that time in 2007, it was feared that the strike would cause electricity in France to become more expensive than in Britain, thus reverting the usual direction of interconnector flows.

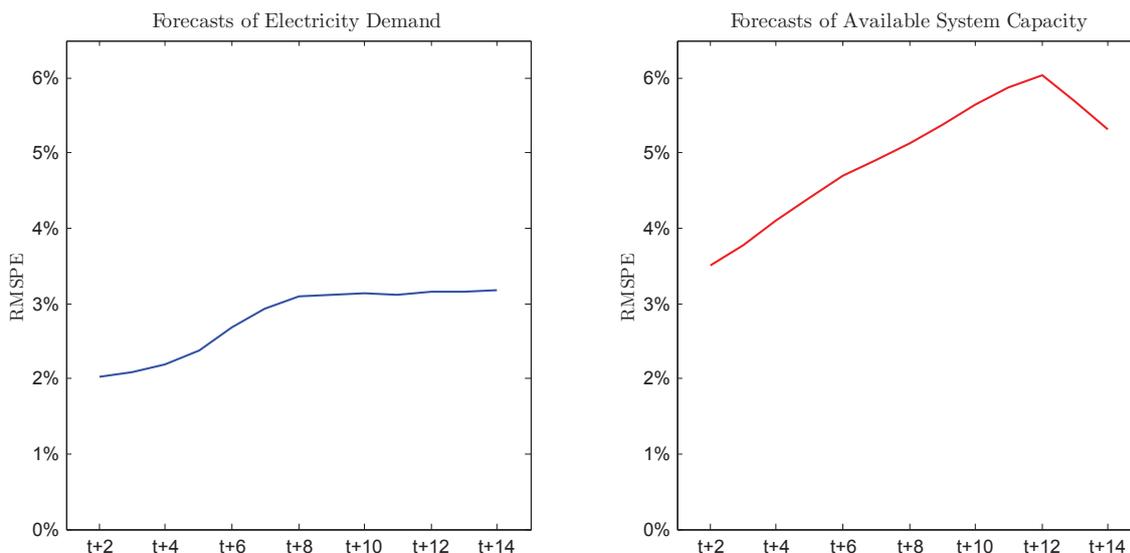
outages. Although likely to further improve pricing performance, starting to integrate *market beliefs* about future available import/export capacity levels would also require us to do so for the rest of our sample, i.e., during times where such market sentiment may be more difficult to infer. Moreover, it is obviously impossible to exactly observe and consistently quantify these beliefs. For instance, it is unknown how long exactly and to what extent market participants would expect the above scenario of strikes in the French energy sector to continue.

In the years 2009–2011, the relative improvement of the forecast-based models is smaller than in previous years. As indicated by the corresponding parameter estimates for  $\beta$  and  $\gamma$ , the influence of demand and capacity as fundamental factors driving power prices has been much reduced during these years, primarily due to growing oversupply in generation capacities leading to permanently higher reserve margin levels. Given that short- to mid-term power prices at that time were almost exclusively driven by natural gas dynamics under these conditions, the impact of incorporating forward-looking information vanishes accordingly. Interestingly, pricing performance of the model for the years 2009–2011 seems to be even slightly better when using demand forecasts only, and leaving capacity forecasts aside. As also stressed by Karakatsani and Bunn (2008), this could be due to the fact that in the British market, the forecasts of available capacity levels (or, equivalently, margins) released by the TSO tend to be received with slight skepticism and, hence, are likely to be adjusted (or not used at all) by market players.

This leads to other, more general problems of capacity forecasts, such as their accuracy in terms of generation from renewables or their consistency in definition with realized data. This is also reflected in Figure 2.4 where prediction errors between forecast and realized demand and capacity levels are summarized.<sup>43</sup> Capturing well the regular consumption patterns that characterize the dynamics of electricity demand, related forecasts are subject to rather low forecast errors only. By contrast, predicted future levels of available capacity are significantly less accurate and this inaccuracy increases more strongly for longer forecast horizons. While this certainly impacts pricing performance during 2009–2011, such generally higher inaccuracy of capacity forecasts nevertheless seems to be of minor importance during times of exceptionally low reserve margins, as shown above.

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<sup>43</sup>Note that especially for forecasts of available capacity, the input capacity data from National Grid has been subject to further adjustments by the authors.



**Figure 2.4: Performance of Demand and Capacity Forecasts**

*This figure shows the root mean squared percentage error (RMSPE) for the 2–14 days-ahead forecasts of electricity demand and available system capacity during the period from 01-Jan-2007 to 31-Dec-2011. Note that especially for capacity forecasts, inputs are based on data released by National Grid plc, after adjustments by the authors.*

The results based on the data obtained from Marex Spectron are presented in Tables 2.4 and 2.5. Again, we observe an improvement in pricing performance when integrating demand and capacity forecasts into our model, as evidenced by relative reductions in total RMSPE of 8% and 15% for 1-month and 2-months ahead forward contracts, respectively. Moreover, the overall pattern of pricing errors for both types of forward contracts is in line with the conclusions drawn from the Bloomberg sample. Notably, integrating demand as well as capacity forecasts into our model again primarily pays off during the years 2007–2008, reducing aggregate RMSE during these years by about £1.20–2.00/MWh. Such economic significance is also confirmed statistically by applying a Wilcoxon signed-rank test which shows that the reductions in errors are significant at the 1%-level. For the remaining years, during which the impact of the fundamental factors  $D_t$  and  $C_t$  has been found to be rather muted, pricing errors can still be reduced by using only demand forecasts as compared to the “no-forecast” case.

Obviously, the differences in error metrics between the models including and excluding forward-looking information are not of the same order of magnitude as those reductions in pricing errors observed for the Bloomberg sample. Importantly, however, the in-sample fitting procedure for the Marex Spectron data sample additionally includes 2-months ahead forward contracts. As such, the fact that the benefits of using forecasts

Table 2.4: **Out-of-Sample Pricing Results: 1-Month Ahead Forward Contracts (OTC)**

*This table reports yearly (and aggregate) pricing errors for 1-month ahead electricity forward contracts from the Marea Spectron (OTC) data sample; implemented models either do not rely on forecasts ("No FC") or incorporate demand and/or capacity forecasts ("FC for  $D_t$ " and "FC for  $D_t$  &  $C_t$ ", respectively). We use weekly subperiods for out-of-sample pricing, with in-sample fitting of the model being performed based on the cross-section of 1-day ahead, 1-month ahead, and 2-months ahead forward quotes collected during the preceding six weeks. Error measures shown are mean percentage error (MPE), mean absolute percentage error (MAPE), root mean squared error (RMSE), and root mean squared percentage error (RMSPE).*

	MPE	MAPE	RMSE	RMSPE	MPE	MAPE	RMSE	RMSPE
	<b>2007</b>				<b>2008</b>			
No FC	-2.73%	6.11%	£4.50	8.97%	-2.78%	7.18%	£9.04	9.51%
FC for $D_t$	-3.60%	6.14%	£4.26	8.57%	-1.88%	8.11%	£9.25	10.23%
FC for $D_t$ & $C_t$	-3.16%	5.77%	£3.90	7.74%	-1.17%	6.71%	£7.49	8.60%
	<b>2009</b>				<b>2010</b>			
No FC	2.44%	3.34%	£1.78	4.65%	0.13%	2.32%	£1.38	3.03%
FC for $D_t$	2.07%	3.15%	£1.63	4.31%	0.12%	2.25%	£1.37	2.98%
FC for $D_t$ & $C_t$	2.52%	3.38%	£1.76	4.63%	0.33%	2.40%	£1.44	3.21%
	<b>2011</b>				<b>2007-2011</b>			
No FC	2.25%	3.18%	£1.87	3.82%	-0.02%	4.30%	£4.52	6.40%
FC for $D_t$	1.81%	3.03%	£1.74	3.61%	-0.19%	4.39%	£4.53	6.42%
FC for $D_t$ & $C_t$	2.36%	3.54%	£2.05	4.19%	0.28%	4.26%	£3.87	5.91%

Table 2.5: Out-of-Sample Pricing Results: 2-Months Ahead Forward Contracts (OTC)

This table reports yearly (and aggregate) pricing errors for 2-months ahead electricity forward contracts from the *Merx Spectron (OTC)* data sample; implemented models either do not rely on forecasts (“No FC”) or incorporate demand and/or capacity forecasts (“FC for  $D_t$ ” and “FC for  $D_t$  &  $C_t$ ”, respectively). We use weekly subperiods for out-of-sample pricing, with in-sample fitting of the model being performed based on the cross-section of 1-day ahead, 1-month ahead, and 2-months ahead forward quotes collected during the preceding six weeks. Error measures shown are mean percentage error (MPE), mean absolute percentage error (MAPE), root mean squared error (RMSE), and root mean squared percentage error (RMSPE).

	MPE	MAPE	RMSE	RMSPE	MPE	MAPE	RMSE	RMSPE
	<b>2007</b>				<b>2008</b>			
No FC	-5.04%	6.86%	£3.75	8.72%	-0.69%	8.61%	£9.76	11.45%
FC for $D_t$	-4.23%	7.26%	£3.95	9.31%	0.66%	7.08%	£8.11	9.56%
FC for $D_t$ & $C_t$	-4.59%	6.97%	£3.64	8.72%	-0.02%	6.21%	£6.81	7.85%
	<b>2009</b>				<b>2010</b>			
No FC	0.49%	2.88%	£1.65	4.46%	-0.84%	2.15%	£1.25	2.74%
FC for $D_t$	0.34%	2.55%	£1.33	3.55%	-0.84%	2.19%	£1.34	2.90%
FC for $D_t$ & $C_t$	0.10%	2.99%	£1.55	4.12%	-0.65%	2.63%	£1.51	3.51%
	<b>2011</b>				<b>2007-2011</b>			
No FC	0.81%	2.19%	£1.48	2.91%	-0.95%	4.34%	£4.57	6.72%
FC for $D_t$	-0.07%	1.76%	£1.20	2.29%	-0.79%	3.99%	£3.96	6.17%
FC for $D_t$ & $C_t$	0.25%	1.78%	£1.22	2.38%	-0.92%	3.96%	£3.47	5.70%

still prevail when calibrating our model to a broader cross-section of forward quotes may clearly be seen as underlining the robustness of our general findings.<sup>44</sup>

Examining the pricing errors in more detail, the year 2008 may again serve as an example to illustrate another and more subtle effect when using forecasts as compared to excluding them. For this year, and based on the *Bloomberg* data sample, pricing performance of the “no-forecast” variant of the model is especially poor, as indicated by an RMSPE of about 20%. For the *Marex Spectron* sample, by contrast, corresponding pricing errors for 1-month ahead contracts are much lower, yielding an RMSPE of less than 10%. In this context, it is important to note that amidst the height of above mentioned capacity shortage in 2008 (that led to the prominent spike in 1-month ahead forward prices in September/October shown in Figure 2.3), supply fears primarily concentrated on the front month. Consequently, 2-months ahead forward contracts at that time were clearly less subject to such strong fluctuations in price levels. Therefore, the broader cross-section of forward quotes in the Marex Spectron sample forces the “no-forecast” variant of our model to simultaneously accommodate such contrary 1-month and 2-months ahead price dynamics, which results in a “mediocre compromise” at best: 1-month ahead contracts are now strongly underestimated (2008 MPE of -2.78% in Table 2.4 vs. 0.98% in Table 2.3), which, however, halves RMSPE to less than 10%, given that underpricing pays off after the sudden “collapse” in post-spike forward pricing levels. Yet, on the other hand, the pronounced spike in 1-month ahead forwards has 2-months ahead contracts become strongly overpriced post-spike (despite an overall 2008 MPE of -0.69%), which alone contributes more than 2% to the overall RMSPE of 11.45%. By contrast, and again comparing Tables 2.3 and 2.4, all pricing errors for the model including demand and capacity forecasts in 2008 are surprisingly similar, irrespective of whether or not 2-months ahead contracts are included in the cross-section.

Put differently, the above example provides evidence of the additional benefits that arise when including forecasts into our model. Forecasting low levels of capacity in the short-term, but higher levels in the mid- to long-term may help govern opposed dynamics of contracts with differing maturities, such as outlined above. This flexibility is also reflected in the implicitly estimated fundamental parameters  $\alpha$ ,  $\beta$ ,  $\gamma$ , and  $\delta$ . In fact, the

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<sup>44</sup>As a further robustness check, the in-sample estimation window was shortened from 6 to 4 weeks, yielding similar pricing results. In order to preserve space, these results are not reported here, yet available from the authors upon request.

implied estimates show clearly higher variation throughout 2007 and 2008 than if demand and/or capacity forecasts are accounted for during the estimation procedure.<sup>45</sup> This appears reasonable given the additional flexibility for the forecast-based model variants in fitting observed prices, whereas the model variant without forecasts always has expected demand and capacity mean-revert to the same long-term levels. As a result, flexibility is reduced, which must be compensated for by higher variation in the set of fundamental parameters. Altogether, this again underlines that excluding forecasts from the pricing procedure not only affects pricing performance, but may also imply using a mis-specified model.

## 2.6 Conclusion

Modeling the dynamics of electricity prices has traditionally been a challenging task for market participants, such as generators/suppliers, traders, and speculators. The strong links between power prices and their fundamental drivers make structural modeling approaches especially appealing in this context, and it can be expected that both current and future developments, such as further integration of geographic markets via market coupling, will even further promote the importance of bottom-up modeling frameworks (albeit at the cost of increasing complexity). At the same time, increasing transparency as well as more reliable outturn and forecast data released by system operators help market participants face these challenges and allow for more informed trading decisions.

In this chapter, we develop and implement a model for electricity pricing that takes these developments into account by integrating forward-looking information on expected levels of electricity demand and available system capacity. Special focus is laid on calibrating the model to market prices of traded electricity contracts and it is shown that the model parameters are easily interpretable in an economic way. Being one of the key advantages of the fundamental approach, this helps to provide deeper insight into the structure of the market than standard reduced-form models could ever do.

Although hard to compare with other pricing studies that focus on different markets or periods, the pricing performance of our model appears very robust and reliable. Importantly, we find that out-of-sample pricing errors can be reduced significantly by

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<sup>45</sup>Table 2.2 provides parameter estimates for the latter case only. To preserve space, parameter estimates implied by estimating the model *without* using forward-looking information are not reported here, yet available from the authors upon request.

making use of forward-looking information. Especially during times of very tight reserve margins, as witnessed for the British market in 2008, capacity forecasts are of crucial importance in order to track sudden outage-induced changes in forward pricing levels and, therefore, significantly reduce pricing errors. However, we have also found that if spare capacities or, equivalently, tightness of the system is not perceived as playing a “fundamental” role, the advantage of employing capacity forecasts reduces and, in some instances, may even lead to marginally lower pricing performance. This is also strongly supported by our findings that capacity forecasts are generally less accurate on average than demand forecasts. Nevertheless, it is still beneficial to keep using *demand* forecasts rather than using no forecasts at all. This is especially true for the pricing of forwards during the years 2009–2011, where the dynamics of natural gas prices are the main fundamental driver so that demand and capacity only play a subordinate role for pricing.

Given the above mentioned challenges and future developments, there is ample room for further research in the field of structural electricity price modeling. First, it would be interesting to conduct empirical pricing studies for other electricity markets as well. Given that structural electricity price models may always appear somewhat “tailored” to capture the characteristics of a specific electricity market, it would be interesting to see how these types of models perform empirically in those markets where merit-order dynamics are different. Second, given that our model is cast in a log-normal setting, it is equally well-suited to option pricing like other previously proposed fundamental models (see, e.g., Carmona et al., 2013). Further *empirical* studies might not only investigate the impact of using forward-looking information on option pricing performance, but also focus on the question of how pricing performance is affected depending on whether a 1- or 2-fuel model is used. Finally, the continued shift towards renewable energy sources in the generation mix of many European power markets poses new and highly complex challenges regarding the forecasting of availability levels of intermittent generation, such as for wind or solar power. These forecasts will play an indispensable role especially when modeling geographic markets that are highly interconnected with each other, so that abundant supplies are likely to “spill over” across borders and impact price levels in neighbouring markets.

## 2.7 Appendix

### 2.7.1 Conditional Expectations Based on Enlarged Filtrations Under the Historical Measure

Let  $(\Omega, \mathcal{F}^D, \mathbb{F}^D = (\mathcal{F}^D)_{t \in [0, T^*]}, \mathbb{P})$  be a filtered probability space and  $q_t$  be specified as in Equation (2.2). Assume that  $\mathbb{E}^{\mathbb{P}}[q_{T_1} \mid \mathcal{G}_t^D]$  and  $\mathbb{E}^{\mathbb{P}}[q_{T_2} \mid \mathcal{G}_t^D]$  (with  $\mathcal{F}_t^D \subset \mathcal{G}_t^D$ ) are available from the system operator. Before computing a forecast of expected electricity demand at time  $T$  with  $t \leq T_1 \leq T \leq T_2$ , we first derive relevant formulae under the assumption that only one forecast point for  $T_1$  is given by the system operator, hence neglecting for the time being the existence of  $\mathbb{E}^{\mathbb{P}}[q_{T_2} \mid \mathcal{G}_t^D]$ , and that a forecast of electricity demand is needed for time  $T$  with  $t \leq T \leq T_1$ . Formally, this can be expressed as follows:

$$\mathbb{E}^{\mathbb{P}}[q_T \mid \mathcal{G}_t^D] = q_t e^{-\kappa^D(T-t)} + \sigma^D \mathbb{E}^{\mathbb{P}} \left[ \int_t^T e^{\varphi(s)} e^{-\kappa^D(T-s)} dB_s^D \mid \mathcal{G}_t^D \right]. \quad (2.21)$$

In order to manipulate the conditional expectation on the RHS of (2.21), a standard approach (see, e.g., Protter, 2004; Biagini and Oksendal, 2005) is to exploit the semi-martingale property of  $B_t^D$  with respect to  $\mathcal{G}_t$ , i.e., to decompose  $B_t^D$  as follows:

$$B_t^D = \hat{B}_t^D + A(t), \quad (2.22)$$

where  $\hat{B}_t^D$  is a  $\mathcal{G}_t^D$ -martingale (standard Brownian motion) and  $A(t)$  a continuous  $\mathcal{G}_t^D$ -adapted process of finite variation, commonly referred to as the "information drift". Following Hu (2011) and Di Nunno et al. (2006),  $\hat{B}_t^D$  in Equation (2.22) can be written more explicitly as:

$$\hat{B}_t^D = B_t^D - \underbrace{\int_0^t b_t(s) B_s^D ds}_{A_1(t)} - \underbrace{\int_0^t a(s) \left( \mathbb{E}^{\mathbb{P}}[Y \mid \mathcal{G}_s^D] - \rho'(s) B_s^D \right) ds}_{A_2(t)}, \quad (2.23)$$

with  $A(t) = A_1(t) + A_2(t)$ . Following Theorem A.1 in Benth and Meyer-Brandis (2009) or, equivalently, Theorem 3.1 in Hu (2011),  $a(s)$  and  $b_t(s)$  in above Equation (2.23) are

given as follows:

$$a(s) = \frac{\rho'(s)}{\tau - \int_0^s (\rho'(u))^2 du}, \quad (2.24)$$

$$b_t(s) = \rho''(s) \int_s^t \frac{\rho'(v)}{\tau - \int_0^v (\rho'(u))^2 du} dv, \quad (2.25)$$

where  $\rho(t) = \mathbb{E}^{\mathbb{P}}[B_t^D Y]$  is twice continuously differentiable,  $\tau = \mathbb{E}^{\mathbb{P}}[Y^2]$  and  $Y$  is a centered Gaussian random variable with  $Y = \int_0^{T_1} e^{\varphi(s)} e^{\kappa^D s} dB_s^D = \int_0^{T_1} e^{\theta \sin(2\pi(ks+\zeta))} e^{\kappa^D s} dB_s^D$ .

Focusing on  $A_1(t)$  and since  $b_s(s) = 0$ , it holds that:

$$\begin{aligned} \int_0^t b_t(s) B_s^D ds &= \int_0^t \int_0^s \frac{\partial b_s}{\partial s}(u) B_u^D du ds \\ &= \int_0^t a(s) \left[ \int_0^s \rho''(u) B_u^D du \right] ds \end{aligned} \quad (2.26)$$

$$= \int_0^t a(s) \left[ \rho'(s) B_s^D - \int_0^s \rho'(u) dB_u^D \right] ds, \quad (2.27)$$

where Equation (2.27) is derived from Equation (2.26) by applying Itô's Lemma to  $\rho'(s) B_s^D$ . Based on the above, Equation (2.23) can now be re-arranged to yield:

$$\hat{B}_t^D = B_t^D - \underbrace{\int_0^t a(s) \left( \mathbb{E}^{\mathbb{P}}[Y | \mathcal{G}_s^D] - \int_0^s \rho'(u) dB_u^D \right) ds}_{A(t)}. \quad (2.28)$$

Given above definition of  $Y$ , and since it can be shown that  $\rho'(t) = e^{\varphi(t)} e^{\kappa^D t}$ , the information drift  $A(t)$  can be further simplified, so that Equation (2.28) now reads:

$$\begin{aligned} \hat{B}_t^D &= B_t^D - \int_0^t a(s) \mathbb{E}^{\mathbb{P}} \left[ \int_s^{T_1} e^{\varphi(u)} e^{\kappa^D u} dB_u^D \mid \mathcal{G}_s^D \right] ds \\ &= B_t^D - \int_0^t a(s) \mathbb{E}^{\mathbb{P}} \left[ \int_s^{T_1} \rho'(u) dB_u^D \mid \mathcal{G}_s^D \right] ds \end{aligned} \quad (2.29)$$

$$= B_t^D - \underbrace{\mathbb{E}^{\mathbb{P}} \left[ \int_t^{T_1} \rho'(u) dB_u^D \mid \mathcal{G}_t^D \right] \int_0^t a(s) \exp \left( - \int_t^s \rho'(v) a(v) dv \right) ds}_{A(t)}, \quad (2.30)$$

where Equation (2.30) is derived from Equation (2.29) based on Proposition A.3 in Benth and Meyer-Brandis (2009). Hence, in our initial setting of Equation (2.21) where a demand forecast  $\mathbb{E}^{\mathbb{P}}[q_T \mid \mathcal{G}_t^D]$  is to be determined that is consistent with the exogenously given

forecast point relating to  $T_1$ , this can now be computed as follows:

$$\mathbb{E}^{\mathbb{P}}[q_T \mid \mathcal{G}_t^D] = q_t e^{-\kappa^D(T-t)} + \underbrace{\sigma^D \mathbb{E}^{\mathbb{P}} \left[ \int_t^T e^{\varphi(s)} e^{-\kappa^D(T-s)} dB_s^D \mid \mathcal{G}_t^D \right]}_{I_G(t,T)} \quad (2.31)$$

$$= q_t e^{-\kappa^D(T-t)} + \sigma^D e^{-\kappa^D T} \mathbb{E}^{\mathbb{P}} \left[ \int_t^T \rho'(s) dB_s^D \mid \mathcal{G}_t^D \right] \quad (2.32)$$

$$\begin{aligned} &= q_t e^{-\kappa^D(T-t)} + \sigma^D e^{-\kappa^D T} \mathbb{E}^{\mathbb{P}} \left[ \int_t^T \rho'(s) d(\hat{B}_s^D + A(s)) \mid \mathcal{G}_t^D \right] \\ &= q_t e^{-\kappa^D(T-t)} + \sigma^D e^{-\kappa^D T} \int_t^T \rho'(s) dA(s) \\ &= q_t e^{-\kappa^D(T-t)} + \sigma^D e^{-\kappa^D T} \mathbb{E}^{\mathbb{P}} \left[ \underbrace{\int_t^{T_1} \rho'(u) dB_u^D}_{(\star)} \mid \mathcal{G}_t^D \right] \underbrace{\int_t^T \rho'(s) a(s) \exp\left(-\int_t^s \rho'(v) a(v) dv\right)}_{f(s)} ds. \end{aligned} \quad (2.33)$$

Note that the term  $I_G(t, T)$  is also referred to as *information premium* which is defined as  $\mathbb{E}^{\mathbb{P}}[q_T \mid \mathcal{G}_t^D] - \mathbb{E}^{\mathbb{P}}[q_T \mid \mathcal{F}_t^D]$ . The term  $(\star)$ , in turn, can be extracted from the given forecast as follows:

$$(\star) = \frac{1}{\sigma^D} \left( e^{\kappa^D T_1} \mathbb{E}^{\mathbb{P}}[q_{T_1} \mid \mathcal{G}_t^D] - q_t e^{\kappa^D t} \right). \quad (2.34)$$

The integral in the second term on the RHS of Equation (2.33) can be further simplified if volatility is constant, as is the case for the dynamics of the capacity process in Equation (2.6). In the case of the seasonal volatility function for the demand process as specified in Equation (2.4), however, no analytic solutions for the integral exist; still, it can be approximated computationally in an efficient way by using standard numerical integration techniques.

Having outlined the general procedure for the case  $T \leq T_1$ , we now turn to the more relevant case where  $\mathbb{E}^{\mathbb{P}}[q_{T_1} \mid \mathcal{G}_t^D]$  and  $\mathbb{E}^{\mathbb{P}}[q_{T_2} \mid \mathcal{G}_t^D]$  (with  $\mathcal{F}_t^D \subset \mathcal{G}_t^D$ ) are released by the system operator and a forecast  $\mathbb{E}^{\mathbb{P}}[q_T \mid \mathcal{G}_t^D]$  needs to be computed with  $t \leq T_1 \leq T \leq T_2$ . We proceed as follows:

$$\mathbb{E}^{\mathbb{P}}[q_T \mid \mathcal{G}_t^D] = \mathbb{E}^{\mathbb{P}} \left[ q_{T_1} + \underbrace{\mathbb{E}^{\mathbb{P}}[q_T - q_{T_1} \mid \mathcal{G}_{T_1}^D]}_{(\star\star)} \mid \mathcal{G}_t^D \right]. \quad (2.35)$$

Re-arranging (\*\*) and taking out what is known, i.e.  $\mathcal{G}_{T_1}^D$ -measurable, we get:

$$\begin{aligned} \mathbb{E}^{\mathbb{P}}[q_T - q_{T_1} \mid \mathcal{G}_{T_1}^D] &= q_{T_1} (e^{-\kappa^D(T-T_1)} - 1) \\ &\quad + \sigma^D \mathbb{E}^{\mathbb{P}} \left[ \int_{T_1}^T e^{\varphi(s)} e^{-\kappa^D(T-s)} dB_s^D \mid \mathcal{G}_{T_1}^D \right]. \end{aligned} \quad (2.36)$$

Combining Equations (2.35) and (2.36) and using iterated conditioning now yields:

$$\begin{aligned} \mathbb{E}^{\mathbb{P}}[q_T \mid \mathcal{G}_t^D] &= \mathbb{E}^{\mathbb{P}}[q_{T_1} \mid \mathcal{G}_t^D] e^{-\kappa^D(T-T_1)} \\ &\quad + \mathbb{E}^{\mathbb{P}} \left\{ \sigma^D \mathbb{E}^{\mathbb{P}} \left[ \int_{T_1}^T e^{\varphi(s)} e^{-\kappa^D(T-s)} dB_s^D \mid \mathcal{G}_{T_1}^D \right] \mid \mathcal{G}_t^D \right\} \\ &= \mathbb{E}^{\mathbb{P}}[q_{T_1} \mid \mathcal{G}_t^D] e^{-\kappa^D(T-T_1)} + \mathbb{E}^{\mathbb{P}}[I_G(T_1, T) \mid \mathcal{G}_t^D]. \end{aligned} \quad (2.37)$$

The term  $\mathbb{E}^{\mathbb{P}}[I_G(T_1, T) \mid \mathcal{G}_t^D]$  in Equation (2.37), however, can be manipulated similarly to Equations (2.31) to (2.33):

$$\begin{aligned} \mathbb{E}^{\mathbb{P}}[I_G(T_1, T) \mid \mathcal{G}_t] &= \mathbb{E}^{\mathbb{P}} \left\{ \sigma^D e^{-\kappa^D T} \mathbb{E}^{\mathbb{P}} \left[ \int_{T_1}^{T_2} \rho'(u) dB_u^D \mid \mathcal{G}_{T_1}^D \right] \int_{T_1}^T f(s) ds \mid \mathcal{G}_t^D \right\} \\ &= \sigma^D e^{-\kappa^D T} \underbrace{\mathbb{E}^{\mathbb{P}} \left[ \int_{T_1}^{T_2} \rho'(u) dB_u^D \mid \mathcal{G}_t^D \right]}_{(\star\star\star)} \int_{T_1}^T f(s) ds. \end{aligned} \quad (2.38)$$

Analogous to Equation (2.34), the term  $(\star\star\star)$  can be backed out from the given forecast points relating to  $T_1$  and  $T_2$ :

$$(\star\star\star) = \frac{1}{\sigma^D} \left( e^{\kappa^D T_2} \mathbb{E}^{\mathbb{P}}[q_{T_2} \mid \mathcal{G}_t^D] - e^{\kappa^D T_1} \mathbb{E}^{\mathbb{P}}[q_{T_1} \mid \mathcal{G}_t^D] \right). \quad (2.39)$$

## 2.7.2 Conditional Expectations Based on Enlarged Filtrations Under an Equivalent Risk-Neutral Measure

For derivatives pricing purposes, and based on Equation (2.14), conditional expectations  $\mathbb{E}^{\mathbb{Q}}[\cdot \mid \mathcal{G}_t]$  and variances  $\mathbb{V}^{\mathbb{Q}}[\cdot \mid \mathcal{G}_t]$  under the enlarged filtration  $(\mathcal{G}_t)_{t \in [0, T^*]}$  and a risk-neutral measure  $\mathbb{Q}$  need to be computed for both demand and capacity processes  $D_t$  and  $C_t$ , respectively.

Defining  $A(t) = \int_0^t \vartheta(s) ds$ , and based on the manipulations in the previous

subsection, the  $\mathcal{G}^D$ -adapted dynamics for  $D_t$  can be stated as below (see Equation (2.2)):

$$dq_t = -\kappa^D \left( q_t - \frac{\sigma^D e^{\varphi(t)}}{\kappa^D} \vartheta(t) \right) dt + \sigma^D e^{\varphi(t)} d\hat{B}_t^D,$$

where  $\hat{B}_t^D$  is a  $\mathcal{G}_t^D$ -adapted standard  $\mathbb{P}$ -Brownian motion.<sup>46</sup> Applying Girsanov's theorem, and given that our market setting is inherently incomplete, we assume that under a suitably chosen risk-neutral measure  $\mathbb{Q}$ ,  $\hat{B}_t^D$  is a semi-martingale and decomposes as follows:

$$\hat{B}_t^D = \check{B}_t^D + \Lambda_{\mathcal{G}}^D(t),$$

where  $\check{B}_t^D$  is a  $\mathcal{G}_t^D$ -adapted standard  $\mathbb{Q}$ -Brownian motion and  $\Lambda_{\mathcal{G}}^D(t) = \int_0^t \lambda_{\mathcal{G}}^D(s) ds$  is a finite variation process governing the change of measure as market price of demand risk. The risk-neutral dynamics for  $D_t$  under the enlarged filtration now are:

$$dq_t = -\kappa^D \left( q_t - \frac{\sigma^D e^{\varphi(t)}}{\kappa^D} (\vartheta(t) + \lambda_{\mathcal{G}}^D(t)) \right) dt + \sigma^D e^{\varphi(t)} d\check{B}_t^D,$$

where conditional expectation  $\mathbb{E}^{\mathbb{Q}}[\cdot \mid \mathcal{G}_t]$  and variance  $\mathbb{V}^{\mathbb{Q}}[\cdot \mid \mathcal{G}_t]$  are then derived in the standard way. As outlined in Section 2.4, the market price of risk will be assumed constant and inferred from price quotes of traded derivative contracts. Depending on whether or not forward-looking information will be used, it will be referred to as  $\lambda_{\mathcal{G}}^D$  or  $\lambda_{\mathcal{F}}^D$ , respectively.

### 2.7.3 Parameter Estimates for Underlying Fundamental Factors

The following tables provide maximum likelihood estimates and corresponding robust standard errors relating to the processes for electricity demand ( $D_t$ ), available capacity ( $C_t$ ), and the price for the generating fuel, i.e., natural gas ( $g_t$ ). For further information on both the derivation of the discrete-time analogues corresponding to Equations (2.2), (2.6), and (2.9), and on the overall (standard) estimation procedure, see, e.g., Ait-Sahalia (2002), Schwartz (1997) or, more generally, Hamilton (1994).

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<sup>46</sup>Recall that we assume the filtration  $(\mathcal{G}_t^C)_{t \in [0, T^*]}$  to be of such nature that  $B_t^D = \hat{B}_t^D + A(t)$  is a semi-martingale.

Table 2.6: Maximum-Likelihood Parameter Estimates for Electricity Demand

This table reports Maximum-Likelihood parameter estimates and robust *t*-statistics (White (1982); [in brackets]) for the discrete-time equivalent of the electricity demand process as specified in Equations (2.2), (2.3), and (2.4). Parameters are estimated using five years of daily electricity demand data (in GW units), and are held constant throughout the following year for pricing purposes. Note that monthly seasonality is measured against January.

Period	2002–2006		2003–2007		2004–2008		2005–2009		2006–2010	
	Estimate	[t-stat]	Estimate	[t-stat]	Estimate	[t-stat]	Estimate	[t-stat]	Estimate	[t-stat]
$a^D$	56.05	[401.72]	56.78	[395.75]	57.25	[429.66]	57.90	[394.80]	57.80	[388.69]
$b^D$	0.36	[13.01]	0.01	[0.47]	-0.27	[-10.38]	-0.69	[-23.81]	-0.67	[-23.28]
$c_{WE}^D$	-6.67	[-76.15]	-6.49	[-73.94]	-6.35	[-74.57]	-6.24	[-71.85]	-6.08	[-70.76]
$c_{PH_1}^D$	-5.70	[-10.99]	-5.56	[-11.37]	-5.17	[-10.69]	-4.90	[-11.34]	-5.51	[-12.99]
$c_{PH_2}^D$	-12.21	[-12.88]	-11.80	[-13.56]	-11.38	[-14.37]	-11.35	[-14.39]	-11.29	[-12.58]
$c_{PH_3}^D$	-7.92	[-15.47]	-7.46	[-14.63]	-7.00	[-12.63]	-6.75	[-10.32]	-5.30	[-12.73]
$c_{PH_4}^D$	-6.82	[-16.45]	-6.41	[-15.13]	-6.05	[-15.70]	-5.63	[-17.39]	-5.37	[-20.13]
$c_2^D$	-1.36	[-7.43]	-1.10	[-5.87]	-1.10	[-6.36]	-1.29	[-6.58]	-1.62	[-7.92]
$c_3^D$	-4.54	[-17.21]	-4.37	[-16.20]	-3.98	[-15.68]	-4.51	[-17.39]	-5.09	[-20.68]
$c_4^D$	-10.95	[-62.34]	-11.00	[-57.07]	-10.63	[-59.30]	-11.04	[-56.50]	-11.88	[-58.90]
$c_5^D$	-12.87	[-81.37]	-12.81	[-79.57]	-12.82	[-85.86]	-13.02	[-83.28]	-13.51	[-83.13]
$c_6^D$	-13.80	[-94.34]	-13.55	[-92.88]	-13.49	[-99.37]	-13.52	[-92.65]	-14.09	[-93.23]
$c_7^D$	-13.83	[-94.59]	-13.66	[-90.73]	-13.57	[-99.13]	-13.68	[-89.38]	-14.19	[-89.93]
$c_8^D$	-14.14	[-94.00]	-14.00	[-92.23]	-13.88	[-101.70]	-14.10	[-95.22]	-14.72	[-96.25]
$c_9^D$	-12.17	[-72.44]	-11.81	[-67.16]	-11.57	[-71.56]	-11.75	[-66.99]	-12.34	[-66.55]
$c_{10}^D$	-7.43	[-34.45]	-7.15	[-35.94]	-7.13	[-39.65]	-7.25	[-37.10]	-7.77	[-37.62]
$c_{11}^D$	-1.72	[-9.22]	-1.30	[-6.75]	-1.24	[-6.57]	-1.53	[-7.58]	-2.15	[-10.53]
$c_{12}^D$	-0.17	[-0.67]	0.26	[1.07]	0.27	[1.08]	0.39	[1.64]	0.50	[1.96]
$\kappa^D$	148.20	[21.47]	137.92	[24.03]	141.93	[23.49]	131.14	[26.53]	125.12	[26.69]
$\sigma^D$	23.72	[40.55]	23.22	[39.64]	22.48	[40.85]	22.37	[41.89]	22.03	[41.90]
$\theta^D$	0.41	[10.43]	0.40	[9.82]	0.41	[10.25]	0.41	[10.48]	0.40	[10.88]
$\zeta^D$	0.21	[17.63]	0.21	[16.33]	0.21	[18.11]	0.22	[19.42]	0.23	[18.33]
LogLik	2984.32		2947.37		2890.18		2879.65		2851.13	

Table 2.7: Maximum-Likelihood Parameter Estimates for Available System Capacity

This table reports Maximum-Likelihood parameter estimates and robust  $t$ -statistics (White (1982); [in brackets]) for the discrete-time equivalent of the capacity process as specified in Equations (2.6) and (2.7). Parameters are estimated using five years of data for daily levels of available capacity (in GW units), and are held constant throughout the following year for pricing purposes. Note that monthly seasonality is measured against January.

Period	2002–2006	2003–2007	2004–2008	2005–2009	2006–2010
$a^C$	64.30 [245.80]	63.96 [243.35]	64.78 [342.91]	64.09 [388.07]	63.32 [391.66]
$b^C$	-0.07 [-1.28]	0.04 [0.56]	-0.28 [-6.00]	-0.37 [-10.60]	-0.07 [-2.14]
$c_{WE}^C$	-4.08 [-40.96]	-3.79 [-37.23]	-3.66 [-36.24]	-3.39 [-31.79]	-3.20 [-28.14]
$c_{PH_1}^C$	-3.97 [-10.19]	-4.11 [-9.77]	-3.45 [-7.80]	-3.23 [-6.85]	-2.91 [-5.10]
$c_{PH_2}^C$	-5.69 [-10.42]	-5.47 [-9.22]	-4.49 [-7.96]	-3.88 [-6.52]	-3.83 [-7.27]
$c_{PH_3}^C$	-1.56 [-2.75]	-0.92 [-1.54]	-0.58 [-1.02]	-0.87 [-1.28]	0.21 [0.35]
$c_{PH_4}^C$	-1.93 [-2.78]	-1.22 [-1.69]	-0.82 [-1.24]	-0.86 [-1.34]	-0.31 [-0.48]
$c_2^C$	-1.14 [-6.70]	-1.33 [-7.91]	-1.25 [-7.66]	-1.22 [-6.46]	-1.02 [-4.89]
$c_3^C$	-3.51 [-16.72]	-3.70 [-18.17]	-3.09 [-15.74]	-2.74 [-12.85]	-2.54 [-11.94]
$c_4^C$	-8.18 [-38.50]	-8.54 [-40.54]	-8.60 [-41.70]	-8.31 [-36.69]	-8.48 [-33.64]
$c_5^C$	-10.89 [-48.52]	-11.40 [-52.22]	-11.58 [-55.40]	-10.97 [-49.18]	-11.32 [-47.63]
$c_6^C$	-12.83 [-69.10]	-13.24 [-72.03]	-13.42 [-77.23]	-12.75 [-63.98]	-12.69 [-65.04]
$c_7^C$	-13.86 [-77.47]	-14.02 [-80.84]	-13.68 [-78.72]	-13.11 [-66.31]	-13.21 [-66.08]
$c_8^C$	-13.33 [-69.21]	-13.57 [-69.54]	-12.97 [-75.28]	-12.42 [-58.00]	-12.24 [-51.63]
$c_9^C$	-11.86 [-61.42]	-11.86 [-61.46]	-12.32 [-61.59]	-11.73 [-51.87]	-11.73 [-52.06]
$c_{10}^C$	-7.68 [-36.33]	-7.98 [-38.31]	-8.28 [-40.26]	-7.81 [-35.42]	-7.81 [-31.77]
$c_{11}^C$	-2.42 [-12.84]	-2.63 [-13.67]	-3.09 [-15.62]	-2.59 [-11.76]	-2.69 [-11.63]
$c_{12}^C$	-0.47 [-2.45]	-0.68 [-3.25]	-1.00 [-4.70]	-0.53 [-2.31]	-0.25 [-0.97]
$c_R^C$	-1.57 [-9.65]	-1.41 [-7.89]	-1.95 [-12.64]	-1.74 [-6.85]	NA [NA]
$\kappa^C$	137.82 [31.50]	157.42 [21.41]	157.42 [20.93]	148.00 [22.62]	138.23 [24.98]
$\sigma^C$	27.37 [55.87]	29.26 [32.27]	29.24 [31.05]	29.94 [31.64]	30.88 [32.86]
LogLik	3245.46	3369.11	3370.00	3411.43	3467.47

Table 2.8: Kalman Filter Parameter Estimates for Natural Gas

This table reports parameter estimates and robust  $t$ -statistics (White (1982); [in brackets]) for the natural gas price process as specified in Equations (2.9) and (2.10), using the Kalman filter and maximum likelihood estimation. Parameters are estimated based on five years of daily natural gas futures price data (using 1-, 2-, 3-, and 4-months ahead contracts), and are held constant throughout the following year for pricing purposes. Note that monthly seasonality is measured against January.

Period	2002–2006		2003–2007		2004–2008		2005–2009		2006–2010	
	Estimate	$t$ -stat								
$a^9$	0.11	[7.70]	-0.09	[-2.77]	-0.11	[-3.88]	0.09	[4.57]	-0.16	[-8.88]
$b^9$	0.56	[9.52]	1.04	[9.07]	-0.08	[-1.26]	-0.08	[-2.91]	-0.31	[-6.22]
$c_2^9$	-0.07	[-34.83]	-0.07	[-33.30]	-0.06	[-29.82]	-0.05	[-32.98]	-0.04	[-24.26]
$c_3^9$	-0.25	[-78.69]	-0.22	[-62.19]	-0.20	[-48.47]	-0.16	[-44.95]	-0.12	[-36.66]
$c_4^9$	-0.41	[-96.57]	-0.38	[-78.62]	-0.34	[-64.59]	-0.31	[-60.07]	-0.24	[-53.62]
$c_5^9$	-0.50	[-107.85]	-0.47	[-91.46]	-0.43	[-75.92]	-0.40	[-74.14]	-0.32	[-60.91]
$c_6^9$	-0.56	[-116.13]	-0.52	[-97.74]	-0.49	[-82.69]	-0.45	[-81.02]	-0.36	[-65.48]
$c_7^9$	-0.59	[-117.36]	-0.56	[-102.07]	-0.53	[-87.58]	-0.50	[-88.41]	-0.40	[-70.95]
$c_8^9$	-0.58	[-108.87]	-0.54	[-92.92]	-0.52	[-81.27]	-0.49	[-84.04]	-0.40	[-66.70]
$c_9^9$	-0.64	[-123.86]	-0.61	[-103.24]	-0.58	[-90.65]	-0.54	[-94.63]	-0.44	[-70.82]
$c_{10}^9$	-0.41	[-80.88]	-0.40	[-67.10]	-0.39	[-62.47]	-0.35	[-64.60]	-0.28	[-55.10]
$c_{11}^9$	-0.20	[-42.95]	-0.18	[-32.71]	-0.17	[-28.83]	-0.13	[-26.62]	-0.09	[-25.51]
$c_{12}^9$	-0.08	[-20.91]	-0.07	[-16.29]	-0.06	[-14.46]	-0.04	[-12.61]	-0.02	[-10.82]
$\kappa^9$	0.26	[5.13]	0.06	[5.87]	0.13	[6.45]	0.29	[10.68]	0.23	[10.80]
$\lambda^9$	3.72	[19.75]	7.54	[7.70]	7.74	[12.46]	5.63	[28.02]	6.38	[24.20]
$\sigma^9$	0.47	[13.28]	0.49	[15.67]	0.49	[15.55]	0.55	[15.34]	0.53	[16.51]
LogLik	11067.23		10535.82		10357.80		10749.18		10972.14	

# Chapter 3

## Electricity Spot and Derivatives Pricing when Markets are Interconnected<sup>\*</sup>

### 3.1 Introduction

In the aftermath of market liberalization, energy markets in the US and Europe have been undergoing a number of significant structural developments and institutional changes that strongly affect the interplay of supply and demand, and hence, the general price formation process in these markets. In the case of electricity, regulatory developments, such as the introduction of emissions trading schemes, but also other aspects of market design, such as power exchanges admitting negative prices, have had a marked and long-lasting impact on the price dynamics of both spot and derivative contracts. Recently, in both US and Western European electricity markets, these evolutions have been overlaid with profound changes in the structure of the supply side that are primarily caused by a general, continued shift towards generation from renewable energy sources.

Since then, in view of rising shares of solar and wind capacities in many countries, the questions of (i) how to best integrate renewable generation into the networks and

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<sup>\*</sup>This chapter is based on the corresponding working paper entitled “*Electricity Spot and Derivatives Pricing when Markets are Interconnected*” co-authored with Roland Füss and Marcel Prokopczuk. Cf. Füss et al. (2013b). The paper has been presented at the SIRE Conference on “Finance and Commodities” 2013, St.Andrews, and in the PiF Seminar 2013 at University of St.Gallen. Status: submitted to *Operations Research*.

(ii) how to mitigate challenges relating to its intermittency and overall variability have ranked high on the agenda of system operators, regulatory authorities, and respective policymakers. However, given that electricity flows are governed by the physical laws of *Kirchhoff* and *Ohm* rather than by the boundaries of geographic markets or otherwise defined price zones, the issue of integrating renewable generation should not be dealt with on a *national* level only, but instead is intimately related to the *interconnectivity* of adjacent electricity markets throughout Europe or the US as a whole. Hence, when assessing future target levels for generation from renewables,<sup>1</sup> it is important to see that these are inherently tied to another aim, i.e., to enable a reliable and efficient transfer of “green” electricity across well-integrated markets and regions in the first place.

Within the EU, reaching this aim coincides with the intended establishment of the “Internal Electricity Market” (IEM), an initiative to support further integration of European electricity markets, with the ultimate goal of achieving full electricity price convergence across member states.<sup>2</sup> Fostering integration between national markets, in turn, requires substantial investments into transmission infrastructure both *within* and *across* national markets. Already in March 2002, the Barcelona European Council agreed on a minimum level of interconnectivity between member states of 10% of installed generation capacities within the respective markets. In this context, the European Network of Transmission System Operators for Electricity (ENTSO-E) estimates that for investment projects of “pan-European significance,” a total capital expenditure of approx. EUR 104bn will be required until 2022 (ENTSO-E, 2012).

However, for the aim of creating a single electricity market across Europe, a well-defined and functioning market model of how to provide access to cross-border transmission capacity is at least as important as further investment into transmission infrastructure. Generally, only few interconnectors in Europe are uncongested<sup>3</sup> and over time, a variety of different congestion management methods have been developed to govern access to scarce transmission capacity for cross-border trade of electricity. In the past, these allocation mechanisms have primarily relied on explicit ex-ante schemes, where traders *first* have to acquire transmission capacity in order to *then* arbitrage

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<sup>1</sup>For instance, the European Union (EU) targets a 20% share for renewable energy sources within its energy mix by the year 2020.

<sup>2</sup>See, e.g., Article 60(2) of Directive 2009/72/EC: “(...) *leading, in the long term, to price convergence.*”

<sup>3</sup>For instance, this is the case between Germany and Austria, where transmission capacity is sufficient and no auctioning of rights for interconnector use is required.

two interconnected, yet institutionally separated electricity markets. In Europe, in their most widespread form, explicit schemes are usually implemented as (sealed-bid) auction processes whereby capacity is allocated for different timeframes.<sup>4</sup> While explicit ex-ante auctions of transmission capacity meet the requirement by the EU that access be provided based on “non-discriminatory market based solutions,”<sup>5</sup> they nevertheless lead to an inefficient market design: amongst a number of deficiencies, it is primarily the timing sequence of capacity and electricity spot markets that forces traders to acquire cross-border transmission capacity for a given direction before the spread in electricity spot prices between the two respective markets is actually determined. Hence, a trader’s decision about both the direction and the amount of transmission capacity to be requested can only be based on an expectation of the spread, and in view of generally high levels of volatility in electricity spot markets, this noisy signal may often cause him to acquire capacity for the wrong direction.

Since these inefficiencies conflict with the objective of (day-ahead) price convergence within a single pan-European electricity market, the more efficient alternative of implicitly allocating cross-border capacity rights via market coupling is increasingly rolled out across European markets nowadays. Here, markets for transmission capacity and spot electricity are integrated and, hence, clear simultaneously, which allows for an optimal allocation of capacities and results in economically efficient cross-border flows. However, although the previous explicit ex-ante design has already been replaced by an implicit mechanism for a number of interconnectors, it will continue to play an important role wherever the harmonization efforts required for market coupling are too high or just infeasible.<sup>6</sup>

In the US, market architecture for most regions is fundamentally different from the uniform (zonal) pricing approach that is prevalent in Europe: instead, nodal pricing (or locational marginal pricing; LMP) has become the standard pricing approach.<sup>7</sup> In

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<sup>4</sup>Prior to the implementation of market coupling, explicit auction schemes prevailed throughout Europe and were (or still are) used to allocate capacity for exchange between, e.g., England – France, France – Spain, France – Belgium, Belgium – the Netherlands, or the Netherlands – Germany.

<sup>5</sup>See Regulations (EC) No. 1228/2003 and its follow-up No. 714/2009. Note that this rules out the use of other cross-border congestion management methods, such as rationing or allocation on a “first-come, first-served”-basis, given their lack of an inherent market-based mechanism.

<sup>6</sup>Also see, e.g., McInerney and Bunn (2013) on this argument. Moreover, should the market coupling algorithm (that determines the optimal cross-border flows) not be available due to technical problems, the explicit ex-ante scheme will be used as default option to allocate capacity.

<sup>7</sup>See, e.g., Bohn et al. (1984), Scheppe et al. (1988), and Hogan (1992) for an overview. Nodal pricing (in different forms) has been implemented, e.g., in California (CAISO), Texas (ERCOT), in the Midwest (MISO), New York (NYISO), and New England (ISO-NE) markets, in the PJM Interconnection, and in the Southwest Power Pool (SPP).

an LMP-based market, the market area is subdivided into numerous pricing points, or nodes, for each of which an individual marginal electricity price is calculated. For instance, in the day-ahead market, participants submit bids and offers for specified point-to-point transactions which are then aggregated and matched by the central market administrator that clears the market. Thus, both day-ahead prices and corresponding flows are determined simultaneously and in the case of no congestion, marginal prices will be equal at each node. However, it is important to note that this approach optimally addresses transmission constraints *within* markets only, whereas the above mentioned problems in efficiently setting up cross-border transactions in Europe also apply for transactions *between* markets in the US. For instance, if a trader wants to arbitrage the PJM and NYISO markets, his transaction still needs to be based on an expectation of the price spread between the two markets so that less-than-optimal or even adverse interconnector flows are a common problem on both sides of the Atlantic.

In the future, interconnectivity within the still fragmented US electricity grid is generally expected to increase further – e.g., through projects such as the planned “Tres Amigas Superstation,” a bundle of three 5-gigawatt (GW) transmission lines that are supposed to unite the three regional grids in the US into a single national grid.<sup>8</sup> While the market design for this emerging huge trading hub is still being worked out, a solution based on implicit allocation, i.e., market coupling, may likely be expected.<sup>9</sup>

In this chapter, we show both empirically and theoretically that the question of explicitly versus implicitly allocating cross-border transmission capacity induces important microstructure effects between interconnected electricity markets,<sup>10</sup> which in turn has significant implications for pricing and hedging in these markets. Both the pace and scope of the above mentioned structural changes in electricity markets pose considerable challenges to market participants and, hence, require to use increasingly sophisticated models that are capable of adequately reproducing such changes in the

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<sup>8</sup>Note that a grid or interconnection is defined as comprising several markets (such as PJM, NYISO, ISO-NE etc. for the Eastern Interconnection) that are electrically tied together and operate at a synchronized frequency. Currently, the three regional grids – the Eastern Interconnection, the Western Electricity Coordinating Council (WECC), and the Electric Reliability Council of Texas (ERCOT) – are only loosely tied together, which impedes, e.g., the transfer of “green” energy generated from wind in Texas or sun in Arizona to centers of high demand within the Eastern Interconnection.

<sup>9</sup>In 2012, Tres Amigas LLC and EPEX Spot have announced a joint cooperation agreement in order to “share mutual expertise in the development and coupling of their respective markets” (EPEX-Spot, 2012). EPEX Spot operates, amongst others, the coupled electricity spot markets in Germany and France.

<sup>10</sup>Unless otherwise stated, when using the term “explicit allocations,” we always refer to explicit *ex-ante* allocations.

“allocation regime.” Moreover, since traditional reduced-form models for electricity pricing (although popular) are unable to capture these effects, model risk increases even further, leaving previously used pricing approaches unreliable in many instances.

As such, we contribute to the literature in the following ways: first, we provide substantial empirical evidence of how spot and futures prices are impacted by the different ways to organize cross-border exchange between electricity markets. Based on both interconnector flows and corresponding electricity prices in adjacent markets, we contrast the above mentioned explicit and implicit allocation schemes and examine how the workings of either mechanism are reflected in empirical price dynamics.

Second, to the best of our knowledge, we are the first to analyze different set-ups for cross-border trade of electricity in a fully stochastic-dynamic setting, based on fundamental state variables such as demand and fuel prices for each market.<sup>11</sup> More precisely, we propose a fundamental two-market model where, due to its granular structure, the influence of the above aspects of market design on the price formation mechanism can be mimicked and re-produced in a realistic way; thus, this setting not only addresses the shortcomings of reduced-form approaches, but also enables us to study in detail the interplay between the different “allocation regimes” and the ensuing dynamics for electricity spot and derivative prices. More generally, our framework allows for analytic pricing formulae for futures (and also options) under both allocation regimes, which adds to its general applicability and helps to retain tractability and ease of use for practitioners.

Third, we show how the most important *stylized facts* of electricity spot price dynamics – price spikes along with high levels of volatility – are altered through the introduction of market coupling in two adjacent markets. More precisely, our model both reproduces and allows to further investigate empirically observed facts such as a “volatility reduction effect” and a general softening of price spikes through market coupling.<sup>12</sup> Taking a risk management perspective, we also show that interconnectivity of electricity markets can strongly impact the term structure of futures prices, such as reversing curves from backwardation into contango and vice versa.

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<sup>11</sup>Note that while different ways of organizing cross-border trade of electricity have been thoroughly examined in the literature (see, e.g., Ehrenmann and Smeers, 2005), these analyses often rely on an exogenous deterministic and/or non-sequential setting.

<sup>12</sup>For instance, on 16 June 2013, given a combination of low demand and high levels of non-flexible generation, negative baseload prices were prevailing throughout the French, Belgian, and German day-ahead electricity markets. Since the surplus was particularly high in France, the market coupling mechanism did not achieve price convergence with neighbouring markets as flows required to equalize prices were exceeding interconnector capacities. However, as noted by EPEX Spot on that day, “(...) *Market Coupling helped to absorb the price peaks despite the low price convergence*” (EPEX-Spot, 2013).

Finally, the basic idea of directly reflecting key aspects of capacity allocation mechanisms in the price dynamics of the respective commodity can easily be transferred into a structural setting where gas pipeline capacities, storage access, or also bandwidth for data transfer have to be acquired prior to (or simultaneously with) commodity markets clearing. Hence, the general structure of our model can also serve as an important benchmark case for other network industries such as natural gas or telecommunications.<sup>13</sup>

The remainder of this chapter is structured as follows: the next section outlines the workings of the explicit and implicit schemes in more detail and provides further institutional background for cross-border trading of electricity. Section 3.3 continues the discussion of the two allocation regimes from an empirical point of view by examining the price dynamics of several coupled and non-coupled electricity markets in Europe and the US. Section 3.4 develops the fundamental two-market model and shows how both explicit ex-ante and implicit allocation mechanisms for capacity rights can be accommodated within this framework. Numerical simulations and comparative-static analyses are employed to confirm that the model is capable to reproduce the key stylized facts of interconnected electricity markets. Section 3.5 concludes.

## 3.2 Institutional Background

The organization of cross-border trade of electricity, by its nature, is characterized by a variety of institutional specificities and operational complexities. In the following, to lay the grounds for a detailed analysis of the above mentioned allocation mechanisms and their impact on electricity price dynamics, we will therefore only focus on the key conceptual ideas of each regime, but leave aside other institutional details and technicalities that are not necessary to motivate our modeling framework.

### 3.2.1 Explicit Allocation Schemes

The generally widespread implementation of explicit allocation mechanisms is mainly due to the fact that they can quickly be set up without requiring a particularly high level

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<sup>13</sup>See, e.g., McDaniel (2003) and Stern and Turvey (2003) for further information and a general overview on capacity auctions in network industries. With respect to natural gas, note moreover that the general idea of electricity market coupling in Europe is currently planned to be transferred to the still fragmented gas markets where first coupling arrangements have recently been proposed (and partially implemented) by several European TSOs and gas exchanges. For instance, for the PEG Nord & Sud zones in France (which are linked by a physical bottleneck), a gas market coupling project was implemented in July 2011.

of institutional harmonization between adjacent electricity markets. Importantly, as is characteristic for explicit schemes, physical transmission capacity markets and electricity (spot) markets are separated from each other, with the first usually clearing ahead of the latter. Hence, enabling cross-border trade in this way only requires a low degree of integration and joint coordination between both transmission system operators (TSOs) and power exchanges in adjacent markets. For instance, this avoids the necessity for a common trading platform and uniform exchange rules including a simultaneous closing of day-ahead auctions for (spot) electricity across markets. However, given that day-ahead prices in both markets are not yet known by the time when traders have to submit their bids for transmission capacity, this generally leads to an inefficient market design.<sup>14</sup> These inefficiencies manifest in interconnectors frequently not being used up to their thermal capacity limits or in adverse physical flows where electricity is directed from a more expensive market into a cheaper connected market. In such a situation, cross-border traders have failed to correctly anticipate the price spread *ex-ante*, and hence end up having acquired transmission capacity for the “wrong” direction.

From an operational point of view, for most interconnectors, explicitly allocated transmission rights are physical in nature and are assigned for a pre-specified direction of flow (e.g., F → GER) and for different timeframes. For instance, prior to the introduction of market coupling at the French–German border, physical transmission rights (PTRs) could be acquired via explicit annual, monthly, and daily auctions. Depending on the timeframe, allocated transmission rights would hence entitle a trader to nominate and physically exchange electricity in a given direction from one market to the other during any hour of the year (month or day, respectively). Figure 3.1 illustrates in more detail the relevant steps to be taken in order to set up such a cross-border transaction, taking as example the explicit *ex-ante* auction scheme at the French–German border before it was replaced by market coupling in 2010.<sup>15</sup> When the holder of a periodic (i.e., monthly or yearly) transmission right wanted to engage in a cross-border transaction on day  $t$ , the TSO had to be notified by 08:15am on day  $t - 1$  and provided with a schedule detailing the respective capacities to be nominated during every hour on day  $t$ .<sup>16</sup> In case these capacity

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<sup>14</sup>This is a well-documented fact in the literature. Inefficient cross-border trade across various European markets has been analyzed by, e.g., Turvey (2006), Kristiansen (2007), Marckhoff and Muck (2009), Bunn and Zachmann (2010), Bunn and Martoccia (2010), or McInerney and Bunn (2013).

<sup>15</sup>Although auction rules may differ across markets, the overall structure and timeline of the allocation procedure described here can generally be seen as representative for other explicit *ex-ante* schemes.

<sup>16</sup>See RTE (2009a), p. 17, for transactions between Germany and France.

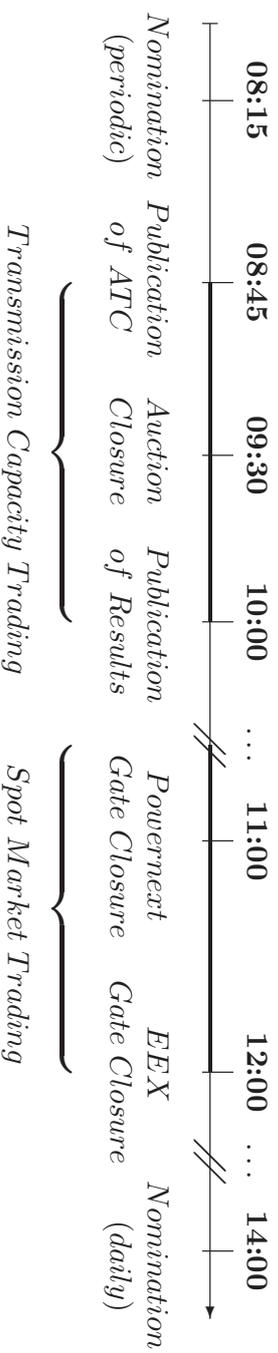


Figure 3.1: **Timeline for the Explicit Ex-Ante Auction Scheme**

*This figure illustrates the timing sequence of the market for cross-border transmission capacity between France and Germany as well as of the day-ahead electricity markets in these two countries. Note that the nomination procedure, the setup of the capacity auction, and the gate closure times of the power exchanges relate to the explicit ex-ante auction scheme which was in place prior to the implementation of the CWE (Central Western Europe) market coupling as per 09-Nov-2010. See RTE (2009a) and RTE (2009b) for further information.*

holders did not want to trade, their non-nominated capacities originally “earmarked” for periodic transactions were implicitly transferred and added to those capacities to be allocated in the daily market. In that case, according to the “use it or sell it” (UIOSI) principle, traders with monthly or yearly rights received as financial compensation for every megawatt (MW) of non-nominated capacity the price as determined during the daily auction for the corresponding hour.<sup>17</sup> Based on the nominations received by the holders of monthly and yearly transmission rights, the TSO then published an update of the available transmission capacity (ATC) that was to be offered on the daily market by 08:45am on day  $t - 1$ ; bids for the daily auction then had to be submitted by 09:30am at the latest, and results were released half an hour later. As already mentioned, this is well ahead of the close of electricity spot (i.e., day-ahead) markets at around noon: 11:00am for French exchange Powernext and 12:00pm (noon) for its German counterpart EEX at that time. Finally, having been awarded a transmission right in the daily market, traders had to notify the TSO by 02:00pm whether they were going to use it and set up a cross-border transaction on the following day  $t$ . As opposed to the UIOSI principle, for holders of daily transmission rights, however, the “use it or lose it” (UIOLI) principle applied instead. I.e., in the case of non-nomination, there was no financial compensation; these unused rights were then, in turn, transferred to the intraday market.

In the US, the institutional design for cross-border (i.e., market-to-market) transactions is less harmonized than in Europe and a variety of different allocation methods coexists, sometimes even at the same border. While a detailed description of these is beyond the scope of this article, we can focus on the following key issues: in view of the nodal pricing system in the US, combining paths of fixed point-to-point transactions would generally allow a trader to ship electricity across states with corresponding path reservations being allocated on a “first-come, first-serve” basis. Given the inherent inefficiency of these fixed allocations, many markets have replaced the physical reservations with a financial transmission rights framework and offer-based export/import scheduling (Spees and Pfeifenberger, 2012). Alternatively, several *merchant* lines have also been awarded transmission rights which are then re-allocated via long-term contracts or shorter-term auctions.<sup>18</sup> Nevertheless, irrespective of the exact institutional setting, bid selection and pricing by the independent system operator (ISO) and the physical

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<sup>17</sup>See RTE (2009b), p. 20.

<sup>18</sup>Examples include the Hudson and Linden VFT interconnectors at the PJM-NYISO border.

scheduling of a market-to-market transaction are still two clearly distinct procedures for most electricity markets in the US. Hence, whereas other non-auction based allocation mechanisms may be more common in the US, these mechanisms share a crucial feature of explicit schemes, i.e. of requiring traders to schedule their transactions ex-ante. Even though there are increasing efforts to move the physical scheduling process closer to real-time,<sup>19</sup> traders still need to rely on an expectation of the price spread between markets, which is – roughly speaking – comparable to the explicit allocation schemes used in Europe.

### 3.2.2 Implicit Allocation Schemes

By contrast, the concept of market coupling avoids the above timing problem for cross-border traders by clearing capacity and electricity markets simultaneously, i.e., by implicitly allocating transmission rights within the spot auction of electricity in each market. Conceptually, a pricing mechanism that is closely related to market coupling has long since been implemented in the Nordic market, Nord Pool, the common electricity market for Norway, Denmark, Sweden, Finland, Estonia, and Lithuania. Here, in a first step, the market clearing system price is determined by equating total demand and supply, as aggregated across each national market, thereby ignoring any potential constraints in transmission capacity between bidding areas.<sup>20</sup> Hence, in case of no congestion, there is a single price for all bidding areas. Otherwise, depending on the location of the transmission bottleneck(s), the market is “split” into several pricing zones that are assigned different area prices, and that comprise one or more bidding areas.

The results of the “market splitting” mechanism are in general similar to what is achieved by the variant of market coupling that is currently implemented between major Western European markets, and that is examined in this study. Yet, in contrast to Nord Pool, it is not a single power exchange that determines (in a “top-down” way) whether to subdivide or retain a single pricing zone across markets. Instead, market

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<sup>19</sup>For instance, the PJM and NYISO markets have recently proposed “Coordinated Transaction Scheduling” (CTS) whereby traders have the option to submit a price spread at which they are willing to set up a market-to-market transaction rather than placing individual bids in each market. While supposed to enhance economically efficient interconnector use, transactions still need to be scheduled based on an expectation of the spread.

<sup>20</sup>In the Nordic market, national markets are further subdivided into one or more *bidding areas*, reflecting transmission bottlenecks both within and across national markets. For instance, the Danish market is subdivided into two bidding areas (Western and Eastern Denmark) which are linked by a 600 MW interconnector that is frequently congested.

coupling is characterized by a more de-centralized (“bottom-up”) approach where the joint coordination effort between all involved national TSOs and power exchanges determines which national markets are “coupled” with each other to form a single pricing zone.

Specifically, for every day and each interconnector, TSOs determine the amount of available transmission capacity (ATC) for which the corresponding transmission rights are no longer allocated explicitly but instead assigned to the respective power exchanges in the adjacent markets. In order to achieve spot price convergence, a joint optimization algorithm considers the net import/export positions of each participating exchange for a given clearing price and thus determines the optimal cross-border flows. As such, a buy-order in a higher-priced market can be matched with a sell-order in a cheaper adjacent market, thus mitigating the price differential with any resulting cross-border flows implicitly covered by the transferred capacity rights. As shown by Meeus et al. (2009) or Weber et al. (2010), the underlying optimization problem can be re-formulated as maximizing overall welfare, constrained by interconnector capacities and other real-time limitations. If one of these constraints is binding – e.g., if flows necessary to equalize prices exceed the ATC –, then prices cannot fully converge and the resulting congestion rent is collected by the owner of the interconnector.

The previously outlined explicit setup at the French–German border was modified when Germany joined the already existing market coupling between France, the Netherlands, and Belgium (the so-called “Trilateral Coupling”) on 09-Nov-2010, marking the starting point for the Central Western Europe (CWE) day-ahead market coupling. As a major change to the pre-existing setting, an explicit allocation of transmission rights for interconnector access is henceforth only held for monthly and yearly contracts, whereas on a day-ahead stage, all cross-border transmission capacities are available to cover the market coupling flows.<sup>21</sup> Following the harmonization of day-ahead gate closure times (12:00pm noon) at all involved power exchanges, the market coupling algorithm then starts to determine the optimal cross-border flows at around 12:05pm. Finally, the resulting day-ahead prices are published by the CWE power exchanges by 12:55pm.<sup>22</sup>

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<sup>21</sup>Note that the transmission capacity of an interconnector is usually “sliced” and allocated for different timescales. Thus, off-exchange transactions, such as customized OTC cross-border trades, can still be executed by traders acquiring monthly or yearly transmission rights, whereas all remaining interconnector capacity can be used by power exchanges for market coupling purposes. Importantly, as pointed out by Meeus (2011), retaining such a split helps to avoid a potential monopolization of the entire organization of cross-border trade of electricity by the power exchanges participating in market coupling.

<sup>22</sup>See <http://www.marketcoupling.com/about-emcc/daily-operations>.

### 3.3 Empirical Analysis:

#### Market Design and Price Dynamics

##### 3.3.1 Ex-Ante Scheduled Transactions in Europe and the US

The general economic inefficiency of explicit ex-ante schemes is analyzed in Figure 3.2 where the LHS graph plots the spread between German and French day-ahead power prices against corresponding net transit nominations as percentage of available interconnector capacity. Clearly, not only are interconnector capacities oftentimes left (partially) unused – although a non-zero price spread in an efficient market setting would call for additional cross-border flow until the price spread is fully exploited or the maximum interconnector capacity is reached. Even worse, out of all nominations during the observation period (26-Oct-2006 to 09-Nov-2010), approx. 32% led to flows in the wrong direction altogether.

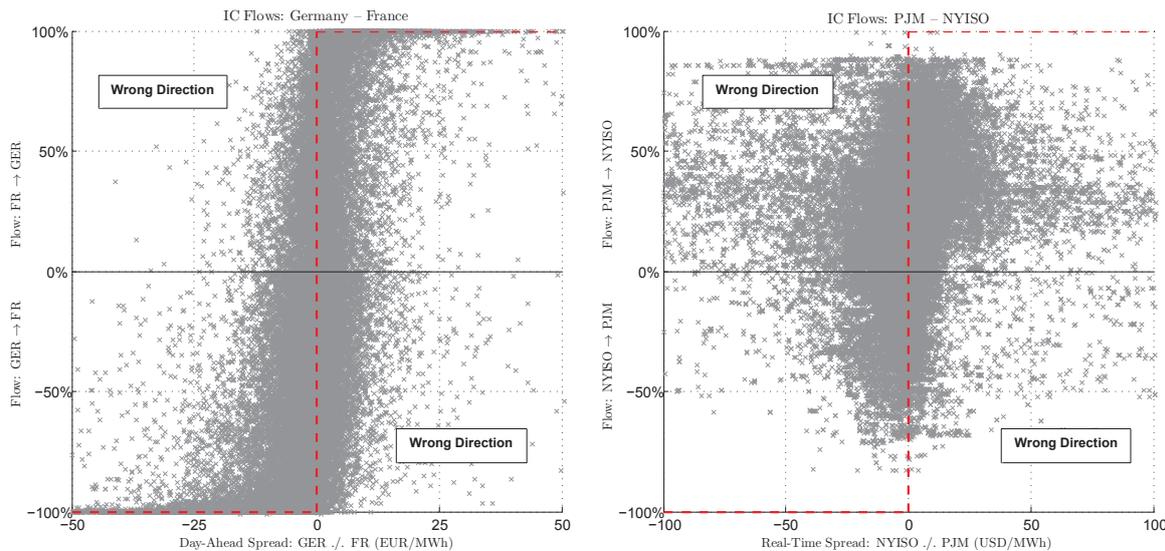
On a more detailed level, the data also reveal that during both peak- and off-peak hours, French day-ahead power prices exceed German prices more frequently than vice versa – or, more precisely, during approx. 58% of all hourly periods.<sup>23</sup> This pattern, in turn, also seems to impact the nomination behavior by cross-border traders: conditional on the spread being negative, i.e. for French day-ahead prices exceeding German prices, net interconnector flows in the economically correct direction (i.e., to the higher-priced market) can be observed during approx. 82% of all hourly peak- and off-peak periods.<sup>24</sup> By contrast, given a positive day-ahead spread (due to German prices being higher than their French counterparts), the share of net interconnector flows in the right direction is disproportionately lower: correct flows can only be observed during approx. 49% of all hours. Hence, market participants tend to bet on a negative spread, with a positive spread not only being less frequently observed but apparently also harder to predict correctly.

This diverse picture also manifests as regards the degree of *net* capacity utilization: on an overall basis, interconnectors are used at slightly less than 50% of their thermal capacity when French power prices exceed German power prices. For the opposite case, average *net* capacity utilization even gets driven down to less than 10% since, as mentioned above, adverse flows are observed more frequently. Disaggregating further, we can observe

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<sup>23</sup>Also note that the French–German interconnection, as a series of multiple alternating current (AC) links, has a higher capacity for GER→F than for F→GER.

<sup>24</sup>Note that in practice, traders may schedule more flows for those hours where they can be sure to properly anticipate the price differential between the markets. Hence, on a volume-weighted basis and conditional on a negative spread, even 93% of all exchanged MWs have flown in the correct direction.



**Figure 3.2: Interconnector Flows: Germany – France and PJM – NYISO**

This figure illustrates inefficient interconnector use in both Europe and the US. The LHS graph plots the day-ahead price spread between Germany and France versus net transit nominations (as percentage of interconnector capacity) for the same hours. Data shown for the period from 26-Oct-2006 to 09-Nov-2010. The RHS graph plots the real-time price spread between the PJM and NYISO markets versus 5-minute scheduled flows (as percentage of interconnector flow limits). Data shown for the period from 01-Jan-2013 to 30-Jun-2013. The red dashed line indicates economically efficient flows (as would have occurred under market coupling) such that there is no flow in the wrong direction, nor less-than-optimal flow in the right direction. Source: RTE (<http://clients.rte-france.com>), NYISO (<http://www.nyiso.com>), and Bloomberg.

that for a price spread of up to 5 EUR/MWh (in either direction) and excluding all adverse flows,<sup>25</sup> French–German interconnectors are still used at only slightly more than 40% of their thermal capacity. For the complementary case where only price spreads larger than 5 EUR/MWh are considered, “gross” capacity utilization for flows in the economically correct direction increases to 69%. Clearly, a comparably lower (absolute-level) spread between day-ahead prices increases the likelihood for traders of having predicted it wrong during the capacity auction and of hence having to engage in an unprofitable trade, which ultimately reduces both gross and net capacity utilization.

Similar results can be found when analyzing interconnector flows between the PJM and NYISO markets in the US during the first half of 2013, as shown in the RHS graph of Figure 3.2.<sup>26</sup> Here, we see that economically inefficient interconnector flows are not

<sup>25</sup>By contrast, the previous two examples related to *net* capacity utilization where adverse flows are included: I.e., for the theoretical case of an interconnector being used up to its full thermal capacity in both directions, resulting net capacity utilization would be 0%.

<sup>26</sup>For the actual real-time price spreads, we use the “NY Keystone” (proxy bus) and “PJM NYIS” prices available via Bloomberg. Five-minute scheduled interconnector flows relate to the PJM/NYISO interface (i.e., excluding the merchant Linden VFT and Hudson lines as well as the Neptune Underwater Transmission Line). Data shown for the period from 01-Jan-2013 until 30-Jun-2013. See [www.monitoringanalytics.com](http://www.monitoringanalytics.com) for further information.

only present in day-ahead markets but also when gate closure is moved closer to real-time (currently 75 minutes prior to the operational hour): in 44% of all 5-minute intervals, inefficient flows from the more expensive into the cheaper market could be observed. As regards (net) capacity utilization, the RHS graph clearly shows that the system operator includes a safety margin of approx. 10% of transfer capacity (which may vary over time and for which our data is not adjusted).<sup>27</sup> However, even abstracting from this, capacity utilization seems to be less endogenously driven by price differentials than in the LHS graph. In fact, imports into NYISO prevail (81% of all flows) although positive price spreads (economically justifying an import) were only observed in 51% of all cases.

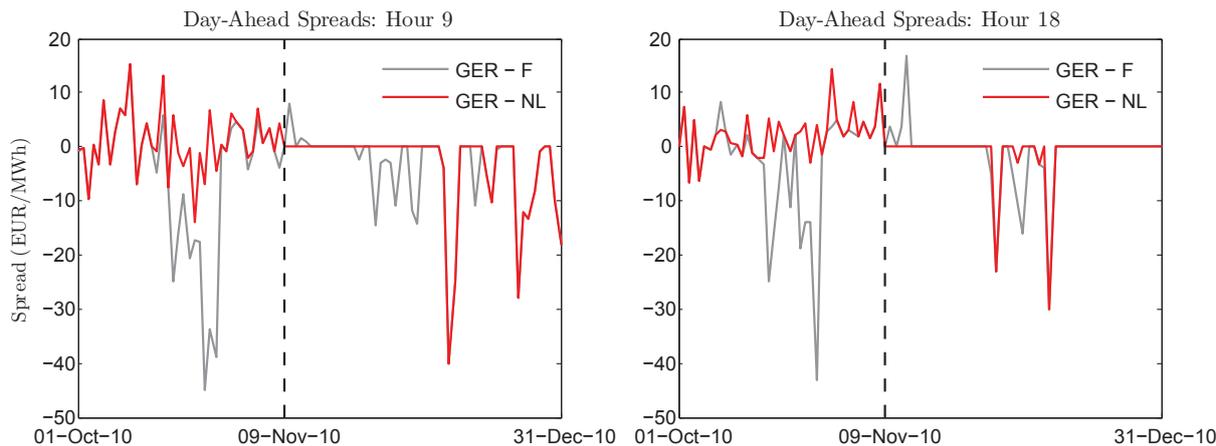
### 3.3.2 Spot and Futures Prices under the CWE Market Coupling

The start of the CWE market coupling has fundamentally changed the dynamics of the day-ahead price spreads between Germany–France and Germany–Netherlands, respectively, as Figure 3.3 clearly illustrates for hours 9 and 18 of the day: for both hourly periods, with few exceptions at mid-November 2010, the spreads are mainly negative or zero, providing clear evidence for interconnector flows henceforth taking economically correct directions – i.e., from the then mainly lower-priced German market to the higher-priced Dutch and French day-ahead markets. The data show that until year-end 2010, *exact* price convergence was reached in 81% of all hourly periods for the German–Dutch day-ahead spread and in approx. 52% of all hours for the German–French spread. Note however, that for specific hours of the day, these percentages can vary significantly. For instance, for the German–French spread over the same period, German day-ahead prices during hour 24 of the day tend to clearly fall below French prices, so that price convergence could only be reached during less than 23% of all hours. By contrast, for hour 17 (18), a zero price spread was observed during 80% (74%) of all periods.

These patterns also manifest when analyzing the *long-term* behavior of the day-ahead spread between the German and the Dutch electricity markets, as illustrated in Figure 3.4. For the spread between the prices of electricity to be delivered in both markets during hour 12 (11:00am - 12:00pm/noon) on the next day, Figure 3.4 shows that since the introduction of the CWE market coupling, positive price spreads were no longer observed at all – with very few, notable exceptions: On 27 March 2011, after the close of the day-ahead auction

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<sup>27</sup>In order to guarantee operational security, a fraction of the interconnector capacity is usually reserved in order to be able to quickly react to contingencies.



**Figure 3.3: Spread in Day-Ahead Prices: GER – F and GER – NL**

*This figure shows the spread in day-ahead electricity prices between Germany and France as well as Germany and the Netherlands. The LHS graph displays day-ahead prices for hour 9 of the day (8:00–9:00am), whereas the RHS graph relates to hour 18 (5:00–6:00pm). The vertical dashed line marks the start of the CWE (Central Western Europe) market coupling between Germany, France, the Netherlands, and Belgium as per 09-Nov-2010. All data sourced from Bloomberg.*

for the following day, the market coupling optimization algorithm could not be run due to a bug in the system which was ultimately caused by the change to daylight-saving time on that day. Consequently, as a fallback mechanism in the case of such a de-coupling, *explicit* (“shadow”) auctions had to be organized in order to allocate the available daily transmission capacity.<sup>28</sup> For the remaining days, however, the spread was either zero or negative, implying that whenever available transmission capacity is insufficient in order to reach price convergence between the two markets, this applies to flows for the direction GER→NL, but not vice versa. More recently, negative spreads have been slightly more prevalent, thus driving down the rate of price convergence, which can be attributed to the steadily growing renewables feed-in in Germany, combined with continued strong levels of natural gas prices affecting the gas-based Dutch electricity market.<sup>29</sup>

Table 3.1 summarizes the above qualitative observations by providing descriptive statistics for the German–Dutch day-ahead spread during selected hours of the day and

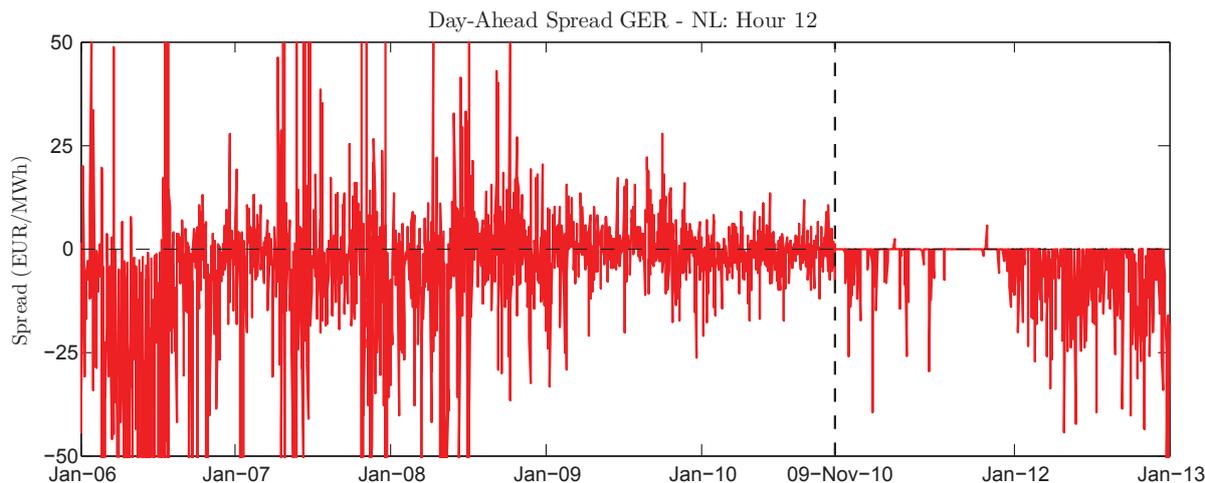
<sup>28</sup>See, e.g., EPEX-Spot (2011) or CREG (2011). A positive spread could also be observed on 29-Oct-2011, which, interestingly, again coincides with the clock change back to winter time.

<sup>29</sup>The role of natural gas as marginal fuel in the Dutch market has been further strengthened since the commissioning of the BritNed subsea cable that links the Dutch with the British market. However, note that there may also be more subtle, technical aspects to be considered: Especially for highly meshed grids, loop flows may lead to physical flows that differ from corresponding commercial point-to-point transactions. See Weber et al. (2010) for an illustrative example. In practice, for instance, high wind generation in the northern part of Germany may lead to unexpected loop flows of electricity into the neighboring Dutch, Czech, and Polish markets, from where it flows back to higher-demand areas in Southern Germany. TSOs need to reflect potential loop flows as contingencies within the safety margins of their interconnectors, which, in turn, reduces capacity available for market coupling flows.

Table 3.1: **Descriptive Statistics: GER – NL Day-Ahead Spread**

This table reports summary statistics for the spread  $s^i$  between day-ahead electricity prices in Germany and the Netherlands for selected hours of the day ( $i = \{6, 9, 12, 15, 18, 21\}$ ). The baseload spread ( $s^{\text{base}}$ ; “Base”) is defined as the (unweighted) arithmetic average of all 24 hourly spreads on a given day. The “Pre Market-Coupling” sample covers the period from 01-Jan-2002 until 08-Nov-2009, the “Post Market-Coupling” sample covers the period thereafter until 31-Dec-2012. Mean<sup>+</sup> and Std. Dev.<sup>+</sup> are defined as  $\mu^+ = \mathbb{E}[s^i | s^i > 0]$  and  $\sigma^+ = \sqrt{\mathbb{E}[(s^i - \mu^+)^2 | s^i > 0]}$ , respectively. Note that spreads beyond a threshold of 500 EUR/MWh were excluded from the samples in order to avoid distortions. All data sourced from Bloomberg.

Hour of Day	6	9	12	15	18	21	Base	
Pre Market-Coupling	n	3234	3234	3225	3229	3205	3234	3234
	$s^i < 0$	33.6%	52.3%	69.2%	73.1%	65.5%	61.9%	68.6%
	Mean	2.01	-2.45	-15.09	-9.10	-12.07	-4.44	-4.84
	Mean <sup>+</sup>	5.34	6.58	7.89	5.64	6.29	3.83	3.30
	Mean <sup>-</sup>	-4.50	-10.66	-25.31	-14.51	-21.75	-9.51	-8.57
	Median	1.81	-0.32	-3.90	-3.02	-2.00	-1.55	-1.83
	Min	-129.47	-464.17	-481.07	-477.09	-480.13	-376.49	-154.98
	Max	43.11	256.84	345.05	430.09	412.31	70.14	74.63
	Std. Dev.	7.13	21.97	46.34	33.84	47.07	15.31	13.47
	Std. Dev. <sup>+</sup>	4.94	13.45	19.82	18.96	19.79	4.92	6.10
	Std. Dev. <sup>-</sup>	6.25	24.83	50.90	36.43	53.93	17.21	14.27
	Skewness	-2.03	-6.83	-4.58	-6.15	-4.97	-8.08	-4.17
	Kurtosis	39.29	141.51	35.15	79.64	40.01	138.19	32.41
	Post Market-Coupling	n	784	784	784	784	784	784
$s^i < 0$		14.7%	15.1%	35.1%	38.5%	24.9%	24.1%	60.5%
$s^i = 0$		83.0%	82.7%	64.5%	61.4%	73.5%	74.4%	30.2%
Mean		-1.77	-1.17	-4.22	-4.92	-3.00	-2.70	-3.11
Mean <sup>+</sup>		2.38	12.15	2.92	0.75	9.10	5.07	0.77
Mean <sup>-</sup>		-12.47	-9.64	-12.05	-12.78	-12.67	-11.53	-5.25
Median		0.00	0.00	0.00	0.00	0.00	0.00	-0.50
Min		-217.23	-40.15	-76.90	-139.76	-57.39	-57.04	-89.43
Max		7.02	30.01	5.41	0.75	28.58	9.86	8.23
Std. Dev.		11.58	5.47	8.39	9.63	7.82	6.88	6.67
Std. Dev. <sup>+</sup>		2.03	10.30	2.25	n.a.	7.26	3.33	1.38
Std. Dev. <sup>-</sup>		28.01	8.80	10.31	11.85	10.63	9.57	7.85
Skewness		-15.45	-2.17	-2.92	-4.83	-2.89	-3.16	-6.43
Kurtosis		273.80	17.34	12.17	50.54	11.09	12.47	69.76



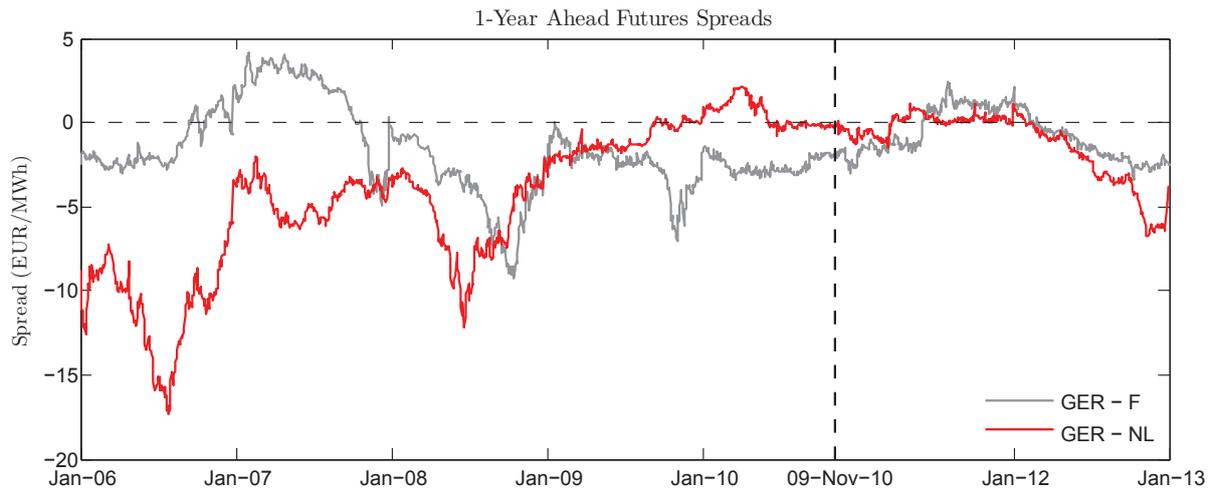
**Figure 3.4: Spread in Day-Ahead Prices: Germany – Netherlands**

*This figure shows the spread in day-ahead electricity prices between Germany and the Netherlands during the period from 01-Jan-2006 to 31-Dec-2012. Underlying day-ahead prices relate to hour 12 of the day (11:00am–12:00pm). The vertical dashed line marks the start of the CWE (Central Western Europe) market coupling between Germany, France, the Netherlands, and Belgium as per 09-Nov-2010. All data sourced from Bloomberg.*

for the corresponding baseload spread, i.e. the average of all 24 hourly spreads. As the key characteristic of market coupling, price convergence – driving down the spread to zero – is reflected in several of the statistics. For instance, note that a zero baseload spread could be observed for approx. 30% of all days since 09-Nov-2010, implying that price convergence was not only reached for several hours, but even for all 24 hours on these days. On a more detailed level, we also see that conditional on the spread being negative, both mean and standard deviation tend to be lower. Consequently, since the introduction of market coupling, not only can negative spreads  $s^i < 0$  be observed less frequently, their spikiness is also reduced in terms of absolute size and variation. This change in the dynamics of the spread and, hence, of its individual components, can be explained by the fact that under market coupling, supply and demand shocks occurring in the interconnected markets can be mitigated more easily – to the extent that these shocks are non-synchronous: The coordination of cross-border flows between coupled markets always helps to reduce the economic scarcity (abundance) of *net* supplies that is signaled by electricity prices in one market spiking upwards (downwards).<sup>30</sup>

Finally, as regards the impact of market coupling on futures prices, both empirical findings and theoretical arguments provide a mixed picture thus far. On the one hand, as

<sup>30</sup>See De Jonghe et al. (2008) and Huisman and Kiliç (2013) for an analysis of such volatility reduction potential in the context of the Trilateral Market Coupling between France, Belgium, and the Netherlands.



**Figure 3.5: Spread in 1-Year Ahead Futures Prices: GER – F and GER – NL**

*This figure shows the spread in (generic) 1-year ahead futures contracts between Germany and France as well as Germany and the Netherlands during the period from 01-Jan-2006 to 31-Dec-2012. The vertical dashed line marks the start of the CWE (Central Western Europe) market coupling between Germany, France, the Netherlands, and Belgium as per 09-Nov-2010. All data sourced from Bloomberg.*

is well known, it is the non-storability of electricity that causes the classic cost-of-carry relationship between spot and futures prices to break down, which has led to the common perception that electricity has a “split personality” with respect to spot and futures prices (Pilipovic, 1998). As such, the argument that futures prices under market coupling need to converge solely “because spot prices do” does not hold. On the other hand, and regardless of the above, the well-known interpretation of futures prices as expected spot prices at maturity under a suitably chosen (possibly non-unique) risk-neutral measure  $\mathbb{Q}$  (Cox and Ross, 1976; and Harrison and Kreps, 1979) still holds – also for electricity. Therefore, the fact that the 1-year ahead German–Dutch futures spread in Figure 3.5 qualitatively tracks the flat and, later on, widening trajectory of the corresponding spot spread during the years 2011 and 2012 in Figure 3.4, may be seen in line with this reasoning.<sup>31</sup> Empirically, the average spread between 1-year ahead German and Dutch futures contracts (from 01-Jan-2006 until the start of market coupling) amounts to -4.49 EUR/MWh, whereas the average post-coupling spread has reduced to -1.23 EUR/MWh (0.05 EUR/MWh in 2011 and -2.61 EUR/MWh in 2012). Yet these results remain slightly inconclusive – especially in view of a generally highly non-constant supply side in each of

<sup>31</sup>This simplified argument assumes any potential risk premia to be constant and sets expected spot spreads equal to realized spreads. Also note that although movements in the German–Dutch spot and futures spreads may coincide as displayed in the two figures, Figure 3.4 only displays the day-ahead spread for hour 12 of the day and not an average baseload spread.

the two markets: for instance, structural changes such as the German nuclear phase-out or a further continued increase in German renewable generation capacity (that outpaces European neighbors) may well have been anticipated and reflected in futures prices at that time already, but not in spot prices. In Section 3.4, we will revisit these aspects from a theoretical point of view and further analyze the question of futures price convergence under market coupling within our proposed modeling framework.

## 3.4 Theoretical Analysis:

### Modeling Two Interconnected Markets

In order to conduct our theoretical analysis of the different market designs, we propose a fundamental two-market framework that is sufficiently granular to reflect the impact both of the most important underlying price drivers as well as of the different allocation mechanisms on price dynamics, while retaining flexibility and mathematical tractability at the same time.<sup>32</sup> Taking the class of fundamental electricity pricing models as starting point,<sup>33</sup> we first adopt a setting similar to the one proposed in Füss et al. (2013a) in order to model electricity spot prices in each market. Similar to Skantze et al. (2004) and Coulon (2013), flows on the interconnector linking the two markets are then derived *endogenously*, depending on whether corresponding cross-border transmission rights are assumed to be allocated explicitly ex-ante or implicitly via market coupling.

#### 3.4.1 Dynamics of Fundamental Variables

In the following, we define a simplified continuous-time setting of two interconnected markets where electricity prices in each market  $i = \{1, 2\}$  are modeled as a function of

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<sup>32</sup>From a technical point of view, it seems obvious that traditional reduced-form approaches are unsuited to include above aspects of market design into the price formation mechanism in a sufficiently detailed manner: irrespective of whether spot prices in two adjacent markets are modeled simultaneously in a *bivariate* reduced-form setting or whether the corresponding spot spread is modeled in a *univariate* setting, the ensuing price dynamics will not reflect the inherent interconnectivity between the markets and, hence, will always ignore the influence of the other market on domestic prices (or of both markets on the corresponding spread). Consequently, price dynamics will not be tied to any underlying economic causality that governs the exchange of cross-border flows, resulting in a general mis-specification of the model. See Mahringer (2013) for an extended critique of the reduced-form approach in a multi-market modeling context.

<sup>33</sup>See, e.g., Eydeland and Wolyniec (2002) for a general overview on the various approaches for electricity pricing, and Carmona and Coulon (2012) for a detailed introduction into the class of fundamental models for electricity pricing.

underlying electricity demand  $D_{i,t}$  as well as the cost of the marginal fuel  $g_{i,t}$  used for electricity generation.

For the dynamics of electricity demand  $D_{i,t}$  on a filtered probability space  $(\Omega, \mathcal{F}^D, \mathbb{F}^D = (\mathcal{F}^D)_{t \in [0, T^*]}, \mathbb{P})$ , a mean-reverting Gaussian Ornstein-Uhlenbeck (OU) process combined with a deterministic seasonal function has generally been considered an adequate modeling choice:<sup>34</sup>

$$D_{i,t} = q_{i,t} + s_{D_i}(t), \quad (3.1)$$

$$dq_{i,t} = -\kappa_{q_i} q_{i,t} dt + \eta_{q_i} dB_{i,t}, \quad (3.2)$$

where  $q_{i,t}$  is an OU-process for market  $i$  with mean-reversion parameter  $\kappa_{q_i}$ ,  $B_{i,t}$  a standard Brownian motion, and  $s_{D_i}(t)$  a deterministic seasonality function in order to capture the distinct seasonal patterns that electricity demand usually exhibits.<sup>35</sup> Note that since we are modeling two geographically neighbouring markets,  $q_{1,t}$  and  $q_{2,t}$  are likely to be correlated, i.e., we allow for  $dB_{1,t}dB_{2,t} = \varrho_q dt$ .

Regarding the type of fuel that is used for electricity generation, we only propose a general specification of the fuel price dynamics  $g_{i,t}$  here since these will strongly depend on (i) the generation park of the respective electricity market to be modeled (e.g., coal- vs. gas-based markets) and (ii) potentially also on the maturity of the electricity (derivative) contract to be priced.<sup>36</sup> Hence, on a filtered probability space  $(\Omega, \mathcal{F}^g, \mathbb{F}^g = (\mathcal{F}^g)_{t \in [0, T^*]}, \mathbb{P})$ , we assume prices for the marginal generating fuel  $g_{i,t}$  to be governed by the commonly used mean-reverting one-factor model analyzed by Schwartz (1997):

$$\ln g_{i,t} = X_{i,t} + s_{g_i}(t), \quad (3.3)$$

$$dX_{i,t} = -\kappa_{X_i} X_{i,t} dt + \eta_{X_i} dW_{i,t}, \quad (3.4)$$

where  $X_{i,t}$  is an OU-process for market  $i$  with mean-reversion parameter  $\kappa_{X_i}$ ,  $W_{i,t}$  a standard Brownian motion, and  $s_{g_i}(t)$  a deterministic seasonality function. Again, we

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<sup>34</sup>See, e.g., Aïd et al. (2009), Aïd et al. (2013), Carmona and Coulon (2012), or Füss et al. (2013a) for further reference.

<sup>35</sup>Commonly employed functional specifications include sine-functions or a combination of monthly dummy variables. See, e.g., Cartea and Villaplana (2008), Aïd et al. (2013), or Füss et al. (2013a).

<sup>36</sup>In case of, e.g., long-term futures contracts, it may be necessary to use a model for generating fuels that adequately captures both short- and long-term fuel price dynamics, i.e., that needs to include two or more factors; see, e.g., Cartea and Williams (2008) for an overview in the case of natural gas. While we refrain from doing so, note that our one-factor setting for the dynamics of the underlying fuel price process could well be extended to also include log-normal multi-factor models for  $g_{i,t}$ .

allow for potential correlation between the marginal fuel price processes in the two markets by setting  $dW_{1,t}dW_{2,t} = \varrho_X dt$ . This also includes the special case that the marginal fuel used in both markets is identical, so that we can set  $\varrho_X = 1$  and  $g_{1,t} = g_{2,t}$ . Note that we assume zero correlation between demand and fuels both within each market as well as across markets, which not only helps to simplify valuation formulae but may also seem justified from an empirical point of view since (short-term) demand is generally inelastic with respect to fuel prices.<sup>37</sup>

Although more recent advances in the field of structural electricity price modeling<sup>38</sup> have proposed to include additional state variables (such as available generation capacity  $C_{i,t}$ ) or to allow for a model-endogenous determination of the merit order in *multi-fuel* set-ups, we confine the model to only include two state variables per market. Although our model could easily be extended to also include a capacity process  $C_{i,t}$  or an additional fuel price process for each market, we refrain from doing so as this would neither change the general structure of the model nor extend the scope of our theoretical analysis, but rather come at the cost of unnecessary complexity.<sup>39</sup> As such, our stylized setting considers available domestic generation capacity as being fixed in both markets; alternatively, if a process for available capacity in each market shall nevertheless be integrated into the model, a straightforward approach would be to model excess capacity, i.e., reserve margins  $M_{i,t} = C_{i,t} - D_{i,t}$ , instead of alternatively treating  $D_{i,t}$  and  $C_{i,t}$  as separate (but possibly correlated) processes.

For the purpose of derivatives pricing further below, we need to recast our four-variate Gaussian setting under the risk-neutral measure  $\mathbb{Q}$  by introducing (possibly time-varying) market prices of demand and fuel price risk,  $\lambda_{q_i,t}$  and  $\lambda_{X_i,t}$ , respectively. However, given that, e.g., electricity demand  $D_{i,t}$  is not a traded asset, this represents a non-hedgeable risk, which in turn yields an incomplete market setting. Consequently, “the” risk-neutral measure  $\mathbb{Q}$  is no longer uniquely defined but instead comes along with an infinite number

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<sup>37</sup>See, e.g., Pirrong and Jermakyan (2008).

<sup>38</sup>See, e.g., Aïd et al. (2013), Carmona et al. (2013), or Füss et al. (2013a).

<sup>39</sup>When empirically implementing our model for an electricity market where the marginal fuel may often change for some time of the day (e.g., coal during off-peak and natural gas during peak hours), a *multi-fuel* structural model might still appear more suitable at first glance. However, note that in electricity spot (day-ahead) markets, products are traded with respect to delivery of electricity during a certain hour of the day. As such, electricity contracts for delivery during night hours and contracts with delivery during, e.g., peak hours are essentially *different* commodities. Therefore, it is still possible to avoid implementing a *multi-fuel* setting by modeling electricity prices on an hourly basis (instead of modeling, e.g., daily (average) electricity prices) and merely fitting the price process  $g_{i,t}$  to different fuel types during peak and off-peak hours.

of alternative equivalent martingale measures.<sup>40</sup> Hence, although straightforward to accommodate into our model, we choose to set  $\lambda_{q_i,t} = \lambda_{X_i,t} = 0$ , implying  $\mathbb{P} = \mathbb{Q}$ . This does not affect the comparative-static analyses further below (which are insensitive to an exact specification of the market prices of demand and fuel price risk) but will rather help to simplify the following analysis.

To sum up, we obtain a four-variate Gaussian setting where conditional on time  $t$ ,  $q_{1,T}$ ,  $q_{2,T}$ ,  $X_{1,T}$ , and  $X_{2,T}$  are distributed as follows:

$$\begin{bmatrix} q_{1,T} \\ q_{2,T} \\ X_{1,T} \\ X_{2,T} \end{bmatrix} \sim N \left( \begin{bmatrix} \mu_{q_1} \\ \mu_{q_2} \\ \mu_{X_1} \\ \mu_{X_2} \end{bmatrix}, \begin{bmatrix} \sigma_{q_1}^2 & \rho_q \sigma_{q_1} \sigma_{q_2} & 0 & 0 \\ \rho_q \sigma_{q_1} \sigma_{q_2} & \sigma_{q_2}^2 & 0 & 0 \\ 0 & 0 & \sigma_{X_1}^2 & \rho_X \sigma_{X_1} \sigma_{X_2} \\ 0 & 0 & \rho_X \sigma_{X_1} \sigma_{X_2} & \sigma_{X_2}^2 \end{bmatrix} \right), \quad (3.5)$$

with

$$\begin{aligned} \mu_{q_i}(t, T) &= q_{i,t} e^{-\kappa_{q_i}(T-t)}, \\ \sigma_{q_i}^2(t, T) &= \frac{\eta_{q_i}^2}{2\kappa_{q_i}} (1 - e^{-2\kappa_{q_i}(T-t)}), \\ \mu_{X_i}(t, T) &= X_{i,t} e^{-\kappa_{X_i}(T-t)}, \\ \sigma_{X_i}^2(t, T) &= \frac{\eta_{X_i}^2}{2\kappa_{X_i}} (1 - e^{-2\kappa_{X_i}(T-t)}), \\ \rho_q(t, T) &= \frac{1}{\sigma_{q_1} \sigma_{q_2}} \frac{\varrho_q \eta_{q_1} \eta_{q_2}}{\kappa_{q_1} + \kappa_{q_2}} (1 - e^{-(\kappa_{q_1} + \kappa_{q_2})(T-t)}), \\ \rho_X(t, T) &= \frac{1}{\sigma_{X_1} \sigma_{X_2}} \frac{\varrho_X \eta_{X_1} \eta_{X_2}}{\kappa_{X_1} + \kappa_{X_2}} (1 - e^{-(\kappa_{X_1} + \kappa_{X_2})(T-t)}). \end{aligned}$$

For ease of notation,  $\mu_{(\cdot)}$ ,  $\sigma_{(\cdot)}$ , and  $\rho_{(\cdot)}$  will always refer to  $\mu_{(\cdot)}(t, T)$ ,  $\sigma_{(\cdot)}(t, T)$ , and  $\rho_{(\cdot)}(t, T)$  unless otherwise stated, e.g., as will be necessary when introducing our modeling setting for the case of an explicit ex-ante allocation of transmission rights.

### 3.4.2 Spot Pricing Formulae

In our setting, the two adjacent electricity markets shall be linked with each other by an interconnection line with capacity  $K$ . Electricity spot prices  $P_{i,t}$  in market  $i$  with

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<sup>40</sup>In case of an empirical implementation,  $\lambda_{q_i,t}$  and  $\lambda_{X_i,t}$  could be inferred by calibrating the model to observed prices of traded electricity contracts, as is done in Cartea and Villaplana (2008) or Füss et al. (2013a), for example. An alternative would be to set  $\mathbb{Q} = \mathbb{Q}^{min}$ , which denotes the *minimal martingale measure* proposed by Foellmer and Schweizer (1991). See, e.g., Aïd et al. (2009) for an application in the context of structural electricity price modeling.

$i = \{1, 2\}$  are then defined as follows:

$$P_{1,t} = \alpha_1 g_{1,t}^{\delta_1} \exp(\beta_1 D_{1,t} - \gamma_1 J(t)), \quad (3.6)$$

$$P_{2,t} = \alpha_2 g_{2,t}^{\delta_2} \exp(\beta_2 D_{2,t} + \gamma_2 J(t)), \quad (3.7)$$

where  $D_{1,t}$  and  $D_{2,t}$  represents completely inelastic electricity demand as given in Equations (3.1) and (3.2), and  $J(t)$  is the flow of electricity on the interconnection line at time  $t$ . Note that as per standard economic reasoning, we assume that  $\alpha_i > 0$ ,  $\beta_i > 0$ ,  $\gamma_i > 0$ , and  $\delta_i > 0$ . Moreover, the market filtration is defined by  $\mathcal{F}_t := \mathcal{F}_t^D \vee \mathcal{F}_t^g$ .

Regarding the above functional specification of the relationship between electricity prices  $P_{i,t}$  and the state variables  $D_{i,t}$  and  $g_{i,t}$ , the following needs to be taken into account: in Equations (3.6) and (3.7), electricity *spot* prices are derived based on an assumed relationship between generating fuels and demand, as is characteristic for the class of fundamental electricity pricing models (see, e.g., Skantze et al., 2000; Cartea and Villaplana, 2008; Pirrong and Jermakyan, 2008; Lyle and Elliott, 2009; or, more generally, Carmona and Coulon, 2012). These models bridge the gap between standard reduced-form settings on the one hand, and dynamic equilibrium models on the other hand, by combining aspects of both approaches: Although derived from an *exogenously* imposed functional specification rather than based on the optimization behavior of individual market participants, electricity prices  $P_{i,t}$  are still derived in a (“reduced”) equilibrium setting where supply equals demand. Using the exponential function to represent the characteristically highly convex curvature of the supply curve (merit-order curve) in electricity markets along with the assumption of completely inelastic demand, Equations (3.6) and (3.7) merely reflect the simple case of an equilibrium in two markets with inelastic demand and where imports (exports) are modeled as reductions (additions) to demand.

Also note that in Equations (3.6) and (3.7), the multiplicative structure of the RHS terms with respect to the generating fuels is clearly in accordance with the empirical observation that fuel prices are generally the main driver of merit-order or bid-stack dynamics in electricity markets (Eydeland and Geman, 1998; Pirrong and Jermakyan, 2008).<sup>41</sup> However, when electricity prices are to be modeled on an hourly basis (as opposed

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<sup>41</sup>Hence, the second part of our spot price formula,  $\exp(\beta_i D_{i,t} \pm \gamma_i J(t))$ , is often interpreted as heat rate function that indicates how many units of generating fuels are required as inputs by generators to produce one unit of electricity.

to modeling daily average/baseload prices), occurrences of negative prices may be much more prominent, in which cases the above fuel price dependence breaks down. Given that in the event of a negative price spike, renewable generation bidding at negative prices tends to abound, state variables for coal or gas prices should not impact the negative price dynamics (although there still may be a link to electricity demand  $D_{i,t}$ ). While we do not focus on the issue of negative prices in order to keep the complexity of the following analysis at a tractable level, note that there are ways to adjust our model for that case.<sup>42</sup>

### 3.4.3 Implicit vs. Explicit Allocation

In the following, for the case when capacity rights are implicitly allocated (“*ia*,” “market coupling”), corresponding electricity spot prices will be denoted  $P_{i,t}^{ia}$  as compared to  $P_{i,t}^{ea}$  when the rights are explicitly allocated (“*ea*,” “connected, but non-coupled markets”). Note that while the two market regimes analyzed in this study share the same foundation, i.e., spot price model, they differ with respect to the determination of the interconnector flow  $J$ . Under either regime,  $J$  will need to be determined based on a simplified allocation rule that leaves out complexities observed in reality, yet still yields a pricing formula that adequately reflects the key characteristics of market coupling (or explicit ex-ante allocation, respectively) in the ensuing price dynamics.

For the concept of market coupling, this allocation rule needs to mimic the economically optimal allocation of transmission capacities so that resulting cross-border flows are always directed from the lower- to the higher-priced market, just until any existing price differential between the two markets is exploited. Hence, we assume in a first step that by the joint effort of power exchanges, TSOs, and the market coupling office in the two markets,  $J$  is set in order to reach perfect price convergence between the markets. The (unconstrained) flow  $\tilde{J}^{ia}(t)$  on the interconnector is hence derived as follows:

$$P_{1,t}^{ia} \stackrel{!}{=} P_{2,t}^{ia} \tag{3.8}$$

$$\alpha_1 g_{1,t}^{\delta_1} \exp\left(\beta_1 D_{1,t} - \gamma_1 \tilde{J}^{ia}(t)\right) = \alpha_2 g_{2,t}^{\delta_2} \exp\left(\beta_2 D_{2,t} + \gamma_2 \tilde{J}^{ia}(t)\right).$$

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<sup>42</sup>As a first “quick fix”, dependence on fuel prices should be removed by setting  $\delta_i = 0$  so that prices could, by way of example, be defined as  $P_{i,t} = -\alpha_i e^{-\beta_i D_{i,t}}$ , such as in Carmona et al. (2013). As a second step, this negative-price regime should be complemented with our standard positive-price regime so that electricity prices will then be determined in a classic regime-switching setting where state probabilities can additionally be dependent on electricity demand, so that prices would not be negative all the time, but only occasionally. See, e.g., Carmona and Coulon (2012) for an application in a one-market setting.

Solving for  $\tilde{J}^{ia}(t)$  then yields:

$$\tilde{J}^{ia}(t) = \frac{1}{\gamma_1 + \gamma_2} [\ln \alpha_1 - \ln \alpha_2 + \delta_1 \ln g_{1,t} - \delta_2 \ln g_{2,t} + \beta_1 D_{1,t} - \beta_2 D_{2,t}]. \quad (3.9)$$

By contrast, we have discussed in Section 3.2 that one of the key differences between implicit and explicit allocation of interconnector rights is the timing disconnect between when a trader is assigned transmission capacity and his physical flows need to be scheduled on the one hand, and when corresponding electricity (spot) markets clear on the other hand. More generally, also note that despite the differences between the respective institutional settings in Europe and the US (e.g., auction-based vs. non-auction based allocation), the explicit schemes are distinctly characterized by the above mentioned time lag which has been clearly identified as the main driver for the inherent economic inefficiency of these schemes in Section 3.3. Hence, in order to derive interconnector flows for the explicit ex-ante regime,  $\tilde{J}^{ea}(t)$ , the sequential nature of capacity and spot market clearance needs to be taken into account: in this case, the allocation of capacity rights is determined *ex-ante* at some point in time  $\tau = t - k$  with  $k$  fixed. Thus, at time  $\tau$ , when being allocated transmission capacity and scheduling flows for time  $t = \tau + k$ , market participants now base their decision on the *expected* price spread between  $P_{1,\tau+k}$  and  $P_{2,\tau+k}$ , serving as best proxy in order to gauge the profitability of their intended cross-border transaction. Consequently, we shall assume that investors seek to schedule cross-border trades up until the (unconstrained) flow  $\tilde{J}^{ea}(t)$  suffices to set *expected* prices in the two markets equal:

$$\begin{aligned} \mathbb{E}_{t-k} [P_{1,t}^{ea}] &\stackrel{!}{=} \mathbb{E}_{t-k} [P_{2,t}^{ea}] \\ \mathbb{E}_{t-k} \left[ \alpha_1 g_{1,t}^{\delta_1} \exp \left( \beta_1 D_{1,t} - \gamma_1 \tilde{J}^{ea}(t) \right) \right] &= \mathbb{E}_{t-k} \left[ \alpha_2 g_{2,t}^{\delta_2} \exp \left( \beta_2 D_{2,t} + \gamma_2 \tilde{J}^{ea}(t) \right) \right]. \end{aligned} \quad (3.10)$$

Solving the above for  $\tilde{J}^{ea}(t)$  yields:

$$\begin{aligned} \tilde{J}^{ea}(t) &= \frac{1}{\gamma_1 + \gamma_2} \left[ \delta_1 \mu_{X_1}(t-k, t) - \delta_2 \mu_{X_2}(t-k, t) + \beta_1 \mu_{q_1}(t-k, t) - \beta_2 \mu_{q_2}(t-k, t) \right. \\ &\quad + \frac{1}{2} \left( \delta_1^2 \sigma_{X_1}^2(t-k, t) - \delta_2^2 \sigma_{X_2}^2(t-k, t) \right) + \frac{1}{2} \left( \beta_1^2 \sigma_{q_1}^2(t-k, t) - \beta_2^2 \sigma_{q_2}^2(t-k, t) \right) \\ &\quad \left. + \ln \alpha_1 - \ln \alpha_2 + \delta_1 s_{g_1}(t) - \delta_2 s_{g_2}(t) + \beta_1 s_{D_1}(t) - \beta_2 s_{D_2}(t) \right], \end{aligned} \quad (3.11)$$

where  $\mu_{(\cdot)}(t-k, t)$  and  $\sigma_{(\cdot)}^2(t-k, t)$  indicate the conditional expectation and variance relating to the respective processes at time  $t$ , yet taken at an earlier point in time (i.e., at  $t-k = \tau$ ).<sup>43</sup>

However, given that both  $\tilde{J}^{ia}(t)$  and  $\tilde{J}^{ea}(t)$ , i.e., the optimal flows required to reach (expected) price convergence, may often surpass the actual capacity  $K$  on the interconnector, the technically feasible flows  $J^{ia}(t)$  and  $J^{ea}(t)$  are both limited ( $-K \leq J^{(\cdot)}(t) \leq K$ ) and given as:

$$J^{(\cdot)}(t) = \max\left(\min\left(\tilde{J}^{(\cdot)}(t), K\right), -K\right).$$

Hence, for either market regime, spot pricing formulae for the two markets need to take into account the above non-linearity in  $J^{(\cdot)}(t)$  and have to distinguish three different scenarios. For markets 1 and 2 under market coupling, these are:

$$P_{1,t}^{ia} = P_{1,t}^{ia,ex} \mathbb{I}_{\{\tilde{J}^{ia}(t) \leq -K\}} + P_{1,t}^{ia,un} \mathbb{I}_{\{-K < \tilde{J}^{ia}(t) < K\}} + P_{1,t}^{ia,im} \mathbb{I}_{\{\tilde{J}^{ia}(t) \geq K\}}, \quad (3.12)$$

$$P_{2,t}^{ia} = P_{2,t}^{ia,im} \mathbb{I}_{\{\tilde{J}^{ia}(t) \leq -K\}} + P_{2,t}^{ia,un} \mathbb{I}_{\{-K < \tilde{J}^{ia}(t) < K\}} + P_{2,t}^{ia,ex} \mathbb{I}_{\{\tilde{J}^{ia}(t) \geq K\}}, \quad (3.13)$$

where  $P_{1,t}^{ia,ex}$  ( $P_{2,t}^{ia,ex}$ ) is the spot price at time  $t$  in market 1 (market 2) if it is exporting electricity to market 2 (market 1). Physical interconnector flows are then constrained by the capacity of the transmission line and an amount of  $K$  gigawatt (GW) units is exported from one market to the other.  $P_{1,t}^{ia,un}$  ( $P_{2,t}^{ia,un}$ ) is the time- $t$  spot price if the interconnection line between the two markets is *un*-constrained. In such case, there is no congestion and market 1 (market 2) may either be exporting or importing at below the capacity limit  $K$ . Correspondingly,  $P_{1,t}^{ia,im}$  ( $P_{2,t}^{ia,im}$ ) is the spot price of electricity in market 1 (market 2) if it is in import-state and  $J^{ia}(t)$  has reached its capacity limit. More explicitly, the above piecewise definition of spot prices under market coupling involves the

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<sup>43</sup>For the explicit scheme, we indirectly assume that the allocation of capacity and the scheduling of physical flows coincide at time  $\tau$ , although for the example illustrated in Figure 3.1, scheduling of *daily* transactions is set *after* spot market clearance. However, note that the determination of  $J^{ea}(t)$  based on *expected* price convergence does not consider the aspect of whether after the close of day-ahead markets, the trader will actually exercise his transmission right. For instance, if the right is out of the money, i.e., if the day-ahead spread turns out to be the opposite of his former belief at time  $\tau = t - k$ , the trader could close out his positions by resorting to the intraday platforms in both markets. Thus, the originally intended cross-border trade would be broken up into a domestic trade in each market, which, however, leaves unaffected the traders' commitments to buy/sell electricity in the *day-ahead* markets. Therefore, net demand  $D_{i,t} - J^{ea}(t)$  in the importing market ( $D_{j,t} + J^{ea}(t)$  in the exporting market) would still be the same, as it is just another counterparty (i.e., the intraday market) to which electricity is delivered or from where it is supplied.

following expressions for market 1 (prices for market 2 are defined analogously):

$$P_{1,t}^{ia,ex} = \alpha_1 g_{1,t}^{\delta_1} \exp(\beta_1 D_{1,t} - \gamma_1(-K)), \quad (3.14)$$

$$P_{1,t}^{ia,un} = \alpha_1 g_{1,t}^{\delta_1} \exp\left(\beta_1 D_{1,t} - \gamma_1 \tilde{J}^{ia}(t)\right), \quad (3.15)$$

$$P_{1,t}^{ia,im} = \alpha_1 g_{1,t}^{\delta_1} \exp(\beta_1 D_{1,t} - \gamma_1 K). \quad (3.16)$$

If market coupling is not in place and transmission rights are allocated explicitly instead, spot prices  $P^{ea,ex}$ ,  $P^{ea,un}$ , and  $P^{ea,im}$  are defined accordingly.<sup>44</sup>

Note, however, that our continuous-time setting generally implies that under the explicit ex-ante regime, the allocation procedure for transmission capacity and/or the related scheduling of transactions would be held during any infinitesimally small period of time, rather than, e.g., once a day in the case of day-ahead markets. Likewise, under market coupling, price convergence at any instance in time is (and will likely remain to be) out of technical reach. Hence, while adhering to a continuous-time framework for mathematical convenience, we follow the common assumption of interpreting electricity prices as discrete-time observations resulting from a price formation process which, in turn, is driven by the continuous-time dynamics of its underlying state variables.<sup>45</sup> Similarly, in our setting, we can interpret  $\tilde{J}^{ia}(t)$  and  $\tilde{J}^{ea}(t)$  as hypothetical electricity flows which would prevail at any moment in time, yet which only materialize when it comes to price formation, i.e., when discretely observed at the scheduling date or at the time when establishing the market coupling flows.

### 3.4.4 Analysis of Spot Prices

In order to illustrate the mechanics of our spot pricing formulae, we employ a simulation with 1,000 time steps for each of the underlying fundamental factors  $D_{1,t}$ ,  $D_{2,t}$ ,  $g_{1,t}$ , and  $g_{2,t}$ . After simulating sample paths from the discretized processes for the state variables, we can then impose our different (exogenous) structural pricing relationships based on Equations (3.6) and (3.7), and depending on whether electricity prices  $P_{1,t}^{(\cdot)}$  and

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<sup>44</sup>To conserve space, we do not state the corresponding differential equations for  $P_{i,t}^{ia}$  and  $P_{i,t}^{ea}$ . Yet it is important to note that with an explicit ex-ante scheme, our spot price process is no longer Markovian given that interconnector flows  $J^{ea}(t)$  were already determined by the values of our state variables at time  $\tau = t - k$ . The corresponding differential equation hence belongs to the class of *stochastic delay differential equations (SDDE)*; see, e.g., Mohammed (1984) or Mao (1997) for further information.

<sup>45</sup>See, e.g., Benth et al. (2008b) or Carmona and Coulon (2012) for a similar discussion.

$P_{2,t}^{(\cdot)}$  are to be derived under an explicit ex-ante or implicit (market coupling) allocation scheme. Comparability of the simulated spread time-series is ensured by using the same variates (and, hence, state variables) in each case, thus allowing us to analyze how shocks from underlying fundamental factors are reflected in electricity prices under the different regimes for cross-border trade.<sup>46</sup> Results are shown in Figure 3.6 where in the top panel, we additionally have plotted the spread  $P_{1,t}^{iso} - P_{2,t}^{iso}$  between two isolated electricity markets with no possibility of cross-border trading. The next two panels show the spreads  $P_{1,t}^{ea} - P_{2,t}^{ea}$  and  $P_{1,t}^{ia} - P_{2,t}^{ia}$ , respectively. Finally, the two panels at the bottom present the corresponding endogenously determined interconnector flows that are related either to the spread under an explicit allocation or market coupling regime, respectively.

As can be seen in the top panel of Figure 3.6, without cross-border trade, the spread  $P_{1,t}^{iso} - P_{2,t}^{iso}$  fluctuates widely around zero, reaching minimum and maximum values of approx. -22 EUR/MWh and 26 EUR/MWh, respectively. As shown in the panel below, allowing for exchange between the markets under an explicit ex-ante scheme generally mitigates the price differential and thus improves price convergence. Taking as an example the characteristic spike of approx. 25 EUR/MWh in the top panel (marked red at simulation step 400), this spike is also reflected in the spread under the explicit ex-ante regime in the panel below, yet only at approximately half of its original magnitude (approx. 13 EUR/MWh). At the same time, the spread series  $P_{1,t}^{ea} - P_{2,t}^{ea}$  still looks slightly spikier as compared to the case of isolated markets, given that the generally smaller spread for explicit allocations (as compared to  $P_{1,t}^{iso} - P_{2,t}^{iso}$ ) comes at the cost of inefficient interconnector flows that cause it to change its sign more frequently. However, if inefficient flows  $J^{ea}(t)$  go in the wrong direction, spikes of the spot spread will even further increase as can be seen when examining the second spike highlighted in red towards the end of the sample period: in case of isolated markets, we have  $P_{1,t}^{iso} - P_{2,t}^{iso} = 14.15$  EUR/MWh, whereas  $P_{1,t}^{ea} - P_{2,t}^{ea}$  increases to 16 EUR/MWh due to an adverse interconnector flow. Examining  $J^{ea}(t)$  in the second to the last panel, we see that these adverse flows primarily

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<sup>46</sup>More precisely, for the simulation, both markets are identically parametrized; the main input parameters to our model were set as follows:  $s_{D_1} = s_{D_2} = 40$  GW (i.e., to simplify, we refrain from incorporating seasonality for the time being), which compares against an interconnector capacity of  $K = 2$  GW. Furthermore, we have set  $\kappa_{q_i} = 0.5$ ,  $\kappa_{X_i} = 0.001$ ,  $\eta_{q_i} = 1.0$ , and  $\eta_{X_i} = 0.02$ . These parameter values are in line with the empirical results of Füss et al. (2013a). Additionally, we assume  $\varrho_X = \varrho_q = 0.5$ . We simulate on a daily basis so that having set  $k = 1$  implies that under explicit allocation, interconnector flows are determined one day ahead. For the more realistic case of  $k < 1$  (to reflect intraday timeframes), we can easily adjust our simulation, yet obtain the same qualitative results.

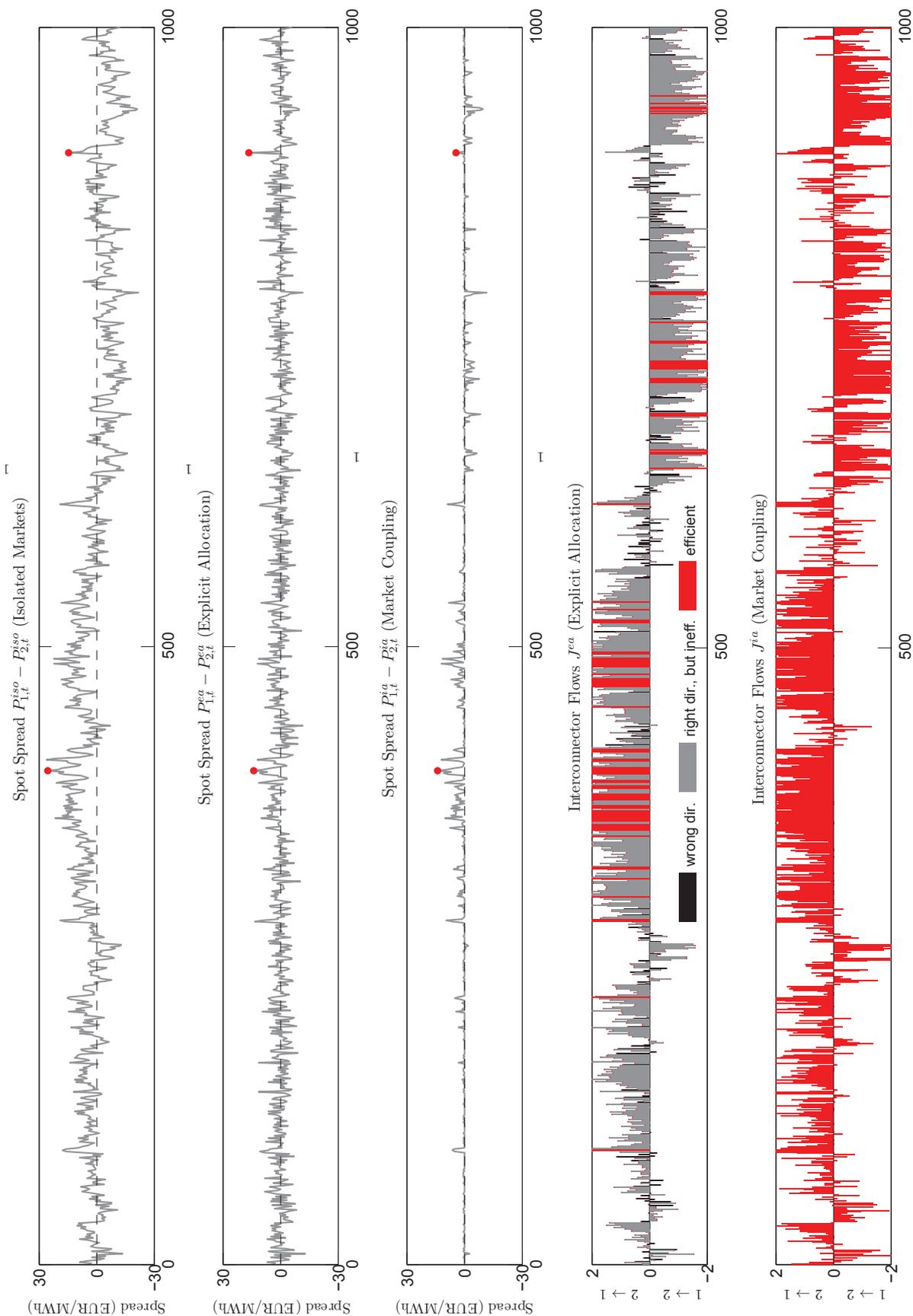


Figure 3.6: Spot Spread Simulations

This figure shows the spot spread  $P_{1,t}^{(\cdot)} - P_{2,t}^{(\cdot)}$  between two electricity markets 1 and 2, for the case that (i) these markets are not interconnected (first panel), (ii) capacity on the interconnector is allocated explicitly ex-ante (second panel), or (iii) the two markets are linked via market coupling (third panel). For each case, the spread is derived from individual electricity prices  $P_{1,t}^{(\cdot)}$  and  $P_{2,t}^{(\cdot)}$  which, in turn, result from simulations of the underlying state variables  $D_{1,t}$ ,  $D_{2,t}$ ,  $g_{1,t}$ , and  $g_{2,t}$  for 1,000 consecutive time steps. Additionally, the two bottom panels show the corresponding endogenously-determined interconnector flows  $J^{ea}$  and  $J^{ia}$  for the two allocation regimes.

occur when the fundamentally implied price differential between the two markets is about to reverse, e.g., as is indicated by the corresponding market coupling flows changing their direction from import to export (or vice versa).<sup>47</sup>

Finally, under market coupling, we see that our pricing model adequately captures the qualitative dynamics of the spot spread as empirically observed, e.g., in Figures 3.3 and 3.4. The “reflected” trajectory of the spread taking on either zero or positive (negative) values during the first (second) half of the sample period thereby merely reflects that in our simulation, spot electricity in market 1 is more expensive (cheaper) during most of that period; hence, a non-zero spread  $P_{1,t}^{ia} - P_{2,t}^{ia}$  can always be observed whenever the price differential between the two markets is too high so that interconnector flows  $J^{ia}(t)$  (that would be required to equalize prices in both markets) exceed the capacity limit  $K$ . This occurs in 219 out of 1000 simulation steps. Out of these 219 cases, in turn, it occurs that  $J^{ea}(t) = J^{ia}(t)$  for 99 cases. In these cases, the fundamentals in the two markets are far apart so that even under an explicit allocation regime, full interconnector use is induced, and we have  $P_{1,t}^{ea} - P_{2,t}^{ea} = P_{1,t}^{ia} - P_{2,t}^{ia}$  (see, e.g., first spike marked red at simulation step 400). In the remaining 120 cases with  $|J^{ia}(t)| = K$ , corresponding flows  $J^{ea}(t)$  are inefficient so that spikes under market coupling can be further mitigated (see, e.g., second spike highlighted red).

Our model also allows to investigate more closely the empirically observed volatility reduction effect of market coupling discussed in Section 3.3. In Figure 3.7, model sensitivities for the (unconditional) variance of log-spot returns of electricity prices under market coupling,  $\ln\left(\frac{P_{i,t+1}^{ia}}{P_{i,t}^{ia}}\right)$ , are provided and compared against the variance in case of isolated markets with no possibility of cross-border trade.<sup>48</sup>

In the LHS graph, we examine how higher variation in electricity prices (or, more precisely, in their underlying fundamental drivers) in market 1 is transmitted into market 2, assuming a market coupling regime with interconnector capacity  $K = \{0, 2, 4, 12\}$ . For

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<sup>47</sup>Note that we strictly define only those interconnector flows  $J^{ea}(t)$  as efficient where  $J^{ea}(t) = J^{ia}(t)$ . This implies that  $J^{ea}(t)$  is only efficient when  $J^{ea}(t) = J^{ia}(t) = \pm K$ , which occurs in 10% of all cases. 76% of all derived flows  $J^{ea}(t)$  are inefficient (yet go in the right direction), whereas the remaining 14% are adverse flows.

<sup>48</sup>For the sensitivity analyses, we use the same parameters as in the simulation study. Also, we have set  $\beta_i = 0.1$  and  $\delta_i = 0.5$ . For the LHS graph, we still assume state variables to be uncorrelated,  $\varrho_q = \varrho_X = 0$ , although correlation obviously only impacts return variance in the two markets for  $K > 0$ . For  $K = 0$ , the unconditional variance of log-spot returns can then be computed as  $2 \times 0.1^2 \times 1^2 \times (1 - 0.6065) + 2 \times 0.5^2 \times 0.2^2 \times (1 - 0.9990) = 0.0080$ , as can be read off from both graphs.

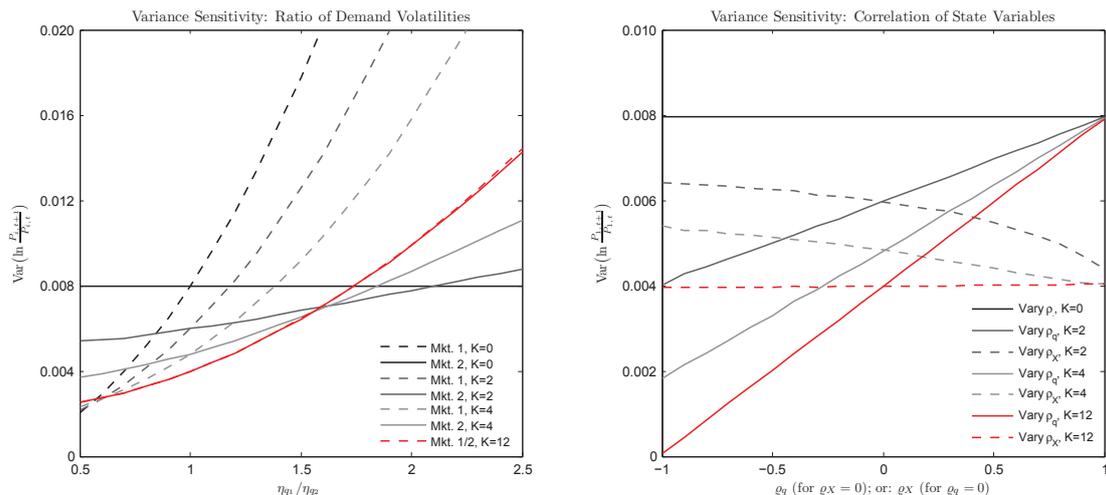


Figure 3.7: Sensitivities for Variance of log-Spot Returns

This figure shows sensitivities of the (unconditional) variance of log-spot returns  $\ln\left(\frac{P_{i,t+1}}{P_{i,t}}\right)$  when varying key input parameters, and assuming given interconnector capacities  $K$  of 0GW, 2GW, 4GW, and 12GW, respectively. In the LHS graph, the variance of log-spot returns in both interconnected markets is plotted against the ratio of (instantaneous) volatilities,  $\frac{\eta_{q1}}{\eta_{q2}}$ , for the processes of electricity demand in both markets (as specified in Equation (3.2)). In the RHS graph, the variance for market 1 is analyzed when varying the correlation either between the processes for electricity demand or for the generating fuels in the two markets.

that purpose, the ratio of instantaneous volatilities of the electricity demand processes,  $\frac{\eta_{q1}}{\eta_{q2}}$ , is varied over a range of 0.5 up to 2.5 (thereby keeping  $\eta_{q2}$  itself fixed). Consequently, for the case of isolated markets, the variance of log-spot returns in market 1 (dashed black line) is steeply increasing, thus merely reflecting the higher variance of the underlying state variable  $D_{1,t}$  that feeds through to spot prices. Given that  $K = 0$ , variance in market 2 remains unaffected, as indicated by the horizontal straight black line. Assuming a modest level of interconnectivity,  $K = 2$ , the dashed dark grey line shows how the increase in variance for market 1 can be mitigated due to the optimal allocation of cross-border capacities under market coupling. The ensuing volatility reduction potential also manifests in the fact that for  $\frac{\eta_{q1}}{\eta_{q2}} = 1.0$ , the two dark grey lines intersect at an approximate level of 0.006: thus, introducing market coupling between these two identically parametrized markets reduces return variance by some 25% alone. Compared to the status quo of isolated markets, for  $K = 2$ , the increased demand volatility in market 1 only starts to affect market 2 for  $\frac{\eta_{q1}}{\eta_{q2}}$  exceeding a ratio of approx. 2.1, so that up to this threshold, any increase in  $\eta_{q1}$  is outweighed by the merits of coupling. Recalling the piecewise definition of spot prices (see Equations (3.12) and (3.13)), we see that spot

prices in markets 1 and 2 will only differ in the import and export states. For higher levels of interconnectivity, the corresponding “weights” for these states,  $\mathbb{I}_{\{\tilde{J}^{ia}(t) \leq -K\}}$  and  $\mathbb{I}_{\{\tilde{J}^{ia}(t) \geq K\}}$ , become smaller whereas  $\mathbb{I}_{\{-K < \tilde{J}^{ia}(t) < K\}}$  for the uncongested state increases, which causes the dynamics of the neighbouring market to increasingly feed through into the other market. This can also be illustrated technically, when inserting  $\tilde{J}^{ia}(t)$  from Equation (3.9) into  $P_{1,t}^{ia,un}$  from Equation (3.15):

$$\begin{aligned} P_{1,t}^{ia,un} &= \alpha_1 g_{1,t}^{\delta_1} \exp\left(\beta_1 D_{1,t} - \gamma_1 \tilde{J}^{ia}(t)\right) \\ &= \left(\alpha_1 g_{1,t}^{\delta_1}\right)^{\frac{\gamma_2}{\gamma_1+\gamma_2}} \left(\alpha_2 g_{2,t}^{\delta_2}\right)^{\frac{\gamma_1}{\gamma_1+\gamma_2}} \exp\left(\frac{\gamma_2}{\gamma_1+\gamma_2} \beta_1 D_{1,t} + \frac{\gamma_1}{\gamma_1+\gamma_2} \beta_2 D_{2,t}\right). \end{aligned} \quad (3.17)$$

Consequently, spot prices effectively become a blend of all *four* state variables and converge towards each other, as do the light grey variance curves in the LHS graph of Figure 3.7. Finally, for high levels of interconnectivity, such as  $K = 12$ , the resulting (perfect) price convergence between the two markets also equalizes return variances, as indicated by the red curves that virtually coincide (except for unrealistically high levels of  $\frac{\eta_{q1}}{\eta_{q2}}$ ).

In the RHS graph of Figure 3.7, the variance of log-spot returns in market 1 is plotted against the correlation between state variables  $\varrho_q$  or  $\varrho_X$ . Note that when varying  $\varrho_q$ , we keep  $\varrho_X$  fixed at zero and vice versa.<sup>49</sup> When examining both straight and dashed lines, we again see that higher levels of interconnector capacity  $K$  generally allow for a higher volatility reduction potential as implied by our model. However, when varying  $\varrho_q$ , the upward-sloping straight lines imply that variance increases along with correlation, up until it reaches its upper bound, i.e., the case of isolated markets.<sup>50</sup> Intuitively, this merely reflects the fact that arising synchronicity of demand shocks in the two markets makes it harder (or even impossible) for the market coupling mechanism to mitigate the resulting price spikes: if net supplies are scarce in both markets, even an economically optimal allocation of cross-border capacities cannot improve the situation. Technically,

<sup>49</sup>Note that the case of  $\varrho_q > 0$  and  $\varrho_X > 0$  should be more realistic from an empirical point of view. However, setting one of the correlation parameters to zero (while varying the other parameter) helps to better disentangle and relate the sensitivities to one or the other factor rather than having to consider mixing effects from both demand and fuel correlations being non-zero at the same time.

<sup>50</sup>Note, however, that although implied by our graph, this bound is not reached exactly. For  $\varrho_q = 1$ ,  $\varrho_X = 0$ , and  $K = 12$ , the difference between the variance of log-returns under market coupling and the variance in isolated markets will approximately amount to  $\frac{1}{2} \delta_i^2 \text{Var}\left(\ln \frac{g_{i,t+1}}{g_{i,t}}\right)$ , where we have assumed aforementioned identical parametrization of markets and where  $\text{Var}\left(\ln \frac{g_{i,t+1}}{g_{i,t}}\right)$  is the (unconditional) variance of log-returns from the fuel price process.

this is primarily due to the increasing contribution of the covariance between the demand processes  $D_{i,t}$  to overall variance, as can be seen from  $P_{1,t}^{ia,un}$  in Equation (3.17). However, there is an additional effect that becomes obvious when instead varying  $\varrho_X$ . Here, it is helpful to recall that the variance of log-returns is always also determined by the indicator functions  $\mathbb{I}_{\{\cdot\}}$  that link the three scenarios in Equations (3.12) and (3.13). Their variance, in turn, is strongly linked to the overall variation in absolute-level interconnector flows  $J^{ia}(t)$ , as stated in Equation (3.9). Yet in contrast to the above, their variation is decreasing for higher levels of correlation  $\varrho_q$  or  $\varrho_X$ , given that the state variables enter Equation (3.9) as pairwise differences. Hence, for a reasonable parametrization of our model, the term  $\delta_1 \ln g_{1,t} - \delta_2 \ln g_{2,t}$  is the primary driver for the variation in  $J^{ia}(t)$ , which is large for  $\varrho_X = -1$  and vanishes for  $\varrho_X = 1$ . For modest levels of interconnectivity, this effect outweighs the “synchronicity” effect observed above and, consequently, causes the dashed lines in the RHS graph of Figure 3.7 to be downward sloping for  $K = 2$  and  $K = 4$ . Finally, for  $K = 12$ , it again suffices to focus on Equation (3.17) where we can see that for varying  $\varrho_X$ , the impact of increasing covariance between fuel prices on overall variance is disproportionately smaller than for varying  $\varrho_q$ , as is also reflected in the graph by the different slopes of the lines highlighted in red.

### 3.4.5 Futures Pricing Formulae

With the bulk of electricity trading generally taking place in liquid forward and futures markets, analyzing the effect of interconnected electricity markets on their price dynamics is at least as important as in the case of spot trading. Furthermore, given the prominent role of electricity forward and futures contracts for hedging purposes, the availability of closed-form pricing formulae for these contracts is particularly crucial from a risk management perspective.

Before starting to derive the pricing formula for a futures contract under both implicit and explicit allocation regimes,  $F_{i,t}^{ia}(T)$  and  $F_{i,t}^{ea}(T)$  with  $i = \{1, 2\}$  and maturity  $T$ , we state a useful result for calculating integrals over multivariate Gaussian densities:<sup>51</sup>

$$\int_{-\infty}^l e^{cx} \Phi\left(\frac{a+bx}{d}\right) \frac{e^{-\frac{1}{2}x^2}}{\sqrt{2\pi}} dx = e^{\frac{1}{2}c^2} \Phi_2\left(l-c, \frac{a+bc}{\sqrt{b^2+d^2}}; \frac{-b}{\sqrt{b^2+d^2}}\right), \quad (3.18)$$

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<sup>51</sup>See, e.g., Carmona and Coulon (2012) for an application of this standard result in their multi-fuel structural electricity pricing model, but also Geske (1979) in the context of pricing compound options.

where  $a, b, c, d$ , and  $l$  are constants,  $\Phi(\cdot)$  and  $\Phi_2(\cdot, \cdot; \rho)$  are the cumulative distribution functions of the univariate and bivariate (correlation  $\rho$ ) standard normal distribution.

Based on the classic result that futures prices equal spot prices expected to prevail at maturity under the risk-neutral measure  $\mathbb{Q}$  (recall our assumption of  $\lambda_{q_i,t} = \lambda_{X_i,t} = 0$ ), and using iterated conditioning, we can explicitly derive the time- $t$  futures price  $F_{i,t}^{ia}(T)$  for market  $i$  and maturity  $T$  under market coupling:

$$\begin{aligned}
F_{1,t}^{ia}(T) &= \mathbb{E}_t [P_{1,T}^{ia}] \\
&= \mathbb{E}_t \left[ P_{1,T}^{ia,ex} \mathbb{I}_{\{\tilde{J}^{ia}(T) \leq -K\}} + P_{1,T}^{ia,un} \mathbb{I}_{\{-K < \tilde{J}^{ia}(T) < K\}} + P_{1,T}^{ia,im} \mathbb{I}_{\{\tilde{J}^{ia}(T) \geq K\}} \right] \quad (3.19) \\
&= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \left( \int_{-\infty}^{lb^{ia}} \alpha_1 g_{1,T}^{\delta_1} \exp(\beta_1 D_{1,T} - \gamma_1(-K)) \phi(q_{1,T}|q_{2,T}) dq_{1,T} \right. \\
&\quad + \int_{lb^{ia}}^{ub^{ia}} \alpha_1 g_{1,T}^{\delta_1} \exp(\beta_1 D_{1,T} - \gamma_1 \tilde{J}^{ia}(T)) \phi(q_{1,T}|q_{2,T}) dq_{1,T} \\
&\quad \left. + \int_{ub^{ia}}^{\infty} \alpha_1 g_{1,T}^{\delta_1} \exp(\beta_1 D_{1,T} - \gamma_1 K) \phi(q_{1,T}|q_{2,T}) dq_{1,T} \right) \\
&\quad \cdot \phi(q_{2,T}) \phi(X_{1,T}|X_{2,T}) \phi(X_{2,T}) dq_{2,T} dX_{1,T} dX_{2,T},
\end{aligned}$$

where  $\phi(q_{1,T}|q_{2,T})$  and  $\phi(X_{1,T}|X_{2,T})$  are the (risk-neutral) conditional densities of  $q_{1,T}$  given  $q_{2,T}$  and of  $X_{1,T}$  given  $X_{2,T}$ , respectively.  $\phi(q_{2,T})$  and  $\phi(X_{2,T})$  are the (risk-neutral) unconditional densities of  $q_{2,T}$  and  $X_{2,T}$ , respectively. The lower and upper bounds  $lb^{ia}$  and  $ub^{ia}$  for the innermost integrals over  $q_{1,T}$  can be found by taking as starting point the corresponding inequations  $\tilde{J}^{ia}(T) \leq -K$  and  $\tilde{J}^{ia}(T) \geq K$ , respectively (where  $J^{ia}(T)$  is given by Equation (3.9)), and then solving for  $q_{1,T}$ . Based on the standard result for Gaussian densities that we have stated above in Equation (3.18), and after some lines of algebra, we finally obtain the following closed-form solution for the futures price  $F_{1,t}^{ia}(T)$  under market coupling:<sup>52</sup>

$$\begin{aligned}
F_{1,t}^{ia}(T) &= \mathbb{E}_t [P_{1,T}^{ia,ex}] \Phi \left( \frac{\mathcal{A}_X^{ia} + \mathcal{A}_q^{ia} - \mathcal{K}}{\sqrt{\mathcal{C}_X^{ia} + \mathcal{C}_q^{ia}}} \right) + \mathbb{E}_t [P_{1,T}^{ia,im}] \left[ 1 - \Phi \left( \frac{\mathcal{A}_X^{ia} + \mathcal{A}_q^{ia} + \bar{\mathcal{K}}}{\sqrt{\mathcal{C}_X^{ia} + \mathcal{C}_q^{ia}}} \right) \right] \\
&\quad + \mathbb{E}_t [P_{1,T}^{ia,un}] \left[ \Phi \left( \frac{\mathcal{B}_X^{ia} + \mathcal{B}_q^{ia} + \bar{\mathcal{K}}}{\sqrt{\mathcal{C}_X^{ia} + \mathcal{C}_q^{ia}}} \right) - \Phi \left( \frac{\mathcal{B}_X^{ia} + \mathcal{B}_q^{ia} - \mathcal{K}}{\sqrt{\mathcal{C}_X^{ia} + \mathcal{C}_q^{ia}}} \right) \right], \quad (3.20)
\end{aligned}$$

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<sup>52</sup>Note that for  $\mathbb{E}_t [P_{1,T}^{ia,ex}]$ ,  $\mathbb{E}_t [P_{1,T}^{ia,un}]$ , and  $\mathbb{E}_t [P_{1,T}^{ia,im}]$ , explicit expressions are stated in the technical Appendix 3.6.1

with:

$$\begin{aligned}
\mathcal{A}_X^{ia} &= \frac{\delta_2}{\beta_1} \mu_{X_2} - \frac{\delta_1}{\beta_1} \mu_{X_1} - \frac{\delta_1^2}{\beta_1} \sigma_{X_1}^2 + \frac{\delta_1 \delta_2}{\beta_1} \rho_X \sigma_{X_1} \sigma_{X_2}, & \mathcal{A}_q^{ia} &= \frac{\beta_2}{\beta_1} \mu_{q_2} - \mu_{q_1} - \beta_1 \sigma_{q_1}^2 + \beta_2 \rho_q \sigma_{q_1} \sigma_{q_2}, \\
\mathcal{B}_X^{ia} &= \frac{\delta_2}{\beta_1} \mu_{X_2} - \frac{\delta_1}{\beta_1} \mu_{X_1} - \frac{\delta_1}{\beta_1} \frac{\delta_1 \gamma_2}{\gamma_1 + \gamma_2} \sigma_{X_1}^2 + \frac{\delta_2}{\beta_1} \frac{\delta_2 \gamma_1}{\gamma_1 + \gamma_2} \sigma_{X_2}^2 + \frac{(\delta_2/\beta_1) \delta_1 \gamma_2 - (\delta_1/\beta_1) \delta_2 \gamma_1}{\gamma_1 + \gamma_2} \rho_X \sigma_{X_1} \sigma_{X_2}, \\
\mathcal{B}_q^{ia} &= \frac{\beta_2}{\beta_1} \mu_{q_2} - \mu_{q_1} - \frac{\beta_1 \gamma_2}{\gamma_1 + \gamma_2} \sigma_{q_1}^2 + \frac{\beta_2}{\beta_1} \frac{\beta_2 \gamma_1}{\gamma_1 + \gamma_2} \sigma_{q_2}^2 + \frac{\beta_2 \gamma_2 - \beta_2 \gamma_1}{\gamma_1 + \gamma_2} \rho_q \sigma_{q_1} \sigma_{q_2}, \\
\mathcal{C}_X^{ia} &= \left( \frac{\delta_1}{\beta_1} \right)^2 \sigma_{X_1}^2 - 2 \frac{\delta_1 \delta_2}{\beta_1^2} \rho_X \sigma_{X_1} \sigma_{X_2} + \left( \frac{\delta_2}{\beta_1} \right)^2 \sigma_{X_2}^2, & \mathcal{C}_q^{ia} &= \sigma_{q_1}^2 - 2 \frac{\beta_2}{\beta_1} \rho_q \sigma_{q_1} \sigma_{q_2} + \left( \frac{\beta_2}{\beta_1} \right)^2 \sigma_{q_2}^2, \\
\underline{\mathcal{K}} &= \frac{\gamma_1 + \gamma_2}{\beta_1} K + \frac{1}{\beta_1} \mathcal{S}, & \overline{\mathcal{K}} &= \frac{\gamma_1 + \gamma_2}{\beta_1} K - \frac{1}{\beta_1} \mathcal{S}, \\
\mathcal{S} &= \ln \alpha_1 - \ln \alpha_2 + \delta_1 s_{g_1}(T) - \delta_2 s_{g_2}(T) + \beta_1 s_{D_1}(T) - \beta_2 s_{D_2}(T).
\end{aligned}$$

It is important to mention that the structure of the above formula merely reflects our 4-variate Gaussian setting where  $F_{1,t}^{ia}(T)$  is simply expressed as an average of the respective futures contracts pertaining to each of the three states. The states are defined by  $J^{ia}(T)$  and weighted by the probability of reaching each such state.<sup>53</sup>

If we are considering an explicit ex-ante allocation of capacity rights, it is again possible to derive analytic futures pricing formulae, even though our piecewise defined spot price process is no longer Markovian, because capacity and electricity spot markets do not clear simultaneously, as mentioned above. However, given that in our setting, the individual state variables  $D_{i,t}$  and  $g_{i,t}$  with  $i = \{1, 2\}$  are still Markovian, and since we only need to condition on a finite number of points in time of the past (i.e., on  $\tau = T - k$  in this case), a closed-form expression for the futures price  $F_{1,t}^{ea}(T)$  is still possible. Note first that, unlike for market coupling,  $J^{ea}(T)$  was already determined at an earlier point in time  $\tau = T - k$  and hence, is known at time  $T$ . At time  $\tau$ , and based on Equation (3.6) above, we then obtain:

$$\begin{aligned}
F_{1,\tau}^{ea}(T) &= \mathbb{E}_\tau [P_{1,T}^{ea}] \\
&= \alpha_1 \exp\left(\delta_1 (\mu_{X_1}(\tau, T) + s_{g_1}(T)) + \beta_1 (\mu_{q_1}(\tau, T) + s_{D_1}(T)) - \gamma_1 J^{ea}(T)\right. \\
&\quad \left. + \frac{1}{2} \delta_1^2 \sigma_{X_1}^2(\tau, T) + \frac{1}{2} \beta_1^2 \sigma_{q_1}^2(\tau, T)\right). \tag{3.21}
\end{aligned}$$

The time- $t$  value of a futures contract under the explicit allocation regime with maturity  $T$ ,  $F_{1,t}^{ea}(T)$ , can then be derived based on Equation (3.21). Using iterated conditioning, it

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<sup>53</sup>However, note that in Equation (3.20), the factors involving the cumulative distribution function  $\Phi(\cdot)$  do not exactly represent these probabilities given that for some state  $z$ ,  $\mathbb{E} [P_{1,T}^{ia,z} \mathbb{I}_{\{A\}}] \neq \mathbb{E} [P_{1,T}^{ia,z}] \mathbb{P}(A)$ .

follows:

$$\begin{aligned}
F_{1,t}^{ea}(T) &= \mathbb{E}_t \left[ \mathbb{E}_\tau \left[ P_{1,T}^{ea} \right] \right] \\
&= \mathbb{E}_t \left[ \mathbb{E}_\tau \left[ P_{1,T}^{ea,ex} \right] \mathbb{I}_{\{\tilde{J}^{ea}(T) \leq -K\}} + \mathbb{E}_\tau \left[ P_{1,T}^{ea,un} \right] \mathbb{I}_{\{-K < \tilde{J}^{ea}(T) < K\}} \right. \\
&\quad \left. + \mathbb{E}_\tau \left[ P_{1,T}^{ea,im} \right] \mathbb{I}_{\{\tilde{J}^{ea}(T) \geq K\}} \right], \tag{3.22}
\end{aligned}$$

where  $\tilde{J}^{ea}(T)$  is  $\mathcal{F}_\tau$ -measurable. The explicit closed-form solution to the above equation is provided in the technical Appendix 3.6.2

### 3.4.6 Analysis of Futures Prices

Based on Equations (3.19) and (3.22), we can easily see that the futures price curves in both markets under either an explicit allocation or market coupling regime are actually a blend of the respective futures price curves for each state, i.e.,  $\mathbb{E}_t \left[ P_{i,T}^{(\cdot),ex} \right]$ ,  $\mathbb{E}_t \left[ P_{i,T}^{(\cdot),un} \right]$ , and  $\mathbb{E}_t \left[ P_{i,T}^{(\cdot),im} \right]$  (as derived in the Appendix), each of which is essentially affine-linear in the underlying state variables (on log-basis). Revisiting our case of two identically parametrized markets, and assuming that the deseasonalized state variables have reverted back to their long-run means, i.e.,  $q_{i,t} = X_{i,t} = 0$ , we can now qualitatively argue that the futures price in an isolated market,  $F_{i,t}^{iso}(T)$ , will always serve as an upper bound to its counterparts  $F_{i,t}^{ea}(T)$  and  $F_{i,t}^{ia}(T)$ . Starting with  $\varrho_X = \varrho_q = 1$ , we can verify from Equations (3.23) and (3.28) that  $\mathbb{E}_t \left[ P_{i,T}^{ia,un} \right] = \mathbb{E}_t \left[ P_{i,T}^{ea,un} \right] = \mathbb{E}_t \left[ P_{i,T}^{iso} \right]$ . In this case, given that the state variables will coincide in any instance, there will be no price differentials between the two markets that would provide an incentive for cross-border trade. Consequently, the weightings for the import and export states in Equations (3.20) and (3.25) must be zero, yielding  $F_{i,t}^{ia}(T) = F_{i,t}^{ea}(T) = F_{i,t}^{iso}(T)$ . With decreasing correlations  $\varrho_X$  and  $\varrho_q$ , two effects must be distinguished: first,  $\mathbb{E}_t \left[ P_{i,T}^{ia,un} \right]$  and  $\mathbb{E}_t \left[ P_{i,T}^{ea,un} \right]$  will always be smaller than  $\mathbb{E}_t \left[ P_{i,T}^{iso} \right]$ , which can again be seen from Equations (3.23) and (3.28) in Appendices A and B. Second, the above mentioned weightings for the import and export states will start to increase, and given the convexity of the exponential function,  $\mathbb{E}_t \left[ P_{i,T}^{(\cdot),ex} \right]$  will have a stronger effect on increasing futures prices than  $\mathbb{E}_t \left[ P_{i,T}^{(\cdot),im} \right]$  on decreasing them. Note, however, that the corresponding weightings in Equations (3.20) and (3.25) are not symmetric but are adjusted by the terms  $\underline{\mathcal{K}}$  and  $\overline{\mathcal{K}}$  ( $\underline{\mathcal{L}}$  and  $\overline{\mathcal{L}}$ , respectively). As a consequence, the import weighting will always be higher than the export weighting,

which – along with the first effect – will ensure that  $F_{i,t}^{(\cdot)}(T) \leq F_{i,t}^{iso}(T)$ .

However, introducing cross-border trade between the two electricity markets may not only lead to a level shift in futures prices  $F_{i,t}^{(\cdot)}(T)$  as compared to  $F_{i,t}^{iso}(T)$ , but may also change the shape of the futures curve, as is shown in Figure 3.8. Here, we have assumed a higher price for the generating fuel in market 1 by setting  $X_{1,t} = 0.4$ , whereas we retain  $q_{i,t} = X_{2,t} = 0$  and fix all other parameters as employed in Subsection 3.4.4. Consequently, for the case of isolated markets, spot prices in market 1 will be higher than the futures prices that reflect the mean-reverting behavior of  $X_{1,t}$  back to its zero-mean in the long run, which altogether results in a backwardated futures curve in market 1. Market 2, given that  $K = 0$ , is not affected by the higher fuel prices in the adjacent market and has its futures curve in contango.

By contrast, with the possibility of cross-border exchange with either explicitly or implicitly allocated capacities, the curve in market 1 now is in contango for short-term maturities, which, in turn, forces the futures curve in market 2 into short-term backwardation. Hence, market 1 clearly benefits from indirectly accessing lower-cost generation in the neighboring market where prices now are correspondingly higher. Disaggregating further, the humped shape of the futures curve in market 1 can be explained by examining its individual components: with  $X_{1,t} = 0.4$  and for very short maturities, the probability of the more expensive market 1 nevertheless being in the *export* state is almost zero. The longer the time horizon, however, the more is  $X_{1,t}$  expected to mean-revert so that the weighting for  $\mathbb{E}_t \left[ P_{1,T}^{(\cdot),ex} \right]$  (see Equations (3.20) and (3.25), respectively) increases. This effect dominates for maturities of up to slightly more than one year. Thereafter, it is outweighed by the facts that (i) both  $\mathbb{E}_t \left[ P_{1,T}^{(\cdot),ex} \right]$  and  $\mathbb{E}_t \left[ P_{1,T}^{(\cdot),im} \right]$  are generally decreasing in  $T$  (given our starting value for  $X_{1,t}$ ) and that (ii)  $\mathbb{E}_t \left[ P_{1,T}^{(\cdot),un} \right]$  – although being approximately flat for any maturity – also exerts downward pressure on the “aggregate” futures curve since its associated weighting factor is decreasing for longer maturities.

Finally, it is interesting to see how the spread between futures prices will react to a change in the allocation mechanism, i.e., when switching from explicit allocations to market coupling (or vice versa). The bottom LHS graph of Figure 3.8 shows the spread between futures prices that corresponds to the top RHS and LHS graphs. As can be seen, switching from the explicit ex-ante mechanism to market coupling leads to a widening

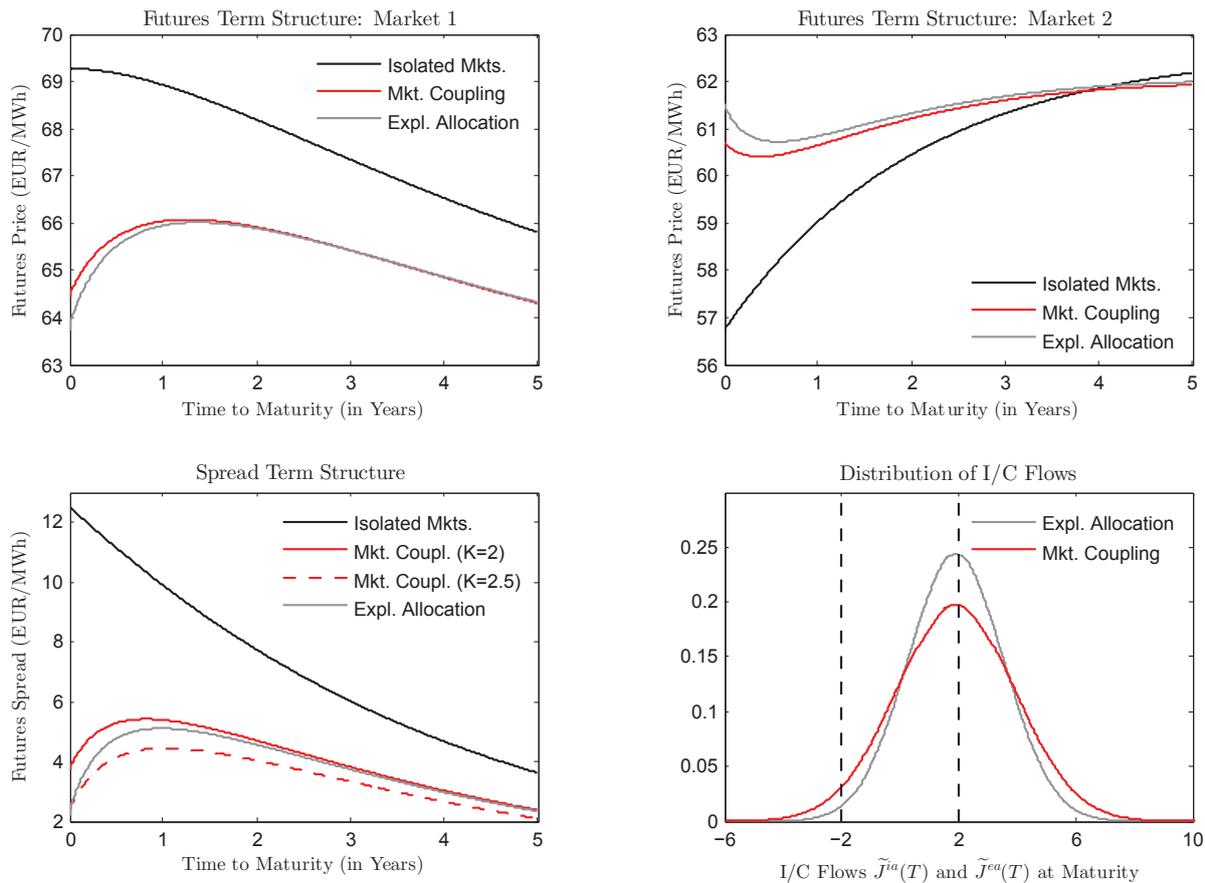


Figure 3.8: **Futures and Futures Spread Term Structure**

The top LHS and RHS graphs show the futures term structure in markets 1 and 2 for the cases that the capacity of the interconnector ( $i$ ) is not available (isolated markets), (ii) is allocated via an explicit ex-ante scheme, or (iii) via an implicit scheme. The bottom LHS graph presents the corresponding spreads between futures prices in both markets. The bottom RHS graph displays the distribution of interconnector flows  $\tilde{J}^{ia}(T)$  and  $\tilde{J}^{ea}(T)$  for an assumed horizon of  $T = 30$  days.

of the futures spread, which seems to be surprising given that spot price convergence under market coupling could be expected to also manifest in futures prices. The reason for this result is as follows: in order to assess how the futures spread will react to a change in the allocation regime, we need to compare Equations (3.19) and (3.22) for both markets. In view of the results from Equations (3.26) and (3.27) in Appendix B, and further simplifying, we shall now only focus on the differences between the indicator variables in the two expressions: in fact, it can be shown that for the more expensive market 1, a change to market coupling will always lead to a higher probability for the export state  $\mathbb{I}_{\{\tilde{J}^{ia}(T) \leq -K\}}$ . To illustrate this fact, the bottom RHS graph of Figure 3.8 shows the distribution of interconnector flows  $\tilde{J}^{ia}(T)$  and  $\tilde{J}^{ea}(T)$  for an assumed maturity of  $T = 30$  days. From Equations (3.9) and (3.11), in turn, we know that  $\tilde{J}^{ia}(T)$  is a

random variable (being itself a function of random variables) up until time  $T$ , whereas  $\tilde{J}^{ea}(T)$  will already be determined at time  $T - k$ . Consequently, the variance of the time- $T$  interconnector flows under market coupling will always be higher than for the case of explicit allocation, as is indicated by the fatter tails of the red distribution in the graph. More precisely, the area below the red curve for flows  $\tilde{J}^{ia}(T) < -K$  essentially reflects the probability of the more expensive market 1 being in the export state. Given our assumption of equal capacity limits in both directions of the interconnector,  $\mathbb{P}(\tilde{J}^{ia}(T) < -K)$  for market 1 always increases as soon as the mean of flows is larger than zero. Hence, the weighting of the three different futures price components under market coupling will shift towards  $\mathbb{E}_t [P_{1,T}^{ia,ex}]$  for market 1 and towards  $\mathbb{E}_t [P_{2,T}^{ia,im}]$  for market 2. As such, roughly speaking, the more expensive market will even see slightly higher futures prices under market coupling, whereas the cheaper market can expect slightly lower futures prices, which altogether leads to a widening of the spread.<sup>54</sup>

However, when comparing these theoretical effects with empirical observations, the following *caveat* applies: in contrast to what is implied by our model, futures prices may nevertheless converge in reality given that actually available interconnector capacities  $K$  under market coupling can be higher than the capacities that are available for the same line under an explicit scheme. In fact, there are two reasons for this. On the one hand, the issue of opposing flow nominations is treated differently under the two regimes; under an explicit scheme, such as explicit ex-ante auctions, transmission rights are allocated separately for each direction and flows nominated in opposite directions cannot be netted at all timescales.<sup>55</sup>

On the other hand, as is analyzed in Mahringer (2013) for the French–German border, the share of explicitly allocated yearly and monthly PTRs that are actually exercised by traders to ship electricity across borders has significantly declined since the introduction of market coupling. In this case, the UIOSI principle leads to a perfect hedging of the spread risk between two markets on a day-ahead basis, which causes market participants

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<sup>54</sup>Note that our simplified reasoning leaves aside other effects such as (i) covariance terms between expectations and indicator functions or (ii) the fact that generally  $\mathbb{E}_t [P_{1,T}^{ia,un}] \leq \mathbb{E}_t [P_{1,T}^{ea,un}]$ , all of which are second-order effects, however, that do not outweigh the indicated directions of how futures prices move when changing from explicit to implicit allocations.

<sup>55</sup>See Höffler and Wittmann (2007) for a detailed analysis of netting in interconnector auctions. With respect to our example represented by the timeline in Figure 3.1, opposing flows nominated by holders of yearly or monthly transmission rights can be netted *prior* to the day-ahead auction of transmission capacity. However, this is not possible on the day-ahead stage itself. For further information, also see Hobbs et al. (2005), Bunn and Zachmann (2010), or Pellini (2012).

to increasingly opt for the financial compensation in case of no-exercise, rather than to set up a physical transaction in order to arbitrage the two markets by themselves.

Hence, these non-nominated capacities become available to the market coupling facilitator on a day-ahead basis, thus increasing  $K$  under market coupling. As can be seen in the bottom LHS graph of Figure 3.8, assuming, by way of example, that the above arguments lead to a 25% increase in actually available interconnector capacity  $K$  under market coupling now clearly leads to a lower futures spread as compared to the case of an explicit ex-ante regime.

### 3.5 Conclusion

Pricing and hedging in electricity markets has traditionally been a challenging and complex field. The complexity is primarily driven by the general non-storability of this commodity, however, additionally complicated by institutional specificities such as the differentiation of hourly electricity products, futures contracts delivering electricity over a certain period rather than at a fixed point in time, longer-term delivery contracts with cascading upon maturity, or other exotic products such as swing options. More recently, the increasing interconnectivity between national markets has further added to these complexities.

As has been shown, different ways of organizing cross-border trade between interconnected markets can significantly alter empirical price dynamics and, hence, render previously used, widespread modeling approaches inapplicable. As such, aspects of market design do and will continue to be of key interest for both practitioners and researchers, especially given two important developments: first, within the foreseeable future, interconnection capacity between markets is projected to remain scarce. Hence, price differentials between markets will persist, and so will different ways to manage ensuing congested cross-border flows. Second, the CWE market coupling focused on in this study is accompanied by a series of other regional coupling initiatives throughout Europe, such as the South-Western Europe (SWE) coupling between France, Spain and Portugal, or the North-Western Europe (NWE) coupling between CWE and the Scandinavian/Baltic countries. Within the “Price Coupling of Regions” (PCR) initiative, these different coupling projects are planned to be integrated in order to finally reach the EU policy goal of a single Internal Electricity Market. By then, the IEM is projected to

be the world's largest electricity market, surpassing other mature markets in both the US and Australia.

Responding to the need for more sophisticated, accurate pricing models in this context, the class of fundamentally driven electricity pricing models has proven to offer a framework that provides sufficient granularity to reflect aspects of market design, while at the same time retaining flexibility and tractability to allow for closed-form solutions for many types of derivative contracts. Specifically, the model proposed in this study does not focus on the different explicit and implicit allocation mechanisms for interconnector capacity *per se*, but rather tries to mimic the outcomes of these different schemes by focusing on the resulting endogenous cross-border flows under each regime. The derived model dynamics and pricing formulae do address the shortcomings of the classic reduced-form approach and provide a rich framework to analyze how the key *stylized facts* of electricity spot prices as well as the term structure of futures prices change when markets are interconnected.

A further extension of our model could be to include a third interconnected market into our setting. While this will increase the complexity of the resulting pricing formulae, the general approach will stay the same, yet will depend on whether the additional market will be linked to both or only one of the two other markets. However, note that it may still be valid to apply a two-market model even if the empirical coupling mechanism involves more than two markets. As a matter of simplification, it may be advisable to reduce the number of markets to be modeled separately by aggregating those neighbouring markets with structurally similar patterns of demand, or similar structures within their generation parks; with only infrequent congestion at their borders, or those markets that are small compared to their neighbor (e.g., Belgium vs. France). For instance, the CWE coupling could be decomposed into the German market on the one hand, and all other markets that are part of the Trilateral coupling on the other hand.

Another avenue for further research could focus on empirically implementing the model to price PTRs under market coupling, e.g., for the German–French border. Released data show that auctions for this border incite high investor interest, and the participation of the trading arms of several investment banks may boost liquidity even further.

## 3.6 Appendix

### 3.6.1 Futures Prices for the Implicit Regime

Expressions for  $\mathbb{E}_t [P_{1,T}^{ia,ex}]$ ,  $\mathbb{E}_t [P_{1,T}^{ia,un}]$ , and  $\mathbb{E}_t [P_{1,T}^{ia,im}]$  in Equation (3.20) can easily be derived based on our piecewise definition of electricity spot prices in Equations (3.14) to (3.14) and using the properties of the lognormal distribution:

$$\begin{aligned}
\mathbb{E}_t [P_{1,T}^{ia,ex}] &= \alpha_1 \exp(\delta_1 (\mu_{X_1} + s_{g_1}(T)) + \beta_1 (\mu_{q_1} + s_{D_1}(T)) + \gamma_1 K + \frac{1}{2} \delta_1^2 \sigma_{X_1}^2 + \frac{1}{2} \beta_1^2 \sigma_{q_1}^2), \\
\mathbb{E}_t [P_{1,T}^{ia,un}] &= \exp\left(\frac{\delta_1 \gamma_2}{\gamma_1 + \gamma_2} (\mu_{X_1} + s_{g_1}(T)) + \frac{\delta_2 \gamma_1}{\gamma_1 + \gamma_2} (\mu_{X_2} + s_{g_2}(T)) + \frac{\beta_1 \gamma_2}{\gamma_1 + \gamma_2} (\mu_{q_1} + s_{D_1}(T)) \right. \\
&\quad + \frac{\beta_2 \gamma_1}{\gamma_1 + \gamma_2} (\mu_{q_2} + s_{D_2}(T)) + \frac{\gamma_2}{\gamma_1 + \gamma_2} \ln \alpha_1 + \frac{\gamma_1}{\gamma_1 + \gamma_2} \ln \alpha_2 \\
&\quad + \frac{1}{2} \left(\frac{\delta_1 \gamma_2}{\gamma_1 + \gamma_2}\right)^2 \sigma_{X_1}^2 + \frac{1}{2} \left(\frac{\delta_2 \gamma_1}{\gamma_1 + \gamma_2}\right)^2 \sigma_{X_2}^2 + \frac{\delta_1 \delta_2 \gamma_1 \gamma_2}{(\gamma_1 + \gamma_2)^2} \sigma_{X_1} \sigma_{X_2} \rho_X \\
&\quad \left. + \frac{1}{2} \left(\frac{\beta_1 \gamma_2}{\gamma_1 + \gamma_2}\right)^2 \sigma_{q_1}^2 + \frac{1}{2} \left(\frac{\beta_2 \gamma_1}{\gamma_1 + \gamma_2}\right)^2 \sigma_{q_2}^2 + \frac{\beta_1 \beta_2 \gamma_1 \gamma_2}{(\gamma_1 + \gamma_2)^2} \sigma_{q_1} \sigma_{q_2} \rho_q\right), \quad (3.23) \\
\mathbb{E}_t [P_{1,T}^{ia,im}] &= \alpha_1 \exp(\delta_1 (\mu_{X_1} + s_{g_1}(T)) + \beta_1 (\mu_{q_1} + s_{D_1}(T)) - \gamma_1 K + \frac{1}{2} \delta_1^2 \sigma_{X_1}^2 + \frac{1}{2} \beta_1^2 \sigma_{q_1}^2).
\end{aligned}$$

### 3.6.2 Futures Prices for the Explicit Ex-Ante Regime

In order to derive an analytic pricing formula for the time- $t$  futures price  $F_{1,t}^{ea}(T)$  with maturity  $T$  in market 1 under the case of explicit allocations, we re-state Equation (3.22):

$$\begin{aligned}
F_{1,t}^{ea}(T) &= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \left( \int_{-\infty}^{lb^{ea}} \mathbb{E}_\tau [P_{1,T}^{ea,ex}] \phi(q_{1,\tau} | q_{2,\tau}) dq_{1,\tau} \right. \\
&\quad + \int_{lb^{ea}}^{ub^{ea}} \mathbb{E}_\tau [P_{1,T}^{ea,un}] \phi(q_{1,\tau} | q_{2,\tau}) dq_{1,\tau} \\
&\quad \left. + \int_{ub^{ea}}^{\infty} \mathbb{E}_\tau [P_{1,T}^{ea,im}] \phi(q_{1,\tau} | q_{2,\tau}) dq_{1,\tau} \right) \\
&\quad \cdot \phi(q_{2,\tau}) \phi(X_{1,\tau} | X_{2,\tau}) \phi(X_{2,\tau}) dq_{2,\tau} dX_{1,\tau} dX_{2,\tau} \quad (3.24)
\end{aligned}$$

where  $\phi(q_{1,\tau} | q_{2,\tau})$  and  $\phi(X_{1,\tau} | X_{2,\tau})$  are the (risk-neutral) conditional densities of  $q_{1,\tau}$  given  $q_{2,\tau}$  and of  $X_{1,\tau}$  given  $X_{2,\tau}$ , respectively.  $\phi(q_{2,\tau})$  and  $\phi(X_{2,\tau})$  are the (risk-neutral) unconditional densities of  $q_{2,\tau}$  and  $X_{2,\tau}$ , respectively. The lower and upper bounds  $lb^{ea}$  and  $ub^{ea}$  for the innermost integrals over  $q_{1,\tau}$  can be found in the same way as for  $lb^{ia}$  and  $ub^{ia}$  in the case of market coupling, i.e., by taking as starting point the corresponding inequations  $\tilde{J}^{ea}(T) \leq -K$  and  $\tilde{J}^{ea}(T) \geq K$ , respectively (where  $J^{ea}(T)$  is given by Equation (3.11)), and then solving for  $q_{1,\tau}$ . To preserve space, we introduce the following

shorthand notation:

$$b_i = e^{-\kappa_{q_i}(T-\tau)} = e^{-\kappa_{q_i}k}, \quad d_i = e^{-\kappa_{X_i}(T-\tau)} = e^{-\kappa_{X_i}k}$$

Simplifying according to Equation (3.18) and after few manipulations, we finally obtain:

$$\begin{aligned} F_{1,t}^{ea}(T) &= \mathbb{E}_t \left[ \mathbb{E}_\tau [P_{1,T}^{ea,ex}] \right] \Phi \left( \frac{\mathcal{A}_X^{ea} + \mathcal{A}_q^{ea} - \underline{\mathcal{L}}}{\sqrt{\mathcal{C}_X^{ea} + \mathcal{C}_q^{ea}}} \right) + \mathbb{E}_t \left[ \mathbb{E}_\tau [P_{1,T}^{ea,im}] \right] \left[ 1 - \Phi \left( \frac{\mathcal{A}_X^{ea} + \mathcal{A}_q^{ea} + \bar{\mathcal{L}}}{\sqrt{\mathcal{C}_X^{ea} + \mathcal{C}_q^{ea}}} \right) \right] \\ &\quad + \mathbb{E}_t \left[ \mathbb{E}_\tau [P_{1,T}^{ea,un}] \right] \left[ \Phi \left( \frac{\mathcal{B}_X^{ea} + \mathcal{B}_q^{ea} + \bar{\mathcal{L}}}{\sqrt{\mathcal{C}_X^{ea} + \mathcal{C}_q^{ea}}} \right) - \Phi \left( \frac{\mathcal{B}_X^{ea} + \mathcal{B}_q^{ea} - \underline{\mathcal{L}}}{\sqrt{\mathcal{C}_X^{ea} + \mathcal{C}_q^{ea}}} \right) \right], \end{aligned} \quad (3.25)$$

where we have:

$$\begin{aligned} \mathcal{A}_X^{ea} &= \frac{\delta_2 d_2}{\beta_1 b_1} \mu_{X_2}(t, \tau) - \frac{\delta_1 d_1}{\beta_1 b_1} \mu_{X_1}(t, \tau) - \frac{(\delta_1 d_1)^2}{\beta_1 b_1} \sigma_{X_1}^2(t, \tau) + \frac{\delta_1 \delta_2 d_1 d_2}{\beta_1 b_1} \rho_X(t, \tau) \sigma_{X_1}(t, \tau) \sigma_{X_2}(t, \tau), \\ \mathcal{A}_q^{ea} &= \frac{\beta_2 b_2}{\beta_1 b_1} \mu_{q_2}(t, \tau) - \mu_{q_1}(t, \tau) - \beta_1 b_1 \sigma_{q_1}^2(t, \tau) + \frac{\beta_2 b_2}{\beta_1 b_1} \beta_1 b_1 \rho_q(t, \tau) \sigma_{q_1}(t, \tau) \sigma_{q_2}(t, \tau), \\ \mathcal{B}_X^{ea} &= \frac{\delta_2 d_2}{\beta_1 b_1} \mu_{X_2}(t, \tau) - \frac{\delta_1 d_1}{\beta_1 b_1} \mu_{X_1}(t, \tau) - \frac{(\delta_1 d_1)^2 \gamma_2}{(\beta_1 b_1)(\gamma_1 + \gamma_2)} \sigma_{X_1}^2(t, \tau) + \frac{(\delta_2 d_2) \gamma_1}{(\beta_1 b_1)(\gamma_1 + \gamma_2)} \sigma_{X_2}^2(t, \tau) \\ &\quad + \frac{\delta_1 \delta_2 d_1 d_2 (\gamma_2 - \gamma_1)}{\beta_1 b_1 (\gamma_1 + \gamma_2)} \rho_X(t, \tau) \sigma_{X_1}(t, \tau) \sigma_{X_2}(t, \tau), \\ \mathcal{B}_q^{ea} &= \frac{\beta_2 b_2}{\beta_1 b_1} \mu_{q_2}(t, \tau) - \mu_{q_1}(t, \tau) - \frac{\beta_1 b_1 \gamma_2}{\gamma_1 + \gamma_2} \sigma_{q_1}^2(t, \tau) + \frac{(\beta_2 b_2)^2 \gamma_1}{\beta_1 b_1 (\gamma_1 + \gamma_2)} \sigma_{q_2}^2(t, \tau) \\ &\quad + \frac{\beta_2 b_2 (\gamma_2 - \gamma_1)}{\gamma_1 + \gamma_2} \rho_q(t, \tau) \sigma_{q_1}(t, \tau) \sigma_{q_2}(t, \tau), \\ \mathcal{C}_X^{ea} &= \left( \frac{\delta_1 d_1}{\beta_1 b_1} \right)^2 \sigma_{X_1}^2(t, \tau) - 2 \frac{\delta_1 \delta_2 d_1 d_2}{(\beta_1 b_1)^2} \rho_X(t, \tau) \sigma_{X_1}(t, \tau) \sigma_{X_2}(t, \tau) + \left( \frac{\delta_2 d_2}{\beta_1 b_1} \right)^2 \sigma_{X_2}^2(t, \tau), \\ \mathcal{C}_q^{ea} &= \sigma_{q_1}^2(t, \tau) - 2 \frac{\beta_2 b_2}{\beta_1 b_1} \rho_q(t, \tau) \sigma_{q_1}(t, \tau) \sigma_{q_2}(t, \tau) + \left( \frac{\beta_2 b_2}{\beta_1 b_1} \right)^2 \sigma_{q_2}^2(t, \tau), \\ \underline{\mathcal{L}} &= \frac{\gamma_1 + \gamma_2}{\beta_1 b_1} K + \frac{1}{\beta_1 b_1} \mathcal{T}, \\ \bar{\mathcal{L}} &= \frac{\gamma_1 + \gamma_2}{\beta_1 b_1} K - \frac{1}{\beta_1 b_1} \mathcal{T}, \\ \mathcal{T} &= \mathcal{S} + \frac{1}{2} (\delta_1^2 \sigma_{X_1}^2(\tau, T) - \delta_2^2 \sigma_{X_2}^2(\tau, T)) + \frac{1}{2} (\beta_1^2 \sigma_{q_1}^2(\tau, T) - \beta_2^2 \sigma_{q_2}^2(\tau, T)). \end{aligned}$$

Finally, regarding explicit expressions for  $\mathbb{E}_t \left[ \mathbb{E}_\tau [P_{1,T}^{ea,ex}] \right]$ ,  $\mathbb{E}_t \left[ \mathbb{E}_\tau [P_{1,T}^{ea,im}] \right]$ , and  $\mathbb{E}_t \left[ \mathbb{E}_\tau [P_{1,T}^{ea,un}] \right]$ , we can see that by using iterated conditioning, we yield the following result for the first two expectations:

$$\mathbb{E}_t \left[ \mathbb{E}_\tau [P_{1,T}^{ea,ex}] \right] = \mathbb{E}_t [P_{1,T}^{ia,ex}], \quad (3.26)$$

$$\mathbb{E}_t \left[ \mathbb{E}_\tau [P_{1,T}^{ea,im}] \right] = \mathbb{E}_t [P_{1,T}^{ia,im}]. \quad (3.27)$$

For  $\mathbb{E}_t \left[ \mathbb{E}_\tau [P_{1,T}^{ea,un}] \right]$ , based on the properties of the lognormal distribution and using iterated conditioning, we obtain after few lines of algebra:

$$\begin{aligned}
\mathbb{E}_t \left[ \mathbb{E}_\tau [P_{1,T}^{ea,un}] \right] &= \exp \left( \frac{\delta_1 \gamma_2}{\gamma_1 + \gamma_2} (\mu_{X_1} + s_{g_1}(T)) + \frac{\delta_2 \gamma_1}{\gamma_1 + \gamma_2} (\mu_{X_2} + s_{g_2}(T)) + \frac{\beta_1 \gamma_2}{\gamma_1 + \gamma_2} (\mu_{q_1} + s_{D_1}(T)) \right. \\
&\quad + \frac{\beta_2 \gamma_1}{\gamma_1 + \gamma_2} (\mu_{q_2} + s_{D_2}(T)) + \frac{\gamma_2}{\gamma_1 + \gamma_2} \ln \alpha_1 + \frac{\gamma_1}{\gamma_1 + \gamma_2} \ln \alpha_2 \\
&\quad + \frac{1}{2} \left( \frac{\delta_1 \gamma_2}{\gamma_1 + \gamma_2} \right)^2 d_1^2 \sigma_{X_1}^2(t, \tau) + \frac{1}{2} \frac{\gamma_2}{\gamma_1 + \gamma_2} \delta_1^2 \sigma_{X_1}^2(\tau, T) \\
&\quad + \frac{1}{2} \left( \frac{\delta_2 \gamma_1}{\gamma_1 + \gamma_2} \right)^2 d_2^2 \sigma_{X_2}^2(t, \tau) + \frac{1}{2} \frac{\gamma_1}{\gamma_1 + \gamma_2} \delta_2^2 \sigma_{X_2}^2(\tau, T) \\
&\quad + \frac{\delta_1 \delta_2 \gamma_1 \gamma_2}{(\gamma_1 + \gamma_2)^2} d_1 d_2 \sigma_{X_1}(t, \tau) \sigma_{X_2}(t, \tau) \rho_X(t, \tau) \\
&\quad + \frac{1}{2} \left( \frac{\beta_1 \gamma_2}{\gamma_1 + \gamma_2} \right)^2 b_1^2 \sigma_{q_1}^2(t, \tau) + \frac{1}{2} \frac{\gamma_2}{\gamma_1 + \gamma_2} \beta_1^2 \sigma_{q_1}^2(\tau, T) \\
&\quad + \frac{1}{2} \left( \frac{\beta_2 \gamma_1}{\gamma_1 + \gamma_2} \right)^2 b_2^2 \sigma_{q_2}^2(t, \tau) + \frac{1}{2} \frac{\gamma_1}{\gamma_1 + \gamma_2} \beta_2^2 \sigma_{q_2}^2(\tau, T) \\
&\quad \left. + \frac{\beta_1 \beta_2 \gamma_1 \gamma_2}{(\gamma_1 + \gamma_2)^2} b_1 b_2 \sigma_{q_1}(t, \tau) \sigma_{q_2}(t, \tau) \rho_q(t, \tau) \right). \tag{3.28}
\end{aligned}$$

# Chapter 4

## Transmission Rights Valuation for Coupled Electricity Markets: An Option-based Approach<sup>\*</sup>

### 4.1 Introduction

In network industries, such as wholesale electricity or natural gas, transactions between adjacent market areas are critically dependent on the institutional framework that coordinates access to the (oftentimes congested) interconnection lines linking the respective networks. The design of this mechanism, in turn, must not only adapt to the specificities of the traded commodity – such as the non-storability of electricity preventing real-time scheduling of flows – but also adequately capture the functional context of why said flow goods shall be exchanged across market borders: be it of limited scope in the case of markets relying on the principle of self-sufficiency, or be it vital for those that primarily serve as a transit hub linking their neighboring markets.

With respect to cross-border trade of *electricity*, its overall economic relevance has gradually changed over time – especially for European markets. Starting in early pre-liberalization times as an “emergency back-up” to help balance unexpected (net) supply shortages between the then primarily isolated European electricity markets, cross-border transactions have since become a crucial instrument to realize the idea of a single pan-

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<sup>\*</sup>This chapter is based on the corresponding single-authored working paper entitled “*Transmission Rights Valuation for Coupled Electricity Markets: An Option-based Approach.*” Cf. Mahringer (2013). The paper has been presented in the PiF Seminar 2013 at University of St.Gallen.

European electricity market, as promoted by the European Union (EU). In this context, as one of the milestones for fostering more competitive and liquid cross-border trade, the EU electricity market directives expressly stipulate third-party access to interconnectors based on a “*non-discriminatory market-based*” allocation mechanism.<sup>1</sup>

From a variety of such allocation mechanisms that comply with this requirement, explicit ex-ante auctions of physical transmission rights (PTRs) have turned out to become the most widely implemented congestion management method,<sup>2</sup> which is primarily due to the fact that they only require a low level of institutional standardization between the respective interconnected markets. Hence, rather than embarking on the arduous task of harmonizing the design of European electricity markets, explicit ex-ante auctions help to avoid this problem by instead separating the markets for cross-border transmission capacity and electricity. Thus, the auction process for PTRs and/or the nomination of corresponding physical cross-border flows are scheduled prior to the clearing of electricity spot markets, so that in general, interconnector regulation does not directly interfere with their individual operational timescales.

However, as has extensively been analyzed in the literature,<sup>3</sup> explicit ex-ante mechanisms are generally prone to producing economically inefficient interconnector flows, given that the above timing disconnect forces cross-border traders to submit bids in the capacity market based on the *expected* rather than realized price differential between the respective two markets. As an alternative, implicit auction mechanisms, such as market coupling, integrate the auction of cross-border capacity into the spot auctions of electricity, thus always yielding economically efficient interconnector flows from the cheaper to the more expensive market. Ultimately, perfect price convergence across all markets involved in the coupling mechanism can be reached when the cross-border flows required to equalize prices are operationally feasible and not constrained by insufficient transmission capacities on the interconnection lines.<sup>4</sup>

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<sup>1</sup>See, e.g., Regulations (EC) No. 714/2009 (Article 12(2)) and its predecessor, No. 1228/2003.

<sup>2</sup>Given that in Europe, most interconnectors at that time used to be (and still are) congested, we will use the terms “capacity allocation mechanism” and “congestion management method” interchangeably.

<sup>3</sup>See, e.g., Ehrenmann and Smeers (2005), Turvey (2006), Zachmann (2008), Bunn and Zachmann (2010), or Füss et al. (2013b).

<sup>4</sup>Note that although prevalent in Europe, market coupling mechanisms are currently planned to also be implemented, e.g., in the US electricity markets: for instance, the Tres Amigas Superstation project has been set up in order to unite the three major regional US electricity grids (the Eastern Interconnection, the Western Electricity Coordinating Council (WECC), and the Electric Reliability Council of Texas (ERCOT)); for the management of cross-border flows across the interconnectors at this trading hub, a market coupling mechanism is currently being considered. See EPEX-Spot (2012).

In this context, these different institutional set-ups for cross-border trade have been shown to significantly impact the dynamics of both spot and futures prices in each of the interconnected markets (Füss et al., 2013b). However, since generic transmission contracts, in their broadest sense, can generally be interpreted as a derivative written on the spread between electricity prices in the two interconnected markets, their value must hence be even more sensitive to a change in the respective “regime” for cross-border trade: clearly, for a transmission *right*, the value of the corresponding underlying spread *option* must be different under a market coupling mechanism (targeting perfect price convergence between markets) as compared to under explicit ex-ante auctioning and scheduling (causing inefficient flows that help sustain price differentials).

In extant literature, however, these institutional details are only rarely taken into account when it comes to the pricing of transmission rights or obligations. Generally, most studies focus on the pricing of PTRs in the context of an explicit ex-ante auction regime,<sup>5</sup> and propose to value the rights as European options on the spread between spot prices in the respective two electricity markets. Important institutional details that characterize the explicit ex-ante auction regime, such as the timing disconnect between the markets for transmission capacity and spot electricity, are left out in most of these analyses, however. For instance, Wobben et al. (2012) model PTRs as options on the spot spread, yet admit that “*in fact these contracts are options on the expected spot prices, because nomination takes place 4h before day-ahead market clearing.*” As such, by assuming that option exercise and determination of its payoff occur simultaneously, most empirical research on the pricing of PTRs – even if only to simplify matters – has thus relied on a framework that, strictly speaking, applies to implicitly auctioned transmission rights rather than to explicit auctions: as will be shown in this chapter, it is only for the former type of auction mechanism that the option payoff (i.e., the spot spread) can be considered realized upon option exercise.

Janssen et al. (2011) follow this line of reasoning and compare the value of PTRs under the two regimes by reducing the problem to an analysis of different times of option exercise. More precisely, given that in the case of explicitly auctioned PTRs, option exercise virtually coincides with the ex-ante scheduling of flows, they propose to value such contract as an option on the *expected* spot price spread between the respective two

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<sup>5</sup>The pricing of explicitly auctioned PTRs is examined by, e.g., Marckhoff (2009), Marckhoff and Muck (2009), Bunn and Martocchia (2010), Wobben et al. (2012), and McNerney and Bunn (2013).

markets, whereas under market coupling, the transmission right shall be valued as an option that is directly written on the same, yet *realized*, spread. However, modeling the spot price spread as a univariate asset fitted to historical data, they do not consider that the spread dynamics under a market coupling regime are essentially different as compared to an explicit ex-ante regime, given that the mechanics of the coupling mechanism will lead to increased spot price convergence between the two markets.<sup>6</sup> Therefore, when comparing the two different types of contracts, it is obvious that an option directly written on the spot spread itself will be worth more than if it were written on an expectation of the spread at maturity and had to be exercised upon nomination – as long as both spreads (erroneously) obey the same dynamics. Nevertheless, the fact that under market coupling, a zero price spread will be obtained more frequently (thus, by contrast, driving down the value of the option on the realized spread) has thus far not yet been considered in literature – neither theoretically, nor empirically.

Hence, it is not only the aspect of market design that makes transmission rights valuation under market coupling a challenging task; finding the best way to adequately capture the intricate dependence structure of the price spread on (i) the most important power price determinants in each of the coupled markets, but also on (ii) other variables such as the interconnector capacity, even further increases the complexity of the required modeling set-up – and clearly questions the widespread use of classic reduced-form approaches in this context: a frequently mentioned example relates to the high share of renewables in the German generation mix. For instance, during times of significant wind energy feed-in, this not only lowers German day-ahead prices but may also spill over into adjacent markets via interconnector flows. This effect tends to be strong for the case of market coupling (see, e.g., Heren, 2011; Carr, 2012a), and often results in primarily negative cross-border spreads (with respect to the German market). By contrast, if the fundamentals in the German and neighboring electricity markets are not too far apart, price spreads can be exactly zero for certain hours of the day and for several days in a row. Obviously, these scenarios cannot be reproduced when using a reduced-form modeling approach – be it univariate to model the spread directly, or be it bivariate when modeling the spot price time series for each market separately.

In this chapter, we address these challenges and investigate the pricing of transmission rights under market coupling, taking the currently prevailing European market design for

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<sup>6</sup>See Füss et al. (2013b) for further information.

coupled electricity markets as a starting point. As such, we contribute to the literature in the following ways: first, we outline the institutional setup for cross-border trading under market coupling and provide empirical evidence that recent changes in market design have clearly affected both pricing levels and exercise behavior for PTRs. In view of the continued implementation of market coupling throughout Europe, spread options between coupled pricing areas are a crucial tool for hedging and, thus, are seeing considerable demand by both physical as well as financial electricity traders. Using data for the French–German interconnector, we see that the apparent undervaluation of PTRs compared to the corresponding futures spreads disappears when changing from an explicit auction set-up with ex-ante scheduling of flows to an implicit set-up such as market coupling.

Second, to the best of our knowledge, we are the first to derive an analytic pricing formula for a spread option on spot electricity prices in two *coupled* markets. Based on the fundamental framework for modeling two interconnected electricity markets proposed by Füss et al. (2013b), we not only take into account the above mentioned institutional details for cross-border trade but also yield a model that adequately reproduces the distinct spread dynamics that are characteristic for a market coupling regime. More generally, the availability of analytic pricing formulae helps to retain tractability and ease of use for practitioners, but also will facilitate calibration to observed PTR prices in case of an empirical implementation.

Finally, we conduct sensitivity analyses for the key input parameters to our spread option formula in order to analyze in detail the main determinants of value for a transmission right under market coupling. Here, it is important to see that since we model electricity prices as a function of fundamental factors, the relationship between option prices and the current value of the underlying spot spread is no longer unique, but must be disaggregated further to the level of underlying state variables. Importantly, differentials in the levels of the individual state variables – that would otherwise result in a non-zero spread between spot prices in each market – can be mitigated and balanced by the market coupling mechanism, which in turn significantly impacts option prices as compared to the situation when the two markets are isolated and not interconnected.

The remainder of this chapter is structured as follows: the next section provides additional background on the institutional framework for cross-border trading under market coupling. Section 4.3 outlines the general modeling framework. Section 4.4 derives

the spread option formula for two coupled markets and provides sensitivity analyses to further illustrate the intricacies of spread option pricing under market coupling. Section 4.5 concludes.

## 4.2 Transmission Rights and Market Coupling

### 4.2.1 Physical and Financial Transmission Contracts

Being a network-bound commodity, the transfer of electricity across markets may be subject to transmission congestion on the respective interconnection line, which prevents price convergence and has traders face locational price risk instead. In order to hedge such transmission risk, instruments and contracts that are available in the areas of cross-border trading and congestion management can generally be categorized according to two criteria: *(i)* enforceability of the contract (rights vs. obligations), and *(ii)* the nature of the settlement mechanism (financial vs. physical).

Transmission *rights* are characterized by a convex payoff profile based on their inherent optionality for the option holder to collect the price difference between two connected markets, to the extent it is non-negative for a pre-specified direction of flow and given hour of the day. For instance, if a generator in market 1 who is contractually obliged to supply electricity to customers in neighboring market 2 hedges his cross-zonal position with a transmission right for the direction  $1 \rightarrow 2$ , he will be entitled to receive the price spread for every transmitted megawatt hour (MWh) of electricity if market 2 is more expensive than market 1 – but not vice versa (since the right is only uni-directional). Transmission *obligations*, by contrast, require their holders to also bear the downside risk of a negative price spread for a given direction of flow – as implied either from the (mandatory) physical scheduling of flows or the payoff from a financially settled transmission obligation.

Regarding the second criterion, *financial* transmission contracts are not directly tied to the scheduling of physical flows of electricity across a possibly congested interconnection line, but instead replicate the same payoff (upon financial settlement) as if the corresponding physical positions had been entered into. By contrast, *physical* transmission contracts are more operational in nature, and are primarily intended to establish an exchange of electricity – be it mandatory or optional – between two connected

markets. As such, while both physical and financial contracts are ultimately linked to the available transmission capacity of the underlying interconnector,<sup>7</sup> only the former class of contracts provide factual network access to the holder. Instead, with an exclusively FTR-based set-up, all cross-border transactions are essentially handled through the involved power exchanges or, e.g., a market coupling facilitator. In this case, and returning to the above example, a generator in market 1 cannot directly serve the customers in market 2 via physically transferring electricity via the interconnector, but can replicate the payoff from such transaction by (i) selling its own generation in market 1, (ii) sourcing the owed volumes in market 2, and (iii) hedging his cross-zonal position with a financial transmission contract.

Out of this classification, physical contracts are prevalent in Europe and are mainly established as rights (i.e., PTRs) which are often also referred to as a “carve-out” of cross-border transmission capacity (Duthaler and Finger, 2008). However, an important contractual feature, the so-called “Use-It-or-Sell-It” (UIOSI) property, also allows these rights to be used as financial transmission rights (FTRs). For instance, given a monthly (yearly) PTR,<sup>8</sup> its holder is entitled to schedule physical flows of electricity across the respective interconnector during any individual hour of that month (year). In case the trader does not want to exercise his right for a certain hour (e.g., because it might be uncertain as to how the price differential between the markets will turn out, thus risking an economically inefficient trade by transferring electricity from a more expensive market into a cheaper market), the TSO needs to be notified one day in advance, usually in the morning hours well ahead of the close of the day-ahead electricity markets.<sup>9</sup>

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<sup>7</sup>In the case of financial contracts, options or obligations issued by the transmission system operator (TSO) need to be funded by the congestion rent that is collected for the respective interconnector concerned, and that depends on (physical) transmission constraints such as the capacity of the line. Hence, given the integrated nature of meshed electricity networks, revenue adequacy (between issued contracts and congestion rents to fund them) is generally ensured by carrying out a simultaneous feasibility test that determines the optimal number of contracts to be issued for all interconnectors within a given network at the same time. Note that in general, a lower number of financial transmission options can be released for a given interconnector than in the case of obligations due to the inability of netting opposite flow directions. See, e.g., ETSO (2006) or, more generally, Kristiansen and Rosellón (2013) for further information.

<sup>8</sup>In Europe, PTRs are usually auctioned off for different timescales, i.e., a certain fraction of the overall interconnector capacity is allocated via yearly and monthly auctions; the remaining capacity either is also auctioned off via the day-ahead capacity market (where transmission rights for all 24 hours of the subsequent day are allocated individually) or, in the case of market coupling, is made available to the power exchanges and/or market coupling office. Hence, in the latter case, OTC traders can still set-up tailor-made, non-exchange based cross-border transactions by acquiring monthly or yearly transmission rights (instead of the daily rights no longer offered to the market).

<sup>9</sup>See, e.g., RTE (2009a) and RTE (2009a) where this process is outlined for cross-border trading at the French–German border.

As a financial compensation in case of non-exercise, the trader then receives the market value of the forfeited option(s) relating to those hour(s) during which he does not want to schedule any cross-border flows. Hence, in the case of explicit auctions, the unused capacity (that was originally allocated to holders of monthly/yearly transmission rights) is returned to the market and added to those capacities that are up for allocation on the day-ahead stage. The prices achieved for the corresponding hours during that auction then yield the financial compensation for traders who chose not to exercise. Yet in the case of market coupling, there are no auctions of *short-term* transmission rights that would help to establish the financial compensation in a similar manner: on a day-ahead stage, these capacities are not allocated to the market but directly passed on to the power exchanges that schedule the market coupling flows intended to achieve price convergence. In this case, the financial compensation instead is defined as the actual spread between the prices resulting from the day-ahead auctions of electricity for the respective coupled markets.

Importantly, this mechanism thus not only always ensures revenue adequacy (Duthaler and Finger, 2008), but also allows for perfect hedging of price differences between adjacent *coupled* markets in Europe by using monthly/yearly PTRs with UIOSI property and not exercising them. As such, this combination of physical and financial features could also be seen as a first step towards transitioning to a fully financial setting comparable to the US, especially given that FTRs were also recently proposed as an alternative to the current physical set-up in Europe by the Agency for the Cooperation of Energy Regulators (ACER).<sup>10</sup>

#### 4.2.2 PTRs and the Impact of the CWE Market Coupling

The implications of the above theoretical concepts and specificities of market design can best be illustrated when examining the introduction of the Central Western Europe (CWE) market coupling and how it changed the valuation of physical transmission rights. Prior to the implementation of CWE market coupling in November 2010, when Germany joined the then already existing “Trilateral Coupling” of the French, Belgian, and Dutch electricity markets, PTRs with UIOSI property for the French–German and Dutch–German borders were allocated via explicit ex-ante auctions on a yearly, monthly,

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<sup>10</sup>For further information, see ACER (2011) and de Maere d’Aertrycke and Smeers (2013).

and day-ahead basis. Under the coupling mechanism, monthly and yearly PTRs are still auctioned explicitly, yet as will be seen, both previous valuation approaches and patterns of exercise for these rights have fundamentally changed.

In the academic literature cited in the previous section, PTRs are generally proposed to be valued as a portfolio of hourly spread options or exchange options (Margrabe, 1978) with payoff  $(P_{1,T} - P_{2,T})^+$  at maturity  $T$  – as implied by the physical positions taken by a cross-border trader who is long in market 2 and short in market 1 whenever electricity is cheaper in the former market than in the latter. Following this line of reasoning, and abstracting from other contractual details for the time being, prices paid for monthly transmission rights should reflect an inherent optionality and, hence, exceed the corresponding spread of 1-month ahead futures prices. Taking the French–German border as an example, as can be seen in the top panel of Figure 4.1, the auction prices for monthly PTRs more or less closely track the spread in 1-month ahead futures contracts when the spread is positive, or are bounded at zero when the spread is negative for a given direction of flow. However, at several instances prior to the start of the CWE market coupling, prices paid were clearly lower than the corresponding futures spread, which contradicts the above valuation argument. This is again illustrated in the middle panel of Figure 4.1, where the absolute-value premia of PTR prices vs. futures spreads are shown for the direction of flow that implies a positive futures spread.<sup>11</sup>

This apparent undervaluation of PTRs especially during the years 2006-09, i.e., *prior* to the German market joining the market coupling mechanism, is due to several reasons. Clearly, cross-border trading in Europe had previously suffered from rather low levels of investor interest and trading activity so that the observed negative price differences between PTRs and futures spreads could merely be due to market participants demanding a premium for the illiquidity of the transmission rights. Importantly, however, the above interpretation of a PTR with payoff  $(P_{1,T} - P_{2,T})^+$  at maturity essentially ignores the fact that under the previous regime, “option exercise” actually had to take place before the spread  $P_{1,T} - P_{2,T}$  was determined: recall that holders of longer-term transmission rights either had to notify their intended transactions to the TSO in the morning or, alternatively, could opt to receive as financial compensation the PTR price at which their

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<sup>11</sup>Specifically, at every auction date, we obtain a PTR price for each direction that could be compared with the corresponding futures spread. However, we refrain from comparing the PTR price for the direction for which the futures spread is negative since the concept of a PTR premium over a negative futures spread might be ill-defined in this context.

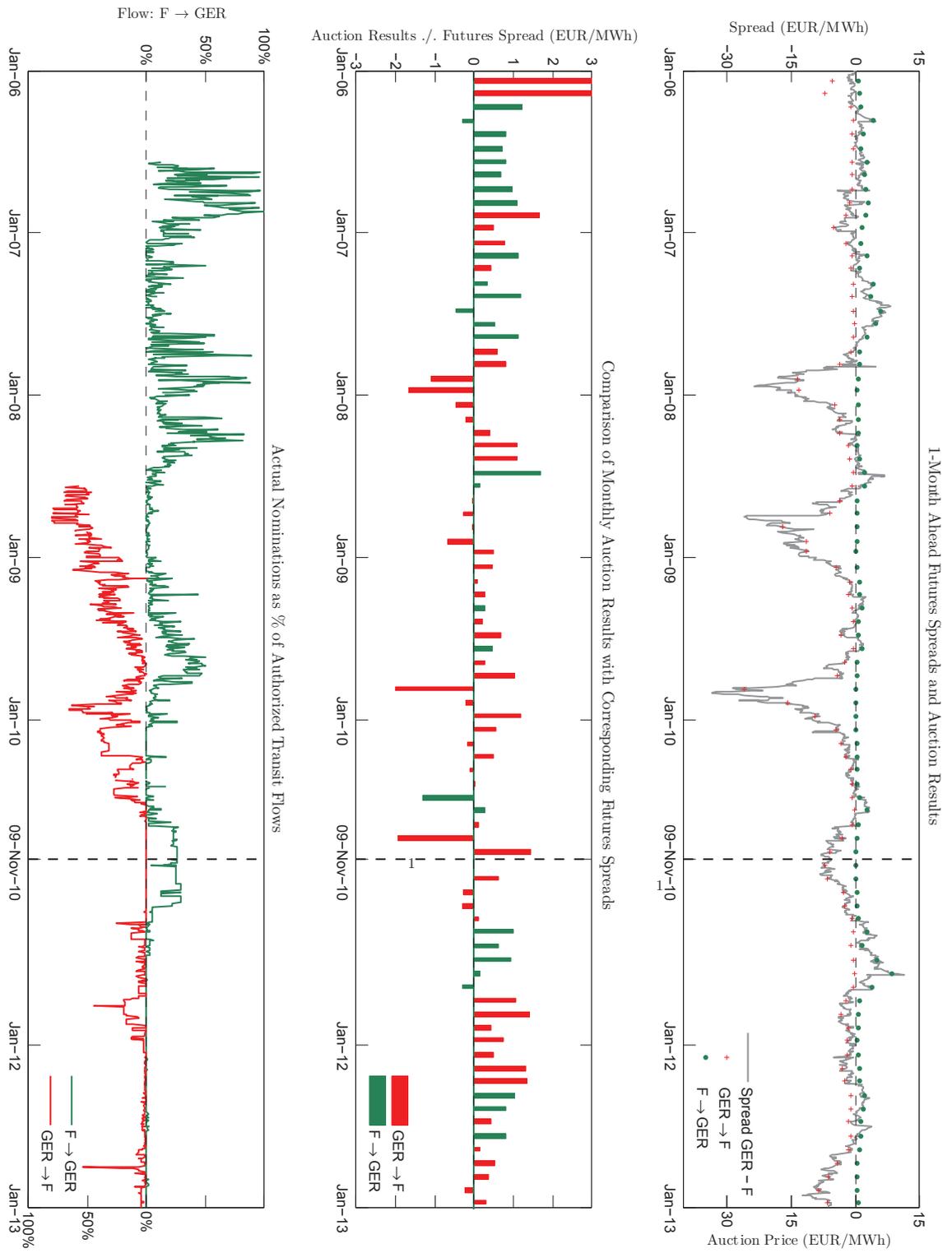


Figure 4.1: Monthly Auction Results and Nominations at the French – German Interconnector

The top panel shows the spread in (generic) 1-month ahead futures contracts between Germany and France as well as corresponding results for monthly auctions of interconnector capacity for  $F \rightarrow GER$  (positive range of secondary y-axis) and  $GER \rightarrow F$  (negative range) during the period from 01-Jan-2006 to 31-Dec-2012. The middle panel illustrates the premium of auction prices over the corresponding futures spread. The bottom panel shows the development of actually nominated transit flows as percentage of authorized flows. The vertical dashed line marks the start of the CWE (Central Western Europe) market coupling between Germany, France, the Netherlands, and Belgium as per 09-Nov-2010. All data sourced from Bloomberg, Ampriion (<http://www.ampriion.net>), and RTE (<http://clients.rte-france.com>).

rights were re-sold in the daily auction. In either case, however, the inefficiencies of the explicit auction setting would affect both the option payoff in case of physical exercise or the price of the re-sold daily PTRs and, hence, of the alternative financial compensation.

Since the start of the CWE market coupling, however, this setting has changed. As outlined above, although *daily* transmission capacity is no longer offered to the market at all, this does not apply to longer-term rights for monthly and yearly transmission capacity which are still allocated explicitly. In addition, according to the UIOSI feature, the financial compensation for non-exercised long-term rights is now directly tied to the actually realized spot price difference during the respective hourly period (to the extent it is positive).<sup>12</sup> Hence, (automatic) exercise of the option takes place whenever the day-ahead spread between the two relevant markets implies a positive intrinsic value of the instrument. Therefore, it is only under market coupling that the underlying to the PTR with a UIOSI feature coincides with the *realized* spread (and not the *expected* spread), and only in this case should valuation of such a contract be based on the payoff profile  $(P_{1,T} - P_{2,T})^+$ .

Moreover, this change to the specifications of the UIOSI principle under market coupling has clearly changed the overall characteristics of these instruments, which may also help to further promote liquidity in both primary and secondary markets. On the one hand, physical traders such as generators now benefit from an increased hedge effectiveness that also allows them to increasingly resort to non-domestic markets in order to hedge forward their production. For instance, recently observed declines in trading volume for Dutch and Belgian futures contracts were attributed to several Dutch and Belgian generators henceforth hedging by using structures based on the more liquid German futures contracts (Carr, 2012b), which can then be complemented with PTRs to account for the remaining spread risk. On the other hand, the possibility to use the PTRs with UIOSI feature as a (purely) financial transmission right is not only appealing to generators seeking effective hedges, but may also help to spark interest from other investor groups. Thus, especially for speculators and other players without physical assets, previous barriers to take part in European cross-border trade may be lowered, which more generally opens up further perspectives for trades to be financially motivated

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<sup>12</sup>See RTE (2009b), pg. 57: “(...) a price equal to (...) the price difference between the relevant day-ahead spot markets in Belgium, France, the Netherlands, and Germany for the considered Hourly Period (which might be zero (0) EUR/MWh), as far as the direction of the Programming Authorization equals the direction of flow resulting from relevant power exchange prices (...).”

rather than physically. These developments are also reflected in Figure 4.1 where in the mid panel, we can qualitatively observe that the undervaluation of PTRs (compared to futures spreads) has clearly been mitigated, given that the pricing of these instruments obviously has become more competitive in general since 09-Nov-2010. Also, the bottom panel indicates that since the start of market coupling, actually nominated capacities have strongly decreased, which provides clear evidence for the above mentioned shift in applicability of PTRs towards hedging (or even speculation) rather than for physical shipping.

## 4.3 Modeling Two Coupled Electricity Markets

### 4.3.1 Reduced-Form Modeling Approaches

Although popular, reduced-form approaches to electricity price modeling increasingly struggle to keep pace with recent developments and regulatory changes in electricity markets, such as the introduction of negative electricity prices at some power exchanges, or the impact of carbon emission allowances as additional price driver.<sup>13</sup> The interconnectivity of European electricity markets adds to these complexities and poses new challenges: unlike under the explicit ex-ante scheduling of cross-border flows that caused the typical oscillating behavior of the day-ahead price spreads between adjacent markets, spread dynamics under market coupling are now fundamentally different, given that prices in coupled markets will usually coincide much more frequently.

In the case of spread options, the dynamics of the underlying price spread have previously most often been modeled either indirectly as the difference between individual spot price time series in a bivariate reduced-form setting (see, e.g., Marckhoff, 2009; or Wobben et al., 2012) or directly as a univariate asset (see, e.g., Cartea and Gonzalez-Pedraz, 2012; or Marckhoff and Muck, 2009). Both approaches, however, will clearly yield a mis-specified model if to be applied to coupled electricity markets, which is due to both structural and practical reasons.

More precisely, not only do reduced-form settings fail, by their inherent nature, to make use of additional information about fundamental factors that strongly drive electricity price dynamics, such as generating fuels, domestic electricity demand, or

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<sup>13</sup>See, e.g., Fanone et al. (2013) and Carmona et al. (2012), respectively, for further information.

available generation capacity. Extending a pricing framework to include this kind of information, especially when combined with forecasts for some of these factors, can help increase pricing performance significantly (Füss et al., 2013a). Consequently, reduced-form models thus also leave aside the same type of information for adjacent markets, which, however, is crucial in view of the increasing interconnectivity of European electricity markets.

Furthermore, against this background, calibration of these models is also problematic and may yield unreliable estimation results. Especially for electricity markets, structural breaks due to a change in market design, such as a first-time roll-out of market coupling or also the case of other national markets joining an existing coupling, may occur relatively often, so that proper and sound model calibration directly based on historic spot (or forward) price data is almost impossible. Note that these problems can be mitigated when instead inferring price dynamics from underlying state variables that may be unaffected by such changes in market design.<sup>14</sup>

In addition, the transmission capacity of an interconnector has a crucial impact on the behavior of the spread between two adjacent markets. Although limited by an upper (thermal) bound, the actually available capacity can fluctuate a lot over time, which need not only be due to varying safety margins as imposed by TSOs, but also due to frequent maintenance and unexpected outages. Kristiansen (2007), for instance, examines cross-border electricity flows between Denmark and Germany during the years 2003-2004, and reports that the interconnector was not operating at all for 23% of the time due to scheduled maintenance. Therefore, when calibrating a reduced-form model – be it univariate to model the spread directly or bivariate for its components – to such *historic* data, the resulting price dynamics would implicitly be driven by an *average* interconnector capacity that may well be too low.<sup>15</sup> Equivalently, the same problem arises when a *future* outage of the interconnector is announced so that the dynamics of the spread would need to be adjusted for this event.

Note that when alternatively implementing a regime-switching approach with the regimes being defined according to available transmission capacity of the interconnector,

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<sup>14</sup>This is especially the case for the state variables that will be used to model electricity prices in our fundamental framework presented further below – i.e., for electricity demand and the generating fuel. Unlike demand and fuel prices, available generation capacity – that is also often used to model electricity prices in a fundamental framework (see, e.g., Cartea and Villaplana, 2008; Aïd et al., 2013; Füss et al., 2013a) – might nevertheless be affected by a change in market design since this may impact the possibly strategic bidding behavior of the generators in each of the interconnected markets.

<sup>15</sup>This is due to the averaging over periods of both full use and outages at the same time.

these problems could potentially be mitigated. However, as can empirically be observed, not only do available capacities fluctuate considerably for many European interconnectors, which might require to factually distinguish more regimes than is technically sensible from a modeling point of view; but also may special cases (e.g., where the interconnector can temporarily be used in one direction only) lead to additional complexities. In any case, the resulting model dynamics would still not be driven by the *economic causality* of cross-border exchange under market coupling, which, however, is prominently incorporated in the fundamental modeling approach that we present for transmission rights valuation further below.

### 4.3.2 Fundamental Modeling Approaches

In order to address the shortcomings of the reduced-form approaches when it comes to transmission rights valuation, we propose to use the modeling framework for two coupled markets that is originally developed in Füss et al. (2013b). This set-up belongs to the class of fundamental (or structural/hybrid) electricity pricing models,<sup>16</sup> and models electricity prices for each market  $i$  as a function of underlying state variables, i.e., domestic electricity demand  $D_{i,t}$  and the price for the domestic generating fuel  $g_{i,t}$ . Having established the dynamics for these fundamental price drivers, we then derive electricity spot prices  $P_{i,t}$  for each market based on an exogenous functional specification that mimics the typical price formation process in electricity spot markets, based on a supply-demand equilibrium with totally inelastic demand and a steeply increasing merit-order curve. Finally, the interconnector flow that links the two markets is introduced into the setting and derived by (i) equating spot prices in each market, and (ii) considering the arising non-linearity in the flow due to the upper/lower bounds to interconnector capacity  $K$ . We briefly re-state the model in the following and refer to Skantze et al. (2004), Coulon (2013), and Füss et al. (2013a, 2013b) for further information.

Regarding the first state variable, the dynamics of  $D_{i,t}$  are modeled based on a mean-reverting Ornstein-Uhlenbeck (OU) process and a deterministic function in order to reflect the distinct seasonalities that can generally be observed for electricity demand

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<sup>16</sup>See, e.g., Pilipovic (1998), Eydeland and Wolyniec (2002), or Benth et al. (2008b) for a general overview on different electricity pricing models, and Carmona and Coulon (2012) for an exhaustive summary of the approaches subsumed under the class of fundamental models.

(see, e.g., Cartea and Villaplana, 2008; or Füss et al., 2013a). On a filtered probability space  $(\Omega, \mathcal{F}^D, \mathbb{F}^D = (\mathcal{F}^D)_{t \in [0, T^*]}, \mathbb{P})$ , demand  $D_{i,t}$  is hence assumed to be governed by the following dynamics:

$$D_{i,t} = q_{i,t} + s_{D_i}(t), \quad (4.1)$$

$$dq_{i,t} = -\kappa_{q_i} q_{i,t} dt + \eta_{q_i} dB_{i,t}, \quad (4.2)$$

where  $q_{i,t}$  is an OU-process for market  $i$  with mean-reversion parameter  $\kappa_{q_i}$ ,  $B_{i,t}$  is a standard Brownian motion, and  $s_{D_i}(t)$  is a deterministic seasonality function. Note that when modeling two geographically neighboring markets, it is natural to assume that  $q_{1,t}$  and  $q_{2,t}$  are correlated, i.e., we allow for  $dB_{1,t}dB_{2,t} = \rho_q dt$ .

Regarding the price process for the generating fuel, we again follow Füss et al. (2013b) and employ the mean-reverting one-factor model by Schwartz (1997) along with a deterministic component; hence, on a filtered probability space  $(\Omega, \mathcal{F}^g, \mathbb{F}^g = (\mathcal{F}^g)_{t \in [0, T^*]}, \mathbb{P})$ , the log fuel price,  $\ln g_t$ , shall be driven by the following dynamics:

$$\ln g_{i,t} = X_{i,t} + s_{g_i}(t), \quad (4.3)$$

$$dX_{i,t} = -\kappa_{X_i} X_{i,t} dt + \eta_{X_i} dW_{i,t}, \quad (4.4)$$

where  $X_{i,t}$  is an OU-process for market  $i$  with mean-reversion parameter  $\kappa_{X_i}$ ,  $W_{i,t}$  is a standard Brownian motion, and  $s_{g_i}(t)$  is a deterministic seasonality function. Again, potential correlation between the marginal fuel price processes in the two markets is considered by allowing for  $dW_{1,t}dW_{2,t} = \rho_X dt$ .

For pricing derivatives, such as spread options in the context of transmission rights valuation, we have to transfer the above  $\mathbb{P}$ -dynamics for  $D_{i,t}$  and  $g_{i,t}$  to a risk-neutral measure  $\mathbb{Q}$  by introducing market prices of demand and fuel price risk,  $\Lambda_{q_i}(t) = \int_0^t \lambda_{q_i,s} ds$  and  $\Lambda_{X_i}(t) = \int_0^t \lambda_{X_i,s} ds$ .<sup>17</sup> Note that in the case of our market setting, however, there is no unique equivalent martingale measure given that non-traded risk factors

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<sup>17</sup>When pricing derivatives in electricity markets, the market price of risk has often empirically been observed to change signs and/or exhibit term structure-like patterns, which can be explained by, e.g., different degrees of hedging pressure of consumers and producers depending on their time horizon for risk diversification (see, e.g., Benth et al., 2008a; Weron, 2008; or, more generally, Bessembinder and Lemmon, 2002). Given that, e.g., in the case of yearly transmission rights, the underlying portfolio of hourly spread options comprises a wide range of maturities, we propose to use time-varying market prices of risk, thus yielding a more flexible parametrization for our class of risk-neutral measures  $\mathbb{Q}$ .

(such as electricity demand  $D_{i,t}$ ) cannot be hedged and, hence, render the market incomplete. Following, e.g., Carmona et al. (2013) or, more generally, Bjork (2009), we thus assume that the market selects a risk-neutral pricing measure  $\mathbb{Q}$  which fulfills  $\mathbb{Q} \in \{\mathbb{Q} \sim \mathbb{P}: \text{the discounted price of each tradable asset is a local } \mathbb{Q}\text{-martingale}\}$ .

Finally, we thus have the following four-variate Gaussian setting where conditional on time  $t$  and under  $\mathbb{Q}$ ,  $q_{1,T}$ ,  $q_{2,T}$ ,  $X_{1,T}$ , and  $X_{2,T}$  are distributed as follows:

$$\begin{bmatrix} q_{1,T} \\ q_{2,T} \\ X_{1,T} \\ X_{2,T} \end{bmatrix} \stackrel{\mathbb{Q}}{\sim} N \left( \begin{bmatrix} \mu_{q_1} \\ \mu_{q_2} \\ \mu_{X_1} \\ \mu_{X_2} \end{bmatrix}, \begin{bmatrix} \sigma_{q_1}^2 & \rho_q \sigma_{q_1} \sigma_{q_2} & 0 & 0 \\ \rho_q \sigma_{q_1} \sigma_{q_2} & \sigma_{q_2}^2 & 0 & 0 \\ 0 & 0 & \sigma_{X_1}^2 & \rho_X \sigma_{X_1} \sigma_{X_2} \\ 0 & 0 & \rho_X \sigma_{X_1} \sigma_{X_2} & \sigma_{X_2}^2 \end{bmatrix} \right), \quad (4.5)$$

with

$$\begin{aligned} \mu_{q_i}(t, T) &= q_{i,t} e^{-\kappa_{q_i}(T-t)} + \eta_{q_i} \int_t^T e^{-\kappa_{q_i}(T-s)} \lambda_{q_i,s} ds, \\ \sigma_{q_i}^2(t, T) &= \frac{\eta_{q_i}^2}{2\kappa_{q_i}} (1 - e^{-2\kappa_{q_i}(T-t)}), \\ \mu_{X_i}(t, T) &= X_{i,t} e^{-\kappa_{X_i}(T-t)} + \eta_{X_i} \int_t^T e^{-\kappa_{X_i}(T-s)} \lambda_{X_i,s} ds, \\ \sigma_{X_i}^2(t, T) &= \frac{\eta_{X_i}^2}{2\kappa_{X_i}} (1 - e^{-2\kappa_{X_i}(T-t)}), \\ \rho_q(t, T) &= \frac{1}{\sigma_{q_1} \sigma_{q_2}} \frac{\varrho_q \eta_{q_1} \eta_{q_2}}{\kappa_{q_1} + \kappa_{q_2}} (1 - e^{-(\kappa_{q_1} + \kappa_{q_2})(T-t)}), \\ \rho_X(t, T) &= \frac{1}{\sigma_{X_1} \sigma_{X_2}} \frac{\varrho_X \eta_{X_1} \eta_{X_2}}{\kappa_{X_1} + \kappa_{X_2}} (1 - e^{-(\kappa_{X_1} + \kappa_{X_2})(T-t)}). \end{aligned}$$

For ease of notation, we introduce the shorthand notation that  $\mu_{(\cdot)}$ ,  $\sigma_{(\cdot)}$ , and  $\rho_{(\cdot)}$  refer to  $\mu_{(\cdot)}(t, T)$ ,  $\sigma_{(\cdot)}(t, T)$ , and  $\rho_{(\cdot)}(t, T)$ .

In order to link state variables and electricity spot prices, we invoke a continuous-time equilibrium by intersecting inelastic demand  $D_{i,t}$  with an exponential supply curve in each market, thereby considering the role of marginal fuel prices  $g_{i,t}$  as primary cost driver when (conventionally) generating electricity. Additionally, the two electricity markets are interconnected where the flow on the corresponding transmission line is denoted by  $J(t)$ .

Electricity spot prices  $P_{i,t}$  in market  $i$  with  $i = \{1, 2\}$  are then defined as follows:

$$P_{1,t} = \alpha_1 g_{1,t}^{\delta_1} \exp(\beta_1 D_{1,t} - \gamma_1 J(t)), \quad (4.6)$$

$$P_{2,t} = \alpha_2 g_{2,t}^{\delta_2} \exp(\beta_2 D_{2,t} + \gamma_2 J(t)), \quad (4.7)$$

where the flow  $J(t)$  on the interconnector results from a simplified allocation rule that is to reproduce the economically efficient allocation of cross-border capacities under market coupling and the resulting price convergence.<sup>18</sup> Hence, in a first step, the (unconstrained) interconnector flow  $\tilde{J}(t)$  is determined by equating the spot prices in the coupled markets:

$$\begin{aligned} P_{1,t} &\stackrel{!}{=} P_{2,t} \\ \alpha_1 g_{1,t}^{\delta_1} \exp(\beta_1 D_{1,t} - \gamma_1 \tilde{J}(t)) &= \alpha_2 g_{2,t}^{\delta_2} \exp(\beta_2 D_{2,t} + \gamma_2 \tilde{J}(t)). \end{aligned} \quad (4.8)$$

Solving for  $\tilde{J}(t)$  then yields:

$$\tilde{J}(t) = \frac{1}{\gamma_1 + \gamma_2} [\ln \alpha_1 - \ln \alpha_2 + \delta_1 \ln g_{1,t} - \delta_2 \ln g_{2,t} + \beta_1 D_{1,t} - \beta_2 D_{2,t}]. \quad (4.9)$$

However, given that flows on the interconnector are limited by its thermal capacity  $K$ , we require for operationally feasible flows  $J(t)$  that  $-K \leq J(t) \leq K$  and, hence, define  $J(t) = \max(\min(\tilde{J}(t), K), -K)$ ;<sup>19</sup> consequently, electricity spot prices in the two markets are piecewise defined according to three different scenarios as implied by the above non-linearity in  $J(t)$ :

$$P_{1,t} = P_{1,t}^{ex} \mathbb{I}_{\{\tilde{J}(t) \leq -K\}} + P_{1,t}^{un} \mathbb{I}_{\{-K < \tilde{J}(t) < K\}} + P_{1,t}^{im} \mathbb{I}_{\{\tilde{J}(t) \geq K\}}, \quad (4.10)$$

$$P_{2,t} = P_{2,t}^{im} \mathbb{I}_{\{\tilde{J}(t) \leq -K\}} + P_{2,t}^{un} \mathbb{I}_{\{-K < \tilde{J}(t) < K\}} + P_{2,t}^{ex} \mathbb{I}_{\{\tilde{J}(t) \geq K\}}, \quad (4.11)$$

where  $P_{1,t}^{ex}$  ( $P_{2,t}^{ex}$ ) is the spot price at time  $t$  in market 1 (market 2) if it is exporting electricity to market 2 (market 1). Physical interconnector flows are then constrained by the capacity of the transmission line and an amount of  $K$  gigawatt (GW) units is exported from one market to the other.  $P_{1,t}^{un}$  ( $P_{2,t}^{un}$ ) is the time- $t$  spot price if the interconnection

<sup>18</sup>Again, we refer to Füss et al. (2013b) for further details.

<sup>19</sup>Note that we could easily accommodate the case of an interconnection line with different levels of transmission capacity for each direction, i.e.,  $K_{1 \rightarrow 2} \neq K_{2 \rightarrow 1}$ , yet refrain from doing so to simplify our analysis.

line between the two markets is *un*-constrained. In such case, there is no congestion and market 1 (market 2) may either be exporting or importing at below the capacity limit  $K$ . Correspondingly,  $P_{1,t}^{im}$  ( $P_{2,t}^{im}$ ) is the spot price of electricity in market 1 (market 2) if it is in import-state and  $J(t)$  has reached its capacity limit. More explicitly, we have for market 1 (prices for market 2 are defined analogously):

$$P_{1,t}^{ex} = \alpha_1 g_{1,t}^{\delta_1} \exp(\beta_1 D_{1,t} - \gamma_1(-K)), \quad (4.12)$$

$$P_{1,t}^{un} = \alpha_1 g_{1,t}^{\delta_1} \exp(\beta_1 D_{1,t} - \gamma_1 \tilde{J}(t)), \quad (4.13)$$

$$P_{1,t}^{im} = \alpha_1 g_{1,t}^{\delta_1} \exp(\beta_1 D_{1,t} - \gamma_1 K). \quad (4.14)$$

## 4.4 Valuation and Analysis of Transmission Rights

### 4.4.1 Valuation as Spread Option

Let  $V_t(T)$  denote the value of a transmission right providing access to an interconnector between two coupled markets at time  $t$  and with maturity  $T$ . As mentioned above, under market coupling,  $V_t(T)$  can be determined by interpreting the right as an option written on the spread between spot electricity in the two markets, with payoff  $(P_{1,T} - P_{2,T})^+ = \max(P_{1,T} - P_{2,T}, 0)$ .  $V_t(T)$  can then be derived as follows:

$$\begin{aligned} V_t(T) = & e^{-r(T-t)} \mathbb{E}_t^{\mathbb{Q}} [(P_{1,T}^{ex} - P_{2,T}^{im})^+ \mathbb{I}_{\{\tilde{J}(T) \leq -K\}} + (P_{1,T}^{un} - P_{2,T}^{un})^+ \mathbb{I}_{\{-K \leq \tilde{J}(T) \leq K\}} \\ & + (P_{1,T}^{im} - P_{2,T}^{ex})^+ \mathbb{I}_{\{\tilde{J}(T) \geq K\}}]. \end{aligned}$$

Given that in the first two states ( $\tilde{J}(T) \leq -K$  and  $-K \leq \tilde{J}(T) \leq K$ ), the option will expire worthless, whereas it will always be in the money for  $\tilde{J}(T) \geq K$ , we now yield:

$$\begin{aligned} V_t(T) = & e^{-r(T-t)} \mathbb{E}_t^{\mathbb{Q}} [(P_{1,T}^{im} - P_{2,T}^{ex}) \mathbb{I}_{\{\tilde{J}(T) \geq K\}}] \\ = & e^{-r(T-t)} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{ub}^{\infty} \left( \alpha_1 g_{1,T}^{\delta_1} \exp(\beta_1 D_{1,T} - \gamma_1 K) - \right. \\ & \left. \alpha_2 g_{2,T}^{\delta_2} \exp(\beta_2 D_{2,T} + \gamma_2 K) \right) \\ & \cdot \phi(q_{1,T}|q_{2,T}) \phi(q_{2,T}) \phi(X_{1,T}|X_{2,T}) \phi(X_{2,T}) dq_{1,T} dq_{2,T} dX_{1,T} dX_{2,T}, \end{aligned} \quad (4.15)$$

where the bound  $ub$  is derived from the inequation  $\tilde{J}(T) \geq K$  (with  $\tilde{J}(T)$ , in turn, similarly defined as in Equation (4.9)) and solving for  $q_{1,T}$ . Furthermore, in order to yield an

analytic pricing formula for the value of a transmission right, we need to compute several integrals over Gaussian densities, which can be simplified by relying on the following standard result:<sup>20</sup>

$$\int_{-\infty}^l e^{cx} \Phi\left(\frac{a+bx}{d}\right) \frac{e^{-\frac{1}{2}x^2}}{\sqrt{2\pi}} dx = e^{\frac{1}{2}c^2} \Phi_2\left(l-c, \frac{a+bc}{\sqrt{b^2+d^2}}; \frac{-b}{\sqrt{b^2+d^2}}\right), \quad (4.16)$$

where  $a, b, c, d$ , and  $l$  are constants,  $\Phi(\cdot)$  and  $\Phi_2(\cdot, \cdot; \rho)$  are the cumulative distribution functions of the univariate and bivariate (correlation  $\rho$ ) standard normal distribution.

Applying Equation (4.16) and leaving out the maths in between, we can derive the following closed-form solution for  $V_t(T)$ :<sup>21</sup>

$$V_t(T) = e^{-r(T-t)} \left[ \mathbb{E}_t^{\mathbb{Q}} [P_{1,T}^{im}] \Phi\left(-\frac{\mathcal{A}_X + \mathcal{A}_q + \bar{\mathcal{K}}}{\sqrt{\mathcal{C}_X + \mathcal{C}_q}}\right) - \mathbb{E}_t^{\mathbb{Q}} [P_{2,T}^{ex}] \Phi\left(-\frac{\mathcal{B}_X + \mathcal{B}_q + \bar{\mathcal{K}}}{\sqrt{\mathcal{C}_X + \mathcal{C}_q}}\right) \right], \quad (4.17)$$

with

$$\begin{aligned} \mathcal{A}_X &= \frac{\delta_2}{\beta_1} \mu_{X_2} - \frac{\delta_1}{\beta_1} \mu_{X_1} - \frac{\delta_1^2}{\beta_1} \sigma_{X_1}^2 + \frac{\delta_1 \delta_2}{\beta_1} \rho_X \sigma_{X_1} \sigma_{X_2}, & \mathcal{A}_q &= \frac{\beta_2}{\beta_1} \mu_{q_2} - \mu_{q_1} - \beta_1 \sigma_{q_1}^2 + \beta_2 \rho_q \sigma_{q_1} \sigma_{q_2}, \\ \mathcal{B}_X &= \frac{\delta_2}{\beta_1} \mu_{X_2} - \frac{\delta_1}{\beta_1} \mu_{X_1} + \frac{\delta_2^2}{\beta_1} \sigma_{X_2}^2 - \frac{\delta_1 \delta_2}{\beta_1} \rho_X \sigma_{X_1} \sigma_{X_2}, & \mathcal{B}_q &= \frac{\beta_2}{\beta_1} \mu_{q_2} - \mu_{q_1} + \frac{\beta_2^2}{\beta_1} \sigma_{q_2}^2 - \beta_2 \rho_q \sigma_{q_1} \sigma_{q_2}, \\ \mathcal{C}_X &= \left(\frac{\delta_1}{\beta_1}\right)^2 \sigma_{X_1}^2 - 2 \frac{\delta_1 \delta_2}{\beta_1^2} \rho_X \sigma_{X_1} \sigma_{X_2} + \left(\frac{\delta_2}{\beta_1}\right)^2 \sigma_{X_2}^2, & \mathcal{C}_q &= \sigma_{q_1}^2 - 2 \frac{\beta_2}{\beta_1} \rho_q \sigma_{q_1} \sigma_{q_2} + \left(\frac{\beta_2}{\beta_1}\right)^2 \sigma_{q_2}^2, \\ \bar{\mathcal{K}} &= \frac{\gamma_1 + \gamma_2}{\beta_1} K - \frac{1}{\beta_1} \mathcal{S}, \\ \mathcal{S} &= \ln \alpha_1 - \ln \alpha_2 + \delta_1 s_{g_1}(T) - \delta_2 s_{g_2}(T) + \beta_1 s_{D_1}(T) - \beta_2 s_{D_2}(T). \end{aligned}$$

We remark that in the case of spread options on electricity spot prices, the underlyings can obviously not be used to set up a hedging portfolio and replicate the option payoff; also, although we model electricity prices as derived from a continuous-time equilibrium between underlying state variables, we do not employ a fully agent-based optimization setting, so that alternatively resorting to utility indifference pricing only for valuing derivatives (but not for determining the underlying price processes as equilibrium outcomes) may be seen as inconsistent.<sup>22</sup> However, even for an incomplete market setting,

<sup>20</sup>For further information on the result in Equation (4.16) (which ultimately is based on a change-of-variables transformation), see Geske (1979) and also Carmona and Coulon (2012).

<sup>21</sup>Note that in below Equation (4.17), we omit to state  $\mathbb{E}_t^{\mathbb{Q}} [P_{1,T}^{im}]$  and  $\mathbb{E}_t^{\mathbb{Q}} [P_{2,T}^{ex}]$  explicitly – both of which, however, can easily be derived based on Equations (4.12) and (4.14), respectively, and the properties of the lognormal distribution.

<sup>22</sup>See, e.g., Kluge (2006) for an introduction to utility indifference pricing in the context of electricity markets and Henderson (2002) for a general overview.

derivative prices have to be consistent within the set of all traded contracts, as required by the (albeit weaker) no-arbitrage arguments that we invoke here for spread option pricing: even though derivative prices will not be unique but depend on the respective equivalent martingale measure chosen, they will exclude the possibility for arbitrage profits based on the set of all other instruments that are actually traded. Note in this context that although modeling electricity prices as a function of underlying state variables increases the number of risk-neutral parameters to be extracted, it can help at the same time to enlarge the set of traded contracts that can be used for calibration purposes – especially when considering electricity derivatives as not being written on spot electricity, but instead as derivatives on underlying electricity demand and the generating fuel. Thus, in an empirical implementation, depending on the scope and number of contracts to be considered in order to extract the parametrization for the risk-neutral measure  $\mathbb{Q}$ , derivatives on only one of these underlying risk factors, such as natural gas options/futures, could additionally be included and allow for an even more robust calibration.

#### 4.4.2 Sensitivity Analysis for Transmission Rights

In order to examine the key determinants of value for transmission rights, Figures 4.2 and 4.3 illustrate payoff profiles  $(P_{1,T} - P_{2,T})^+$  and option values  $V_t(T)$  for given interconnector capacities  $K$  of 0 GW, 2 GW, and 5 GW.<sup>23</sup> Note, however, that unlike for a “classic” payoff profile of a plain vanilla spread option, we here cannot directly plot the option payoff  $(P_{1,T} - P_{2,T})^+$  against the underlying spot spread as measured on the abscissa. Recalling that for  $K > 0$ , the spot spread consists of the components in Equations (4.10) and (4.11), we see that plotting option values  $V_t(T)$  obtained for different interconnector capacities  $K$  against the same spot spread obviously would lead to distortions: notably, the probability of observing a given level of the spread in the market will not be the same across different capacities  $K$ . Instead, in the below analysis, we vary the fundamental state variables that are at the origin of changes in the spot spread, which not only avoids the above problem, but also allows for a more detailed picture on how  $V_t(T)$  is impacted

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<sup>23</sup>We assume two identically parametrized markets where the main input parameters to our model are set as follows:  $s_{D_i} = 40$  GW,  $s_{g_i} = 0.5$  (thus leaving out any seasonalities in electricity demand or fuel prices as a matter of simplification),  $\kappa_{q_i} = 0.5$ ,  $\kappa_{X_i} = 0.001$ ,  $\eta_{q_i} = 1.0$ ,  $\eta_{X_i} = 0.02$ ,  $\beta_i = 0.1$ , and  $\delta_i = 0.5$ . Furthermore, even though from an empirical point of view, the case of  $\rho_q > 0$  and  $\rho_X > 0$  may be more relevant, we assume state variables to be uncorrelated when analyzing their individual impact on option prices (instead of having to separate “overlapping effects” arising from non-zero correlations). Finally, we also assume  $\lambda_{q_i,t} = \lambda_{X_i,t} = 0$  and  $r = 0$ .

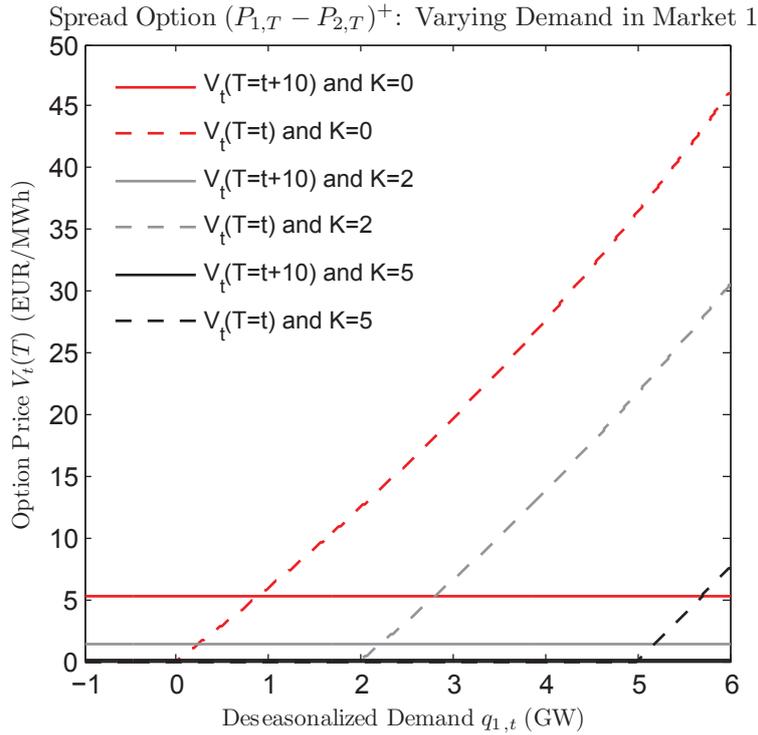
differently by movements in electricity demand and fuel prices.

In Figure 4.2, we analyze how both payoff and value of the transmission right,  $V_t(T)$ , depend on deseasonalized demand in market 1,  $q_{1,t}$  (thereby keeping the initial values of the other state variables fixed at zero, i.e.,  $q_{2,t} = X_{i,t} = 0$ ). Interestingly, the kink in the payoff profiles (indicating where the option starts to be in the money) is shifting to the right for higher levels of interconnector capacity  $K$ , which is due to the increased price convergence through the market coupling mechanism: for  $K > 0$ , the interconnector helps to meet higher demand in market 1 (as indicated by  $q_{1,t}$ ) with additional supplies from the neighboring market and thus creates a “deadband” of differentials in the underlying state variables that can still be balanced in order to maintain a zero price spread between electricity markets. In the LHS graph, we furthermore see that irrespective of the specific level of  $K$ , all transmission right values for an assumed time-to-maturity of  $\tau = T - t = 10$  days are virtually straight horizontal lines and insensitive to the initial value chosen for  $q_{1,t}$ . Hence, if today’s variation in electricity demand does not impact the final distribution of the option payoff upon maturity, all demand shocks away from the long-term (seasonal) mean must reverse by then, as is ensured by a correspondingly high speed of mean reversion for our OU-process (as specified in Equations (4.1) and (4.2)): In fact, having set  $\kappa_{q_i} = 0.5$  implies a half-life for demand shocks of only slightly more than one day, as is characteristic for electricity demand.<sup>24</sup> Consequently,  $q_{1,t}$  will not impact the option value  $V_t(T)$  except for very short-term maturities. By contrast, when instead varying the deseasonalized log-fuel price  $X_{1,t}$  in Figure 4.3, it is obvious that for the same maturity, the generally lower speed of mean reversion clearly causes option values to be more sensitive to variation in fuel prices. Since positive (negative) deviations of  $X_{1,t}$  from the long-term mean will decay only very slowly, they will still be persistent at maturity and, compared to the horizontal lines in the LHS graph, will lead to a relative increase (decrease) in option value.

With respect to the question how interconnector capacity  $K$  affects option prices, it is obvious to see from both figures that higher levels of  $K$  facilitate price convergence, thus strongly reducing the likelihood of reaching the export state  $\tilde{J}(T) \geq K$ . Therefore, taking the case of  $K = 5$  GW in Figure 4.3 as an example, and with all state variables expected

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<sup>24</sup>Note that the parameter values used for our sensitivity analyses are in line with the empirical values as estimated in Füss et al. (2013a). High levels for the speed of mean reversion for electricity demand are also confirmed by, e.g., Pirrong and Jermakyan (2008) or Coulon et al. (2013).



**Figure 4.2: Transmission Right Values and Payoff Profiles: Varying Demand**

*This figure shows the payoff profiles for an option written on the spot spread between two coupled electricity markets,  $(P_{1,T} - P_{2,T})^+$ , and assuming given interconnector capacities  $K$  of 0 GW, 2 GW, and 5 GW. Additionally, the spread option value  $V_t(T)$  is plotted for a time-to-maturity of  $\tau = T - t = 10$  days. Note that since the underlying of the option, i.e., the spot spread  $P_{1,t} - P_{2,t}$ , is itself sensitive to the interconnector capacity  $K$ , we instead vary de-seasonalized demand  $q_{1,t}$  in market 1 in order to avoid distortions (thereby keeping the other state variables in market 1 and 2 fixed at zero).*

to mean-revert to their unconditional moments by maturity at time  $T$ ,  $V_t(T = t + 10)$  is already close to zero and coincides with the horizontal axis. At the same time, this underlines the degree of model risk if transmission rights under market coupling were (erroneously) to be valued as spread options within a classic reduced-form setting. Given that interconnectivity and ensuing frequent price convergence between the markets would be ignored (i.e.,  $K = 0$ ), option values would generally be overstated: for instance,  $V_t(T = t + 10)$  for  $K = 0$  is almost four times as high as with  $K = 2$  – and in fact, we can show that  $V_t(T)$  for  $K = 0$  will always serve as an upper bound to option prices.

More precisely, for the case of two isolated markets with no possibilities to import/export electricity from each other, Equation (4.17) above entails Margrabe's formula (see Margrabe, 1978) as a special case and reduces to the well-known case of an exchange option on two lognormally-distributed assets,  $\ln(P_{1,T}^{iso})$  and  $\ln(P_{2,T}^{iso})$  with mean  $\mu_{P_i}$ , variance  $\sigma_{P_i}^2$ , and correlation  $\rho_{P_1, P_2}$ . As is straightforward to show, for  $K = 0$ ,

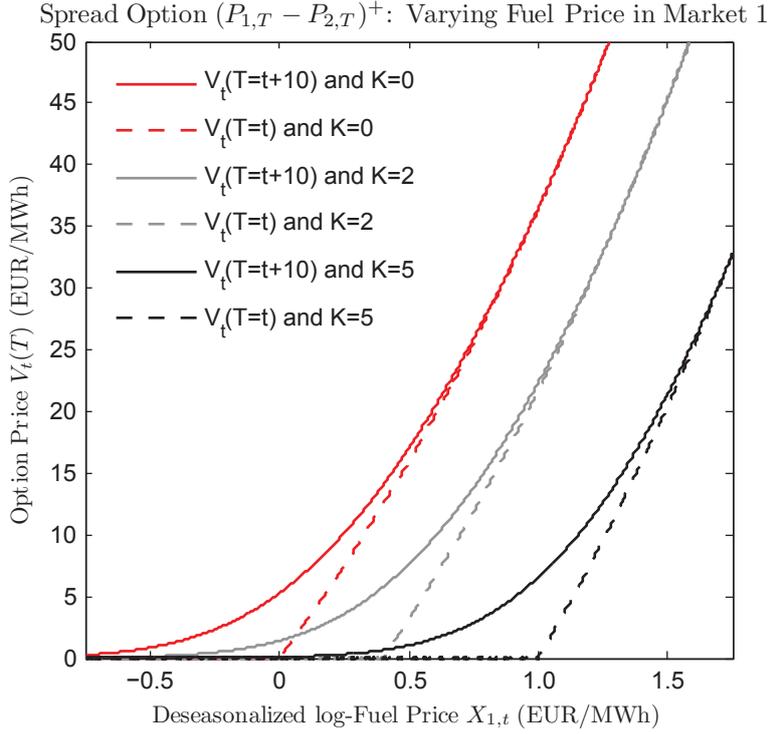


Figure 4.3: **Transmission Right Values and Payoff Profiles: Varying Fuel Price**

This figure shows the payoff profiles for an option written on the spot spread between two coupled electricity markets,  $(P_{1,T} - P_{2,T})^+$ , and assuming given interconnector capacities  $K$  of 0 GW, 2 GW, and 5 GW. Additionally, the spread option value  $V_t(T)$  is plotted for a time-to-maturity of  $\tau = T - t = 10$  days. Note that since the underlying of the option, i.e., the spot spread  $P_{1,t} - P_{2,t}$ , is itself sensitive to the interconnector capacity  $K$ , we instead vary the log-fuel price  $X_{1,t}$  in market 1 in order to avoid distortions (thereby keeping the other state variables in market 1 and 2 fixed at zero).

our option pricing formula then can be summarized as follows:

$$V_t(T) = e^{-r(T-t)} [F_{1,t}^{iso}(T)\Phi(\mathcal{D}_1) - F_{2,t}^{iso}(T)\Phi(\mathcal{D}_2)], \quad (4.18)$$

with

$$\mathcal{D}_1 = \frac{\ln\left(\frac{F_{1,T}^{iso}}{F_{2,T}^{iso}}\right) + \frac{1}{2}\sigma_{P_1,P_2}^2}{\sigma_{P_1,P_2}}, \quad \mathcal{D}_2 = \frac{\ln\left(\frac{F_{1,T}^{iso}}{F_{2,T}^{iso}}\right) - \frac{1}{2}\sigma_{P_1,P_2}^2}{\sigma_{P_1,P_2}},$$

where  $F_{i,t}^{iso}(T) = \mathbb{E}_t^{\mathbb{Q}}[P_{i,T}^{iso}]$  and  $\sigma_{P_1,P_2}^2 = \sigma_{P_1}^2 - 2\rho_{P_1,P_2}\sigma_{P_1}\sigma_{P_2} + \sigma_{P_2}^2$ . Even though it may be intuitively clear that  $V_t(T)$  must increase when interconnector capacity gets smaller, it is helpful to realize how this effect is reproduced by the mechanics of our model: notably, for  $K \rightarrow 0$ , we can already see that the arguments to the cumulative distribution functions (cdf) in Equation (4.17) converge towards  $\mathcal{D}_1$  and  $\mathcal{D}_2$ , respectively.

For identically parametrized markets, we then have  $F_{1,T}^{iso} = F_{2,T}^{iso}$  and  $\mathcal{D}_1 = -\mathcal{D}_2$ . This, in turn, implies that for  $K = 0$ , the *difference* between the “weightings”  $\Phi(\mathcal{D}_1)$  and  $\Phi(\mathcal{D}_2)$  will be highest and, given the curvature of the standard normal cdf, will start to become smaller with increasing  $K$  (as a consequence of both functional arguments to the cdf in Equation (4.17) becoming smaller themselves). At the same time, it is only for  $K = 0$  that  $\mathbb{E}_t^{\mathbb{Q}} [P_{1,T}^{im}]$  and  $\mathbb{E}_t^{\mathbb{Q}} [P_{2,T}^{ex}]$  coincide, with the latter term otherwise exceeding the former; consequently, for non-zero  $K$ , not only will the probability weightings converge to each other, but  $\mathbb{E}_t^{\mathbb{Q}} [P_{1,T}^{im}]$  and  $\mathbb{E}_t^{\mathbb{Q}} [P_{2,T}^{ex}]$  will also move in opposite directions, causing  $V_t(T)$  in Equation (4.17) to become even smaller and, hence, establishing the Margrabe formula in Equation (4.18) as an upper bound.

Finally, Figure 4.4 shows additional sensitivities of the option value  $V_t(T)$  when varying (i) the interconnector capacity  $K$ , and (ii) the ratio of unconditional (long-term) demand between the two markets. Additionally, the probability of the spread option being “in the money” is shown in the panels on the right, where we have computed the risk-neutral probability  $\mathbb{Q}(A)$  with  $\mathbb{Q}(A) = \mathbb{E}_t^{\mathbb{Q}} [\mathbb{I}_A]$  (where  $A$  refers to the event  $\tilde{J}(T) \geq K$ ).

As can be seen in the top row of Figure 4.4, the option value is very sensitive to the interconnector capacity  $K$  given that an increase in  $K$  strongly facilitates price convergence between the two markets, in which case the option will expire worthless. In fact, for our example, an interconnector capacity of 5 GW is already sufficient to drive both option value and probability of being in the money down to zero. Also note that for  $K = 0$ , i.e., for above mentioned Margrabe case, price convergence cannot be reached (unless by coincidence); thus, given that all other parameters were chosen to be equal in the two markets, the probability of exercising the option is exactly 1/2.

Varying the ratio of long-term demand in the two markets  $\mu_{q_1}/\mu_{q_2}$  for  $T \gg t$ , the option value is zero for ratios less than 1.0: in this case, higher demand in market 2 always induces higher electricity prices  $P_{2,T}$  at maturity, so that  $(P_{1,T} - P_{2,T})^+$  will be zero. On the other hand, a small increase of the ratio beyond 1.0 implies *ceteris paribus* a strong increase in option value. Finally, note that although the two markets are again identically parametrized, the probability  $\mathbb{Q}(A)$  for a long-term demand ratio of exactly 1.0 is now below 1/2. For a fixed interconnector capacity (which here has been set to  $K = 2$ ), price convergence between the two markets can now be reached more easily than in the previous

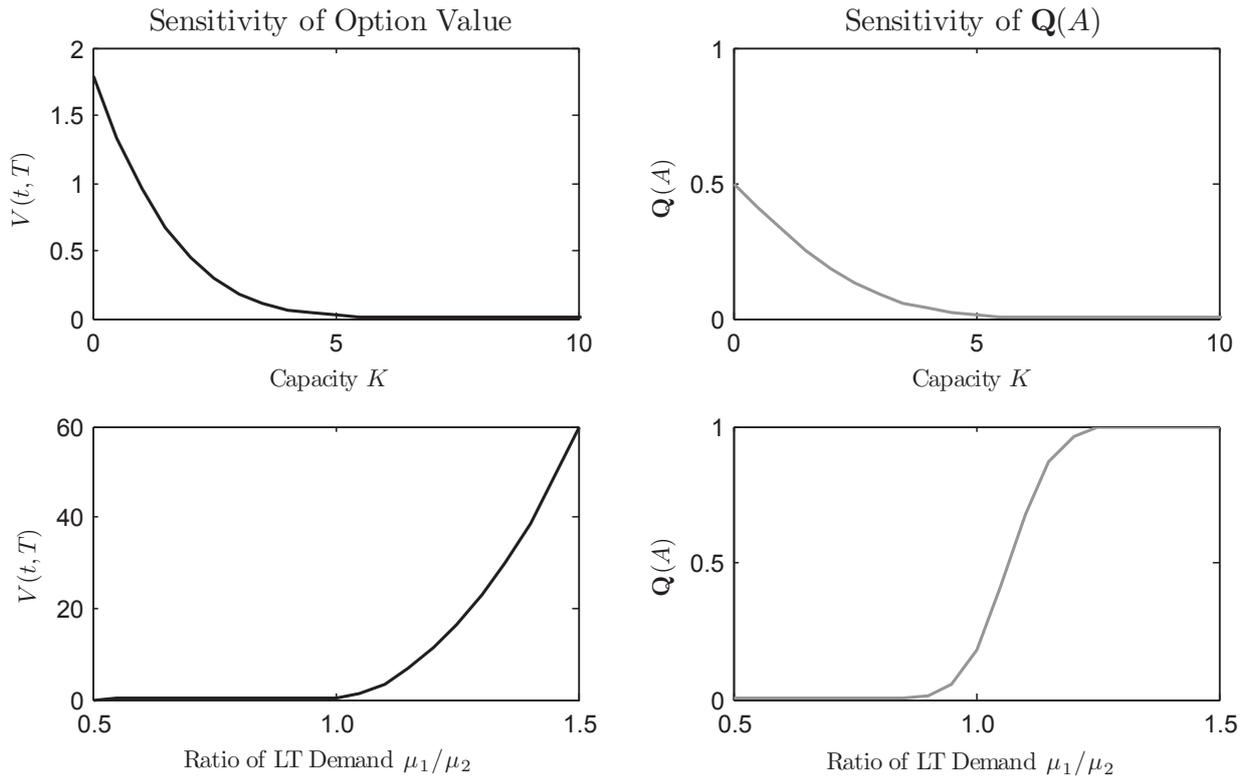


Figure 4.4: **Transmission Right Value Sensitivities**

This figure shows sensitivities of the transmission right value  $V_t(T)$  (with payoff  $(P_{1,T} - P_{2,T})^+$  at time  $T$ ) as well as of the risk-neutral probability  $Q(A)$  (where  $Q(A) = \mathbb{E}_t^Q[\mathbb{I}_A]$  with  $A$  referring to the event  $\tilde{J}(T) \geq K$ ), when varying key determinants: (i) interconnector capacity  $K$ , and (ii) the ratio of unconditional (long-term) demand between the two markets.

case outlined above, which must be compensated by a higher demand ratio in order to reach a probability of  $1/2$ .

## 4.5 Conclusion

Driven by continuing efforts to foster electricity market integration across Europe, cross-border trading has gradually evolved out of a primarily operational into a more liquid setting where transactions are increasingly becoming financial rather than physical. Following the recent roll-out of market coupling across the central European markets, and accompanied by a small but important change with respect to the UIOSI feature of physical transmission rights, longer-term PTRs now benefit from increased hedge effectiveness – given that the common inefficiencies when scheduling flows ex-ante no longer prevail. At the same time, the possibility of not exercising PTRs (but using them as FTRs instead) has sparked further investor interest, e.g., by financial players such as

the trading arms of major investment banks. As preliminary empirical evidence confirms, these changes have resulted in more competitive auction bids for PTRs that entail a premium above the corresponding futures spread, which in turn is in accordance with the common spread option approach to transmission rights valuation.

However, the intricate price dynamics of the spot spread between two coupled markets render previously used reduced-form approaches unreliable and call for more granular modeling frameworks: the fundamental setting presented in this chapter incorporates a simplified representation of a market coupling mechanism and, thus, adequately reproduces the stylized facts of electricity prices in coupled markets – which hence also applies to the corresponding spread dynamics. Our log-normal setting furthermore allows us to derive a closed-form pricing formula for transmission rights by interpreting the right as an option on the spot spread between the two markets, which is important both for computing the related option Greeks and other risk management measures, but even more so when it comes to an empirical implementation of the model.

Aspects for further research are abundant and, hence, this chapter could be taken as a starting point for further research centering on the analysis of transmission rights valuation in a fundamental modeling context: First of all, our setting could be extended to include additional coupled markets, which – albeit at the cost of increasing complexity – will clearly enhance its applicability and scope of use given the continued spreading (and linking) of both new and existing coupling initiatives. In this context, the availability of analytic pricing formulae will be crucial: not only because it usually allows for a better understanding of the model mechanics, but also since computational speed becomes even more important the more markets are to be modeled at the same time. Otherwise, simulations may likely suffer from the curse of dimensionality, making calibration to observed PTR or other derivatives prices – which is a frequently used calibration method used in an incomplete market setting – even less viable. Also, given the disproportionately high influence of renewable generation sources on spot prices in Germany, the value of transmission rights for neighboring markets could start to become sensitive to the potential occurrence of negative prices (e.g., at least for few selected hours of the day during which negative prices are most likely to occur). Hence, it would be interesting to extend our model to allow for negative-price regimes, as could be achieved with, e.g., a regime-switching set-up.<sup>25</sup>

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<sup>25</sup>See, e.g., Coulon et al. (2013) for an application in the context of structural electricity pricing models.

From an empirical point of view, it would be important to test the pricing performance of our fundamental setting, using publicly available PTR auction prices for one or several borders at the same time. Although the small sample size – PTRs are auctioned off only once a month/year – may affect the results, this also poses new challenging questions when it comes to calibrating our model. In order to yield stable and robust calibration results, trade prices of other electricity spot and/or derivatives contracts may need to be considered, too, and it would be interesting to see whether our setting is sufficiently flexible to adequately price all these contracts at the same time. Finally, note that although not discussed explicitly in this chapter, the *market splitting* mechanism used in the Nordic electricity market, NordPool, is conceptually similar to the market coupling mechanism analyzed here. Within NordPool, transmission risk between pricing areas is not managed via transmission rights but instead with Contracts for Difference (CfD), which represents an entirely financial-market based solution.<sup>26</sup> Given that our model could easily be adapted to this slightly different institutional framework, CfDs could be valued as the price of a forward spread between the local area price and the system price, thereby again taking into account that the market splitting mechanism facilitates prices convergence.

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<sup>26</sup>See, e.g., Kristiansen (2004) or Marckhoff and Wimschulte (2009) for further information.



# Chapter 5

## Concluding Remarks

In this thesis, we have focused on modeling electricity spot and forward prices in a structural framework, as opposed to the class of reduced-form models that are traditionally and widely used for other types of commodities, but also for other asset classes such as equities or FX. Although the basic idea of the latter class of models, i.e., to disentangle electricity price dynamics into a seasonal component and one (or more) stochastic processes, generally yields well-tested, pragmatic, and tractable modeling frameworks, these approaches do not fully take account of both the specificities and recent developments present in today’s liberalized electricity markets.

First and foremost, the transparency of the price formation mechanism for (spot) electricity, described in Chapter 1, allows for a more detailed and “informed” modeling of prices than is feasible for other commodity markets.<sup>1</sup> The class of pricing models presented in this thesis takes up this idea by going one step beyond mimicking the stochastic behavior of electricity *prices* and instead focusing on the interaction of fundamental price *drivers* such as supply and demand or the underlying fuel used for electricity generation.

In Chapter 2, we accordingly present a structural pricing framework and, based on the concept of *enlarged filtrations*, extend it to integrate forecasts of electricity demand and supply. These forecasts are available for many electricity markets globally and, as is shown in the empirical part of Chapter 2, clearly help to reduce pricing errors for electricity forward contracts compared to the case in which such information is discarded. While we thus strongly recommend using forward-looking information for electricity derivatives pricing whenever available, our results also show that the relative importance of the

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<sup>1</sup>See, e.g., Pirrong (2012).

fundamental factors (e.g., fuel vs. demand/capacity) may change over time, and so will the advantage of using forecasts for derivatives pricing purposes.

Apart from increasing amounts of (forward-looking) data that is publicly available to market participants and that reliable pricing models must be able to reflect in their forecasts, further – new – challenges for electricity pricing also stem from the high degree of cross-border integration and interconnectedness of electricity markets today. As outlined in Chapter 3, the organization of cross-border trade of electricity via interconnection lines across Europe not only raises many interesting questions of institutional market design and (micro-) structure; above all, it also requires to adapt electricity pricing models to the respective allocation scheme under which corresponding interconnector capacities are made available to the market.

Most notably, this applies to the continued roll-out of market coupling schemes across the European electricity markets which eliminate previous inefficiencies emanating from explicit ex-ante allocation methods and, hence, accordingly foster price convergence across neighboring markets. Importantly, however, we show that this change in market structure significantly impacts price dynamics for interconnected electricity markets, even more so for cross-border spreads, which altogether can hardly be reflected with the standard reduced-form models. Hence, we extend our fundamental pricing framework presented in Chapter 2 into a two-market setting in which, thanks to our structural approach, both said allocation schemes can be invoked and tested for their implications on price dynamics.

Whereas Chapter 3 especially focuses on spot and forward contracts, the impact of market coupling on transmission rights valuation is focused on in Chapter 4. Here we present the current regulatory framework for interconnector access in Europe and derive a new analytic valuation formula for transmission rights providing access to the interconnection lines, when valued as spread options under the case of market coupling. Continuing the discussion and extending the results from the previous chapter, we highlight how the coupling mechanism significantly affects both option values and premia versus corresponding forward spreads, and show how our results relate to the well-known Margrabe formula for spread option pricing.

Having given a rigorous treatment of the class of fundamental electricity pricing models, their fields of application, their advantages, and their limitations, it should also

be borne in mind that a reliable and powerful pricing model will always need to be tailored to and reflect currently prevailing market foundations and paradigms. This starts with the respective “target market model”<sup>2</sup> and corresponding network code for wholesale market design, and is overlaid with regulatory policies such as on carbon emission allowances or renewables feed-in tariffs. In addition, our set of given “market axioms” also subsumes other exogenous factors, such as the state of technology, according to which efficient large-scale storage facilities for electricity are currently still neither technically nor economically feasible. At the same time, the obvious state of flux that electricity markets – especially in Europe – are facing nowadays has probably never been as pronounced and dynamic since early liberalization efforts have started to unfold.

Therefore, while we have illustrated in this thesis how structural modeling approaches can help to address many of the new challenges around electricity pricing, the relationship between (market) structure and (pricing/risk management) strategy will continue to change, and so will the above “pillars” that define the current status quo in electricity markets. As such, our fundamental modeling frameworks will also finally have to follow suit. However, be it the continued increase in renewable generation and the related move towards a more decentralized generation concept that will change the structure of the supply side and its cost function; be it upon further technological revolution that smart grids, electronic cars and other battery-based solutions will impact the demand side and its degree of (in-) elasticity: maintaining a structural view on the electricity price formation process, its drivers, and their interplay, rather than on the (historical) trajectory of prices itself, should help to capture the impact of these coming innovations in a more holistic way in the first place – and, as will be crucial for electricity pricing and risk management purposes in the future, it is only on these informed grounds that modelers then should decide about which abstractions to impose on their model in order to balance accuracy and tractability.

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<sup>2</sup>In Europe, for instance, the “Florence Forum” was set up in 1998 in order to promote the development of the European coupling-based target model and monitor the overall process of market integration. The basic day-ahead target model for the Internal Electricity Market in Europe was presented at the 17th meeting of the Florence Forum in 2009, and laid out the basic principles and guidelines for today’s market coupling arrangements.



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# Curriculum Vitae

Since 2013	<b>Goldman Sachs AG</b> – Frankfurt, Germany Investment Banking Division (IBD) Associate
2011 - 2014	<b>University of St.Gallen (HSG)</b> , Switzerland Doctoral Studies in Business Administration/Finance
2009 - 2011	<b>Goldman, Sachs &amp; Co. oHG</b> – Frankfurt, Germany Investment Banking Division (IBD) Analyst
2005 - 2009	<b>University of Mannheim</b> , Germany Diploma in Business Administration ( <i>Diplom-Kfm.</i> )
2004	<i>Abitur</i> at Moll-Gymnasium Mannheim, Germany
1985	Born in Mannheim, Germany