#### Essays in Macroeconomics and Monetary Policy

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submitted by

#### Daniel Kienzler

from

Germany

Approved on the application of

#### Prof. Paul Söderlind, PhD

and

Prof. Dr. Christian Merkl

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# Summary

My dissertation consists of three self-contained essays in macroeconomics. Each essay deals with a different topic, namely (i) the influence of the output gap on potential output, (ii) long-term unemployment and skill loss, and (iii) leverage-sensitive bank financing costs. For each topic, implications for monetary policy are examined.

In the first essay (joint work with Kai D. Schmid), we present a business cycle model where — contrary to the basic New Keynesian model potential output is subject to hysteresis. That is, potential output is influenced by the output gap (the deviation of actual output from potential output). This mechanism has gained relevance in the course of the recent recession as many researchers and policy institutions have stressed the negative implications of the economic downturn for the development of potential output. We contrast simulation outcomes of the model with hysteresis to those of the basic model. Taking hysteresis into consideration allows for a stronger amplification of and a persistent output response after monetary shocks. This is in accordance with US data. Stability and welfare analyses suggest a more prominent role for economic activity in the central bank's interest rate rule if hysteresis effects are taken into account.

In the second essay, I document stylized business cycle facts about longterm unemployment in several European countries. I develop a New Keynesian business cycle model featuring labor market frictions and skill loss during unemployment and show that the model succeeds in reproducing the empirical observations. I find that the skill loss mechanism helps to explain volatility patterns across macroeconomic variables, negative duration dependence, and the behavior of the long-term unemployment proportion among total unemployment around business cycle turning points. Optimal monetary policy in the presence of skill loss allows for a higher volatility of inflation after productivity shocks to reduce skill deterioration and mitigate consumption losses by means of a lower volatility of long-term unemployment. In this way, optimal monetary policy takes into account the skill loss mechanism that private agents do not consider when they make hiring and firing decisions.

The third essay (joint work with Johannes Fritz) is about the role of bank financing costs in the transmission of monetary policy. Recently, policy makers in Europe have conjectured that the monetary policy stance is not transmitted undisturbed to bank financing costs and hence to lending rates for the real economy. This scenario cannot be described by conventional macroeconomic models. We present a New Keynesian model with a variable spread between bank financing costs and the central bank's policy rate. The sign and size of this spread depends on balance sheet conditions in the banking sector. The balance sheet conditions, in turn, can be influenced by deteriorating bank net worth arising from loan losses in economic downturns. This setup allows us to study scenarios in which policy rate movements are not fully passed on to bank financing costs. The effects of this impairment on aggregate variables are small for shocks that emanate in the real sector but sizable for a direct shock to bank net worth. An optimal policy rule in the presence of leverage-sensitive bank financing costs requires a decrease (increase) in the policy rate in response to tightening (easing) bank financing conditions.

# Zusammenfassung

Meine Dissertation besteht aus drei eigenständigen Aufsätzen im Bereich der Makroökonomie. Jeder Aufsatz behandelt ein unterschiedliches Thema, und zwar (i) die Beeinflussung des Produktionspotenzials durch die Produktionslücke, (ii) Langzeitarbeitslosigkeit und Fähigkeitsverluste sowie (iii) vom Verschuldungsgrad abhängige Bankfinanzierungskosten. In jedem Aufsatz werden die jeweiligen Implikationen dieser Sachverhalte für die Geldpolitik untersucht.

Im ersten Aufsatz (gemeinsam mit Kai D. Schmid) beschreiben wir ein Konjunkturmodell, in dem das Produktionspotenzial, im Gegensatz zum neukeynesianischen Standardmodell, Hysterese-Effekten unterliegt. Das bedeutet, dass das Produktionspotenzial von der Produktionslücke (der Abweichung der tatsächlichen Produktion vom Produktionspotenzial) beeinflusst wird. Dieser Mechanismus hat im Zuge der jüngsten Rezession an Bedeutung gewonnen, da viele Wirtschaftsforscher und Politikinstitutionen die negativen Auswirkungen des wirtschaftlichen Abschwungs für das Produktionspotenzial betont haben. Wir vergleichen Simulationsergebnisse unseres erweiterten Modells mit Ergebnissen des Standardmodells. Die Berücksichtigung von Hysterese-Effekten ermöglicht eine stärkere Amplifikation von und eine persistente Reaktion der Produktion nach geldpolitischen Schocks. Dies steht in Einklang mit US-amerikanischen Daten. Stabilitäts- und Wohlfahrtsanalysen legen eine stärkere Berücksichtigung der realwirtschaftlichen Aktivität in der geldpolitischen Zinsregel nahe wenn Hysterese-Effekte in die Analyse miteinbezogen werden.

Im zweiten Aufsatz dokumentiere ich stilisierte Fakten zur Langzeitarbeitslosigkeit in einigen europäischen Ländern. Ich entwickle ein neukeynesianisches Konjunkturmodell mit Arbeitsmarktfriktionen und Fähigkeitsverlusten während der Arbeitslosigkeit und zeige, dass das Modell die empirischen Beobachtungen nachbilden kann. Die Berücksichtigung von Fähigkeitsverlusten hilft dabei, Volatilitätsmuster von makroökonomischen Variablen, eine negative Abhängigkeit der Einstellungswahrscheinlichkeit von der Arbeitslosigkeitsdauer sowie das Verhalten des Anteils der Langzeitarbeitslosigkeit an der gesamten Arbeitslosigkeit an Wendenpunkten des Konjunkturzykluses zu erklären. Optimale Geldpolitik unter Einbeziehung von Fähigkeitsverlusten erlaubt eine höhere Inflationsvolatilität nach Produktivitätsschocks, um durch eine niedrigere Volatilität von Langzeitarbeitslosigkeit den Verfall von Fähigkeiten zu reduzieren und somit Konsumverluste abzufedern. Auf diese Weise berücksichtigt optimale Geldpolitik die Auswirkungen von Fähigkeitsverlusten, welche private Akteure nicht miteinbeziehen wenn sie Einstellungs- und Entlassungsentscheidungen treffen.

Der dritte Aufsatz (gemeinsam mit Johannes Fritz) befasst sich mit der Rolle von Bankfinanzierungskosten in der Transmission von Geldpolitik. In letzter Zeit haben geldpolitische Entscheidungsträger die Vermutung geäußert, dass der geldpolitische Kurs nicht störungsfrei an die Bankfinanzierungskosten und somit an die Kreditsätze für die Realwirtschaft übermittelt wird. Dieses Szenario kann von herkömmlichen makroökonomischen Modellen nicht abgebildet werden. Wir entwickeln ein neukeynesianisches Konjunkturmodell mit einer variablen Differenz zwischen Bankfinanzierungskosten und dem Zinssatz der Zentralbank. Das Vorzeichen und die Größe dieser Differenz hängt von der Verfassung der Bankbilanzen ab, welche wiederum von abnehmendem Eigenkapital der Banken im Zuge von Kreditausfällen in wirtschaftlichen Abschwüngen beeinflusst wird. Dieser Modellrahmen erlaubt es uns Szenarien zu analysieren, in denen Änderungen der Zentralbankzinsen nicht vollständig an die Bankfinanzierungskosten weitergereicht werden. Die Auswirkungen dieser Störungen auf makroökonomische Variablen sind klein bei Schocks, die dem realen Sektor entspringen, jedoch beträchtlich bei einem direkten Schock auf das Eigenkapital von Banken. Eine optimale Zinsregel der Zentralbank unter Berücksichtigung von verschuldungsgradabhängigen Bankfinanzierungskosten impliziert eine Zinssenkung (Zinserhöhung) bei strafferen (lockereren) Bankfinanzierungskonditionen.

# Chapter 1

# Hysteresis in potential output and monetary policy

Joint work with Kai D. Schmid<sup>1</sup>

## 1.1 Introduction

Due to the recent recession, the political and academic debate has experienced a revival of the so-called hysteresis phenomenon. In the face of the severe crisis, a large number of researchers and economic policy institutions such as, for example, DeLong and Summers (2012), Furceri and Mourougane (2009), the European Commission (2009), or the International Monetary Fund (2009) have addressed the negative implications of the economic downturn for the development of potential output. From this debate, the important questions arises if hysteresis has economic policy implications and if so, how economic policy should react to hysteresis.

<sup>&</sup>lt;sup>1</sup>A version of this article is published in the *Scottish Journal of Political Economy*, 61(4), 371-396, under the same title.

Generally speaking, hysteresis means that pronounced changes in aggregate demand exhibit procyclical, persistent real supply-side effects. While several facets of hysteresis have been documented in the macroeconomic literature since the early 1970s (see section 1.2 for a short overview), the question of how to consider such effects in terms of monetary or fiscal policy strategies has hardly been addressed in macroeconomic models. Specifically, currently used standard models designed for monetary policy research fade out the stimulus of demand-determined actual output on an economy's potential output. This holds for the New Keynesian model à la Galí (2008) or Woodford (2003) as well as for rather pragmatic models within the inflation targeting context such as Svensson (2000).<sup>2</sup> However, as Orphanides et al. (2000) argue, ignoring hysteresis effects may involve substantial misjudgment for the conduct of fiscal or monetary policies.

We address this shortcoming by examining the consequences of hysteresis in potential output for monetary policy. To this end, we extend the basic New Keynesian model in Galí (2008) by hysteresis, that is, by allowing the path of potential output to be influenced by the lagged output gap. The output gap describes the difference between actual output and potential output. It represents a high demand for positive realizations and a low demand for negative realizations. To work out the relevance of hysteresis for monetary policy, we contrast simulation outcomes of the extended model with the basic New Keynesian model in Galí (2008) and with empirical second moments. Moreover, we examine the implications for the conduct of monetary policy with respect to stability and welfare considerations.

We find that the extended model produces more realistic adjustment patterns than the basic New Keynesian model after monetary shocks hit the economy. Specifically, hysteresis helps to reproduce empirically well-documented persistence patterns of output. Furthermore,

 $<sup>^{2}</sup>$ A notable exception to this is chapter 5 in Woodford (2003), where he extends the basic model-setup by capital investment and illustrates that productive capacity as well as the equilibrium real rate of interest are affected by monetary policy.

our model exhibits a number of features that assign a more important role to active output gap stabilization in monetary policy rules if the economy is subject to hysteresis. First, if the central bank applies a monetary policy rule and the degree of hysteresis is large enough, achieving a unique stable equilibrium requires a reaction to the output gap for certain ranges of the reaction parameter for inflation. Second, if a welfare loss criterion based on the variability of inflation and the output gap is applied, reacting to the output gap in the monetary policy rule yields welfare loss reductions beyond those that would arise without hysteresis effects. The reason for these results lies in the dynamics of the output gap. If the central bank wants to fight inflation, the procyclical behavior of potential output requires a balancing reaction to the output gap in order to maintain downward pressure on inflation. At the same time, this downward pressure helps to reduce inflation variability.

As a robustness check we also show that, depending on the degree of hysteresis, actively stabilizing the output gap can still be welfare improving even if there is measurement uncertainty with respect to the output gap. Furthermore, if hysteresis is in effect, shifting the focus on output instead of the output gap in the monetary policy rule is not necessarily welfare deteriorating as is the case in the basic New Keynesian model.

The relevance of hysteresis for stabilization policy has been mentioned by several authors such as Ball (1999), DeGrauwe and Costa Storti (2007), Lavoie (2004) or Solow (2000). However, hysteresis has not played a meaningful role in standard macroeconomic models so far. Exceptions are DeLong and Summers (2012), Fritsche and Gottschalk (2006), and Mankiw (2001) who basically share a common reduced form specification for hysteretic adjustment that was originally proposed by Hargreaves Heap (1980). However, these are either partial equilibrium models focusing, for example, on the labor market or do not address monetary policy implications. Our study also refers to the aforementioned reduced form specification but puts it in the context of a monetary model, enabling us to examine implications of hysteresis for monetary policy.

The analysis closest to ours is Kapadia (2005) who also examines hysteresis in a New Keynesian model. However, there are a number of aspects which distinguish our approach from his study. First, Kapadia (2005) analyzes cost-push shocks, while we consider productivity and monetary shocks (which is more common in the business cycle literature and facilitates comparisons with the basic New Keynesian model). Second, Kapadia (2005) strongly focuses on different specifications of the Phillips curve. While this is a useful robustness check, it also clouds somewhat the role of hysteresis in the model. Our approach involves the basic New Keynesian Phillips curve but varies the degree of hysteresis to get better insights into the hysteresis mechanism. This is also an important robustness check since little work has been done to quantify the degree of hysteresis so far.<sup>3</sup> Third, while Kapadia (2005) focuses on optimal adjustment paths, we provide a more policy focused view by analyzing stability and welfare issues in the framework of interest rate rules that could potentially be adopted by policy makers. This also involves an analysis of the implications of output gap mismeasurement by the central bank.

The remainder of this paper is organized as follows: Section 1.2 briefly summarizes the basic mechanisms constituting hysteresis in potential output. Section 1.3 introduces our model. Section 1.4 examines the model dynamics after macroeconomic shocks, with a focus on monetary shocks. Section 1.5 investigates the implications of hysteresis in potential output for monetary policy. Section 1.6 examines the robustness of our policy implications under output gap uncertainty. Section 1.7 discusses the plausibility of the magnitude of hysteresis in potential output. Section 1.8 concludes.

 $<sup>^3\</sup>mathrm{See}$  section 1.7 for a discussion of this issue.

# 1.2 Hysteresis in potential output

This section summarizes the most important channels of hysteresis in potential output. Subsection 1.2.1 addresses the underlying theoretical mechanisms. Subsection 1.2.2 points to the respective empirical evidence.

#### 1.2.1 Theoretical mechanisms

From a theoretical perspective, pronounced changes in aggregate demand can impact potential output in several ways. As pointed out by Blanchard (2008), DeLong and Summers (2012), Schmid (2010) or Solow (2000), according to the factors within a conventional production function (capital stock, employment and total factor productivity) one may categorize three major channels of hysteretic adjustment: First, varying net capital formation, second, labor market hysteresis, and third, investment-induced efficiency gains. These mechanisms have been documented extensively in the literature and therefore will only briefly be addressed in the following.

First, as a very basic insight from the theory of economic growth, capital investment not only affects aggregate income (multiplier dynamics) but also changes an economy's productive capacity (accelerator mechanism). Hence, for example, in case of a severe recession the investment shortfall reduces the future productive potential of an economy.

Second, as pointed out by Phelps (1972) and also addressed by, for example, Ball and Mankiw (2002) or Spahn (2000), labor market hysteresis captures the procyclical impact of recent cyclical unemployment upon the current natural rate of unemployment. Thereby, cyclical changes of the demand for employees lead to the adjustment of the effective labor supply. The most prominent channels behind this phenomenon are insider-outsider mechanisms as highlighted by Blanchard and Summers (1987) and Lindbeck and Snower (1988) and dequalification processes during unemployment as addressed by Hargreaves Heap (1980) and Pissarides (1992).

Third, as pioneered by Arrow (1962), Kaldor (1966), Solow (1960) and Young (1928), changes in aggregate demand may stimulate the growth of labor productivity. Thereby, market expansion during economic upturns pushes the division of labor and stimulates industrial specialization and intersectoral spillovers which raise productivity on a macroeconomic level. Furthermore, the application of innovative production techniques and the use of new machinery — which is itself stimulated by cyclical capacity adjustment — promote learning-by-doing effects and initiate intersectoral knowledge spillovers.

#### 1.2.2 Empirical evidence

On the empirical side, many studies such as Cerra and Saxena (2008), DeLong and Summers (2012), European Commission (2009), Furceri and Mourougane (2009), Miles (2012), or Pisani-Ferry and van Pottelsberghe (2009) have provided evidence for hysteresis in potential output in the context of the Great Recession. Most of these analyses go beyond the recent experience and thus also cover earlier time periods. Thereby, it has become evident that hysteresis, although difficult to quantify, not only occurs in times of severe economic downturns but also during economic upswings (positive hysteresis). This is in line with the underlying theoretical considerations. Within this literature the adjustment of potential output to cyclical fluctuations is normally explained by the above mentioned hysteresis channels.

Focusing on the empirical relevance of these specific mechanisms, there has been a variety of studies since the late 1970s exploring the scope of labor market hysteresis and the procyclical character of productivity growth. For example, Blanchard and Summers (1988) as well as Layard and Bean (1989) state empirical evidence for de-qualification and decreasing re-employment options of long-term unemployed work-

ers. Hargreaves Heap (1980) and McGregor (1978) observe a positive relationship between average unemployment duration and the level of unemployment. Daly et al. (2011) find that during the Great Recession the natural rate of unemployment has risen substantially. Regarding investment-induced efficiency gains, Léon-Ledesma and Thirlwall (2002) and Cornwall and Cornwall (2002), among others, provide broad cross-country evidence of positive effects of aggregate demand on labor productivity.

### 1.3 Model

Our modelling framework to address the implications of hysteresis in potential output for monetary policy is the basic New Keynesian setup. It consists of a dynamic IS-equation, a forward-looking Phillips curve and a central bank reaction function. For a detailed derivation of the basic model, we refer to chapter 3 in Galí (2008).

The dynamic IS-equation of the model reads

$$y_t = E_t \{ y_{t+1} \} - \frac{1}{\sigma} (i_t - E_t \{ \pi_{t+1} \} - \rho), \qquad (1.1)$$

where  $y_t$  is (log) output,  $i_t$  is the central bank's (nominal) interest rate,  $\pi_t$  is the inflation rate in period t, and  $\rho$  is the discount rate. This ISequation is a log-linearized version of the household's Euler equation combined with the market clearing condition that consumption equals output.

Inflation dynamics are captured by a forward-looking Phillips curve given by

$$\pi_t = \beta E_t \{ \pi_{t+1} \} + \kappa (y_t - y_t^*), \tag{1.2}$$

where  $y_t^*$  is (log) potential output in period t. Its derivation involves aggregating the log-linearized optimal price-setting rules of monopolisticcompetitive firms facing a constant probability of resetting prices, in a neighborhood of the zero inflation steady state. In the context of this model, potential output is the equilibrium level of output if prices were completely flexible.

The central bank employs the interest rate rule

$$i_t = \rho + \gamma \pi_t + \psi (y_t - y_t^*) + \nu_t, \qquad (1.3)$$

where  $\nu_t$  is an exogenous component of monetary policy following the AR(1)-process  $\nu_t = \rho_{\nu}\nu_{t-1} + u_t^{\nu}$ .  $u_t^{\nu}$  is an error term with mean zero and variance  $\sigma_{\nu}$ .  $\gamma$  and  $\psi$  are parameters determining the strength of the central bank's reaction to inflation and the output gap.

In the basic New Keynesian model, potential output evolves according to  $y_t^* = a_t$ , where  $a_t$  is (log) productivity.<sup>4</sup> Assuming an AR(1)-process for productivity yields

$$y_t^* = \rho_a y_{t-1}^* + u_t^a, \tag{1.4}$$

where  $u_t^a$  is a productivity shock with mean zero and variance  $\sigma_a$ . We deviate from this specification to allow for hysteresis effects. Specifically, we assume that potential output is influenced by the output gap in the last period. The process for potential output is thus

$$y_t^* = \rho_a y_{t-1}^* + \eta (y_{t-1} - y_{t-1}^*) + u_t^a, \qquad (1.5)$$

where  $\eta$  is the degree of hysteresis. Equation 1.5 can be rearranged to

$$y_t^* = (\rho_a - \eta)y_{t-1}^* + \eta y_{t-1} + u_t^a.$$
(1.6)

Potential output is thus not only a function of productivity and past potential output but also a function of past actual output. This formulation of hysteresis is meant to capture the various channels described in section 1.2 on an aggregate level. The higher (lower)  $\eta$ , the more

 $<sup>\</sup>overline{{}^{4}\text{We have neglected the constant}}$  since it plays no role for the model dynamics.

(less) potential output is affected by actual output. Note that the basic New Keynesian model is nested in our specification for  $\eta = 0$ .

## 1.4 Hysteresis and monetary policy shocks

Since we ultimately aim at analyzing monetary policy issues, we need an indication of whether our model is reliable. Therefore, we compare model generated second moments to empirical second moments and show that our model can improve on some dimensions compared to the basic New Keynesian model. The two variables we can evaluate are inflation and output. Unfortunately, such a comparison is not possible for potential output. In the model, the variance of potential output heavily depends on innovations in productivity, while in the data potential output is some kind of smoothed trend of actual output.<sup>5</sup>

Nevertheless, we can still infer from the characteristics of inflation and output if our specification for potential output adds realism to the model. As we will show, hysteresis helps to reconcile model outcomes with well-established stylized facts. For US data, Christiano et al. (2005) document a persistent response of output to monetary shocks which cannot be reproduced by the basic New Keynesian model. Our model with hysteresis is particularly suited for studying the effects of monetary shocks since they have an impact on potential output in our specification. This in turn implies richer dynamics of output and inflation in response to monetary shocks than in the basic New Keynesian model. In contrast, we do not expect substantial deviations regarding productivity shocks because, as in the basic model, these impact potential output directly, outweighing the hysteresis effect.

The fact that hysteresis in potential output mainly has consequences for monetary and hence demand side shocks does not compromise the

 $<sup>^5\</sup>mathrm{This}$  also holds if potential output is calculated using the production function approach.

relevance of the analysis. There is a large literature documenting the importance of demand shocks in general and monetary shocks in particular for real variables. Romer and Romer (1989, 1994) use a narrative approach to show that monetary shocks account for more than a fifth of the variation in real economic activity in the US. Using a structural VAR, Christiano et al. (2005) estimate that the fraction of the variance in US output due to monetary shocks is between 15% and 27% depending on the time horizon after the shock. Using the same methodology, Bouakez et al. (2005) get even higher numbers, especially in the short run. More generally, Romer and Romer (1989, 1994) conclude that demand shocks are the primary source of business cycle fluctuations. This result is supported by Smets and Wouters (2007) who find that demand shocks account for more than 50% of the variation in US GDP in the short run and approximately 40% in the long run.

To simulate second moments, we have to calibrate the model's parameters. Following Galí (2008), we set  $\beta = 0.99$ ,  $\rho = -log(\beta)$ ,  $\kappa = 0.13$ and  $\sigma = 1$ , which implicitly assumes an annual steady state real interest rate of 4%, a log utility function of consumers, a degree of price stickiness of four quarters, a Frisch elasticity of labor supply of 1, an elasticity of substitution of 6 and a labor elasticity of output of 1/3. For these parameters, we stick to this calibration throughout the rest of the paper. This facilitates direct comparisons of our model outcomes to the basic New Keynesian model in Galí (2008). Moreover, we have to decide on the values of the policy reaction parameters. Like much of the literature that aims at describing a realistic behavior of the Federal Reserve, we set  $\gamma = 1.5$  (see Taylor (1993)). For the reaction to the output gap, we set  $\psi = 0.3$  since it is the smallest value for which stability is ensured for all considered degrees of hysteresis.<sup>6</sup> The degree of hysteresis is varied between  $\eta = 0$  — in which case our model coincides with the basic New Keynesian model — and  $\eta = 0.5$ . This parameter range for  $\eta$  will be discussed in section 1.7. For the specification of the exogenous processes, we follow Smets and Wouters (2003) and set

 $<sup>^{6}</sup>$ The issue of stability is separately analyzed and discussed in subsection 1.5.1.

 $\rho_a = 0.81$ ,  $\sigma_a = 0.5\%$  and  $\sigma_{\nu} = 0.15\%$ . The last two values are also found in Lechthaler et al. (2010). Moreover, we use different values for  $\rho_{\nu}$  to illustrate the persistence properties of our model in response to monetary shocks.

Table 1.1 contrasts empirical and model generated business cycle statistics for output y and inflation  $\pi$ . The first data row shows empirical unconditional moments for the US. Data are taken from the OECD and range from 1955Q1 to 2012Q4. For calculating empirical moments we follow Stock and Watson (1999). Empirical moments for output are obtained using the cyclical component of US real GDP, calculated as percentage deviation from the HP-filtered trend. Inflation is calculated as the quarter-on-quarter percentage change in the CPI at annual rates. Empirical moments for inflation are calculated using the cyclical component of inflation obtained by an HP filter.

The other second moments in table 1.1 are model generated moments for joint, monetary, and productivity shocks for different degrees of hysteresis. Looking at productivity shocks, the only notable difference between the various hysteresis specifications and the basic New Keynesian model ( $\eta = 0$ ) is the somewhat larger amplification of the productivity shock as hysteresis effects increase. This is documented by increasing standard deviations for output as  $\eta$  increases, bringing the model predictions closer to the empirical counterpart on this account. Otherwise, as noticed above, hysteresis does not make much of a difference for productivity shocks.

Consequently, as the productivity shock is the dominating shock for our calibration,<sup>7</sup> the effects of hysteresis for joint shocks seem to be small and only apparent for the reported standard deviations. Note, however, that this is only the case because our relatively small model features only one demand shock (the monetary shock). In medium scale

<sup>&</sup>lt;sup>7</sup>The productivity shock is approximately 3.5 times stronger than the monetary shock in line with the estimates of Smets and Wouters (2003).

	Standa	rd dev.	Autocorr.		Corr.
	y	$\pi$	y	$\pi$	$y, \pi$
US data	0.0155	0.0215	0.85	0.32	0.27
Joint shock					
$\eta = 0$	0.0106	0.0013	0.89	0.86	-0.88
$\eta = 0.1$	0.0107	0.0013	0.89	0.85	-0.87
$\eta = 0.2$	0.011	0.0013	0.89	0.85	-0.86
$\eta = 0.3$	0.0114	0.0013	0.9	0.85	-0.85
$\eta = 0.4$	0.0121	0.0013	0.91	0.85	-0.84
$\eta = 0.5$	0.0135	0.0014	0.92	0.85	-0.83
Monetary shock					
$\eta = 0$	0.0016	0.0004	0.5	0.5	1
$\eta = 0.1$	0.0021	0.0004	0.64	0.45	0.89
$\eta = 0.2$	0.0027	0.0004	0.76	0.43	0.62
$\eta = 0.3$	0.0037	0.0005	0.84	0.46	0.28
$\eta = 0.4$	0.0052	0.0006	0.89	0.55	-0.07
$\eta = 0.5$	0.0073	0.0007	0.93	0.65	-0.37
Productivity Shock					
$\eta = 0$	0.0104	0.0012	0.9	0.9	-1
$\eta = 0.1$	0.0105	0.0012	0.9	0.9	-1
$\eta = 0.2$	0.0106	0.0012	0.9	0.9	-1
$\eta = 0.3$	0.0108	0.0012	0.91	0.91	-1
$\eta = 0.4$	0.11	0.0012	0.91	0.91	-1
$\eta = 0.5$	0.0114	0.0012	0.91	0.91	-1

Table 1.1: Business cycle statistics for a productivity shock, a persistent monetary policy shock ( $\rho_{\nu} = 0.5$ ), and a joint shock. Notes: The first data row shows empirical second moments. Quarterly US data from 1955Q1 to 2012Q4 are obtained from the OECD Quarterly National Accounts database and the Main Economic Indicators database. y is the cyclical component of real GDP as percentage deviation from the HP-filtered trend.  $\pi$  is the cyclical component of inflation rate obtained by an HP filter. The inflation rate is calculated as quarterly percentage change of consumer price index at an annual rate. The other data rows show model generated second moments for joint, monetary, and productivity shocks, respectively, and for different degrees of hysteresis  $\eta$ .

models with several demand shocks, the productivity shock would no longer be as dominant as it is here.

The main effects of hysteresis come to light when we look at monetary shocks. Table 1.1 shows model moments for an autocorrelated monetary shock with  $\rho_{\nu} = 0.5$ . This is often assumed to incorporate a realistic amount of persistence into the model, as, for example, in Galí (2008). As usual, in the basic model ( $\eta = 0$ ) the autocorrelation of the monetary shock is passed on to the autocorrelation of output and inflation, and the correlation of these two variables is one. As hysteresis kicks in, the correlation between output and inflation decreases with  $\eta$  and standard deviations rise, which is a desirable feature of the model considering the respective empirical counterparts. More importantly, the persistence of output increases substantially, which is in line with the empirical evidence on monetary shocks. The persistence of inflation first decreases when hysteresis effects become relevant and then increases with the strength of hysteretic adjustment. Note that this feature also brings the model predictions closer to the empirical counterpart for moderate degrees of hysteresis.

To elucidate the implications of hysteresis for the persistence characteristics of the model for a monetary shock, we also consider a one-off (transitory) monetary policy shock. This is illustrated in table 1.2. Without externally induced persistence, the autocorrelation for output and inflation in the basic model is zero. When hysteresis effects are considered, the autocorrelation in output and inflation increases substantially, up to 0.72 and 0.45, respectively, for a high degree of hysteresis. For output, even small to medium degrees of hysteresis bring about notable improvements of the model's internal persistence properties.

The improvements for the amplification of shocks and the internal persistence are well apparent when we look at impulse response functions (IRFs) for a one-off monetary policy shock. Figure 1.1 shows IRFs for output, inflation, potential output, and the policy rate. The different

	Standard dev.		Aut	Corr.	
	y	$\pi$	y	$\pi$	$y, \pi$
US data	0.0155	0.0215	0.85	0.32	0.27
Joint shock					
$\eta = 0$	0.0105	0.0012	0.89	0.89	-0.98
$\eta = 0.1$	0.0106	0.0012	0.89	0.89	-0.98
$\eta = 0.2$	0.0107	0.0012	0.89	0.89	-0.98
$\eta = 0.3$	0.0109	0.0012	0.9	0.89	-0.97
$\eta = 0.4$	0.011	0.0012	0.9	0.9	-0.97
$\eta = 0.5$	0.0116	0.0012	0.91	0.9	-0.97
Monetary shock					
$\eta = 0$	0.001	0.0001	0	0	1
$\eta = 0.1$	0.0011	0.0001	0.13	-0.0489	0.9172
$\eta = 0.2$	0.0013	0.0001	0.29	0	0.68
$\eta = 0.3$	0.0015	0.0002	0.46	0.12	0.35
$\eta = 0.4$	0.0018	0.0002	0.6	0.29	0.185
$\eta = 0.5$	0.0023	0.0002	0.72	0.45	-0.26

**Table 1.2:** Business cycle statistics for a one-off (transitory) monetary policy shock  $(\rho_{\nu} = 0)$  and a joint shock. Notes: The explanations for table 1.1 carry over to this table. Model second moments for the productivity shock are not reported since they are the same as in table 1.1.

IRFs for each variable refer to different degrees of hysteresis. While there is no output persistence at all for  $\eta = 0$ , persistence gradually increases for higher degrees of hysteresis. We also see why the autocorrelation coefficient for inflation is negative for a small degree of hysteresis (see table 1.2) in case of a monetary shock. The reason is that inflation exhibits an "overshooting" behavior. After inflation decreases on impact of the monetary policy shock, the hysteretic adjustment induces a relatively quick decrease of the output gap, attenuating the downward pressure on inflation. For a small value of  $\eta$ , this effect dominates the slow adjustment to equilibrium in the subsequent periods. We also see that potential output is responding quite heavily in the hysteresis case, while it is constant after a monetary policy shock in the basic New Keynesian model ( $\eta = 0$ ). Moreover, the IRFs illustrate well the stronger



**Figure 1.1:** Model generated impulse response functions for output, inflation, potential output, and the policy rate for a one-off (transitory) monetary policy shock. Notes: Different colors represent different degrees of hysteresis  $\eta$ . Units on the vertical axes are % of standard error of the underlying shock.

amplification of shocks when hysteresis is in effect which is apparent in the magnitude of the responses.

# 1.5 Implications of hysteresis for monetary policy

We study the implications of hysteresis for a monetary authority using an interest rate rule as a policy guideline. That is, the central bank decides on the nominal interest rate according to a reaction function of endogenous variables. The monetary policy decision then boils down to setting the reaction parameters on these endogenous variables.

In this setting, we address two questions: First, how can monetary policy achieve a stable economy when hysteresis effects are in place? In particular, we are looking for constellations of reaction parameters which yield a unique stable equilibrium. Second, for the set of parameter constellation that yields unique stationary equilibria, which policy yields a minimal welfare loss? In addition, we elucidate how policies that yield stable outcomes and minimal welfare losses under hysteresis differ from the respective policies in the baseline model without hysteresis effects.

For the analysis of stability and welfare issues it is convenient to rewrite the model so as to present it in matrix form. As appendix 1.A shows, equations (1.1), (1.2), (1.3) and (1.6) can be summarized as follows:

$$\begin{pmatrix} \nu_{t+1} \\ y_{t+1}^* \\ E_t \{y_{t+1}\} \\ E_t \{\pi_{t+1}\} \end{pmatrix} = A \begin{pmatrix} \nu_t \\ y_t^* \\ y_t \\ \pi_t \end{pmatrix} + B \begin{pmatrix} u_{t+1}^{\nu} \\ u_{t+1}^{a} \end{pmatrix},$$
(1.7)

where

$$A = \begin{pmatrix} \rho_{\nu} & 0 & 0 & 0\\ 0 & (\rho_a - \eta) & \eta & 0\\ \frac{1}{\sigma} & \frac{(-\beta * \psi - \kappa)}{(\beta * \sigma)} & \frac{\beta * (\sigma + \psi) + \kappa)}{\sigma * \beta} & \frac{\beta * \gamma - 1}{\sigma * \beta}\\ 0 & \frac{\kappa}{\beta} & -\frac{\kappa}{\beta} & \frac{1}{\beta} \end{pmatrix}; \quad B = \begin{pmatrix} 1 & 0\\ 0 & 1\\ 0 & 0\\ 0 & 0 \end{pmatrix}.$$

#### 1.5.1 Stability in a hysteretic economy

Model (1.7) has two predetermined variables  $(y_t^* \text{ and } \nu_t)$  and two nonpredetermined variables  $(y_t \text{ and } \pi_t)$ . Hence, according to Blanchard and Kahn (1980) a stationary unique solution will exist if and only if A has two eigenvalues inside and two eigenvalues outside the unit circle.<sup>8</sup> Since checking this condition analytically is not possible in our model, we apply a numerical procedure to show that the determinacy of the equilibrium depends on the central bank's reaction parameters given a certain degree of hysteresis  $\eta$ .

Assuming that the central bank can adjust its reaction parameters  $\gamma$  and  $\psi$  in 0.1-steps, figure 1.2 illustrates the determinacy and indeterminacy regions in the  $(\gamma, \psi)$ -space for different degrees of hysteresis. We look at positive values for  $\gamma$  up to 5 and for  $\psi$  up to 2. A wider range would not yield different results. Recall that for  $\eta = 0$ , we are back to the basic New Keynesian model, so we can readily compare the hysteresis to the non-hysteresis case.

Figure 1.2(a) depicts the determinacy region for the basic New Keynesian model and represents the well-known Taylor principle: to achieve a unique stable equilibrium, the central bank has to adjust the interest rate overproportionally in response to a change in inflation. This requires  $\gamma > 1$ , at least when the central bank is not reacting at all to changes in the output gap. As figure 1.2(b) and 1.2(c) show, this principle carries over to economies with mild hysteresis effects.

However, for higher degrees of hysteresis the indeterminacy region expands. In particular, the overproportional change in the interest rate is not a sufficient condition any more. Figures 1.2(d), 1.2(e), and 1.2(f) show that for certain ranges of  $\gamma > 1$ , a reaction to the output gap is required in order to achieve determinacy. These ranges expand as the degree of hysteresis increases. In addition, the required reaction to output gap variations increases with  $\eta$ . For example, for  $\eta = 0.5$  and  $\gamma \in [1.7; 2.2]$  the reaction parameter for the output gap,  $\psi$ , has to be above 0.3, while for the same range of  $\gamma$  and  $\eta = 0.4$ ,  $\psi > 0.2$  suffices for a stable equilibrium.

<sup>&</sup>lt;sup>8</sup>The required rank conditions are satisfied for all stable parameter constellations, see Blanchard and Kahn (1980).



**Figure 1.2:** Model generated stability regions. Notes: Graphs (a)-(f) represent different degrees of hysteresis in potential output  $\eta$ . The vertical axes show the reaction parameter for the output gap  $\psi$ , while the horizontal axes show the reaction parameter for inflation  $\gamma$ .

An additional important observation is that the ranges of the inflation reaction parameter requiring a reaction to the output gap include  $\gamma = 1.5$ , a value often associated with a good description of the actual behavior of major central banks. In this sense, our model can provide an explanation why it could be reasonable for a monetary authority to react to economic activity, which is considered to be common practice among central banks, as, for example, Taylor (1999a) mentions.

The pattern of the instability regions in figures 1.2(d), 1.2(e), and 1.2(f) for  $\gamma > 1$  is quite distinctive. The required reaction to the output gap first increases and then (gradually) goes back to zero as  $\gamma$  increases. A possible explanation is the following: Suppose inflation rises and the central bank, according to its policy rule, reacts with an interest rate increase. The ensuing downward pressure on inflation is induced by a negative output gap. If potential output is subject to hysteresis, it adjusts downward in the subsequent periods, mitigating the pressure on inflation. A balancing reaction to the output gap — which goes in the opposite direction of the initial interest rate increase — can maintain the pressure on inflation emanating from the output gap and help to bring inflation back to a unique equilibrium. For very high values of the inflation reaction parameter, the initial effect of a reaction to inflation on the output gap is strong enough to ensure a unique equilibrium. For values of the inflation reaction parameter bigger than but close to one, the initial decrease in the output gap by a reaction to inflation implies only a small hysteretic adjustment, thus also making the balancing effect of a reaction to the output gap unnecessary.

## 1.5.2 Hysteresis and welfare implications for monetary policy

Knowing the set of feasible parameter combinations for different degrees of hysteresis, we proceed by analyzing optimal monetary policy when the central bank uses an interest rate rule. Therefore, we require a

criterion to assess welfare implications of monetary policy. Much of the literature has adopted a welfare loss criterion based on a second-order approximation of the household's utility function, as in Rotemberg and Woodford (1999). This has the advantage that the welfare criterion is consistent with the specific model at hand. However, the disadvantage is that the policy maker has to know the model in order to employ the "correct" welfare loss function. Paez-Farrell (2012) points out that if this is not the case, using an exogenous quadratic welfare loss function might be less detrimental than using a micro-founded loss function. So far, not much work has been done regarding hysteresis effects in business cycle models, suggesting high model uncertainty. Furthermore, the micro-founded approach would result in different welfare criteria for different models. This makes it difficult to conduct meaningful comparisons across models, which is one of our intentions. Therefore, we use an exogenous quadratic loss function to evaluate welfare consequences of monetary policy.<sup>9</sup> It expresses the welfare loss in terms of a weighted average of the variance of inflation and the variance of the output gap:

$$L = [\phi var(y_t - y_t^*) + var(\pi_t)].$$
(1.8)

Here,  $\phi$  is the relative weight of the output gap variance in the welfare loss. Note that this loss function has the same form as the welfare loss function in the basic New Keynesian model of Galí (2008). There,  $\phi$  takes on the value 0.02, and we adopt this calibration for the subsequent simulations.<sup>10</sup>

For calculating the welfare loss associated with different policies, we apply the following procedure: For parameter constellations of  $\gamma$  and  $\psi$  which yield a determinate system (the ranges for these parameter values

<sup>&</sup>lt;sup>9</sup>Other papers that use exogenous welfare criteria include Angeloni et al. (2003), Davis and Huang (2011), Orphanides et al. (2000), or Taylor (1979).

<sup>&</sup>lt;sup>10</sup>The choice of  $\phi$  is not critical for our qualitative results. To illustrate this, we report the variance of both inflation and the output gap separately in the following.

correspond to the analysis presented in subsection 1.5.1), we calculate the welfare loss based on the implied variances of the output gap and inflation according to equation (1.8). Again, we alter the values for the reaction parameters  $\gamma$  and  $\psi$  in 0.1-steps. We then check which reaction parameter constellation yields the minimum welfare loss. In this way, we obtain an optimized monetary policy rule. As before, we consider various degrees of hysteresis throughout our analysis. For the sake of exposition, we fix  $\gamma$  to 1.5, but the results for different  $\gamma$ 's do not change qualitatively.

Figure 1.3 shows the variances for inflation and the output gap for varying output gap reaction parameters, whereas figure 1.4 shows the values for the loss function (1.8) for varying output gap reaction parameters.<sup>11</sup> We can see in figure 1.3 that the variability of both inflation and the output gap declines as the central bank's reaction to the output gap increases. This translates to the loss function in figure 1.4.



Figure 1.3: Model implied variances for inflation (a) and the output gap (b) for different values of output gap reaction parameter  $\psi$ . Notes: Colors represent different degrees of hysteresis  $\eta$ . The inflation reaction parameter  $\gamma$  is fixed to 1.5.

For all degrees of hysteresis, the minimal welfare loss is attained for a strong reaction to the output gap; in this dimension, the hysteresis

<sup>&</sup>lt;sup>11</sup>Due to the non-determinacy for small  $\psi$ 's given comparably high values of  $\eta$ , the smallest value for  $\psi$  in figures 1.3 and 1.4 is 0.2 and 0.3 for  $\eta = 0.4$  and  $\eta = 0.5$ , respectively.

 $(\eta > 0)$  and the non-hysteresis  $(\eta = 0)$  case do not differ. However, we see that there are higher costs for not reacting to the output gap when the economy is subject to hysteresis. Especially in the region of relatively low  $\psi$ 's, the benefits of a stronger reaction to the output gap are higher for higher degrees of hysteresis. Put differently, the marginal loss reduction for reacting to the output gap increases when the economy is subject to hysteresis.



Figure 1.4: Model-implied welfare loss for different values of the output gap reaction parameter  $\psi$ . Notes: Colors represent different degrees of hysteresis  $\eta$ . The inflation reaction parameter  $\gamma$  is fixed to 1.5.

Therefore, output gap stabilization becomes relatively more important if hysteresis effects are in place. When an expansionary shock hits the economy, both the output gap and inflation increase. Therefore, a reaction to the output gap helps to reduce the volatility of both the output gap and inflation, thus reducing welfare losses. This holds for both the hysteresis and the non-hysteresis case. However, as noticed in section 1.4, hysteresis increases the amplification of shocks. Hence, the stronger the hysteresis effect, the stronger the endogenous interest rate reaction for given shock magnitudes and values of the output gap parameter. As a consequence, an increase in the value of the output
gap parameter implies a stronger decrease in the volatilities of inflation and the output gap for higher degrees of hysteresis.

### 1.6 Dealing with output gap uncertainty

Our analysis so far makes a case for active output gap stabilization rather than only focusing on inflation. However, a popular critique of output gap stabilization is that in reality the output gap is measured with error and thus may not be a suitable variable to base policy decisions upon. Due to incomplete information about the current state of the economy and the unobservability of potential output, the central bank faces uncertainty with regard to output gap dynamics. As output gap data is only available with a considerable time lag, monetary policy has to rely on estimated values. Diverging estimation results provided by different measuring techniques as well as the frequent and considerable data revisions extensively illustrate the disputable reliability of output gap measures. Thus, as, for example, pointed out by the European Central Bank (2000) or Orphanides (1999), the usefulness of output gap measures for monetary policy might be questionable.

In particular, overestimation of potential output in times of downturns and underestimation during macroeconomic expansion bears the risk of procyclical overreaction regarding interest rates. Therefore, central banks may have to fear an unanticipated path of potential output that ultimately classifies the original interest rate reaction as inadequate. Hence, an interest rate policy that attributes less or no weight to output gaps in interest rate rules is suggested by authors such as McCallum (2001), Onatski (2000), or Willems (2009).

In light of this critique, we analyze whether we can maintain our finding that reacting to economic activity is desirable from a welfare point of view when hysteresis effects are in place. We approach this issue in two ways: First, we examine our model's welfare properties when output gap uncertainty is explicitly taken into account in the form of a measurement error and check if output gap stabilization remains a desirable feature of monetary policy.

Second, Taylor (1999b) suggests to consider output itself (or deviations of output from steady state) instead of the output gap in monetary policy rules. A well-known result — as described, for example, in Galí (2008) — is that in the absence of real frictions, this specification of monetary policy suggests to refrain completely from reacting to output since it drives up welfare losses. We examine if this is still true when hysteresis effects are in place, as our previous results advocate for a more prominent role of directly stabilizing economic activity.

### 1.6.1 Taking uncertainty explicitly into consideration

To evaluate the effective risk of suboptimal policy reactions to output gap mismeasurement in the context of hysteresis, we first analyze the robustness of our model's welfare implications when the output gap is measured with error.<sup>12</sup> As proposed by Orphanides et al. (2000), we capture output gap mismeasurement by an additive observation distortion  $\xi_t$  within the central bank's reaction function:

$$i_t = \rho + \gamma \pi_t + \psi[(y_t - y_t^*) + \xi_t] + \nu_t, \qquad (1.9)$$

where  $\xi_t = \rho_{\xi}\xi_{t-1} + \epsilon_t^{\xi}$ .  $\xi_t$  can be thought of as the process describing the ex-post revisions with regard to the ex-ante estimate of the output gap.  $\rho_{\xi}$  represents the persistence of the observation distortion and  $\epsilon_t^{\xi}$ is assumed to be white noise with variance  $\sigma_{\xi}$ .

<sup>&</sup>lt;sup>12</sup>The concept of robustness examines the ability of a central bank's strategy to guarantee desirable results for different macroeconomic specifications. Given the uncertainty regarding the exact state of macroeconomic aggregates robust policy rules are preferable, as described in McCallum (1988, 1997).

Within this framework, we can examine if the advantages of active output gap stabilization that arise if hysteresis is in effect are outweighed by the disadvantages that come along with output gap measurement errors. We employ the following procedure: Based on the estimates of Orphanides et al. (2000), we look at a "best case", a "base case" and a "worst case" with a relatively low ( $\rho_{\xi} = 0.8$ ), medium ( $\rho_{\xi} = 0.84$ ), and high ( $\rho_{\xi} = 0.96$ ) persistence of the measurement error process, respectively. Within each case, we fix the central bank's reaction parameter for inflation  $\gamma$  to 1.5 and calculate the output gap reaction parameter that yields the lowest welfare loss,  $\psi^*$ , for varying intensities of the measurement error shock  $\sigma_{\xi}$ . Again, we consider different degrees of hysteresis between  $\eta = 0$  and  $\eta = 0.5$ .<sup>13</sup>

Figure 1.5 illustrates the results; figures 1.5(a), 1.5(b), and 1.5(c) refer to the best, base, and worst case described above. The loss-minimizing reaction to the output gap is shown on the vertical axis, while the intensity of the measurement error is shown on the horizontal axis.

As expected, as the effect of the measurement error kicks in, the importance of output gap stabilization declines with higher values of  $\sigma_{\xi}$ . The higher the persistence of the measurement error, the faster the optimal strength of output gap stabilization falls. However, while for the basic New Keynesian model ( $\eta = 0$ ) and mild degrees of hysteresis ( $\eta = 0.1$  and  $\eta = 0.2$ ) the optimal reaction to the output gap remains at zero for increasing measurement error shocks,  $\eta \geq 0.3$  suggests a positive reaction to the output gap in order to minimize welfare losses. This is true for all persistence patterns of the measurement error. The required reaction to the output gap rises with the degree of hysteresis.

In particular, for the values of  $\sigma_{\xi}$  estimated by Orphanides et al. (2000) — indicated by the vertical lines in the graphs — a reaction to the output gap can be optimal depending on the persistence of the measurement error and the degree of hysteresis. In the best case, that is,

<sup>&</sup>lt;sup>13</sup>Note that the analysis in Orphanides et al. (2000) and our examination are based on the same welfare loss function.



Figure 1.5: Optimal output gap reaction parameter  $\psi^*$  plotted against intensity of measurement error. Notes: Figures (a), (b), and (c) show low, medium and high persistence of measurement error, respectively. Vertical lines indicate estimates for the standard deviations of the measurement error shock by Orphanides et al. (2000). Different colors refer to different degrees of hysteresis  $\eta$ .

for a relatively small persistence in the measurement error, even small hysteresis effects suffice to render a reaction to the output gap beneficial. Depending on the strength of hysteresis, the optimal  $\psi$  ranges from 0.1 to 0.3. In the base and worst case, medium to strong, but not small hysteresis effects require active output gap stabilization. The range of  $\psi^*$  is again between 0.1 and 0.3.

Thus, we find that even when the output gap is measured with error, hysteresis effects can imply a beneficial role for active output gap stabilization. It depends on the strength of the hysteresis effect and the size of the measurement error to what extent the central bank should target the output gap.

### 1.6.2 Dispensing with the output gap in the monetary policy rule

At least since the influential work of Taylor (1993), many researchers have studied monetary policy rules in which the policymaker reacts to output (or the deviation from output from its steady state) rather than the output gap. While, in the context of the New Keynesian model, the monetary authority would like to employ the output gap in the reaction function (since it is the variable that influences the inflation process). it is not feasible to do so. The argument is that the output gap is not directly observable and should therefore be replaced by output in the monetary policy rule. As our previous results suggest a beneficial role for the stabilization of the output gap when hysteresis is in effect, the question appears whether we can generalize our results to the active stabilization of economic activity (be it output or the output gap). This question is particularly interesting because the standard result is that a reaction to output inevitably reduces the economy's welfare performance in the absence of real imperfections.<sup>14</sup> In the following, we examine if this finding can be maintained when hysteresis is considered.

 $<sup>^{14}</sup>$ For a detailed exposition of this result, see Galí (2008), chapter 4.

The analysis can be viewed as a further robustness check for our model implications.

We proceed similar as in subsection 1.5.2, except that the monetary policy rule contains output instead of the output gap.<sup>15</sup> Figure 1.6 shows the variances of the output gap and inflation for this case. Again, since the weight on inflation is high in the welfare loss function, the pattern of inflation variances translates into welfare losses, shown in figure 1.7.



**Figure 1.6:** Model implied variances for inflation (a) and the output gap (b) for different values of the output gap reaction parameter  $\psi$ . Notes: Colors represent different degrees of hysteresis  $\eta$ . The inflation reaction parameter  $\gamma$  is fixed to 1.5. The central bank reacts to output instead of the output gap.

The black line in figure 1.7 reproduces the above mentioned result of the basic New Keynesian model with a strong (but slightly diminishing) marginal increase in the welfare loss as the reaction to output increases. When hysteresis is considered, the slope of the loss curve for every  $\psi$  decreases substantially for small degrees of hysteresis and disappears completely for  $\eta \geq 0.3$ . For  $\eta = 0.2$ , the increase is very small. That is, a reaction to output only involves small or no welfare losses if hysteresis is in effect.

<sup>&</sup>lt;sup>15</sup>Note that the stability regions for  $\gamma > 1$  are similar to those in subsection 1.5.1. Therefore, we do not discuss them here again.



**Figure 1.7:** Welfare loss for different values of the output gap reaction parameter  $\psi$ . Colors represent different degrees of hysteresis  $\eta$ . The inflation reaction parameter  $\gamma$  is fixed to 1.5. The central bank reacts to output instead of the output gap.

Intuitively, when the central bank stabilizes output, the output gap the variable which determines inflation dynamics — could in principle fluctuate heavily due to movements in potential output. These fluctuations would then be passed on to inflation via the Phillips curve. This is exactly what happens in the basic model. However, in the hysteresis case, potential output depends positively on lagged actual output, as equation (1.6) illustrates. For this reason, output and potential output cannot drift apart strongly if the central bank stabilizes output. Consequently, for small degrees of hysteresis, the central bank only induces little variation in the output gap and inflation by stabilizing output. For larger degrees of hysteresis ( $\eta \geq 0.3$ ), no additional loss is created by reacting to output.

Hence, while reacting to output does not yield welfare gains as is the case for output gap stabilization, it produces small or no welfare losses if the economy exhibits hysteresis effects.

## 1.7 Discussion: Plausible degrees of hysteresis

So far, we have considered the degree of hysteresis  $\eta$  to range from 0 to 0.5. This has been done to obtain the best possible insights into the characteristics of our model, that is, to learn how the dynamics in the economy change when different intensities of hysteresis are in effect. Clearly, the question arises what could actually be a plausible degree of hysteresis. We address this issue from three perspectives: First, we summarize the values of  $\eta$  used in similar models. Second, we point to empirical evidence for the degree of hysteresis in potential output. Third, we deduct plausible parameter values for  $\eta$  from the comparison of our model dynamics with the data.

First, studies that use a similar specification for hysteresis as our model are Fritsche and Gottschalk (2006), Mankiw (2001) and Kapadia (2005). The former two studies work with  $\eta = 0.1$ , the latter applies a value of  $\eta = 0.25$ .

Second, empirically, the degree of hysteretic adjustment in potential output is difficult to quantify. This is for several reasons: (a) Economic up- and downturns may themselves be triggered by long-lasting changes in the economy. Hence, it is hard to identify demand-side developments that are due to hysteresis and not partly driven by technological impulses or exogenous shifts in labor force participation. (b) Time series information of potential output is usually obtained by filtering some kind of cyclically changing data on production. Thus, the impact of changes in the output gap upon potential output cannot be measured in a straight forward way as data on potential output are merely a trend component of output data. (c) In the course of economic downturns, it is hard to abstract from the stabilizing impact of policy responses upon the pure hysteresis mechanism, that is, the adjustment of future potential output to actual output. The degree of hysteretic adjustment is likely to be moderated by mitigating demand-side policies. One way to approach the magnitude of hysteresis in potential output despite these troubles has recently been suggested by DeLong and Summers (2012). Taking a production function perspective, these authors approximate hysteresis in potential output by the procyclical adjustment of the capital stock and the labor supply. Their study covers US data for the adjustment of the capital stock from 1967 to 2012 as well as labor market dynamics for France, Germany, Italy and the UK since 1970 and for the US since 1990. The authors provide evidence that a 1% output shortfall may induce a reduction of potential output by up to 0.3%. Furthermore, there is evidence from studies that focus on the adjustment of the natural unemployment rate to changes in lagged cyclical unemployment. For example, Logeay and Tober (2006) report labor market hysteresis for the euro area from 1973-2002, suggesting a value of  $\eta = 0.26$ . Jäger and Parkinson (1994) measure a value of 0.18  $\leq \eta \leq 0.22$  for the UK and for West Germany from 1961-1991.

Third, a comparison of our model's second moments with the empirical second moments for inflation and output (see tables 1.1 and 1.2 in section 1.4) suggests that values of  $0.2 \le \eta \le 0.3$  seem plausible. Considering the correlation between output and inflation for monetary policy shocks, the degree of hysteresis matching the empirical data best is  $\eta = 0.3$ . Although the propagation of shocks is still too weak to match the empirical moments, increasing values of  $\eta$  lead to a somewhat better approximation of the empirical standard deviations. This holds for a persistent as well as for a transitory monetary policy shock. Regarding the autocorrelation of output (inflation), the model matches the data fairly well for  $\eta = 0.3$  ( $\eta = 0.2$ ) in case of a persistent and for  $\eta = 0.5$  ( $\eta = 0.4$ ) in case of a transitory monetary policy shock. However, while it seems that for some statistics a high degree of hysteresis seems to be favorable, there are several indications that  $\eta > 0.3$ is not plausible. For example, the correlation between inflation and output for a monetary policy shock becomes negative for high values of hysteresis, which contradicts common empirical and theoretical considerations. Furthermore, the autocorrelations of inflation and output become too high compared to the empirical data.

Summarizing these different viewpoints, a value of  $\eta$  around 0.25 seems to be a reasonable assumption for the degree of hysteresis in potential output. The fact that we also consider lower and higher degrees of hysteresis can be understood as a robustness check in the light of uncertainty about the true value for  $\eta$ . Against the background of our analysis, a magnitude of  $\eta > 0.2$  indicates that hysteretic adjustment of potential output indeed exhibits important implications for the conduct of monetary policy.

### 1.8 Conclusion

Due to the severe economic downturn in the recent recession, the topic of hysteresis has re-entered the economic agenda. However, most standard models designed for monetary policy research do not consider hysteresis effects and are of little help for the assessment of policy strategies when potential output is subject to hysteresis.

Our paper addresses this shortcoming by examining the consequences of hysteresis in potential output for monetary policy within the basic New Keynesian framework. We model hysteresis by allowing the path of potential output to be influenced by the lagged output gap. To work out the empirical relevance of hysteresis, we contrast simulation outcomes of our model with empirical second moments for output and inflation. Furthermore, we examine the implications of hysteresis for the conduct of monetary policy with respect to stability and welfare considerations.

We find that hysteresis helps to improve the model's performance: The amplification of macroeconomic shocks increases and the adjustment of output after monetary shocks is more persistent. Moreover, our model exhibits a number of features that assign a more important role to the stabilization of economic activity by the central bank if the economy is subject to hysteresis. First, for a sufficiently high degree of hysteresis and certain, empirically plausible ranges for the inflation parameter in the central bank's interest rate rule, a reaction to the output gap is required to obtain a unique stable equilibrium. Second, the marginal reduction of welfare losses by reacting to the output gap is particularly high when the economy is subject to hysteresis. Robustness checks show that actively stabilizing the output gap can reduce welfare losses even when the output gap is measured with error. Furthermore, reacting to output instead of the output gap does not necessarily increase welfare losses, as is inevitably the case in the basic New Keynesian model.

We consider our analysis as a first step towards a better understanding of the consequences of hysteresis for monetary policy. Our findings point out that hysteresis in potential output bears important implications for the conduct of monetary policy and that ignoring hysteresis effects may be costly. Thus, more research is required to enhance the reliability of policy recommendations. Future research in this field could consider hysteretic adjustment in medium scale models. This would shed more light on the different implications of hysteresis regarding supply vs. demand shocks, and on the validity of our results. In addition, as also pointed out by DeLong and Summers (2012), further empirical evidence for the quantification of the degree of hysteresis in potential output is another important step to learn more about the hysteresis mechanism and its implications for economic policy. Thereby, as mentioned by O'Shaughnessy (2011), the potential asymmetry of hysteretic adjustment with respect to the direction of shock impulses might be an important issue. Further research could thus differentiate between expansionary and contractionary demand shocks with regard to the magnitude and the timing of the hysteretic adjustment.

# Appendix 1.A Matrix representation of the model

Plugging (1.3) into (1.1) and rearranging yields

$$E_t\{y_{t+1}\} = \frac{\sigma + \psi}{\sigma} y_t - \frac{1}{\sigma} E_t\{\pi_{t+1}\} + \frac{\gamma}{\sigma} \pi_t - \frac{\psi}{\sigma} y_t^* + \frac{1}{\sigma} \nu_t.$$
(1.10)

Rearranging (1.2) gives

$$E_t\{\pi_{t+1}\} = \frac{1}{\beta}\pi_t - \frac{\kappa}{\beta}(y_t - y_t^*).$$
(1.11)

Plugging this in (1.10) and collecting terms gives

$$E_t\{y_{t+1}\} = \frac{\beta(\sigma+\psi)+\kappa}{\sigma\beta}y_t + \frac{\beta\gamma-1}{\sigma\beta}\pi_t - \frac{\beta\psi+\kappa}{\sigma\beta}y_t^* + \frac{1}{\sigma}\nu_t. \quad (1.12)$$

Additionally, we can iterate (1.6) one period forward to obtain

$$y_{t+1}^* = (\rho_a - \eta)y_t^* + \eta y_t + u_{t+1}^a.$$
(1.13)

We assume an autoregressive process for the exogenous monetary policy component according to

$$\nu_{t+1} = \rho_{\nu}\nu_t + u_{t+1}^{\nu}.$$
(1.14)

We can now summarize equations (1.11), (1.12), (1.13), and (1.14) compactly by the following matrix representation:

$$\begin{pmatrix} \nu_{t+1} \\ y_{t+1}^* \\ E_t \{y_{t+1}\} \\ E_t \{\pi_{t+1}\} \end{pmatrix} = A \begin{pmatrix} \nu_t \\ y_t^* \\ y_t \\ \pi_t \end{pmatrix} + B \begin{pmatrix} u_{t+1}^{\nu} \\ u_{t+1}^{a} \end{pmatrix},$$
(1.15)

where

$$A = \begin{pmatrix} \rho_{\nu} & 0 & 0 & 0\\ 0 & (\rho_a - \eta) & \eta & 0\\ \frac{1}{\sigma} & \frac{(-\beta \ast \psi - \kappa)}{(\beta \ast \sigma)} & \frac{\beta \ast (\sigma + \psi) + \kappa)}{\sigma \ast \beta} & \frac{\beta \ast \gamma - 1}{\sigma \ast \beta} \\ 0 & \frac{\kappa}{\beta} & -\frac{\kappa}{\beta} & \frac{1}{\beta} \end{pmatrix}; \quad B = \begin{pmatrix} 1 & 0\\ 0 & 1\\ 0 & 0\\ 0 & 0 \end{pmatrix}.$$

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### Chapter 2

# Cyclical long-term unemployment, skill loss, and monetary policy

### 2.1 Introduction

In this paper, I present a sticky price business cycle model which tracks durations in unemployment and introduces duration dependent skill loss during unemployment. I show that the model can reproduce several stylized facts about the cyclical component of long-term unemployment (LTU hereafter), defined as unemployment that lasts longer than four quarters. In particular, (i) hiring rates for long-term unemployed are lower than those for shorter-term unemployed (often called duration dependence); (ii) LTU is more volatile than total unemployment, which in turn is more volatile than the LTU proportion (the share of LTU among total unemployment), which in turn is more volatile than output; (iii) while the unconditional correlation between total unemployment and the LTU proportion is positive, the correlation around business cycle turning points is temporarily negative, meaning that the LTU proportion lags behind total unemployment. While stylized fact (i) is documented in the literature — for example, in Jackman and Layard (1991), Machin and Manning (1999), or Nickell (1979) — I use quarterly European data on LTU, total unemployment, and output to document the stylized facts (ii) and (iii).

In the model I use to reproduce those facts — which is based on the model in Lechthaler et al. (2010) — unemployed workers have different skill levels, depending on the duration of their unemployment spell. Unemployed workers are randomly assigned to firms at the beginning of a period. For their employment decisions, intermediate goods producers take into account skill differences between previously unemployed workers, a random operating cost shock hitting each firm-worker pair as well as hiring and firing costs. The influence of skill differences on the marginal revenue products of firms leads to different hiring thresholds for the operating cost shock depending on the previous duration of unemployment. This results in lower hiring rates for long-term unemployed workers and a lower volatility of the LTU proportion, in line with empirical data.

The unconditional correlation between total unemployment and the LTU proportion in the model is positive. As total unemployment increases during a recession, LTU increases stronger because hiring rates for all durations decrease. This implies that fewer people leave LTU and more and more shorter-term unemployed fall into LTU over time. As a consequence, the LTU proportion increases with total unemployment. However, at the beginning of a recession, an increasing firing rate creates a larger pool of shorter-term unemployed workers and hence a lower LTU proportion. Additionally, because of the effect of skill loss on the hiring thresholds, LTU lags total unemployment over the business cycle. Both effects imply that total unemployment and the LTU proportion exhibit a negative correlation around business cycle turning points, as also pointed out by Machin and Manning (1999).

Based on the observation that skill loss helps to explain the stylized facts and the model bears empirical relevance, I examine optimal monetary policy in the presence of skill loss. Faia et al. (2014) show that implementing the flexible price allocation is not optimal if firms face hiring and firing costs because of inefficient unemployment fluctuations. Hence, the monetary authority faces a trade-off between stabilizing inflation and stabilizing unemployment. I show that skill loss accentuates this trade-off. In response to a productivity shock, the monetary authority accepts more inflation variability to reduce skill deterioration in the workforce and losses in production and consumption possibilities.

To analyze these policy issues, I put the above described labor market framework in a general equilibrium context. The household consists of the different types of workers and decides on its optimal consumption and saving behavior. Households supply labor to intermediate firms which produce intermediate goods. Monopolistically competitive retailers buy these intermediate goods, transform them into final goods and sell these at a markup over marginal costs. Sticky prices are introduced through price adjustment costs. They allow for a meaningful role of monetary policy in the model. To reproduce the stylized facts, I assume that the central bank follows a Taylor rule. I calibrate the model to German data and solve it using perturbation methods.

The motivation to study the cyclical behavior of LTU and its policy implications comes from the recent developments in LTU. While high levels of LTU have long been an issue in European countries, the sharp increases in some countries in the course of the recent economic crisis suggest a substantial cyclical component in LTU. Figure 2.1 shows total unemployment rates in several European countries while figure 2.2 shows the corresponding LTU proportions. It is evident that a strong increase in total unemployment — as, for example, in Spain or Greece — is accompanied by a strong increase in LTU proportions, while a rather moderate development in total unemployment — as, for example, in Germany or France — is accompanied by a moderate development in the LTU proportion, illustrating the cyclicality of the latter.



Figure 2.1: Total unemployment rates in Belgium (BEL), Spain (ESP), France (FRA), Germany (GER), Greece (GRC), Italy (ITA), Netherlands (NL) and Portugal (PT). Notes: Quarterly data are taken from the OECD and range from 2006Q1 to 2011Q4.

This cyclicality in LTU has been recognized by researchers and policymakers. For example, Katz (2010, p. 4) remarks that "[...] cyclical problems swamp structural problems in terms of the source of unacceptably high overall and long-term unemployment [...]" and Bernanke (2012, p. 1-2) notes that "[...] while both cyclical and structural forces have doubtless contributed to the increase in long-term unemployment, the continued weakness in aggregate demand is likely the predominant factor." If this is indeed the case, monetary policy can play a role in mitigating the negative impact of LTU on the economy.

The paper is related to several strands of the literature. Shimer (2005) compares the cyclical properties of the search and matching model in the tradition of Mortensen and Pissarides (1994) to the data and concludes that the model cannot reproduce the high volatility of labor mar-



Figure 2.2: LTU proportion in Belgium (BEL), Spain (ESP), France (FRA), Germany (GER), Greece (GRC), Italy (ITA), Netherlands (NL) and Portugal (PT). Notes: Quarterly data are taken from the OECD and range from 2006Q1 to 2011Q4. The LTU proportion is the proportion of long-term unemployed workers among total unemployment.

ket variables in the US (the "Shimer-puzzle"). Gartner et al. (2012) verify these findings for Germany. Hall (2005) proposes sticky wages as a mechanism to overcome this deficiency of the standard search and matching labor market model. Gertler and Trigari (2009) endogenize wage stickiness by assuming that not all firms and workers can renegotiate wages each period. An alternative solution to the Shimer-puzzle is presented by Brown et al. (forthcoming). These authors show that hiring and firing costs, in conjunction with operating cost shocks for firm-worker pairs, drive a wedge between the hiring and the retention rate, and in this way introduce realistic amplification patterns for macroeconomic shocks. Because of the good performance of their framework regarding empirical relevance, I use the same labor market

setup. My contribution is to analyze the cyclical behavior of LTU in this framework.

A number of recent papers, such as Krause and Lubik (2007), Thomas (2008), or Walsh (2003), have incorporated labor market frictions into general equilibrium monetary models in order to study labor market reactions to monetary shocks and policy. These papers adopt search and matching frictions as the basic labor market framework. Lechthaler et al. (2010) instead use the labor turnover cost approach of Brown et al. (forthcoming) described above. They show that, while the desirable properties for labor market variables generated by labor turnover costs are retained in general equilibrium, they additionally help to explain persistence and amplification properties of output in response to monetary and productivity shocks. Thus, the general equilibrium framework presented in section 2.3 will be based on Lechthaler et al. (2010). My paper adds the distinction between different unemployment durations and introduces skill loss during unemployment.

The idea that skill loss is a distinguishing feature between short- and long-term unemployment is not new. Ljungqvist and Sargent (1998) use skill loss to explain how welfare states can slip into high unemployment regimes during turbulent times.<sup>1</sup> While their study is targeted at unemployment developments in the long run, I focus on cyclical movements in LTU. Pissarides (1992) shows in an overlapping generations model that skill loss during unemployment creates persistence in employment after macroeconomic shocks. A recent study by Laureys (2012) examines the implications of skill depreciation for optimal labor market policy. What sets my analysis apart from these studies is the nominal general equilibrium framework in which I can ultimately investigate monetary policy issues. Moreover, my research object is different from Laureys (2012) and Pissarides (1992). I seek to explain stylized business cycle facts about LTU with the help of skill loss. By contrast, Pissarides (1992) focuses on the persistence of employment

<sup>&</sup>lt;sup>1</sup>This view is challenged by den Haan et al. (2005).

and does not analyze the cyclical behavior of LTU. Similarly, Laureys (2012) does not relate the effects of skill loss to the behavior of LTU; in fact, her study does not distinguish between short- and long-term unemployment. Moreover, as described above, with the search and matching framework used in these papers, it is problematic to reproduce empirical observations about unemployment dynamics.

A recent study by Esteban-Pretel and Faraglia (2010) combines search and matching frictions and skill loss in a sticky price model and examines the role of skill loss for aggregate variables when monetary shocks hit the economy. Again, my analysis distinguishes itself from their paper regarding the labor market setting which allows for more realistic amplification patterns. I provide a systematic comparison of model outcomes to business cycle statistics. Moreover, I draw on Khan et al. (2003), Levin et al. (2006), and Schmitt-Grohé and Uribe (2004) to examine Ramsey optimal monetary policy in the presence of skill loss. To the best of my knowledge, this has not been done before.

The paper proceeds as follows: In the next section, I present stylized facts about the cyclical behavior of LTU. Section 2.3 presents the model and section 2.4 the calibration. Sections 2.5 and 2.6 compare the model predictions with the stylized facts from the data and examine the role of skill loss in explaining the stylized facts, respectively. Section 2.7 examines optimal monetary policy in the presence of skill loss. Section 2.8 concludes.

# 2.2 Stylized facts on long-term unemployment

Table 2.1 shows relative standard deviations, correlations and autocorrelations for output, total unemployment, the LTU proportion and LTU in several continental European countries based on quarterly OECD

	BEL	ESP	FRA	GER	GRC	ITA	NL	PT
Rel. std.								
y	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
$u^L/u$	3.94	7.24	3.20	3.80	1.46	3.02	8.32	4.07
$u^L$	8.07	13.35	5.53	10.29	5.33	8.28	12.82	10.35
u	6.81	7.93	4.85	7.01	4.76	5.02	9.15	7.09
Corr.								
$_{y,u}$	-0.52	-0.93	-0.76	-0.33	-0.83	-0.82	-0.56	-0.90
$y,(u^L/u)$	0.37	-0.91	0.27	0.38	0.36	-0.21	0.44	-0.34
$y, u^L$	-0.06	-0.94	-0.17	-0.14	-0.63	-0.67	-0.15	-0.39
$u,(u^L/u)$	0.13	0.83	0.24	0.49	0.25	0.49	0.16	0.48
$u, u^L$	0.78	0.86	0.75	0.92	0.97	0.87	0.73	0.57
Autocorr.								
y	0.92	0.98	0.97	0.89	0.95	0.95	0.95	0.96
u	0.73	0.98	0.83	0.89	0.83	0.91	0.93	0.91
$u^L/u$	0.81	0.98	0.91	0.98	0.97	0.97	0.95	0.91
$u^L$	0.91	0.98	0.96	0.96	0.97	0.98	0.94	0.96

Table 2.1: Unconditional moments of labor market variables and output in Belgium (BEL), Spain (ESP), France (FRA), Germany (GER), Greece (GRC), Italy (ITA), Netherlands (NL) and Portugal (PT). Notes: Quarterly data with different starting dates until 2011Q4 are taken from the OECD. u is the total unemployment rate and  $u^L/u$  is the LTU proportion among total unemployment. Unemployed workers are classified as LTU if their unemployment spell lasts more than four quarters. Output y is real GDP in USD with fixed PPPs. Variables are seasonally adjusted. All statistics are calculated using the cyclical component of the respective variable, defined as the deviation from the trend component of the HP filter. For relative standard deviations (rel. std.), percentage deviations from the trend component of the HP filter were used. Relative standard deviations are standard deviations divided by the standard deviation of output. The smoothing parameter for the HP filter is set to 100000 as in Shimer (2005).

data.<sup>2</sup> I present European data since my model in section 2.3 will feature characteristics often associated with European economies.

<sup>&</sup>lt;sup>2</sup>For each country the end of the sample is 2011Q4. The beginning of the sample for output and total unemployment is 1995Q1; for LTU and the LTU proportion, it depends on data availability. Specifically, the first observations, respectively, are: Belgium (BEL): 1999Q1; Spain (ESP): 1992Q1; France (FRA): 2003Q1; Germany (GER): 1999Q1; Greece (GRC): 1998Q1; Italy (ITA): 1992Q4; Netherlands (NL): 2002Q2; Portugal (PT): 1992Q1. All variables were obtained seasonally adjusted, except LTU which I seasonally adjusted with the Census X-12 procedure. Following Shimer (2005), I apply the HP filter with smoothing parameter 10<sup>5</sup> to extract the cyclical component of the variables.

We can see several characteristic features of the data: Looking at relative standard deviations,<sup>3</sup> we see a volatility pattern for all countries. The most volatile variable is LTU, followed by total unemployment. The LTU proportion, in turn is less volatile than total unemployment but still more volatile than output.

Turning to correlations, we see a less consistent picture. The correlations between output and total unemployment (negative), output and LTU (negative), total unemployment and the LTU proportion (positive) as well as total unemployment and LTU (positive) have the the same sign across all countries. In contrast, the signs of the correlation between output and the LTU proportion vary across countries. Regarding autocorrelations, all variables exhibit a similarly high degree of persistence across countries.

Moreover, there is an interesting feature in the relationship between total unemployment and the LTU proportion over the business cycle, pointed out by Machin and Manning (1999) for annual data. While the general relationship — as seen in row 4 of the correlations panel in table 2.1 — is positive, there seems to be a temporary negative relationships at the turning points of the business cycle. This means that when the economy starts to contract (expand), total unemployment rises (decreases) while the LTU proportion still declines (increases). In other words, the LTU proportion lags behind the total unemployment rate. Plotting and connecting the observations of these two variables results in a counter-clockwise movement when following the line through time.

Figure 2.3 shows such a scatter plot for the most recent recession around 2008 and the following recovery.<sup>4</sup> The described temporary negative correlation between total unemployment and the LTU proportion is most clearly visible for the turn into a recessionary period in or after the beginning of 2008. Note that the negative correlation is again

 $<sup>{}^{3}</sup>$ A value bigger than 1 indicates higher a volatility in the respective variable than in output.

 $<sup>^4{\</sup>rm The}$  pattern is also visible for other turning points but for the sake of lucidity only the most recent period is shown.



Figure 2.3: LTU proportion and total unemployment around business cycle turning points in Belgium (BEL), Spain (ESP), France (FRA), Germany (GER), Greece (GRC), Italy (ITA), Netherlands (NL) and Portugal (PT). Notes: Quarterly data are taken from the OECD. Shown are scatter plots of the cyclical components of the LTU proportion and the total unemployment rate from 2006Q1 to 2011Q4. This time period is meant to include the most recent recession and recovery, where applicable.

visible for the turn into the recovery for countries which actually had one by the end of 2011 like France or Germany. For countries without a recovery like Greece or Spain, the slope stays positive after 2008.

Another characteristic of LTU is what the literature calls negative duration dependence. There are several studies which confirm that unemployed workers' hiring probability declines with higher duration in unemployment. Examples are Nickell (1979) and Jackman and Layard (1991) for the UK, Machin and Manning (1999) for several European countries and Aaronson et al. (2010) for the US. There has been a debate in this literature whether the lower hiring probabilities for longerterm unemployed reflect "true" duration dependence or merely unobserved heterogeneity. Machin and Manning (1999) make the argument that unobserved heterogeneity may itself be responsible for duration dependence by means of stigmatization of long-term unemployed on behalf of employers.

### 2.3 Model

In this section I present a model which explicitly takes into account LTU and duration dependent skill loss during unemployment. The model is then used to extract cyclical properties of LTU, total unemployment, and output, and compare them to the stylized facts presented above.

As a basic framework, I use a discrete time sticky price model with a frictional labor market. One time period in the model refers to one quarter. There are four types of agents: households, intermediate firms, retail firms and the monetary authority.

The labor market features frictions in the form of hiring and firing costs as in Brown et al. (forthcoming) and Lechthaler et al. (2010). These authors show that labor turnover costs can account better for labor market and business cycle stylized facts than the basic matching function approach in the tradition of Mortensen and Pissarides (1994). The main reason for the improved amplification and persistence effects is that hiring and firing costs drive a wedge between the job-finding rate and the job retention rate. In contrast, the search and matching framework implies that these two rates are equal, conditional on a match. In a partial equilibrium setting, Brown et al. (2011) show that the labor market framework with linear hiring and firing costs is particularly useful for considering heterogeneity in the duration of unemployment. I extend their work to a general equilibrium setting.

Households consume differentiated goods, save in bonds, and supply labor to intermediate firms. Members of the household are either employed or unemployed. If they are unemployed, they can be short-term unemployed (that is, they have been unemployed for up to four quarters) or long-term unemployed (that is, they have been unemployed for more than four quarters). If workers are employed, they can be incumbents (that is, they were employed during the previous period), hired out of short-term unemployment, or hired out of LTU. Employed workers are more productive than short-term unemployed workers, who in turn are more productive than long-term unemployed workers. These differences in productivities reflect skill loss processes during unemployment spells as emphasized in Pissarides (1992) or Ljungqvist and Sargent (1998).

Intermediate firms employ labor to produce intermediate goods. Unemployed workers are assigned randomly to intermediate firms at the beginning of a period. Worker-firm pairs are subject to idiosyncratic random operating costs each period which affect the profit generated by a worker. Intermediate firms make hiring and firing decisions according to the expected stream of profits taking into account hiring and firing costs and the skill level of workers.

Retail firms act under monopolistic competition and face price adjustment costs as in Rotemberg (1982). They sell their differentiated products to the households. For the comparison of model outcomes to the data, the central bank is guided by a standard Taylor rule. Later, in section 2.7, the Taylor rule is replaced by Ramsey optimal policy.

### 2.3.1 Households

There is a continuum of agents in the economy represented by the unit interval. I follow Merz (1995) and assume that all agents belong to a large family-household. In this way, household members insure themselves against income risks arising from heterogeneous employment statuses across agents and time.

There is a continuum of differentiated final goods in the interval [0, 1] produced by retail firms. The quantity of the *i*th good consumed in period *t* is represented by  $C_t(i)$ . Agents consume a Dixit-Stiglitz aggregator  $C_t$  of these differentiated goods, where  $C_t \equiv \int_0^1 [C_t(i)^{\frac{\epsilon-1}{\epsilon}} di]^{\frac{\epsilon}{\epsilon-1}}$ .  $\epsilon > 1$  is the elasticity of substitution between final goods. The house-hold faces the following problem in each *t*:

$$\max_{C_t(i)} C_t \quad \text{s.t. } Q_t = \int_0^1 P_t(i) C_t(i) di,$$
(2.1)

where  $P_t(i)$  is the price of the differentiated good *i*. That is, the household maximizes consumption by deciding on the optimal expenditure on the differentiated goods given an expenditure level  $Q_t$ . Combining the first order conditions  $\left(\frac{C_t}{C_t(i)}\right)^{\frac{1}{\epsilon}} = \lambda P_t(i)$  ( $\lambda$  is the Lagrange multiplier) with the expenditure constraint and defining the aggregate price level of the economy as  $P_t = \left(\int_0^1 P_t(i)^{1-\epsilon} di\right)^{\frac{1}{1-\epsilon}}$  yields  $C_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\epsilon} \frac{Q_t}{P_t}$ . Using this expression and the definition of  $C_t$ , total expenditure of the household can be stated in terms of the aggregates:

$$\int_{0}^{1} P_t(i)C_t(i)di = P_tC_t.$$
(2.2)

This implies the following demand functions for the differentiated goods that is relevant for retail firms:

$$C_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\epsilon} C_t.$$
(2.3)

In addition to the optimal consumption bundle, the infinitely-lived household with discount factor  $\beta$  has to decide on the optimal consumption/saving behavior. For this purpose, it maximizes the expected value of the sum of discounted period utility functions

$$U_t = E_t \left\{ \sum_{t=0}^{\infty} \beta^t \frac{C_t^{1-\sigma}}{1-\sigma} \right\},\tag{2.4}$$

where  $\sigma$  is the coefficient of relative risk aversion, subject to the real budget constraint

$$C_t + \frac{B_t}{P_t} \le N_t w_t + \frac{B_{t-1}}{P_t} (1 + i_{t-1}) + U_t b + \Theta_t.$$
(2.5)

Each period, the household can buy nominal bonds  $B_t$  which pay a gross interest rate  $(1 + i_t)$  in the next period.  $N_t$  is the number of employed workers. Unemployed household members  $U_t$  receive real revenues from home production b.  $w_t$  represents the real wage. I assume that short- and long-term unemployed workers do not differ in terms of their home production. Since firms are owned by households, total firm profits  $\Theta$  are declared to the households.

The first-order conditions of the household's problem are:

$$C_t^{-\sigma} = \lambda_t \tag{2.6}$$

and

$$E_t \left\{ \lambda_{t+1} \beta \frac{1}{P_{t+1}} \right\} (1+i_t) = \lambda_t \frac{1}{P_t}.$$
(2.7)

Combining (2.6) and (2.7) gives the Euler equation

$$E_t \left\{ C_{t+1}^{-\sigma} \frac{1}{\pi_{t+1}} \right\} \beta(1+i_t) = C_t^{-\sigma}, \qquad (2.8)$$

where  $\pi_{t+1} = \frac{P_{t+1}}{P_t}$  is the gross inflation rate in period t+1. The Euler equation says that optimality requires the expected ratio of marginal utilities of consumption in the future and today to be equal to the price ratio of consumption today and in the future.

### 2.3.2 Firms

Intermediate firms employ labor and produce a homogeneous intermediate good. Retail firms take intermediate goods as input and produce differentiated final goods. The separation of intermediate goods producers and final goods producers is standard in the literature and avoids interactions between price setting and wage bargaining at the firm level, as Blanchard and Galí (2010) point out.

#### 2.3.2.1 Intermediate goods firms and the labor market

I follow Faia et al. (2014) and assume that there is a large number of workers and firms, and a lot more workers than firms. This ensures that intermediate firms face identical decision problems. The timing of the labor market is as follows: First, unemployed workers are randomly assigned to firms. Second, worker-firm pairs draw a random operating cost  $\epsilon_t$ , where  $E(\epsilon_t)$  is normalized to zero. This random costs serves as a tool to endogenize both hiring and firing decisions. Third, wages  $w_t$ are negotiated. Fourth, hiring and firing decisions are made.

Intermediate firms operate under the production function

$$Z_t = A_t (a^I n_t^I + a^S n_t^S + a^L n_t^L), (2.9)$$

where  $Z_t$  is the quantity of the intermediate good;  $n_t^I$  is the number of employed workers who were employed in the previous period and exhibit individual productivity  $a^I$ ;  $n_t^S$  is the number of employed workers who were short-term unemployed in the previous period and exhibit individual productivity  $a^S$ ;  $n_t^L$  is the number of employed workers who were long-term unemployed in the previous period and exhibit individual productivity  $a^L$ .  $A_t$  denotes the aggregate level of productivity in the economy. Intermediate firms sell their output to the retail firms in a perfectly competitive environment for the price  $P_z$ , so that real marginal costs  $mc_t$  must equal the real price:  $mc_t = \frac{P_{z,t}}{P_t}$ .

In the following, I describe the derivation of the firing rate for incumbent workers, the hiring rate for short-term unemployed workers and the hiring rate for long-term unemployed workers.

The contemporaneous revenue of an incumbent worker for the intermediate firm is the marginal product of the worker times the additional revenue of one more unit of output. Additionally, with retention rate  $1 - \phi_{t+1}$  the worker is not fired at the beginning of the next period and generates a future revenue stream. The costs for the firm are the wage and the random operating costs in the current and in future periods. If the worker is fired with rate  $\phi_{t+1}$  in the next period, the firm has to pay firing costs f. In line with Krause and Lubik (2007), I assume that the firing rate consists of an external component  $\phi^x$  and an internal component  $\phi^n$  so that

$$\phi_t = \phi^x + (1 - \phi^x)\phi_t^n.$$
(2.10)

A recursive formulation for the profits generated by an incumbent worker,  $\Theta_t^I$ , is

$$\Theta_{t}^{I} = A_{t}a^{I}mc_{t} - w_{t} - \epsilon_{t} + E_{t} \left\{ \beta_{t+1} \left( \left( 1 - \phi_{t+1} \right) \int_{-\infty}^{v_{f,t+1}} \frac{\Theta_{t+1}^{I}g(\epsilon_{t+1})}{G(v_{f,t+1})} d\epsilon_{t+1} - \phi_{t+1}f \right) \right\}.$$
(2.11)
$\beta_{t+1}$  denotes the effective discount factor and is defined as  $\beta_{t+1} = \beta \left(\frac{C_{t+1}}{C_t}\right)^{-\sigma}$ . It converts future profits — which eventually accrue to the households — in terms of current utility.  $G(\cdot)$  and  $g(\cdot)$  denote the cumulative distribution function and the probability density function, respectively, of the operating cost shock. Since incumbent workers are only employed in future periods if it is profitable for the firm to continue employment, future profits are conditional on the shock being below a certain threshold  $v_{f,t+1}$ , which will be determined below.

If future expected profits are expressed as

$$E_t\{\Theta_{t+1}^I\} = E_t\{(1 - \phi_{t+1})(A_{t+1}a^Imc_{t+1} - w_{t+1} - E_\epsilon\{\epsilon_{t+1} \mid \epsilon_{t+1} \le v_{f,t+1}\} + E_{t+1}\{\beta_{t+2}\Theta_{t+2}^I\}) - \phi_{t+1}f\},$$
(2.12)

the profits generated by an incumbent worker for an intermediate firm can be written as

$$\Theta_t^I = A_t a^I m c_t - w_t - \epsilon_t + E_t \{\beta_{t+1} \Theta_{t+1}^I\}.$$
(2.13)

Employing the incumbent worker only pays for the firm if these profits are higher than the profits from firing the worker, -f. Hence, the employment relationship is ended if the realization of the random cost shock  $\epsilon_t$  implies  $\Theta_t^I < -f$ . Accordingly, the firing threshold is defined by

$$v_{f,t} = A_t a^I m c_t - w_t + f + E_t \{ \beta_{t+1} \Theta_{t+1}^I \}$$
(2.14)

and the firing rate is the probability that the random cost shock is above the firing threshold:

$$\phi_t^n = 1 - G(v_{f,t}). \tag{2.15}$$

An expression for the firm's profits  $\Theta_t^S$  generated by a worker who was previously short-term unemployed is

$$\Theta_{t}^{S} = A_{t}a^{S}mc_{t} - w_{t} - \epsilon_{t} + E_{t} \left\{ \beta_{t+1} \left( \left( 1 - \phi_{t+1} \right) \int_{-\infty}^{v_{f,t+1}} \frac{\Theta_{t+1}^{I}g(\epsilon_{t+1})}{G(v_{f,t+1})} d\epsilon_{t+1} - \phi_{t+1}f \right) \right\}.$$
(2.16)

The worker generates contemporaneous revenue but the firm has to pay a wage and operating costs. If the worker is not fired in the next period, she is an incumbent worker and accordingly generates the profits of an incumbent worker conditional on the operating costs being below the firing threshold. If the worker is fired in the next period, firing costs have to be paid by the firm. Expression (2.16) shows that the expected future profits generated by a previously short-term unemployed worker are equal to the expected future profits of a previously employed worker. Hence,

$$\Theta_t^S = A_t a^S m c_t - w_t - \epsilon_t + E_t \{\beta_{t+1} \Theta_{t+1}^I\}.$$
 (2.17)

A previously short-term unemployed worker is hired if the random cost shock is low enough to generate positive profits taking into account the hiring costs h, that is, if  $h < \Theta_t^S$ . Thus, the hiring threshold for short-term unemployed workers,  $v_{h,t}^S$  is defined by

$$v_{h,t}^{S} = A_{t}a^{S}mc_{t} - w_{t} - h + E_{t}\{\beta_{t+1}\Theta_{t+1}^{I}\}$$
(2.18)

and the hiring rate for short-term unemployed workers is

$$\eta_t^S = G(v_{h,t}^S). \tag{2.19}$$

Profits  $\Theta_t^L$  generated by a worker previously long-term unemployed can be written as

$$\Theta_{t}^{L} = A_{t}a^{L}mc_{t} - w_{t} - \epsilon_{t} + E_{t} \left\{ \beta_{t+1} \left( \left( 1 - \phi_{t+1} \right) \int_{-\infty}^{v_{f,t+1}} \frac{\Theta_{t+1}^{I}g(\epsilon_{t+1})}{G(v_{f,t+1})} d\epsilon_{t+1} - \phi_{t+1}f \right) \right\}.$$
(2.20)

Similar explanations as for the profits generated by a previously shortterm unemployed worker apply. Hence, profits can be written as

$$\Theta_t^L = A_t a^L m c_t - w_t - \epsilon_t + E_t \{\beta_{t+1} \Theta_{t+1}^I\}, \qquad (2.21)$$

where  $E_t\{\Theta_{t+1}^L\} = E_t\{\Theta_{t+1}^I\}$ . If the operating costs are low enough to generate positive profits despite hiring costs, that is,  $h < \Theta_t^L$ , a long-term unemployed worker is hired. The hiring threshold  $v_{h,t}^L$  is defined as

$$v_{h,t}^{L} = A_{t}a^{L}mc_{t} - w_{t} - h + E_{t}\{\beta_{t+1}\Theta_{t+1}^{I}\}$$
(2.22)

and the hiring rate for long-term unemployed workers is

$$\eta_t^L = G(v_{h,t}^L). \tag{2.23}$$

#### 2.3.2.2 Employment and unemployment dynamics

Let  $n_t$  be the employment rate, that is, employment divided by the labor force, and  $u_t$  the total unemployment rate. The share of the labor force which is employed after being employed is

$$n_t^I = (1 - \phi_t) n_{t-1}. \tag{2.24}$$

The share of the labor force which is employed after being short-term unemployed is

$$n_t^S = \eta_t^S u_{t-1}^S, (2.25)$$

where  $u_t^S$  is the short-term unemployment rate. Similarly, the share of the labor force which is employed after being long-term unemployed is

$$n_t^L = \eta_t^L u_{t-1}^L, (2.26)$$

where  $u_t^L$  is the LTU rate. The short-term unemployment rate consists of the share of the labor force which is unemployed for less than or equal to four quarters. Hence, to determine the short-term unemployment rate I have to track the duration of unemployment. Let  $_du_t$  denote the share of the labor force which has been unemployed for d periods. Then

$$_{1}u_{t} = \phi_{t}n_{t-1}$$
 (2.27)

$${}_{2}u_{t} = (1 - \eta_{t}^{S}) {}_{1}u_{t-1}$$
(2.28)

$${}_{3}u_{t} = (1 - \eta_{t}^{S}) {}_{2}u_{t-1}$$
(2.29)

$$_{4}u_{t} = (1 - \eta_{t}^{S})_{3}u_{t-1}.$$
 (2.30)

The short-term unemployment rate can then be determined by

$$u_t^S = {}_1u_t + {}_2u_t + {}_3u_t + {}_4u_t \tag{2.31}$$

and the LTU rate consists of those who are unemployed for more than four quarters:

$$u_t^L = (1 - \eta_t^L)u_{t-1}^L + (1 - \eta_t^S)_4 u_{t-1}.$$
 (2.32)

Consequently, the employment and the total unemployment rate of the economy are

$$n_t = (1 - \phi_t)n_{t-1} + \eta_t^S u_{t-1}^S + \eta_t^L u_{t-1}^L$$
(2.33)

and

$$u_t = u_t^S + u_t^L, (2.34)$$

respectively. Since the labor force is normalized to 1, it holds that

$$n_t + u_t = 1. (2.35)$$

#### 2.3.2.3 Wages

I follow Brown et al. (2011) and assume that the wage is the outcome of a Nash bargain between the intermediate firm and the median incumbent worker with operating costs  $\bar{\epsilon}$  who faces no risk of dismissal at the negotiated wage.<sup>5</sup> As Lechthaler et al. (2010) point out, this kind of wage determination is especially suited for European economies where unions play a crucial role in wage negotioations. In this setting, the fallback position is not unemployment and a vacancy for the workers and the firm, respectively, but disagreement. During disagreement, the worker receives a fallback income (e.g. support out of a union fund or from family members), which for simplicity is assumed to be equal to unemployment benefits, while the firm incurs costs *s* which can be seen as strike costs for example.<sup>6</sup>

Let  $V_t^N$ ,  $V_t^U$ , and  $V_t^J$  be the value of a job for the worker, the value of unemployment for the worker, and the value of a job for an intermediate firm, respectively, under agreement in period t. Let  $\tilde{V}_t^N$  and  $\tilde{V}_t^J$  be the value of a job for the median worker and the firm under disagreement in period t. Then, the expected present value of a job for the median worker under agreement is

$$V_t^N = w_t + E_t \left\{ \beta_{t+1} \left[ (1 - \phi_{t+1}) V_{t+1}^N + \phi_{t+1} V_{t+1}^U \right] \right\}, \quad (2.36)$$

whereas under disagreement it is

$$\tilde{V}_t^N = b + E_t \left\{ \beta_{t+1} \left[ (1 - \phi_{t+1}) V_{t+1}^N + \phi_{t+1} V_{t+1}^U \right] \right\}.$$
 (2.37)

<sup>&</sup>lt;sup>5</sup>This assumption implies that all workers receive the same wage.

<sup>&</sup>lt;sup>6</sup>Brown et al. (forthcoming) show that individual wage bargaining leads to similar results as the centralized procedure.

Under agreement, the worker receives the wage, under disagreement she receives fallback income. The continuation values are the same under agreement and disagreement since it is assumed that disagreement does not affect future returns. Either the worker is not fired in the next period and continues to receive the value of employment, or she is fired and receives the value of unemployment. The value of a job for the firm under agreement is

$$V_t^J = A_t a^I m c_t - w_t - \bar{\epsilon} + E_t \left\{ \beta_{t+1} \left[ (1 - \phi_{t+1}) V_{t+1}^J - \phi_{t+1} f \right] \right\}, \quad (2.38)$$

whereas under disagreement it is

$$\tilde{V}_t^J = -s + E_t \left\{ \beta_{t+1} \left[ (1 - \phi_{t+1}) V_{t+1}^J - \phi_{t+1} f \right] \right\}.$$
(2.39)

Under agreement, the firm receives the marginal profit generated by the median incumbent, while under disagreement it incurs the strike costs. In the next period the firm gets the value of a job or has to pay firing costs, depending on whether the worker is fired or not. It follows that the insiders bargaining surplus is  $V_t^N - \tilde{V}_t^N$  and the firms bargaining surplus is  $V_t^J - \tilde{V}_t^J$ . Firms and workers bargain over the wage to maximize the Nash product

$$\Upsilon = (V_t^N - \tilde{V}_t^N)^{\gamma} (V_t^J - \tilde{V}_t^J)^{(1-\gamma)}$$

$$\Leftrightarrow \Upsilon = (w_t - b)^{\gamma} (A_t a^I m c_t - w_t - \bar{\epsilon} + s)^{(1-\gamma)}.$$

$$(2.40)$$

Rearranging the first-order condition of this maximization problem gives the wage

$$w_t = (1 - \gamma)b + \gamma (A_t a^I m c_t - \bar{\epsilon} + s).$$
(2.41)

#### 2.3.2.4 Retail sector

There is a continuum of monopolistic competitive firms in the retail sector indexed by  $i \in (0; 1)$ . Retailers buy inputs from intermediate firms and transform them one-to-one to differentiated final consumption goods  $Y_t(i)$ . They face quadratic price adjustment costs as in Rotemberg (1982). Firm *i* in the retail sector chooses the price in order to maximize its expected discounted profit stream subject to the household's demand function (2.3). Hence, the optimization problem for retailers reads

$$\max_{\{P_t(i)\}} \Theta_t^R = E_t \left\{ \sum_{t=0}^{\infty} \beta_t \left[ \frac{P_t(i)}{P_t} Y_t(i) - mc_t Y_t(i) - \frac{\psi}{2} \left( \frac{P_t(i)}{P_{t-1}(i)} - \pi \right)^2 Y_t \right] \right\}$$
  
s.t.  $Y_t(i) = \left( \frac{P_t(i)}{P_t} \right)^{-\epsilon} Y_t$ 

where  $\pi$  denotes steady state inflation,  $Y_t \equiv \int_0^1 [Y_t(i)^{\frac{\epsilon-1}{\epsilon}} di]^{\frac{\epsilon}{\epsilon-1}}$  and  $\frac{\psi}{2} \left(\frac{P_t(i)}{P_{t-1}(i)} - \pi\right)^2 Y_t$  are the price adjustment cost. The first order condition reads

$$Y_{t}P_{t}^{\epsilon-1}(1-\epsilon)P_{t}(i)^{-\epsilon} + \epsilon mc_{t}Y_{t}P_{t}^{\epsilon}P_{t}(i)^{-\epsilon-1} -\psi Y_{t}\left(\frac{P_{t}(i)}{P_{t-1}(i)} - \pi\right)\frac{1}{P_{t-1}} + \psi E_{t}\left\{Y_{t+1} \\ \left(\frac{P_{t+1}(i)}{P_{t}(i)} - \pi\right)\frac{P_{t+1}(i)}{P_{t}(i)^{2}}\right\} = 0.$$
(2.42)

By symmetry, the *i*'s can be neglected. Multiplying by  $P_t$  and dividing by  $Y_t$  yields the supply equation

$$(1-\epsilon) + mc_t\epsilon - \psi(\pi_t - \pi)\pi_t + \psi E_t \left\{ (\pi_{t+1} - \pi)\pi_{t+1} \frac{Y_{t+1}}{Y_t} \right\} = 0. \quad (2.43)$$

#### 2.3.3 Monetary authority and aggregate productivity process

The central bank follows a Taylor-type rule, that is, it reacts to deviations of output from steady state output and to deviations of inflation from steady state inflation. Additionally, and as is common in the literature, it is assumed that the monetary authority engages in interest rate smoothing, that is, it also reacts to the past interest rate level. The monetary policy rule is thus described by

$$\frac{1+i_t}{1+i} = \left(\frac{1+i_{t-1}}{1+i}\right)^{\gamma_i} \left[ \left(\frac{\pi_t}{\pi}\right)^{\gamma_\pi} \left(\frac{y_t}{y}\right)^{\gamma_y} \right]^{1-\gamma_i} e^{\mu_t^i}.$$
 (2.44)

 $\gamma_i$ ,  $\gamma_{\pi}$  and  $\gamma_y$  are the central banks interest rate, inflation and output reaction parameters, respectively, and variables without time subscript are steady state values.  $\mu_t^i$  is a monetary shock term.

Aggregate productivity is assumed to follow the AR(1) process

$$A_t = A^{1-\rho} A_{t-1}^{\rho} \mathrm{e}^{\mu_t^A}, \qquad (2.45)$$

where  $\mu_t^A$  is an aggregate productivity shock.

#### 2.3.4 Aggregation and equilibrium

Real profits for intermediate firms  $\Theta_t^F$  are real revenues minus total costs, where the latter comprise wage payments, operating costs and

hiring and firing costs:

$$\Theta_t^F = mc_t A_t (a^I n_t^I + a^S n_t^S + a^L n_t^L) - w_t n_t$$
  
-  $n_t^I \left( \frac{\int_{-\infty}^{v_{f,t}} \epsilon_t g(\epsilon_t) d\epsilon_t}{1 - \phi_t} \right) - n_t^S \left( \frac{\int_{-\infty}^{v_{h,t}^S} \epsilon_t g(\epsilon_t) d\epsilon_t}{\eta_t^S} \right)$ (2.46)  
-  $n_t^L \left( \frac{\int_{-\infty}^{v_{h,t}^L} \epsilon_t g(\epsilon_t) d\epsilon_t}{\eta_t^L} \right) - u_{1,t} f - n_t^S h - n_t^L h.$ 

Retailers make real revenues  $Y_t$ , have to pay for their inputs and incur price adjustment costs. Hence profits for the retailers are

$$\Theta_t^R = Y_t - mc_t A_t (a^I n_t^I + a^S n_t^S + a^L n_t^L) - \frac{\psi}{2} (\pi_t - \pi)^2 Y_t. \quad (2.47)$$

Aggregate profits in the economy are intermediate firm profits plus retailer profits:

$$\Theta_t = Y_t - w_t n_t - n_t^I \left( \frac{\int_{-\infty}^{v_{f,t}} \epsilon_t g(\epsilon_t) d\epsilon_t}{1 - \phi_t} \right) - n_t^S \left( \frac{\int_{-\infty}^{v_{h,t}^S} \epsilon_t g(\epsilon_t) d\epsilon_t}{\eta_t^S} \right) - n_t^L \left( \frac{\int_{-\infty}^{v_{h,t}^L} \epsilon_t g(\epsilon_t) d\epsilon_t}{\eta_t^L} \right) - u_{1,t}f - n_t^S h - n_t^L h - \frac{\psi}{2} (\pi_t - \pi)^2 Y_t.$$
(2.48)

Combining (2.48) with the budget constraint (2.5) yields

$$C_{t} = Y_{t} - n_{t}^{I} \left( \frac{\int_{-\infty}^{v_{f,t}} \epsilon_{t} g(\epsilon_{t}) d\epsilon_{t}}{1 - \phi_{t}} \right) - n_{t}^{S} \left( \frac{\int_{-\infty}^{v_{h,t}^{L}} \epsilon_{t} g(\epsilon_{t}) d\epsilon_{t}}{\eta_{t}^{S}} \right) - n_{t}^{L} \left( \frac{\int_{-\infty}^{v_{h,t}^{L}} \epsilon_{t} g(\epsilon_{t}) d\epsilon_{t}}{\eta_{t}^{L}} \right) - u_{1,t} f - n_{t}^{S} h - n_{t}^{L} h - \frac{\psi}{2} (\pi_{t} - \pi)^{2} Y_{t}.$$

$$(2.49)$$

For given interest rate and aggregate productivity processes  $\{A_t, i_t\}_{t=0}^{\infty}$ , a competitive equilibrium in this economy is defined as a sequence of variables

$$\{C_t, mc_t, w_t, \Theta_t^I, \phi_t, \phi_t^n v_{f,t}, v_{h,t}^S, v_{h,t}^L, \\\eta_t^S, \eta_t^L, n_t^I, n_t^S, n_t^L, u_t, 2u_t, 3u_t, 4u_t, u_t^S, u_t^L, Y_t, \pi_t\}_{t=0}^{\infty}$$

which satisfy the household optimality condition (2.8), the profit process (2.12), the hiring and firing thresholds and rates (2.10), (2.14), (2.15), (2.18), (2.19), (2.22), and (2.23), the labor market flow processes (2.24), (2.25), (2.26), (2.27), (2.28), (2.29), (2.30), (2.31), and (2.32), the wage equation (2.41), the supply equation (2.43), the economy budget constraint (2.49), and the production function (2.9). A collection of the equilibrium conditions is provided in appendix 2.A.

### 2.4 Calibration and simulation

I calibrate the model to German data and simulate second moments and dynamic responses to shocks hitting the economy. I focus on Germany because Lechthaler et al. (2010) provide a good reference point for calibrating hiring and firing costs in Germany, and because certain characteristics of the model, like the type of labor market frictions or centralized wage bargaining, are often associated with European economies.

Table 2.2 gives an overview of the calibration. The household's discount rate  $\beta$  is assumed to be 0.99, implying an annual real interest rate of 4%. Following Lechthaler et al. (2010), the elasticity of substitution between final consumption goods,  $\epsilon$ , is set to 10, the coefficient of risk aversion  $\sigma$  to 2, and the parameter of price adjustment,  $\psi$ , to 104.85 to match an average price duration of four quarters (see e.g. Nakamura and Steinsson (2008)). As no definitive evidence on the bargaining power of workers is available in the literature, I follow Krause and Lubik (2007) and set  $\gamma = 0.5$ .

Bentolila and Bertola (1990) report that firing costs in Germany amount to approximately 75% of the annual wage, which implies f = 2.6. Mortensen and Pissarides (1999) use hiring costs amounting to 15% of annual output leading to h = 0.54. As in Lechthaler et al. (2010), unemployment benefits are chosen to be 65% of the level of productivity. Keane and Wolpin (1997) estimate the rate of skill loss to be approximately 30% after one year of unemployment. I thus normalize  $a^{I}$  to 1 and set  $a^{L}$  to 0.7 and  $a^{S}$  to 0.92. These numbers will be changed later on to show how the results depend on the relative productivities of the heterogeneous labor force.

Following Brown et al. (forthcoming, 2011) and Lechthaler et al. (2010), I assume that the random operating cost is logistically distributed with cumulative distribution function  $G(\epsilon; sd) = \frac{1}{1+e^{-\epsilon/sd}}$  and probability density function  $g(\epsilon; sd) = \frac{e^{-\epsilon/s}}{s(1+e^{-\epsilon/s})^2}$  (recall that  $E(\epsilon)$  is normalized to 0). The distributional parameter sd and the strike cost parameter s are calibrated so that the steady state values of the total unemployment rate and the LTU proportion match the average of the respective series over the period 1998Q1-2011Q4 in Germany. Thus, to obtain u = 0.085and  $\frac{u^L}{u} = 0.51$ , s is set to 0.1 and sd to 0.6.

Steady state aggregate productivity is assumed to be 1 and the inflation and output reaction coefficients are set to  $\gamma_{\pi} = 1.5$  and  $\gamma_y = 0.125$ . Söderlind et al. (2005) and Belke and Polleit (2007) report an estimated smoothing degree of approximately 0.7, so  $\gamma_i = 0.7$ . The autocorrelation coefficient of productivity,  $\rho$ , is 0.94. This value is in line with estimates from Smets and Wouters (2005). The monetary shock is a one-off shock, that is, it does not exhibit autocorrelation.

Parameter	Description	Value	Source
β	Discount rate	0.99	Lechthaler et al. (2010), standard value
e	Elasticity of substitution	10	Lechthaler et al. $(2010)$ , standard value
σ	Coefficient of risk aversion	2	Lechthaler et al. $(2010)$ , standard value
$\psi$	Parameter of price adjustment	104.85	Equiv. to avg. price dur. of 1 year
7	Bargaining power of workers	0.5	Krause and Lubik $(2007)$
f	Firing costs	2.6	Bentolila and Bertola (1990)
h	Hiring costs	0.54	Mortensen and Pissarides (1999)
$a^{I}$	Prod. of incumbent worker	1	Normalized
$a^L$	Prod. of worker prev. in LTU	0.7	Keane and Wolpin (1997)
$a^S$	Prod. of worker prev. in STU	0.92	Assuming linear skill loss
sd	Distributional parameter.	0.6	To match avg. $u, u^L/u$ in data
S	Disagreement costs for firm	0.1	To match avg. $u, u^L/u$ in data
$\gamma_{\pi}$	Inflation reaction coefficient	1.5	Standard value
$\gamma_y$	Output reaction coefficient	0.125	Standard value
$\gamma_i$	Degree of interest rate smoothing	0.7	Söderlind et al. $(2005)$
θ	Autocorr. coefficient of productivity	0.94	Smets and Wouters $(2005)$

 Table 2.2:
 Calibration and source of calibrated value. Notes: STU abbreviates short-term unemployment.

Finally, I follow Lechthaler et al. (2010) and set the standard deviation of the productivity and the monetary shock to 0.5% and 0.15%, respectively.<sup>7</sup>

The model is solved using perturbation methods. A solution of the model is defined as a set of decision rules for the endogenous variables expressed as policy functions of the lagged state variables and aggregate shocks of the economy. Technically, the deterministic steady state of the model economy is computed to obtain linear approximations of the policy functions about the steady state.

The model moments reported below are produced as follows: In each simulation run, 250 time series observations are generated by drawing monetary and productivity shocks from a normal distribution with means zero and standard deviations as reported above. The first 100 observations are discarded and the remaining 150 are used to calculate relative standard deviations, correlations and autocorrelations for each variable. This procedure is repeated 1000 times. The mean of the resulting sample with size 1000 is the statistic of interest.

## 2.5 Comparing model outcomes to the data

I now compare the model-generated moments with the stylized facts presented in section 2.2. Note that in this section, I look at moments for the model with skill loss, while the next section compares these outcomes with those of the model without skill loss.

The model is in line with the evidence of negative duration dependence. That is, in the model the hiring rate for long-term unemployed workers, which is 14.23%, is lower than the hiring rate for short-term unemployed workers, which is 25.66%. The resulting average job finding rate of around 20% is in line with the value found in Lechthaler et al. (2010).

<sup>&</sup>lt;sup>7</sup>See also Smets and Wouters (2005, 2003)

Second moments are presented in table 2.3. For ease of comparison, the first data column repeats the stylized facts for Germany (see table 2.1). The second, fourth and sixth data columns in table 2.3 contain moments obtained by simulating the model with skill loss ("sl") for joint, productivity and monetary shocks, respectively. It is apparent that productivity shocks exert the main influence on the overall performance of the model as the results for joint shocks are very close to those for productivity shocks alone.

Looking at the panel with relative standard deviations in table 2.3, the model can reproduce the volatility pattern of labor market variables and output. LTU is the most volatile variable, followed by total unemployment. The LTU proportion is less volatile than total unemployment but still more volatile than output. This is true for joint, productivity, and monetary shocks.

When it comes to correlations, the model captures the correct sign in all but one case, namely the correlation between output and the LTU proportion. This correlation seems hard to capture as it switches signs across countries as shown in section 2.2. In the other cases where the signs are correct, the correlations produced by the model tend to be too high in absolute value.

While the fourth row in the correlations panel in table 2.3 shows the positive correlation between total unemployment and the LTU proportion, figure 2.4(a) inspects the model-generated correlation between these two variables more closely. It is the simulated counterpart for figure 2.3.<sup>8</sup> While the overall positive correlation is obvious, the two variables are negatively correlated around business cycle turning points. The model can thus reproduce the counterclockwise movement of the line connecting the single observations through time.

There are several other findings which I do not seek to explain explicitly in this paper, but can still be connected to the existing literature.

<sup>&</sup>lt;sup>8</sup>The figure shows the case of joint shocks. For the sake of lucidity, the scatter plot shows only 30 observations over approximately one cycle.

		Joint	$\operatorname{shock}$	Productiv	vity shock	Moneta	ry shock
	GER	$\mathbf{sl}$	$\operatorname{nsl}$	sl	$\operatorname{nsl}$	sl	nsl
Rel. std.							
y	1.000	1.000	1.000	1.000	1.000	1.000	1.000
$n^T/n$	3.800	4.067	9.044	4.038	8.986	7.199	21.043
$n^{T}$	10.294	8.343	14.084	8.294	13.989	13.312	32.402
n	7.014	4.282	4.906	4.257	4.874	8.823	13.023
Corr.							
y,u	-0.333	-0.963	-0.966	-0.963	-0.967	-0.994	-1.000
$y,(u^L/u)$	0.381	-0.857	-0.882	-0.857	-0.881	-0.452	-0.798
$y, u^L$	-0.138	-0.921	-0.918	-0.921	-0.918	-0.903	-0.921
$u,(u^L/u)$	0.495	0.955	0.968	0.959	0.969	0.378	0.798
$u, u^L$	0.920	0.989	0.986	0.990	0.987	0.866	0.921
Autocorr.							
у	0.885	0.964	0.948	0.963	0.947	0.779	0.821
n	0.892	0.987	0.982	0.989	0.983	0.707	0.821
$n_T/n$	0.976	0.991	0.989	0.992	0.989	0.832	0.941
$n^{T}$	0.962	0.992	0.990	0.992	0.989	0.928	0.924

Table 2.3: Data and model moments. Notes: The first data column shows empirical moments for Germany (see table 2.1). The rest of the data columns show simulated moments generated using the model in section 2.3 in the case of joint, productivity, and monetary shocks. The model is simulated without skill loss (nsl) and with skill loss (sl).



Figure 2.4: Model-generated plot of LTU proportion and total unemployment over the business cycle. Notes: The graphs show scatter plots for simulated data (joint shocks) with the LTU proportion and the total unemployment rate on the axes. Panel (a) is the result of the model including skill loss and panel (b) is the result of the model without skill loss for the same sequence of shocks.

First, as shown in data row four of the relative standard deviation panel in table 2.3, the model is able to produce a higher volatility for total unemployment than for output, which reproduces one of the basic findings in Lechthaler et al. (2010). However, the magnitude of total unemployment volatility for joint shocks is still not large enough to reproduce that of the empirical observations. The very high volatility of total unemployment relative to output in Germany is also documented in Gartner et al. (2012). As we see in column six and row four of the relative standard deviations panel in table 2.3, monetary shocks are in principle capable of producing such high volatilities. This indicates that models with several demand shocks might be able to produce a higher overall volatility of total unemployment. Second, the model is able to generate large degrees of persistence for all types of shocks. For joint and productivity shocks, these are even a bit bigger than those found in the data.

#### 2.6 The role of skill loss

The skill loss mechanism is the main distinguishing feature of the model in this paper compared to most existing models in the literature. This section aims at illustrating the importance of skill loss in reproducing stylized facts. To this end, I switch off the skill loss mechanism in the model presented in section 2.3, that is I set  $a^I = a^S = a^L$ . The resulting model is one with a homogeneous work force. I can now compare moments of the model with and without skill loss.

As a first result, and contrary to the model with skill loss, the model without the skill loss mechanism is not able to reproduce a lower hiring rate for workers in LTU than for workers with shorter durations in unemployment. The hiring rate for both types of unemployed is around 19%. This result is a direct consequence of the influence of skill loss on the hiring thresholds of short- and long-term unemployed workers, as seen in equations (2.19) and (2.23).

Turning to second moments, data columns three, five and seven in table 2.3 show the outcomes for the model without skill loss ("nsl"). The most striking result is that the skill loss mechanism is crucial for reproducing the volatility pattern in the data. With a homogeneous work force, LTU and the LTU proportion are far too volatile. In addition, the model without skill loss predicts, counterfactually, that the LTU proportion is more volatile than the total unemployment rate. Both of these results are true for joint, productivity as well as monetary shocks. The skill loss mechanism mutes the impact of the shock on the hiring threshold for long-term unemployed workers more than on the hiring threshold for short-term unemployed workers, leading to a volatility ordering which is in line with the empirical evidence.

In terms of unconditional correlations, there is little difference between the model with heterogeneous and homogeneous skills. If anything, skill loss improves the model's performance by reducing the absolute value of the correlation between output and the LTU proportion as well as between total unemployment and the LTU proportion, moving the model moments in the direction of the empirical moments. These improvements are especially noticeable when it comes to the specific correlation pattern between total unemployment and the LTU proportion. Figure 2.4(b) shows a scatter plot of simulated values of those variables for the model without skill loss. The shock sequence is the same as in 2.4(a) with skill loss present. While the positive correlation is again clearly visible, the model without skill loss does not exhibit the temporary negative correlation around business cycle turning points (bottom left and upper right corner of the graph) as clearly as the model with skill loss. In this sense, LTU lags the development in total unemployment over the business cycle more when workers lose skills during unemployment.

When it comes to autocorrelations, we see that for joint shocks, skill loss adds persistence to all variables compared to the model without skill loss. However, this is due to the dominating nature of the productivity shock. For monetary shocks, the autocorrelation even declines somewhat for the skill loss model vis-a-vis the model excluding skill loss. This result corroborates findings in related studies: Using real labor market models, Merkl and Snower (2008) and Pissarides (1992) show that labor market persistence increases for heterogeneous productivities in the labor force, while the monetary model described in Esteban-Pretel and Faraglia (2010) yields a decrease in persistence when skill loss is present.

To gain further insight into the dynamics of the model, figures 2.5 and 2.6 show the reaction of several variables to a monetary and productivity shock, respectively. The horizontal axes show quarters, while the vertical axes show percentage deviations of the respective variable from the steady state.<sup>9</sup> The solid lines represent impulse response functions for the model with skill loss, while the dashed lines are impulse responses for the model without skill loss.

 $<sup>^9\</sup>mathrm{For}$  inflation and the nominal interest rate, percentage point deviations from steady state are shown.



Figure 2.5: Impulse response functions for a contractive monetary shock. Notes: The graphs show a comparison of IRFs generated by the model with (solid lines) and without (dashed lines) skill loss, respectively, where the central bank follows a Taylor rule. Time periods correspond to quarters.

As shown in figure 2.5, output decreases in response to a contractive monetary shock. This is due to a decrease in demand since consumers postpone consumption into the future. As can be seen in equations (2.14), (2.18) and (2.22), lower marginal costs decrease the hiring and firing thresholds due to lower unitary profits, causing the hiring rates for short- and long-term unemployment durations to drop and the firing rate to increase.<sup>10</sup> This translates into higher short- and long-term unemployment and consequently into higher total unemployment. Note

<sup>&</sup>lt;sup>10</sup>Note that wages also decrease when a contractive monetary shock hits, but this decrease is weaker than the decrease in marginal costs.

that the drop in the hiring rate for long-term unemployed is smaller than that for the short-term unemployed.

With skill loss present in the economy, LTU and the LTU proportion show a more muted response, in line with the data. Note that with skill loss, the LTU proportion even decreases initially as the increase in total unemployment is stronger than the increase in LTU. Only in the subsequent quarters the LTU proportion increases before converging back to the steady state in accordance with total unemployment. This pattern illustrates again the overall positive but temporary negative correlation between total unemployment and the LTU proportion seen in figures 2.3 and 2.4(a).

In response to an adverse productivity shock, as shown in figure 2.6, output decreases on impact, reaching its strongest reaction with a lag. Total unemployment and LTU also respond with a lag and reach their peak reaction after eight quarters. The lagged reaction of the unemployment variables can be explained by offsetting effects of productivity and wages (negative) on the one hand and marginal costs (positive) on the other hand on the hiring and firing thresholds. It is noteworthy that the output reaction to a productivity shock peaks later when skill loss is present. Since the responses of short-term unemployment are similar in magnitude and timing for both cases this is due to the delayed peak reaction in LTU in the case of skill loss.

Moreover, the lower volatilities for LTU and the LTU proportion are evident in the impulse-response functions. The adverse productivity shock has a direct impact on the marginal revenue product of workers as shown in equations (2.18) and (2.22). Basically, skill loss has a dampening effect on the marginal revenue product of labor in the sense that movements in productivity and marginal costs in response to shocks are not fully transmitted into unitary profits and hence hiring and firing thresholds.



Figure 2.6: Impulse response functions for an adverse productivity shock. Notes: The graphs show a comparison of IRFs generated by the model with (solid lines) and without (dashed lines) skill loss, respectively, where the central bank follows a Taylor rule. Time periods correspond to quarters.

# 2.7 Optimal monetary policy in the presence of skill loss

As shown above, modeling the skill loss mechanism is important for capturing stylized facts from the data. Consequently, the implications of skill loss for monetary policy are examined. In a recent study, Faia et al. (2014) show that labor turnover costs induce a trade-off for the central bank. Hiring and firing costs imply a waste of resources. Hence, implementing the flexible price allocation is not optimal anymore because of inefficient unemployment fluctuations. Consequently, the monetary authority has to strike a balance between stabilizing inflation and smoothing out unemployment fluctuations. This trade-off is also at play here. However, an additional factor relevant for welfare considerations enters the picture as firms do not account for skill loss processes during unemployment.

The optimal policy plan is determined by a Ramsey planner who maximizes the family household's utility function (2.4) subject to the equilibrium conditions of the competitive economy. The Taylor rule is replaced by the Ramsey planner's first-order conditions, shown in appendix 2.B. Similar as in section 2.4, the deterministic steady state of the Ramsey problem's first-order conditions is computed to obtain linear approximations of the policy functions around that steady state.

It is instructive to first look at the difference between the Ramsey policy and the Taylor rule. Figure 2.7 looks at impulse response functions after an adverse productivity shock for the Ramsey monetary policy and the Taylor rule policy.<sup>11</sup> The solid line shows the Ramsey policy, the dashed line the Taylor rule policy. As expected, the Ramsey policy tolerates more short-term, long-term, and hence total unemployment, compared to the policy of a Taylor rule. This comes at the benefit of a much lower inflation volatility.

However, skill loss during unemployment accentuates the unemploymentinflation trade-off. This becomes clear by comparing the Ramsey policies for the case with and without skill loss. Figure 2.8 shows the optimal path of model variables in response to an adverse productivity shock. Here, the solid lines represent impulse responses for the model with skill loss, while the dashed lines show impulse responses without skill loss.

In the case of an adverse productivity shock, output declines while short-term, long-term and total unemployment as well as inflation increase. The rise in the unemployment variables is driven by declining

<sup>&</sup>lt;sup>11</sup>Note that monetary shocks cannot be examined since the interest rate rule is replaced by the interest rate path supporting the optimal allocation.



Figure 2.7: Ramsey monetary policy vs. Taylor rule for an adverse productivity shock. Notes: The graphs show impulse response functions generated by the model with skill loss. The dashed and solid lines correspond to the model with Taylor rule and Ramsey policy, respectively. Inflation and the interest rate are annualized percentage point deviations from the steady state, the rest is shown in percentage deviations from the steady state. Time periods correspond to quarters.

hiring rates and an increasing firing rate. As in Faia et al. (2014), the Ramsey planner can use inflation to reduce inefficient unemployment fluctuations. However, in the setting with skill loss, this trade-off is more accentuated than in the setting without skill loss. The lower left graph in figure 2.8 shows that the Ramsey planner allows for a bigger increase in inflation in the presence of skill loss. Note that the unemployment variables increase less in this case. In particular, the response of LTU is more muted and peaks later. This is also reflected in the smoothed response of the LTU proportion. In this way, the Ram-



Figure 2.8: Ramsey monetary policy with and without skill loss for an adverse productivity shock. Notes: The graphs compare impulse response functions for the Ramsey optimal monetary policy with (solid line) and without (dashed line) skill loss. Inflation and the interest rate are annualized percentage point deviations from the steady state, the rest is shown in percentage deviations from the steady state. Time periods correspond to quarters.

sey planner reduces skill loss processes during unemployment at the cost of higher inflation volatility and hence mitigates production and consumption losses due to a lower average productivity. Consequently, the implied interest rate processes that support the optimal adjustment paths of inflation and the unemployment variables in the two scenarios proceed lower than if the monetary authority employs a Taylor rule but the shift is relatively more pronounced in the scenario with skill loss.

A look at the model-generated moments shows that the optimal inflation volatility approximately doubles when skill loss is present. The relative standard deviation of inflation, measured as described in section 2.2, is 0.14 in the model with heterogeneous skills compared to 0.08 in the model with homogeneous skills.

### 2.8 Conclusion

LTU exhibits substantial variations over the business cycle. The cyclical components of LTU and the LTU proportion are several times more volatile than the cyclical component of output. The same is true for total unemployment, but its volatility is lower than that of LTU and higher than that of the LTU proportion. Total unemployment and the LTU proportion are generally positively correlated but exhibit a temporary negative correlation around the turning points of the business cycle. Furthermore, the literature documents that unemployment is subject to negative duration dependence, that is, the unemployed workers' hiring rate declines with higher duration in unemployment.

A New Keynesian business cycle model with a frictional labor market and skill loss during unemployment is able to match these stylized facts. The skill loss mechanism makes it relatively less attractive for firms to hire workers from the LTU pool, which explains negative duration dependence. The impact of skill loss on the marginal revenue product of firms and hiring and firing thresholds helps to reproduce the empirical evidence on the volatility pattern of output and labor market variables, as well as the behavior of the LTU proportion around business cycle turning points.

Due to hiring and firing costs, the monetary authority faces a trade-off between stabilizing inflation and stabilizing unemployment. Skill loss accentuates this trade-off. Optimal monetary policy in the presence of skill loss therefore accepts more inflation after adverse productivity shocks to reduce skill deteriorations and mitigate production and consumption losses.

### Appendix 2.A Equilibrium equations

Define  $\Xi^{I}$  as the expected operating costs for incumbent workers conditional on not being fired and  $\Xi^{S}$  and  $\Xi^{L}$  as the expected operating costs for short- and long-term unemployed workers, respectively, conditional on being hired. Then, there are 28 endogenous variables: c, y, $i, v_{h}^{S}, v_{h}^{L}, \eta^{S}, \eta^{L}, v_{f} \phi, \phi^{n}, n^{S}, n^{L}, n^{I}, {}_{1}u, {}_{2}u, {}_{3}u, {}_{4}u, u^{L}, u^{S}, w,$  $mc, \Theta^{I}, \Theta^{S}, \Theta^{L}, \pi, \Xi^{I}, \Xi^{S}, \text{ and } \Xi^{L}$ . The competitive economy can be characterized by the following 27 equilibrium conditions plus the interest rate rule:

$$c_t^{-\sigma} = \mathcal{E}_t \left\{ c_{t+1}^{-\sigma} \beta \, \frac{i_t}{\pi_{t+1}} \right\} \tag{2.50}$$

$$0 = 1 - \epsilon + \epsilon m c_t - \pi_t \psi (\pi_t - 1) + \psi \beta E_t \left\{ \left( \frac{c_{t+1}}{c_t} \right)^{-\sigma} (\pi_{t+1} - 1) \pi_{t+1} \frac{y_{t+1}}{y_t} \right\}$$
(2.51)

$$w_t = (1 - \gamma) \ b + \gamma \ \left(mc_t \ A_t \ a^I + s - \bar{\epsilon}\right)$$
(2.52)

$$\phi_t^n = 1 - \frac{1}{1 + \exp\left(\frac{-v_{f,t}}{sd}\right)} \tag{2.53}$$

$$\phi_t = \phi^x + \phi_t^n \ (1 - \phi^x) \tag{2.54}$$

$$v_{f,t} = mc_t A_t a^I - w_t + f + \beta \operatorname{E}_t \left\{ \left(\frac{c_{t+1}}{c_t}\right)^{-\sigma} \Theta_{t+1}^I \right\} \quad (2.55)$$

$$\Theta_t^I = (1 - \phi_t) \left( mc_t A_t a^I - w_t - \Xi_t^I + \beta \operatorname{E}_t \left\{ \Theta_{t+1}^I \left( \frac{c_{t+1}}{c_t} \right)^{-\sigma} \right\} \right) - \phi_t f$$
(2.56)

$$\Xi_t^I = \frac{v_{f,t} - \frac{v_{f,t}}{1 + \exp\left(\frac{v_{f,t}}{sd}\right)} - sd \log\left(1 + \exp\left(\frac{v_{f,t}}{sd}\right)\right)}{1 - \phi_t}$$
(2.57)

$$\Theta_t^S = (1 - \phi_t) \left( mc_t A_t a^I - w_t - \Xi_t^I + \beta \operatorname{E}_t \left\{ \Theta_{t+1}^I \left( \frac{c_{t+1}}{c_t} \right)^{-\sigma} \right\} \right) - \phi_t f$$
(2.58)

$$\Theta_t^L = (1 - \phi_t) \left( mc_t A_t a^I - w_t - \Xi_t^I + \beta \operatorname{E}_t \left\{ \Theta_{t+1}^I \left( \frac{c_{t+1}}{c_t} \right)^{-\sigma} \right\} \right) - \phi_t f$$
(2.59)

$$v_{h,t}^{S} = mc_t A_t a^{S} - w_t - h + \beta E_t \left\{ \left(\frac{c_{t+1}}{c_t}\right)^{-\sigma} \Theta_{t+1}^{S} \right\}$$
(2.60)

$$\eta_t^S = \frac{1}{1 + \exp\left(\frac{-v_{h,t}^S}{sd}\right)} \tag{2.61}$$

$$v_{h,t}^{L} = mc_t A_t a^{L} - w_t - h + \beta E_t \left\{ \left(\frac{c_{t+1}}{c_t}\right)^{-\sigma} \Theta_{t+1}^{L} \right\}$$
(2.62)

$$\eta_t^L = \frac{1}{1 + \exp\left(\frac{-v_{h,t}^L}{sd}\right)} \tag{2.63}$$

$$\Xi_t^S = \frac{v_{h,t}^S - \frac{v_{h,t}^S}{1 + \exp\left(\frac{v_{h,t}^S}{sd}\right)} - sd \log\left(1 + \exp\left(\frac{v_{h,t}^S}{sd}\right)\right)}{\eta_t^S}$$
(2.64)

$$\Xi_t^L = \frac{v_{h,t}^L - \frac{v_{h,t}^L}{1 + \exp\left(\frac{v_{h,t}^L}{sd}\right)} - sd \log\left(1 + \exp\left(\frac{v_{h,t}^L}{sd}\right)\right)}{\eta_t^L}$$
(2.65)

$$n_t^L = \eta_t^L \, u_{t-1}^L \tag{2.66}$$

$$n_t^S = \eta_t^S \, u_{t-1}^S \tag{2.67}$$

$$n_t^I = (1 - \phi_t) \left( n_{t-1}^L + n_{t-1}^S + n_{t-1}^I \right)$$
(2.68)

$${}_{1}u_{t} = \phi_{t} \left( n_{t-1}^{L} + n_{t-1}^{S} + n_{t-1}^{I} \right)$$
(2.69)

$${}_{2}u_{t} = \left(1 - \eta_{t}^{S}\right) {}_{1}u_{t-1} \tag{2.70}$$

$$_{3}u_{t} = \left(1 - \eta_{t}^{S}\right) _{2}u_{t-1}$$
 (2.71)

$$_{4}u_{t} = \left(1 - \eta_{t}^{S}\right)_{3}u_{t-1} \tag{2.72}$$

$$u_t^L = \left(1 - \eta_t^S\right) \,_4 u_{t-1} + u_{t-1}^L \left(1 - \eta_t^L\right) \tag{2.73}$$

$$u_t^S = 1 - u_t^L - n_t^I - n_t^S - n_t^L$$
(2.74)

$$y_t = A_t \left( a^I n^I t + a^S n_t^S + a^L n_t^L \right)$$
(2.75)

$$c_{t} = y_{t} - \Xi_{t}^{I} n^{I} t - \Xi_{t}^{L} n_{t}^{L} - \Xi_{t}^{S} n_{t}^{S} - f_{1} u_{t} - h n_{t}^{S} - h n_{t}^{L} - y_{t} \frac{\psi}{2} (\pi_{t} - 1)^{2}$$

$$(2.76)$$

$$\frac{1+i_t}{1+i} = \left(\frac{1+i_{t-1}}{1+i}\right)^{\gamma_i} \left[ \left(\frac{\pi_t}{\pi}\right)^{\gamma_\pi} \left(\frac{y_t}{y}\right)^{\gamma_y} \right]^{1-\gamma_i} \exp\left(\mu_t^i\right). \quad (2.77)$$

Total unemployment is defined as  $u_t = u_t^L + u_t^S$  and aggregate employment as  $n_t = n_t^L + n_t^S + n_t^I$ . The exogenously driven process for technology is given by  $A_t = A^{1-\rho}A_{t-1}^{\rho}e^{\mu_t^A}$ .

#### Appendix 2.B Ramsey policy

The Ramsey planner maximizes the expected discounted sum of period utility functions conditional on information at time 0 subject to the equilibrium conditions of the competitive economy. For a given stochastic process for productivity, the Ramsey planner chooses  $c_t$ ,  $y_t$ ,  $i_t$ ,  $v_{h,t}^S$ ,  $v_{h,t}^L$ ,  $\eta_t^S$ ,  $\eta_t^L$ ,  $v_{f,t}$ ,  $\phi_t^n$ ,  $n_t^S$ ,  $n_t^L$ ,  $n_t^I$ ,  $_{1t}u_t$ ,  $_{2u_t}$ ,  $_{3u_t}$ ,  $_{4u_t}$ ,  $u_t^L$ ,  $u_t^S$ ,  $w_t$ ,  $mc_t$ ,  $\Theta_t^I$ ,  $\Theta_t^S$ ,  $\Theta_t^L$ ,  $\pi_t$ ,  $\Xi_t^I$ ,  $\Xi_s^S$ , and  $\Xi_t^L$  to maximize

$$E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \frac{C_t^{1-\sigma}}{1-\sigma} \right\}$$
  
s.t. (2.50)-(2.76).

Let  $\lambda i$ , i = 1, 2, ..., 27 be the Lagrange multipliers on constraints (2.50)-(2.76) of the Lagrangian L. The first order conditions of the maximization problem are (2.50)-(2.76) and

$$\begin{aligned} \frac{\partial \mathbf{L}}{\partial \eta_t^L} &= \lambda \mathbf{14}_t - u_{t-1}^L \,\lambda \mathbf{17}_t + u_{t-1}^L \,\lambda \mathbf{24}_t \\ &- \frac{\lambda \mathbf{16}_t \,\left( sd \,\log\left(\mathbf{1} + \exp\left(\frac{v_{h,t}^L}{sd}\right)\right) + \frac{v_{h,t}^L}{\mathbf{1} + \exp\left(\frac{v_{h,t}^L}{sd}\right)} - v_{h,t}^L \right)}{(\eta_t^L)^2} &= 0 \end{aligned}$$

$$(2.78)$$

$$\begin{aligned} \frac{\partial \mathbf{L}}{\partial \eta_t^S} &= \lambda 12_t + {}_1u_{t-1}\,\lambda 21_t + {}_2u_{t-1}\,\lambda 22_t \\ &+ {}_3u_{t-1}\,\lambda 23_t + {}_4u_{t-1}\,\lambda 24_t - u_{t-1}^S\,\lambda 18_t \\ &- \frac{\lambda 15_t \left(sd\,\log\left(1 + \exp\left(\frac{v_{h,t}^S}{sd}\right)\right) + \frac{v_{h,t}^S}{1 + \exp\left(\frac{v_{h,t}^S}{sd}\right)} - v_{h,t}^S\right)}{(\eta_t^S)^2} = 0 \end{aligned}$$

$$(2.79)$$

$$\frac{\partial \mathcal{L}}{\partial i_t} = \mathcal{E}_t \left\{ \frac{c_t^\sigma \beta \,\lambda \mathbf{1}_t}{\pi_{t+1}} \right\} = 0 \tag{2.80}$$

$$\frac{\partial \mathbf{L}}{\partial mc_t} = (\phi_t - 1) A_t a^I \lambda 7_t - A_t a^I \lambda 6_t - A_t a^L \lambda 13_t - A_t a^S \lambda 11_t - \epsilon \lambda 2_t + (\phi_t - 1) A_t a^I \lambda 9_t + (\phi_t - 1) A_t a^I \lambda 10_t - \gamma A_t a^I \lambda 3_t = 0$$
(2.81)

$$\frac{\partial \mathbf{L}}{\partial n_t^I} = \lambda 19_t + \lambda 25_t + \Xi_t^I \lambda 27_t + \beta \operatorname{E}_t \{\lambda 19_{t+1} \ (\phi_{t+1} \quad (2.82) - 1)\} - A_t a^I \lambda 26_t - \beta \operatorname{E}_t \{\phi_{t+1} \lambda 20_{t+1}\} = 0$$

$$\begin{split} \frac{\partial \mathbf{L}}{\partial c_{t}} &= \lambda 27_{t} + \frac{1}{c_{t}^{\sigma}} + \frac{\sigma c_{t}^{\sigma-1} \lambda \mathbf{1}_{t-1}}{\beta} - \mathbf{E}_{t} \left\{ \frac{\Theta_{t+1}^{I} \sigma \beta c_{t}^{\sigma-1} \lambda \mathbf{0}_{t}}{c_{t+1}^{\sigma}} \right\} \\ &- \mathbf{E}_{t} \left\{ \frac{\Theta_{t+1}^{S} \sigma \beta c_{t}^{\sigma-1} \lambda \mathbf{1}_{t}}{c_{t+1}^{\sigma}} \right\} - \mathbf{E}_{t} \left\{ \frac{\Theta_{t+1}^{L} \sigma \beta c_{t}^{\sigma-1} \lambda \mathbf{1}_{s}}{c_{t+1}^{\sigma}} \right\} \\ &- \frac{\sigma i_{t} \beta c_{t}^{\sigma-1} \lambda \mathbf{1}_{t}}{\pi_{t+1}} + \frac{\Theta_{t}^{I} \sigma c_{t-1}^{\sigma} \lambda \mathbf{0}_{t-1}}{c_{t}^{1+\sigma}} + \frac{\Theta_{t}^{S} \sigma c_{t-1}^{\sigma} \lambda \mathbf{1}_{t-1}}{c_{t}^{1+\sigma}} \\ &+ \frac{\Theta_{t}^{L} \sigma c_{t-1}^{\sigma} \lambda \mathbf{1}_{s-1}}{c_{t}^{1+\sigma}} + \mathbf{E}_{t} \left\{ \frac{\Theta_{t+1}^{I} \sigma c_{t}^{\sigma-1} \beta \lambda \mathbf{7}_{t} (\phi_{t}-1)}{c_{t+1}^{\sigma}} \right\} \\ &+ \mathbf{E}_{t} \left\{ \frac{(\phi_{t}-1) \Theta_{t+1}^{I} \sigma c_{t}^{\sigma-1} \beta \lambda \mathbf{9}_{t}}{c_{t+1}^{\sigma}} \right\} \\ &+ \mathbf{E}_{t} \left\{ \frac{(\phi_{t}-1) \Theta_{t+1}^{I} \sigma c_{t}^{\sigma-1} \beta \lambda \mathbf{1}_{t}}{c_{t+1}^{\sigma}} - \frac{(\phi_{t-1}-1) \Theta_{t}^{I} \sigma c_{t-1}^{\sigma} \lambda \mathbf{9}_{t-1}}{c_{t}^{1+\sigma}} \\ &- \frac{\Theta_{t}^{I} \sigma c_{t-1}^{\sigma} \lambda \mathbf{7}_{t-1} (\phi_{t-1}-1)}{c_{t}^{1+\sigma}} - \frac{(\phi_{t-1}-1) \Theta_{t}^{I} \sigma c_{t-1}^{\sigma-1} \lambda \mathbf{9}_{t-1}}{c_{t}^{1+\sigma}} \\ &- \mathbf{E}_{t} \left\{ \frac{(\pi_{t+1}-1) y_{t+1} \sigma \psi \pi_{t+1} \beta c_{t}^{\sigma-1} \lambda \mathbf{2}_{t}}{c_{t+1}^{\sigma}} \right\} \\ &+ \frac{(\pi_{t}-1) y_{t} \sigma \psi \pi_{t} c_{t-1}^{\sigma} \lambda \mathbf{2}_{t-1}}{c_{t}^{1+\sigma}} = 0 \end{split}$$

$$\frac{\partial \mathbf{L}}{\partial n_t^L} = \beta \mathbf{E}_t \left\{ \lambda \mathbf{19}_{t+1} \left( \phi_{t+1} - 1 \right) \right\} + \lambda \mathbf{17}_t + \lambda \mathbf{25}_t \qquad (2.84)$$
$$+ \lambda \mathbf{27}_t \left( h + \Xi_t^L \right) - A_t a^L \lambda \mathbf{26}_t - \beta \mathbf{E}_t \left\{ \phi_{t+1} \lambda \mathbf{20}_{t+1} \right\} = 0$$

$$\frac{\partial \mathbf{L}}{\partial n_t^S} = \beta \mathbf{E}_t \left\{ \lambda \mathbf{19}_{t+1} \left( \phi_{t+1} - 1 \right) \right\} + \lambda \mathbf{18}_t + \lambda \mathbf{25}_t \qquad (2.85)$$
$$+ \lambda \mathbf{27}_t \left( h + \Xi_t^S \right) - A_t a^S \lambda \mathbf{26}_t - \beta \mathbf{E}_t \left\{ \phi_{t+1} \lambda \mathbf{20}_{t+1} \right\} = 0$$

$$\frac{\partial \mathbf{L}}{\partial \phi_t} = \lambda \mathbf{5}_t + \lambda \mathbf{19}_t \left( n_{t-1}^S + n_{t-1}^L + n^I t - 1 \right) - \left( n_{t-1}^S + n_{t-1}^L \right) \\ + n^I t - 1 \right) \lambda \mathbf{20}_t + \lambda \mathbf{7}_t \left( mc_t A_t a^I + f - w_t \right) \\ - \Xi_t^I + \Xi_t \left\{ \frac{\beta c_t^\sigma \Theta_{t+1}^I}{c_{t+1}^\sigma} \right\} + \lambda \mathbf{9}_t \left( mc_t A_t a^I + f - w_t - \Xi_t^I + \Xi_t \right) \\ \left\{ \frac{\beta c_t^\sigma \Theta_{t+1}^I}{c_{t+1}^\sigma} \right\} + \lambda \mathbf{10}_t \left( mc_t A_t a^I + f - w_t - \Xi_t^I + \Xi_t \left\{ \frac{\beta c_t^\sigma \Theta_{t+1}^I}{c_{t+1}^\sigma} \right\} \right) \\ + \frac{\lambda \mathbf{8}_t \left( sd \log \left( 1 + \exp \left( \frac{v_{f,t}}{sd} \right) \right) + \frac{v_{f,t}}{1 + \exp \left( \frac{v_{f,t}}{sd} \right)} - v_{f,t} \right)}{\left( \phi_t - 1 \right)^2} = 0$$

$$(2.86)$$

$$\frac{\partial \mathcal{L}}{\partial \phi_t^n} = \lambda 4_t + \lambda 5_t \ (\phi^x - 1) = 0 \tag{2.87}$$

$$\frac{\partial \mathcal{L}}{\partial \pi_t} = \lambda 2_t \left( \psi \left( \pi_t - 1 \right) + \psi \pi_t \right) - \lambda 2_{t-1} \left( \frac{(\pi_t - 1) y_t \psi c_{t-1}^{\sigma}}{c_t^{\sigma} y_{t-1}} \right) + \frac{y_t \psi \pi_t c_{t-1}^{\sigma}}{c_t^{\sigma} y_{t-1}} + \frac{y_t \psi \pi_t c_{t-1}^{\sigma}}{c_t^{\sigma} y_{t-1}} \right) + y_t \psi \lambda 27_t (\pi_t - 1) + \frac{\lambda 1_{t-1} c_{t-1}^{\sigma} i_{t-1}}{\pi_t^2} = 0$$
(2.88)

$$\frac{\partial \mathbf{L}}{\partial \Theta_t^I} = \lambda 7_t + \frac{c_{t-1}^{\sigma} \lambda 7_{t-1} (\phi_{t-1} - 1)}{c_t^{\sigma}} + \frac{c_{t-1}^{\sigma} (\phi_{t-1} - 1) \lambda 9_{t-1}}{c_t^{\sigma}} + \frac{c_{t-1}^{\sigma} (\phi_{t-1} - 1) \lambda 10_{t-1}}{c_t^{\sigma}} - \frac{c_{t-1}^{\sigma} \lambda 6_{t-1}}{c_t^{\sigma}} = 0$$
(2.89)

$$\frac{\partial \mathcal{L}}{\partial \Theta_t^L} = \lambda 10_t - \frac{c_{t-1}^{\sigma} \lambda 13_{t-1}}{c_t^{\sigma}} = 0$$
(2.90)

$$\frac{\partial \mathcal{L}}{\partial \Theta_t^S} = \lambda 9_t - \frac{c_{t-1}^{\sigma} \lambda 11_{t-1}}{c_t^{\sigma}} = 0$$
(2.91)

$$\frac{\partial \mathcal{L}}{\partial_1 u_t} = \lambda 20_t + f \,\lambda 27_t + \beta \,\mathcal{E}_t \left\{\lambda 21_{t+1} \left(\eta_{t+1}^S - 1\right)\right\} = 0 \quad (2.92)$$

$$\frac{\partial \mathcal{L}}{\partial_2 u_t} = \lambda 2 \mathbf{1}_t + \beta \mathcal{E}_t \left\{ \left( \eta_{t+1}^S - 1 \right) \lambda 2 \mathbf{2}_{t+1} \right\} = 0$$
(2.93)

$$\frac{\partial \mathcal{L}}{\partial_3 u_t} = \lambda 22_t + \beta \mathcal{E}_t \left\{ \left( \eta_{t+1}^S - 1 \right) \lambda 23_{t+1} \right\} = 0$$
(2.94)

$$\frac{\partial \mathcal{L}}{\partial_4 u_t} = \lambda 23_t + \beta \mathcal{E}_t \left\{ \left( \eta_{t+1}^S - 1 \right) \lambda 24_{t+1} \right\} = 0$$
(2.95)

$$\frac{\partial \mathbf{E}}{\partial u_t^L} = \lambda 24_t + \lambda 25_t - \beta \mathbf{E}_t \left\{ \eta_{t+1}^L \lambda 17_{t+1} \right\} + \beta \mathbf{E}_t \left\{ \lambda 24_{t+1} \left( \eta_{t+1}^L - 1 \right) \right\} = 0$$
(2.96)

$$\frac{\partial \mathcal{L}}{\partial u_t^S} = \lambda 25_t - \beta \mathcal{E}_t \left\{ \eta_{t+1}^S \lambda 18_{t+1} \right\} = 0$$
(2.97)

$$\frac{\partial \mathcal{L}}{\partial v_{f,t}} = \lambda 6_t - \frac{\lambda 8_t \left(\frac{\exp\left(\frac{v_{f,t}}{sd}\right)}{1+\exp\left(\frac{v_{f,t}}{sd}\right)} + \frac{1}{1+\exp\left(\frac{v_{f,t}}{sd}\right)} - \frac{v_{f,t} \exp\left(\frac{v_{f,t}}{sd}\right)}{sd\left(1+\exp\left(\frac{v_{f,t}}{sd}\right)\right)^2} - 1\right)}{\phi_t - 1} + \frac{\exp\left(\frac{(-v_{f,t})}{sd}\right)\lambda 4_t}{sd\left(1+\exp\left(\frac{(-v_{f,t})}{sd}\right)\right)^2} = 0$$
(2.98)

$$\frac{\partial \mathcal{L}}{\partial v_{h,t}^{L}} = \lambda 13_{t}$$

$$+ \frac{\lambda 16_{t} \left( \frac{\exp\left(\frac{v_{h,t}^{L}}{sd}\right)}{1+\exp\left(\frac{v_{h,t}^{L}}{sd}\right)} + \frac{1}{1+\exp\left(\frac{v_{h,t}^{L}}{sd}\right)} \right)}{\eta_{t}^{L}}$$

$$- \frac{\lambda 16_{t} \left( \frac{v_{h,t}^{L} \exp\left(\frac{v_{h,t}^{L}}{sd}\right)}{sd\left(1+\exp\left(\frac{v_{h,t}^{L}}{sd}\right)\right)^{2}} + 1 \right)}{\eta_{t}^{L}}$$

$$- \frac{\exp\left(\frac{\left(-v_{h,t}^{L}\right)}{sd}\right)\lambda 14_{t}}{sd\left(1+\exp\left(\frac{\left(-v_{h,t}^{L}\right)}{sd}\right)\right)^{2}} = 0 \qquad (2.99)$$

$$\frac{\partial \mathbf{L}}{\partial v_{h,t}^{S}} = \lambda 11_{t}$$

$$+ \frac{\lambda 15_{t} \left( \frac{\exp\left(\frac{v_{h,t}^{S}}{sd}\right)}{1+\exp\left(\frac{v_{h,t}^{S}}{sd}\right)} + \frac{1}{1+\exp\left(\frac{v_{h,t}^{S}}{sd}\right)} \right)}{\eta_{t}^{S}}$$

$$- \frac{\lambda 15_{t} \left( \frac{v_{h,t}^{S} \exp\left(\frac{v_{h,t}^{S}}{sd}\right)}{sd \left(1+\exp\left(\frac{v_{h,t}^{S}}{sd}\right)\right)^{2}} - 1 \right)}{\eta_{t}^{S}}$$

$$- \frac{\exp\left(\frac{\left(-v_{h,t}^{S}\right)}{sd}\right) \lambda 12_{t}}{sd \left(1+\exp\left(\frac{\left(-v_{h,t}^{S}\right)}{sd}\right)\right)^{2}} = 0 \qquad (2.100)$$

$$\frac{\partial \mathbf{L}}{\partial w_t} = \lambda \mathbf{13}_t + \lambda \mathbf{11}_t + \lambda \mathbf{6}_t + \lambda \mathbf{3}_t - \lambda \mathbf{7}_t \ (\phi_t - 1) - (\phi_t - 1) \ \lambda \mathbf{9}_t \quad (2.101) \\ - (\phi_t - 1) \ \lambda \mathbf{10}_t = \mathbf{0}$$

$$\frac{\partial \mathbf{L}}{\partial \Xi_t^I} = \lambda 8_t + n_t^I \lambda 27_t - \lambda 7_t (\phi_t - 1) - (\phi_t - 1) \lambda 9_t - (\phi_t - 1) \lambda 10_t = 0$$
(2.102)

$$\frac{\partial \mathcal{L}}{\partial \Xi_t^L} = \lambda 16_t + n_t^L \,\lambda 27_t = 0 \tag{2.103}$$

$$\frac{\partial \mathcal{L}}{\partial \Xi_t^S} = \lambda 15_t + n_t^S \,\lambda 27_t = 0 \tag{2.104}$$

$$\frac{\partial \mathbf{L}}{\partial y_t} = \lambda 26_t + \lambda 27_t \left( \frac{\psi (\pi_t - 1)^2}{2} - 1 \right) 
- \frac{(\pi_t - 1) \psi \pi_t c_{t-1}^{\sigma} \lambda 2_{t-1}}{y_{t-1} c_t^{\sigma}} 
+ \mathbf{E}_t \left\{ \frac{(\pi_{t+1} - 1) y_{t+1} \psi \pi_{t+1} c_t^{\sigma} \beta \lambda 2_t}{c_{t+1}^{\sigma} y_t^2} \right\} = 0.$$
(2.105)

Note that  $\lambda j$ ,  $j = \{1, 2, 6, 7, 9, 10, 11, 13\}$  are state variables. From a timeless perspective, the  $\lambda j_{-1}$  take on their steady state values.

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# Chapter 3

# The role of bank financing costs in the transmission of monetary policy

Joint work with Johannes Fritz

# 3.1 Introduction

In this paper, we present a macroeconomic model with a variable spread between bank financing costs and the central bank's policy rate. The sign and size of this spread depends on balance sheet conditions in the banking sector. The balance sheet conditions, in turn, can be influenced by deteriorating bank net worth arising from loan losses in economic downturns. This setup allows us to study scenarios in which the transmission of monetary policy is impaired in the sense that endogenous policy rate movements are not fully passed through to bank financing costs and hence credit costs for the real economy.

The motivation for modeling such a scenario comes from recent developments in the euro area. European Central Bank (ECB) President Mario Draghi has repeatedly expressed his concern about "[...] the proper transmission of [the] policy stance to the real economy [...]" (ECB, 2012b). In his view, "[o]ne reason [...] is that the cost of bank credit to firms is inevitably linked to the cost of market funding for the banks themselves. If there are fears about potential destructive scenarios, the cost of funding for banks can be affected [...]. It is that distortion in financing costs that hinders the smooth functioning of credit markets and the transmission of monetary policy" (ECB, 2012a).

According to this view, a model that seeks to reproduce the possibility of an impaired transmission mechanism in this vein has to incorporate banks whose financing costs can deviate from the monetary policy rate. Conventional macroeconomic models are not able to do so, either because there is no financial sector or because the financial sector does not involve autonomous financial intermediaries. In the latter case, bank financing costs always equal the risk-less policy rate, for the relation between banks and their funding sources is frictionless. The transmission of monetary policy thus cannot be disrupted in the banking sector by assumption. This is the case, for example, in the financial accelerator model of Bernanke et al. (1999) (BGG hereafter) — one of the most widely used frameworks for credit frictions in the literature — where credit frictions only have consequences for firms but not for financial intermediaries.

In our model, we introduce a financial friction that makes depositors demand a compensation for the perceived riskiness of banks. The perceived riskiness of banks is captured by the banking sector's leverage ratio, that is, the ratio of bank assets to bank net worth. Accordingly, aggregate bank financing costs fluctuate around the risk-less rate depending on the leverage ratio of the banking system. By incorporating risk concerns into bank financing conditions, our model allows for a meaningful analysis of disruptions in the monetary policy transmission process in the banking sector. Another feature of our model is that we allow for endogenous loan losses on behalf of banks. This provides a mechanism for connecting bank balance sheets to aggregate economic conditions.

We focus on scenarios where the monetary authority reacts endogenously to deteriorating economic conditions. In contrast, an exogenous policy rate cut induces better economic conditions. This implies different consequences of endogenous and exogenous policy rate movements for the banking sector's leverage ratio and hence the monetary policy transmission process.

The model suggests that leverage-sensitive bank financing costs can be an effective disturbance for the transmission of monetary policy. If the central bank cuts interest rates in a recession, the contemporaneous deterioration of banks' balance sheets — and hence the increasing risk component in bank financing costs — has the potential to either render the pass-through of policy rates to bank financing costs less effective, or even counteract policy rate movements. This, in turn, tightens lending conditions in the real sector, thus reducing investment and output beyond the reductions that would occur without leverage-sensitive bank financing costs. However, for shocks originating in the real sector like exogenous spending shocks, preference shocks and shocks to the riskiness of intermediate goods producers who engage in credit contracts with banks — we find that the effects on aggregate outcomes are small. We can explain this finding with the small amount of endogenously generated loan losses, implying that bank net worth deteriorates only little after real sector shocks. In contrast, a shock that depletes bank net worth directly has a sizable impact on aggregate outcomes. In light of the recent developments in several European banking sectors, such a shock seems empirically relevant.

The potential disruption of the monetary policy transmission mechanism in the presence of leverage-sensitive bank financing costs raises the question about the adequate reaction of the central bank. Therefore, we consider a policy rule that allows the central bank to react to bank financing conditions and examine whether an optimal policy rule assigns a non-zero weight on the associated reaction coefficient. We find that it is optimal for the central bank to negatively respond to bank financing conditions. That is, if bank financing conditions tighten, the central bank should lower the policy rate. We also find that a strong reaction to inflation is the most important component of monetary policy. This result is in line with the literature on optimal monetary policy which assigns a superior role to inflation stabilization.

We round off our analysis with some empirical evidence for two of our model's main mechanisms using aggregate euro area data: First, our estimations confirm a positive relationship between bank financing costs and lending rates to the real economy. Second, we find that bank financing costs are positively related to the bank leverage ratio.

The rest of this paper is organized as follows. The next section provides a verbal outline of the model and sets it in the context of the related literature. A detailed description of the model follows in section 3.3. Section 3.4 presents the calibration strategy and section 3.5 describes the model results. Section 3.6 deals with the implications of leverage-sensitive bank-financing costs for the conduct of monetary policy. Section 3.7 presents empirical evidence for two main implications of our model. Section 3.8 concludes and discusses avenues for future research.

# 3.2 Descriptive model outline and related literature

This paper presents a New Keynesian model with leverage-sensitive bank financing costs. To reflect the research object — the influence of bank financing costs on the transmission of monetary policy — our model requires a credit-constrained private sector, a credit-constrained banking sector and depositors as the ultimate source of funds. The basis of our work is the model in BGG. In line with their setup, we assume that intermediate goods producers need to borrow from banks to realize their investment projects. To this end, intermediate goods producers and banks engage in a credit contract. This credit contract is characterized by a financial friction. The intermediate goods sector is the only client of the banking sector. We thus do not differentiate between different asset classes for banks (e.g. loans, bonds, equities etc.) or distinguish different loan varieties (e.g. non-financial company loans, consumer loans, mortgages etc.). Furthermore, as in BGG the only depositor in our setup is the household. We thus abstract from other existing bank funding sources such as interbank lending or debt instruments.

We extend BGG's model in two ways: First, we model banks as entities that are independent from households. In particular, banks accumulate their own net worth and engage in a credit contract with households which determines the terms of households' deposits. In contrast, BGG assume that banks are merely a veil; they are ultimately a part of the household and play no active role in the economy. Establishing an independent banking sector allows us to introduce a BGG-type friction between depositors and banks into the model, besides the canonical friction between banks and intermediate goods producers. This additional friction is vital for the central mechanism of the model, namely the deviation of bank financing costs from the risk-free policy rate. Our conjecture is that bank financing costs are a function of the perceived riskiness of the banking system. This riskiness is captured by the bank leverage ratio.

Our second deviation from the original BGG model concerns the credit contract between the banks and the intermediate goods producers. BGG assume that the intermediate goods sector fully insures the banking sector against any loss from loan default. The insurance is modeled as a state contingent credit contract between the bank and the intermediate goods sector. This contract is signed in the period of loan origination and contains a lending rate for each possible state of the economy in the following period when the loan is repaid. Thus, for example, in an economic downturn with higher debt default, the surviving intermediate goods producers pay a higher lending rate on their pre-existing loans in order to cover the bankruptcy costs of their former colleagues. In our model, we maintain this insurance structure in the credit contract between banks and depositors. In the spirit of a deposit insurance scheme, banks guarantee the risk-free rate to the households through the described state contingent contract. However, contrary to BGG, we do not assume this insurance scheme to hold between banks and the intermediate goods sector. In our model, banks and the intermediate goods producers agree on a non-state contingent lending rate in the period of loan origination. The consequence of this change is that the bank is now exposed to aggregate risk. If its assumptions about future economic conditions turn out to be too optimistic, part of its loan book turns sour. This loss transmits into a reduction of bank net worth and an increase in the bank leverage ratio. Given that our model features leverage-sensitive depositors, bank financing costs will increase.

In sum, this setup provides the basis for the sought feedback between an economic downturn, bank balance sheet and lending rates to the real sector. The rise in bank financing costs is endogenous.

Besides households, banks, and intermediate goods producers, our model also contains capital producers and retailers. While the former three are autonomous, capital goods producers and retailers belong to the household, and hence their proceeds are rebated to the household which will be reflected in the household's budget constraint. Since banks and intermediate goods producers are autonomous, they consume in their own right. The way in which this happens is that they accumulate net worth in the lending business which is consumed when they exit the market with a constant probability. Put differently, accumulated net worth disappears with the accumulator. As will be explained in sections 3.3.2 and 3.3.3, it is assumed that exiting banks and intermediate goods producers are replaced by an equal measure of new agents so that the total number of banks and intermediate goods producers stays constant.

Our model is not the first that emphasizes the role of banks in the economy. One strand of the literature that features a meaningful banking sector is built on the model of Holmstrom and Tirole (1997).<sup>1</sup> The key difference between this strand of the literature and our work lies in the nature of the financial friction and resulting implications for bank financing costs. Following Holmstrom and Tirole (1997), the presence of moral hazard on the side of the borrower reduces their capacity to take on debt. However, this constraint only works along the borrowing quantity, but not along the cost of credit. While lenders will adjust the loan volume, the lending rate is always equal to the risk-free rate. This specification is thus unsuitable for our purposes as bank financing costs remain insensitive to the leverage ratio of the bank.

Other authors extended the work of BGG with a banking sector. The model closest to ours is the one in Hirakata et al. (2009) (HSU hereafter).<sup>2</sup> In what they refer to as "chained credit contracts", HSU feature a model where banks play a dual role as both lenders and debtors. Both relationships are subject to a financial frictions built on the costly state verification mechanism put forth in BGG. Contrary to our model, HSU assume that banks are the monopolistic supplier of loans to a group of entrepreneurs. Given its status as a monopolist, banks are at the core of the HSU model and solve the maximization problem for all agents in the financial market.

<sup>&</sup>lt;sup>1</sup>See e.g. Aikman and Paustian (2006), Chen (2001), Gertler and Karadi (2011), Gertler and Kiyotaki (2010), or Meh and Moran (2004, 2010) for single-friction versions.

 $<sup>^2 \</sup>rm See$  also Hirakata et al. (2013), Hirakata et al. (2011) and Ueda (2012) for applications of this model. See Badarau and Levieuge (2011) for a similar two-country model.

The most important difference between our model and HSU concerns the nature of the debt contract between banks and intermediate goods producers. HSU maintain the state contingent nature of all credit contracts. In their model, entrepreneurs thus insure banks against all aggregate risk. However, the absence of bank losses implies that the variation of bank funding costs is only a function of final loan demand, rather than the health of the bank balance sheet. Hence, the model is incapable of generating loan volume reductions or lending rate increases due to decreasing bank net worth. In the HSU model, the causality goes into the opposite direction. There, a decreased loan demand reduces bank revenues and thus bank net worth. Given their full insurance through the entrepreneurs, it is impossible in this setup to generate bank losses endogenously.

The specification of non-state contingent interest rates in the credit contract between banks and the producing sector used in this paper has been proposed by Benes and Kumhof (2011) (BK hereafter). In their model, lending rates between banks and the real economy are set in the contracting period and cannot be altered thereafter. Given this deviation, banks still operate under a zero-profit condition ex ante. However, once the aggregate state is realized, banks make a profit or a loss ex post. Due to this alteration, it is possible that banks take a loss that impairs their ability to repay depositors. In order to induce flows from risk-averse households, one has to find a safety net to credibly ensure that households will be made whole in the next period.

The way BK approach this issue is to impose capital adequacy requirements for banks. If banks breach the capital adequacy regulation, they have to pay a penalty. The threat of the penalty entices banks to hold an equity buffer in excess of the statutory minimum.<sup>3</sup> BK choose the amount of the penalty such that the buffer is big enough to ensure that banks on average never deplete their net worth to an extent that

 $<sup>^{3}</sup>$ See Kollmann et al. (2011) for a similar setup in an two-country model with one global bank and a single financial friction.

would endanger the repayment of deposits. Thus, there is no need for depositors to punish their bank for risky behavior.

The implementation of capital adequacy rules is unsuitable for our model as bank financing costs are not sensitive to the leverage ratio in this scenario. Contrary to BK, we thus introduce a BGG-type financial contract between households and banks subject to costly state verification. As the deposit rate is state contingent, this contract can be interpreted as imposing a deposit insurance scheme: Banks acknowledge that a share of their sector may fail to deliver the promised interest rate and thus go bankrupt. However, non-bankrupt banks will adjust interest payments ex post in a way that the average interest earned on all deposits is equivalent to the risk-free rate. The benefit of imposing the canonical BGG credit contract for the relationship between depositors and banks is that it implies deposit rates which reflect the leverage of banks.

## 3.3 Model

#### 3.3.1 Households

Households supply labor  $h_t^H$  to intermediate goods producers. Households maximize the expected sum of discounted period utility functions subject to their real budget constraint:

$$\max_{c_t, h_t^H, \frac{D_t}{P_t}} E_t \left\{ \sum_{t=0}^{\infty} \beta^t \left[ \zeta_t^c \log\left(c_t\right) - \psi \frac{h_t^{1+\nu}}{1+\nu} \right] \right\}$$
  
s.t.  $(\lambda_t) \quad c_t + \frac{D_t}{P_t} = \frac{W_t^H}{P_t} h_t^H - g_t + \frac{R_{t-1}}{\pi_t} \frac{D_{t-1}}{P_{t-1}} + \frac{\Pi_t^{cap}}{P_t} + \frac{\Pi_t^{ret}}{P_t}$ 

where  $c_t$  is consumption,  $\beta$  is the household's discount factor,  $\zeta_t^c$  is a preference shock,  $\psi$  is a scaling parameter for the disutility of labor and

 $\nu$  is the elasticity of labor supply. The right hand side of the budget constraint represents household income.  $P_t$  is the price level,  $W_t^H$  is the nominal wage,  $g_t$  are lump sum real taxes that the government consumes and  $\pi_t = \frac{P_t}{P_{t-1}}$  is the inflation rate. Being the owner of both the retailer and the capital producer, the household is the final recipient of their nominal profits  $\Pi_t^{cap}$  and  $\Pi_t^{ret}$ . Moreover, the household receives the nominal gross risk-free interest rate  $R_{t-1}$  for nominal deposits  $D_{t-1}$ made in the prior period. Deposits are the only savings vehicle available to the households in this economy. For expositional clarity, we assume that deposits are administered by designated household members called investors. Investors deposit household savings at banks under the condition to generate the nominal gross risk-free interest rate. The left hand side of the budget constraint shows that households can use their income for consuming retail goods or making deposits. Denoting the Lagrange multiplier as  $\lambda_t$ , the equilibrium conditions are

$$\zeta_t^c \frac{1}{c_t} = \lambda_t \tag{3.1}$$

$$\psi \left( h_t^H \right)^{\nu} = \lambda_t \frac{W_t^H}{P_t} \tag{3.2}$$

$$\lambda_t = \beta R_t E_t \left\{ \frac{\lambda_{t+1}}{\pi_{t+1}} \right\}$$
(3.3)

#### 3.3.2 Banks

Banks take deposits from investors. Banks finance nominal loans  $L_t$  to intermediate goods producers using nominal deposits and nominal bank net worth  $N_t^B$ .<sup>4</sup> The bank balance sheet is thus

$$L_t = N_t^B + D_t. aga{3.4}$$

<sup>&</sup>lt;sup>4</sup>The banking sector is composed of a continuum of banks. However, we present the model relations in aggregate terms right away. As Fernández-Villaverde (2010) explains, this is possible because the contract relations are such that all banks choose to charge the same deposit rate and take the same amount of deposits irrespective of their individual net worth. After presenting the banks' maximization problem, footnote 6 goes into some detail about this point.

To avoid confusion, it is useful to discuss the timing convention of this model explicitly at this point. Note that all loans  $L_t$  originated in period t are used to finance capital purchases  $k_t$  in period t. As is common in the literature, capital is used in production only in period t+1. We denote all variables in this model with a time subscript that identifies their settlement period, that is, the period in which their final magnitude has been settled. As neither loan nor capital volumes can be adjusted after the transaction at the end of period t, we denote them  $k_t$  and  $L_t$ . However, as the return to a unit of bank loans and a unit of capital is only known after the realization of aggregate uncertainty at the beginning of the next period, we denote these returns  $R_{t+1}^B$  and  $R_{t+1}^k$ , respectively.

Following BGG, we assume that households are risk-averse and banks are risk-neutral. In order to entice deposit flows, banks have to guarantee investors a return equal to the risk-free rate  $R_t$ . Banks make this guarantee by proposing a state contingent debt contract that includes a menu of ex post nominal gross deposit rates  $R_{t+1}^{D}$ .<sup>5</sup> On this menu, one deposit rate is chosen for every possible state of the economy in the next period. The chosen deposit rate ensures a return equal to the current risk-free rate for households. Banks thus fully ensure households against all risk in the economy.

In particular, banks insure the depositors against aggregate and bankspecific risk. For reasons that will be explained below, the return on banking  $R_{t+1}^B$ , that is, the gross nominal interest rate earned on a unit of loans, fluctuates with aggregate risk. Furthermore, bank-specific risk arises through a partial default of the banking sector in every state of the economy. The share of defaulting banks fluctuates with aggregate risk, but is positive in all states of the economy.

<sup>&</sup>lt;sup>5</sup>Note that while the menu of possible deposit rates is contracted in period t, the final deposit rate is only settled after aggregate uncertainty vanishes at the beginning of the following period. In line with the rationale of the timing convention explained above, we thus denote the return on deposits made in period t with  $R_{t+1}^D$ .

We introduce this bank-specific risk by transplanting the setup of BGG into the relationship between investors and banks. In our model, banks receive a periodic i.i.d. draw of idiosyncratic productivity  $\chi_t$ . That is, the banks' total gross revenue from lending is  $\chi_{t+1}R_{t+1}^BL_t$ . The idiosyncratic productivity is log-normally distributed with mean 1 and variance  $\sigma_{\chi}^2$ . The individual productivity draw is private information to the bank.

After idiosyncratic productivity has been drawn and aggregate uncertainty is realized, banks decide whether to default on their depositors or not. Banks that receive a draw that does not allow them to repay their depositors in full have no choice but to declare bankruptcy immediately. Declaring default demands handing over all assets to the depositors. Note, however, that as long as the idiosyncratic productivity of the defaulting bank is private information, the defaulting bank may in principle transfer only a fraction of their assets to the depositors. Banks that received a draw that allows them to honor their debts in full also have the choice to declare default and transfer a chosen volume of assets to the depositors.

A setup introduced by Townsend (1979) reduces the default choice to an intuitive outcome. First, depositors can learn the idiosyncratic productivity — and thus the total revenue — of the defaulted bank by paying monitoring costs corresponding to a proportion  $\mu^B$  of the bank's assets. Second, the depositor of the defaulted bank may repossess all assets found in the monitoring process. A monitored bank is thus left with zero assets, while the monitoring depositor receives the value of repossessed assets minus monitoring costs. As established by Townsend (1979), the outcome of these arrangements is that only insolvent banks declare default and depositors only monitor defaulted banks. Hence, the cut-off value  $\bar{\chi}_{t+1}$  for the idiosyncratic productivity below which banks declare default coincides with the value for  $\chi_{t+1}$  that just generates sufficient revenue to repay the depositors and is determined by

$$\bar{\chi}_{t+1} R^B_{t+1} L_t = R^D_{t+1} D_t.$$
(3.5)

Positive bank default reflects on the deposit rates agreed upon in the debt contract between investor and bank. When the contract is signed in period t, the realized value of the return on banking is still unknown as it is a function of the aggregate state of the economy in period t+1. To ensure sufficient deposit revenues, banks propose a contract with a menu of deposit rates that satisfy the investors' participation constraint

$$R_{t}D_{t} = [1 - F_{\chi}(\bar{\chi}_{t+1})]R_{t+1}^{D}D_{t} + (1 - \mu^{B})F_{\chi}(\bar{\chi}_{t+1})\int_{0}^{\bar{\chi}_{t+1}}\frac{\chi_{t+1}f_{\chi}(\chi_{t+1})}{F_{\chi}(\bar{\chi}_{t+1})}d\chi_{t+1}R_{t+1}^{B}L_{t},$$
(3.6)

where  $f_{\chi}$  and  $F_{\chi}$  are the probability and cumulative density function, respectively, of  $\chi$ . The left hand side of equation (3.6) represents the payoff required by investors. The first term on the right hand side is the total interest payment received from the fraction  $1-F_{\chi}(\bar{\chi}_{t+1})$  of solvent banks. The second term on the right hand side is the repossession value from the fraction  $F_{\chi}(\bar{\chi}_{t+1})$  of banks that went bankrupt, less the monitoring costs. For this equation to hold ex post, banks will choose the ex post deposit rate  $R_{t+1}^D$  after aggregate uncertainty and hence the return on banking — is realized in period t+1. One can rationalize such a contract as a deposit insurance scheme that shields bank creditors from bank default.

Before we characterize the optimization problem of the banking sector, it is useful to introduce additional notation. Let  $1-\Psi(\bar{\chi}_{t+1})$  denote the share of total bank revenue that remains in the banking sector after all deposits have been repaid. This share is constructed as one minus the share of bank revenues spent on depositor repayment or lost in bank default. Furthermore, let  $M(\bar{\chi}_{t+1})$  be the share of total bank assets belonging to defaulted banks. Then,

$$1 - \Psi(\bar{\chi}_{t+1}) = 1 - \left[ \bar{\chi}_{t+1} \int_{\bar{\chi}_{t+1}}^{\infty} f(\chi) \, d\chi + \int_{0}^{\bar{\chi}_{t+1}} \chi f(\chi) \, d\chi \right], (3.7)$$

$$M(\bar{\chi}_{t+1}) = \int_{0}^{\chi_{t+1}} \chi f(\chi) \, d\chi.$$
 (3.8)

Recall that the investor loses a share  $\mu^B$  of repossessed bank assets due to monitoring activities. Using the above definitions, the net proceeds of the investor are thus given by  $\left[\Psi(\bar{\chi}_{t+1}) - \mu^B M(\bar{\chi}_{t+1})\right] R^B_{t+1} L_t$ . In turn, the share of aggregate bank returns that stays in the banking sector can be expressed as  $\left[1 - \Psi(\bar{\chi}_{t+1})\right] R^B_{t+1} L_t$ .

Using (3.4), (3.5), (3.7) and (3.8), the investors' participation constraint (3.6) can be expressed as

$$\left[\Psi\left(\bar{\chi}_{t+1}\right) - \mu^{B}M\left(\bar{\chi}_{t+1}\right)\right]R_{t+1}^{B}L_{t} = R_{t}\left(L_{t} - N_{t}^{B}\right).$$
 (3.9)

Banks maximize their expected share of the aggregate return on banking subject to the investors' participation constraint:

$$\max_{\bar{\chi}_{t+1},L_t} E_t \left\{ \left[ 1 - \Psi \left( \bar{\chi}_{t+1} \right) \right] R_{t+1}^B \right\} L_t$$
  
s.t.  $\left( \Upsilon_t^B \right) \quad \left[ \Psi \left( \bar{\chi}_{t+1} \right) - \mu^B M \left( \bar{\chi}_{t+1} \right) \right] R_{t+1}^B L_t = R_t \left( L_t - N_t^B \right).$ 

The first-order conditions for this problem are

$$0 = \Psi'(\bar{\chi}_{t+1}) - \Upsilon_t^B \left[ \Psi'(\bar{\chi}_{t+1}) - \mu^B M'(\bar{\chi}_{t+1}) \right], \quad (3.10)$$
  

$$0 = E_t \left\{ \frac{R_{t+1}^B}{R_t} \left[ 1 - \Psi(\bar{\chi}_{t+1}) \right] + \Upsilon_t^B \left[ \frac{R_{t+1}^B}{R_t} \left[ \Psi(\bar{\chi}_{t+1}) - \mu^B M(\bar{\chi}_{t+1}) \right] - 1 \right] \right\}, \quad (3.11)$$

and the participation constraint (3.9).<sup>6</sup>

We conclude the description of the banking sector with the evolution of bank net worth. The main source of bank net worth is end-of-period equity  $V_t^B$ , generated from lending funds to the intermediate goods producers:

$$V_t^B = [1 - \Psi(\bar{\chi}_t)] R_t^B L_{t-1}$$

$$= R_t^B L_{t-1} - R_{t-1} (L_{t-1} - N_{t-1}^B) - \mu^B M(\bar{\chi}_t) R_t^B L_{t-1}.$$
(3.12)

The first term on the right-hand side after the second equality represents total revenues of the banking sector. The second and third term are the value of the total debt repayment to depositors and the value lost in bankruptcy, respectively.

To circumvent scenarios where banks accumulate enough net worth to be self-financing, we follow BGG and assume that banks survive a period only with probability  $\gamma^B < 1.^7$  Banks that do not survive the period consume all their equity. Banks' real consumption  $c_t^B$  is thus represented by

$$c_t^B = (1 - \gamma^B) \frac{V_t^B}{P_t}.$$
(3.13)

<sup>6</sup>Note that we can express equation (3.9) as

$$\left[\Psi\left(\bar{\chi}_{t+1}\right) - \mu^{B}M\left(\bar{\chi}_{t}\right)\right]\frac{R_{t+1}^{B}}{R_{t}} = \frac{(\kappa_{t}-1)}{\kappa_{t}}$$

where  $\kappa_t = \frac{L_t}{N_t^B}$  and  $(\kappa - 1) = \frac{L_t - N_t^B}{N_t^B}$ . Thus, banks will have the same leverage ratio  $\kappa_t$  irrespective of their level of net worth and the maximization problem of banks can be written in terms of the leverage ratio. We apply the same type of argument later on for the intermediate goods producers and present their sector in aggregate terms right away as well.

<sup>&</sup>lt;sup>7</sup>In the original BGG model, this assumption is made only with respect to the real sector. See Gertler and Karadi (2011) or Gertler and Kiyotaki (2010) for models that also feature a constant exit rate for banks. One may rationalize such an assumption as banking sector that is myopic relative to the infinitely-lived households.

Exiting banks are replaced in equal numbers by new banks. As explained by BGG, a second source of net worth is required for technical reasons: It is necessary that banks start off their first lending activity with own net worth. In order to provide them with start-up capital, banks inelastically supply one unit of labor  $h^B$  for the production of intermediate goods.<sup>8</sup> Thus, besides their profits from lending activities, banks generate a small nominal wage income  $W_t^B$ . The determination of this wage is outlined in section 3.3.3. As will be explained in the calibration section 3.4, we choose the share of labor provided by banks as well as the magnitude of the associated wage payments to be negligible in order to avoid a significant effect on the model dynamics.

The evolution of the banking sector's end-of-period nominal net worth  $N_t^B$  is the sum of surviving banks' equity plus the wage income:

$$N_t^B = \gamma^B V_t^B + W_t^B + \epsilon_t^{N^B}.$$
 (3.14)

Note that the net worth equation is augmented by  $\epsilon_t^{N^B}$ , an exogenous net worth shock.<sup>9</sup> We will use the exogenous net worth shock later to illustrate the consequences of a direct impact on bank net worth.

#### 3.3.3 Intermediate goods producers

Real output is generated by intermediate goods producers. They are also the ultimate borrowers in our model.  $^{10}\,$ 

Intermediate goods producers combine labor  $h_t$  and capital  $k_t$  to produce intermediate goods  $x_t$ . They purchase capital from capital produc-

<sup>&</sup>lt;sup>8</sup>For simplicity, we assume that all banks, not only new ones, supply labor.

<sup>&</sup>lt;sup>9</sup>This specification resembles the ones in Gilchrist and Leahy (2002) and Hirakata et al. (2013). See e.g. Christiano et al. (2008) for an exogenous decrease in net worth induced by a shock on the survival rate  $\gamma$ .

<sup>&</sup>lt;sup>10</sup>There is a continuum of intermediate goods producers. However, we present the relations for intermediate goods producers in aggregate terms right away. For intermediate goods producers, we apply the same type of argument as in footnote 6 for banks.

ers at a nominal price  $Q_t$  and use it for production in the next period. The production technology takes the usual Cobb-Douglas form:

$$x_t = a_t k_t^{\alpha} h_t^{1-\alpha}. aga{3.15}$$

 $a_t$  is aggregate productivity and follows the process

$$\frac{a_t}{a} = \left(\frac{a_{t-1}}{a}\right)^{\rho_a} e^{\epsilon_{a,t}}, \qquad (3.16)$$

where a denotes steady state aggregate productivity,  $\rho_a$  is a persistence parameter, and  $\epsilon_{a,t} \sim N(0, \sigma_{\epsilon,a}^2)$  is an aggregate productivity shock.

Intermediate goods producers sell their output to retail firms in a competitive environment at the nominal price  $P_{x,t}$ . Accordingly, real marginal costs in the economy are  $p_{x,t} = \frac{P_{x,t}}{P_t}$ . Thus, the real marginal return to capital from producing intermediate goods in period t is  $p_{x,t}\alpha \frac{x_t}{k_{t-1}}$ . Taking into account that capital used in t + 1 is bought at price  $Q_t$  and undepreciated capital is sold after production at price  $Q_{t+1}$ , we can express the expost nominal gross return to capital purchased in period t as

$$R_{t+1}^{k} = \pi_{t+1} \frac{p_{x,t+1} \alpha \frac{x_{t+1}}{k_t} + (1-\delta) q_{t+1}}{q_t}, \qquad (3.17)$$

where  $q_t = \frac{Q_t}{P_t}$  is the real price of capital and  $\delta$  is the depreciation rate of capital.

To finance their capital purchases, intermediate goods producers use their end-of-period nominal net worth  $N_t^I$  and nominal loans  $L_t$  from banks. The balance sheet equation of intermediate goods producers thus reads

$$Q_t k_t = N_t^I + L_t. (3.18)$$

Intermediate goods producers are subject to aggregate and idiosyncratic uncertainty. Aggregate uncertainty enters the intermediate goods sector via the aggregate productivity shock. Idiosyncratic uncertainty enters through a periodic i.i.d. draw of idiosyncratic productivity,  $\omega_t$ , which transforms the return to capital into  $\omega_{t+1}R_{t+1}^k$  and is private information to intermediate goods producers.  $\omega_t$  is log-normally distributed with mean 1 and variance  $\sigma_{\omega,t}^2$ . We assume that the standard deviation  $\sigma_{\omega,t}$  follows the stochastic process

$$\frac{\sigma_{\omega,t+1}}{\sigma_{\omega}} = \left(\frac{\sigma_{\omega,t}}{\sigma_{\omega}}\right)^{\rho_{\sigma_{\omega}}} e^{\epsilon_{\sigma_{\omega},t+1}}, \qquad (3.19)$$

where  $\sigma_{\omega}$  is the steady state value of  $\sigma_{\omega,t}$  and  $\epsilon_{\sigma_{\omega},t+1} \sim N(0, \sigma_{\epsilon,\omega})$  is a shock to the riskiness of intermediate goods producers. The assumption of time-varying intermediate goods producer risk has been empirically validated by Christiano et al. (2013). Intuitively, they describe a risk shock as fluctuating degrees of uncertainty regarding the payoffs of business investments. When uncertainty is high, that is, the dispersion of  $\omega_t$  is increased, credit is only extended at a higher price and a lower volume. Having estimated their model using US data, Christiano et al. (2013) find that risk shocks are an important source of business cycle fluctuations.

The debt contract between banks and intermediate goods producers deviates from the debt contract between households and banks. Specifically, we do not assume that intermediate goods producers take on all the risk to insure banks. In our setup, banks and intermediate goods producers agree on a non-state contingent contract. That is, the lending rate for loans originated in period t is determined in the same period. We denote this non-state contingent nominal gross interest rate  $R_t^L$ . In line with the Townsend (1979)-framework discussed above, banks can monitor intermediate goods firms that declare default and seize the remaining assets  $\omega_{t+1}R_{t+1}^kQ_tk_t$  by paying monitoring costs corresponding to a fraction  $\mu^I$  of these assets. The cut-off value  $\omega_{t+1}$  for the idiosyncratic productivity below which intermediate goods producers declare default is thus determined by

$$\bar{\omega}_{t+1} R_{t+1}^k Q_t k_t = R_t^L L_t.$$
(3.20)

Note that the timing of the cut-off equation (3.20) is slightly different than that of the cut-off equation (3.5) in section 3.3.2 due to the non-state contingent nature of the lending rate.

Since banks in our model carry part of the aggregate risk, the return on banking  $R_{t+1}^B$  is uncertain at the time of loan origination. Denote the cumulative and the probability density function of  $\omega$  by  $F_{\omega}$  and  $f_{\omega}$ , respectively. A share  $1 - F_{\omega}(\bar{\omega}_{t+1})$  of intermediate goods producers pay back their loans plus interest, while a share  $F_{\omega}(\bar{\omega}_{t+1})$  cannot pay back and the bank repossesses a fraction  $1 - \mu^I$  of the remaining assets. Thus,

$$E_{t} \{ R_{t+1}^{B} \} L_{t} = E_{t} \left\{ [1 - F_{\omega} (\bar{\omega}_{t+1})] R_{t}^{L} L_{t} + (1 - \mu^{I}) F_{\omega} (\bar{\omega}_{t+1}) \right.$$
$$\cdot \int_{0}^{\bar{\omega}_{t+1}} \frac{\omega_{t+1} f_{\omega} (\omega_{t+1})}{F_{\omega} (\bar{\omega}_{t+1})} d\omega R_{t+1}^{k} Q_{t} k_{t} \right\}.$$
(3.21)

The expected value of the return on banking will only be realized if the aggregate shock turns out as expected. If a negative aggregate shock hits the economy, banks suffer a revenue shortfall  $\Delta_t$  which can be expressed as

$$\Delta_t = \left[ E_{t-1} \left\{ R_t^B \right\} - R_t^B \right] L_{t-1}.$$
 (3.22)

Given that the banking sector guarantees a fixed repayment amount to depositors, a revenue shortfall directly depletes bank net worth  $N_t^B$ and thus affects bank leverage.

Before describing the maximization problem for intermediate goods producers, it is again useful to introduce additional notation. We define the share of intermediate goods producers' gross revenues kept by intermediate goods producers as  $1 - \Gamma(\bar{\omega}_{t+1})$ , where

$$\Gamma(\bar{\omega}_{t+1}) = \bar{\omega}_{t+1} \int_{\bar{\omega}_{t+1}}^{\infty} f_{\omega}(\omega_{t+1}) d\omega_{t+1} + \int_{0}^{\bar{\omega}_{t+1}} \omega_{t+1} f_{\omega}(\omega_{t+1}) d\omega_{t+1}.$$
(3.23)

Recall that only a fraction  $(1 - \mu^I)$  of the assets of bankrupt intermediate goods producers can be repossessed. Hence, the share of intermediate goods producers' gross revenue lost in bankruptcy is  $\mu^I G(\bar{\omega}_{t+1})$ with

$$G(\bar{\omega}_{t+1}) = \int_0^{\bar{\omega}_{t+1}} \omega_{t+1} f_\omega(\omega_{t+1}) \, d\omega_{t+1}, \qquad (3.24)$$

and the share of intermediate goods producers' gross revenue ceded to banks is  $\Gamma(\bar{\omega}_{t+1}) - \mu^I G(\bar{\omega}_{t+1})$ . Using (3.18), (3.20), (3.24), (3.23) in (3.21) yields

$$E_t \{ R_{t+1}^B \} (Q_t k_t - N_t^I) = E_t \{ [\Gamma(\bar{\omega}_{t+1}) - \mu^I G(\bar{\omega}_{t+1})] R_{t+1}^k \} Q_t k_t.$$
(3.25)

The optimization problem for intermediate goods producers is thus

$$\max_{k_t,\bar{\omega}_{t+1}} E_t \left\{ \begin{bmatrix} 1 - \Gamma\left(\bar{\omega}_{t+1}\right) \end{bmatrix} R_{t+1}^k \right\} Q_t k_t$$
  
s.t.  $\left(\Upsilon_t^I\right) = E_t \left\{ \begin{bmatrix} \Gamma\left(\bar{\omega}_{t+1}\right) - \mu^I G\left(\bar{\omega}_{t+1}\right) \end{bmatrix} R_{t+1}^k \right\} Q_t k_t$   
 $= E_t \left\{ R_{t+1}^B \right\} \left( Q_t k_t - N_t^I \right).$ 

The first-order conditions are

$$0 = \Gamma'(\bar{\omega}_{t+1}) - \Upsilon_{t}^{I} \left[ \Gamma'(\bar{\omega}_{t+1}) - \mu^{I} G'(\bar{\omega}_{t+1}) \right], \quad (3.26)$$
  

$$0 = E_{t} \left\{ \frac{R_{t+1}^{k}}{R_{t+1}^{B}} \left[ 1 - \Gamma(\bar{\omega}_{t+1}) \right] + \Upsilon_{t}^{I} \left[ \frac{R_{t+1}^{k}}{R_{t+1}^{B}} \left[ \Gamma(\bar{\omega}_{t+1}) - \mu^{I} G(\bar{\omega}_{t+1}) \right] - 1 \right] \right\}, \quad (3.27)$$

and (3.25).

Intermediate goods producers' end-of-period net worth  $N_t^I$  stems from two sources. The main source is end-of-period equity  $V_t^I$ , generated from intermediate goods production:

$$V_t^I = [1 - \Gamma(\bar{\omega}_t)] R_t^k Q_{t-1} k_{t-1}$$

$$= R_t^k Q_{t-1} k_{t-1} - R_t^B (Q_{t-1} k_{t-1} - N_{t-1}^I) - \mu^I F_\omega(\bar{\omega}_t) E(\omega | \omega < \bar{\omega}_t) R_t^k Q_{t-1} k_{t-1}.$$
(3.28)

The first term on the right-hand side after the second equality represents total revenues of the intermediate goods sector. The second and third term are the value of the total debt repayment to banks and the value lost in bankruptcy, respectively.

For the reasons expressed above, intermediate goods producers survive a period only with probability  $\gamma^{I} < 1$ . Those that do not survive the period consume their equity. Thus, intermediate goods producers' real consumption  $c_{t}^{I}$  is represented by

$$c_t^I = \left(1 - \gamma^I\right) \frac{V_t^I}{P_t}.$$
(3.29)

Again, we assume that exiting intermediate goods producers are replaced in equal number with newly entering intermediate goods producers. The second source of net worth is necessary to equip new intermediate goods producers with start-up capital. For this reason, we assume that intermediate goods producers work at (other) intermediate goods producers with a constant labor supply  $h^I \equiv 1$  to generate a wage income  $W_t^I$ .<sup>11</sup> Thus, the law of motion for the end-of-period intermediate goods sector net worth is

$$N_t^I = \gamma^I V_t^I + W_t^I. aga{3.30}$$

<sup>&</sup>lt;sup>11</sup>For simplicity, we assume that all intermediate goods producers, not only new ones, supply labor.

The derivation of the wage payments concludes the description of the intermediate goods sector. Total labor input is given by

$$h_t = \left(h_t^H\right)^{1-\Omega^I - \Omega^B} \left(h_t^I\right)^{\Omega^I} \left(h_t^B\right)^{\Omega^B}, \qquad (3.31)$$

where  $\Omega^B$  and  $\Omega^I$  are the shares of bankers and intermediate goods producers in the total labor supply. Assuming a competitive labor market, the wages of the agents are equal to the marginal product of their labor services. Thus, the wage expressions are as follows:

$$\frac{W_t^H}{P_t} = (1-\alpha) \Omega p_{x,t} \frac{x_t}{h_t^H}$$
(3.32)

$$\frac{W_t^I}{P_t} = (1-\alpha) \Omega^I p_{x,t} \frac{x_t}{h^I}$$
(3.33)

$$\frac{W_t^B}{P_t} = (1-\alpha) \Omega^B p_{x,t} \frac{x_t}{h^B}.$$
(3.34)

#### 3.3.4 External finance premia

In the described setup, banks and intermediate goods producers cannot fully self-finance their desired quantity of assets. This implies that in equilibrium, the marginal return of an additional unit of assets is equal to the marginal cost of external finance.<sup>12</sup> The marginal cost of external

$$"E_t(R_{t+1}^k) = s\left(\frac{N_{t+1}^j}{Q_t K_{t-1}^j}\right) R_{t+1}, s(\cdot) < 0.$$
(3.9)

For an entrepreneur who is not fully self-financed, in equilibrium the return to capital will be equated to the marginal cost of external finance. Thus equation (3.9) expresses the equilibrium condition that the ratio s of the cost of external finance to the safe rate - which

<sup>&</sup>lt;sup>12</sup>We deduce this terminology from two passages in the handbook article by BGG. The first passage makes explicit reference to "the cost of external finance" and is taken from the model description. The second passage is part of the calibration section and, in our reading, implicitly equalizes  $E_t(R_{t+1}^k)$  with the term "cost of external finance". BGG introduce the term "cost of external finance" on page 1354, in the paragraph below their equation (3.9):

finance of the intermediate goods producer is thus given by  $E_t \{R_{t+1}^k\}$ and that of the bank by  $E_t \{R_{t+1}^B\}$ .

The marginal cost of external finance fluctuates with the leverage ratios of the borrowers. As in BGG, we can combine equations (3.9), (3.10) and (3.11) to obtain

$$E_t \left\{ R_{t+1}^B \right\} = \Phi^B \left( \frac{L_t}{N_t^B} \right) R_t, \qquad (3.35)$$

where  $\Phi^B$  is an increasing function in the banks' leverage ratio  $\frac{L_t}{N_t^B}$ . Similarly, equations (3.25), (3.26) and (3.27) can be combined to give

$$E_t \left\{ R_{t+1}^k \right\} = \Phi^I \left( \frac{Q_t k_t}{N_t^I} \right) E_t \left\{ R_{t+1}^B \right\}, \qquad (3.36)$$

From the first sentence of the above quotation, we deduce that the "cost of external finance" is written on the RHS of equation (3.9). In equilibrium, it is equal to the expected return on assets on the LHS of BGG's equation (3.9) which is the primary interpretation of  $E_t(R_{t+1}^k)$ . We interpret the second sentence of the above quotation as a re-arrangement of (3.9) into  $\frac{E_t(R_{t+1}^k)}{R_{t+1}} = s\left(\frac{N_{t+1}^i}{Q_t K_{t-1}^j}\right)$ . In our reading, the first part of the second sentence ("the ratio s of the cost of external finance to the safe rate") refers to the numerator  $E_t(R_{t+1}^k)$  and the denominator  $R_{t+1}$  respectively.

Furthermore, we build our interpretation on the calibration strategy of BGG. They write on page 1368:

"Specifically, we choose parameters to imply the following three steady state outcomes: (1) a risk spread,  $R^k - R$ , equal to two hundred basis points, approximately the historical average spread between the prime lending rate and the six-month Treasury bill rate; [...]"

This calibration strategy equalizes steady state value of  $E_t(R_{t+1}^k)$  with "the prime lending rate". Although BGG do not go into further detail about what constitutes the data on the prime lending rate, we believe this data is consistent with what one would refer to as "the cost of external finance".

we have called the discounted return to capital but may be equally well interpreted as the external finance premium - depends inversely on the share of the firm's capital investment that is financed by the entrepreneur's own net worth."

where  $\Phi^{I}$  is an increasing function in the intermediate goods producers' leverage ratio  $\frac{Q_{t}k_{t}}{N_{t}^{I}}$ . Appendix 3.A gives details on the derivation of the expressions for the external finance premia. The intuition for these representations is that asymmetric information between lenders and borrowers increases the agency costs for lower values of the borrower's net worth, thereby increasing the premium that borrowers have to pay to their lenders.

We further adopt the terminology of the original BGG model and refer to the external finance premia as  $efp^{I} = \frac{E_t \{R_{t+1}^k\}}{E_t \{R_{t+1}^B\}}$  for intermediate goods producers, and  $efp^{B} = \frac{E_t \{R_{t+1}^B\}}{R_t}$  for banks.

#### 3.3.5 Capital producers

Capital producers are perfectly competitive. They buy back undepreciated capital from intermediate goods producers after intermediate goods production. Moreover, they buy the quantity  $i_t$  of final goods as inputs to capital production. Capital producers sell capital for the real price  $q_t$  to intermediate goods firms.

As is standard in the literature, we assume convex capital adjustment costs. In particular, an investment of  $i_t$  yields  $i_t - \frac{\varphi_k}{2} \left(\frac{i_t}{k_{t-1}} - \delta\right)^2 k_{t-1}$  units of new physical capital  $k_t$ . Thus, the law of motion for capital is

$$k_t = (1-\delta) k_{t-1} + i_t - \frac{\varphi_k}{2} \left(\frac{i_t}{k_{t-1}} - \delta\right)^2 k_{t-1}. \quad (3.37)$$

The capital producer's real profit  $\frac{\Pi_t^{cap}}{P_t}$  is the revenue from capital sales to the intermediate goods producer net of her costs from repurchasing undepreciated capital and buying investment goods:

$$\frac{\Pi_t^{cap}}{P_t} = q_t \left( k_t - (1 - \delta) \, k_{t-1} \right) - i_t. \tag{3.38}$$

Using the law of motion for capital, we can eliminate  $k_t$  from the profit expression:

$$\frac{\Pi_t^{cap}}{P_t} = q_t \left[ i_t - \frac{\varphi_k}{2} \left( \frac{i_t}{k_{t-1}} - \delta \right)^2 k_{t-1} \right] - i_t.$$
(3.39)

Hence, the capital producer's maximization problem in each period is:

$$\max_{i_t} \frac{\Pi_t^{cap}}{P_t}.$$

The first-order condition is

$$1 = q_t \left[ 1 - \varphi_k \left( \frac{i_t}{k_{t-1}} - \delta \right) \right]. \tag{3.40}$$

#### 3.3.6 Retailers

There is a continuum of monopolistically competitive retailers in the model economy indexed by *i*. They buy homogeneous intermediate goods at real price  $p_{x,t}$  and transform them one-to-one into differentiated retail goods  $x_t(i)$ . The agents do not use the differentiated retail goods separately but a Dixit-Stiglitz aggregator of the differentiated goods  $y_t \equiv \int_0^1 [x_t(i)^{\frac{\theta-1}{\theta}} di]^{\frac{\theta}{\theta-1}}$ , where  $\theta > 1$  is the elasticity of substitution between retail goods. We call  $y_t$  final goods. Choosing the optimal expenditure level for each of the differentiated retail goods implies a demand function of the form

$$x_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\theta} y_t.$$
(3.41)

 $P_t(i)$  is the price of retail good *i* and  $P_t = \left(\int_0^1 P_t(i)^{1-\theta} di\right)^{\frac{1}{1-\theta}}$  is the aggregate price level in the economy. Retailers take (3.41) as a constraint in their profit maximization problem.

Retail goods are sold for real price  $\frac{P_t(i)}{P_t}$ . Due to their market power, retailers can choose the price of their retail goods but price changes are subject to adjustment costs as in Rotemberg (1982). Firm *i* maximizes its discounted profit stream subject to the demand function for retail good *i*:

$$\max_{P_{t}(i)} E_{t} \left\{ \sum_{t=0}^{\infty} \beta^{t} \frac{\lambda_{t+1}}{\lambda_{t}} \left[ \frac{P_{t}(i)}{P_{t}} \left( \frac{P_{t}(i)}{P_{t}} \right)^{-\theta} y_{t} - p_{x,t} x_{t}(i) - \frac{\varphi_{p}}{2} \left( \frac{P_{t}(i)}{P_{t-1}(i)} - \pi \right)^{2} y_{t} \right] \right\}$$
  
s.t.  $\left( \frac{P_{t}(i)}{P_{t}} \right)^{-\theta} y_{t} = x_{t}(i),$ 

where  $\pi$  represents steady state inflation,  $y_t \equiv \int_0^1 \left[ x_t(i)^{\frac{\theta-1}{\theta}} di \right]^{\frac{\theta}{\theta-1}}$ , and the last term in the maximization expression are the price adjustment costs. The first-order condition reads

$$0 = (1 - \theta) y_t + p_{x,t} \theta y_t - \varphi_p (\pi_t - \pi) \pi_t y_t + \beta E \left\{ \frac{\lambda_{t+1}}{\lambda_t} \varphi_p (\pi_{t+1} - \pi) \pi_{t+1} y_{t+1} \right\}.$$
(3.42)

# 3.3.7 Exogenous spending, monetary policy, and resources

The government consumes a fraction  $\tau_t$  of the retail goods aggregator. This fraction fluctuates around its steady state value  $\bar{\tau}$  due to an exogenous spending shock  $\epsilon_{q,t}$ :

$$g_t = \tau_t y_t, \tag{3.43}$$

where  $\frac{\tau_t}{\bar{\tau}} = \left(\frac{\tau_{t-1}}{\bar{\tau}}\right)^{\rho_g} e^{\epsilon_{g,t}}.$ 

Monetary policy follows the rule

$$\frac{R_t}{R} = \left(\frac{R_{t-1}}{R}\right)^{\rho_R} \left[ \left(\frac{\pi_t}{\pi}\right)^{\phi_p} \left(\frac{y_t}{y}\right)^{\phi_y} \right]^{1-\rho_R} e^{\epsilon_{R,t}}, \quad (3.44)$$

where R and y are the steady state values of the nominal gross interest rate and output, respectively. That is, the central bank reacts to deviations of inflation and output from their respective steady states. Moreover, as is common in the literature, we assume interest rate smoothing on behalf of the central bank, where  $\rho_R$  is the smoothing parameter.  $\phi_{\pi}$  and  $\phi_y$  are the reaction parameters for output and inflation deviations, respectively.  $\epsilon_{R,t}$  denotes a non-persistent monetary policy shock with mean zero and variance  $\sigma_R^2$ . We only use this shock to demonstrate the different consequences of endogenous and exogenous policy rate movements.

Final goods are consumed by households, invested by capital producers or purchased by the government. Moreover, part of the final goods are lost in monitoring activities. The resource constraint is

$$y_{t} = c_{t} + c_{t}^{I} + c_{t}^{B} + g_{t} + i_{t} + \frac{\omega_{p}}{2} (\pi_{t} - 1)^{2} y_{t} + \mu^{B} F(\bar{\chi}_{t}) E(\chi | \chi < \bar{\chi}_{t}) R_{t}^{B} l_{t-1} + \mu^{I} F(\bar{\omega}_{t}) E(\omega | \omega < \bar{\omega}_{t}) R_{t}^{k} q_{t-1} k_{t-1},$$
(3.45)

where  $l_t = \frac{L_t}{P_t}$ .

## 3.4 Calibration and steady state

The calibration for the financial sector builds on three steady state values for each borrower: (i) the leverage ratio, (ii) the bankruptcy rate and (iii) the external finance premium. Our model economy features two borrowers, namely banks and intermediate goods producers. The strategy is to find empirical averages for these variables and use them as steady state values. In turn, these steady state values imply parameter values which support that steady state.

The leverage ratio of the intermediate good sector is taken from recent empirical work by Kalemli-Ozcan et al. (2012). Using firm-level data from the ORBIS database, they find a leverage ratio of 4.5 for European non-financial companies. For the bank leverage ratio in our model, we use data from the ECB and estimate a leverage ratio of 18.<sup>13</sup> Kalemli-Ozcan et al. also report the European median bank leverage ratio to vary between 17.5 and 15 in their sample. Given that the empirical section at the end of this paper is based on euro area data, we choose to simulate our model with the data taken from the ECB.<sup>14</sup>

To the best of our knowledge, there is no aggregate data for firm and bank bankruptcy in the euro area. We thus rely on BGG for the assumed firm bankruptcy rate of 3 percent per year (0.75 percent per quarter). The bankruptcy rate for banks is taken from FDIC data.<sup>15</sup> We find an annual bank bankruptcy rate of .554 percent (.14 percent per quarter).

Finally, we rely again on the mentioned ECB data to construct the costs of external finance for both the bank and the intermediate good producer. Specifically, we take the average value of the lending rate to non-financial companies as the costs of external finance for the intermediate good producer and the average interest rate on deposits with fixed maturity as a measure of bank financing costs. As further explained in the empirical section below, neither measure is a perfect description of external funding costs, but we are unaware of more accurate and publicly available data sets. To construct a measure of the external finance premium, we use the average yield on German government bonds with

 $<sup>^{13}\</sup>mathrm{See}$  appendix 3.C for the data description.

 $<sup>^{14}{\</sup>rm A}$  robustness simulation with a bank leverage ratio of 10 (the leverage ratio used by HSU) did not affect our results qualitatively.

<sup>&</sup>lt;sup>15</sup>Calculated as the average ratio of "Number of Failed Institutions" to "Total Number of Banks (incl. savings and commercial)" taken from the FDIC's Historical Trends Series. Annual data covering 1990 to 2012.

a remaining maturity of one year as the risk-free rate. The bond data are taken from the Bundesbank.  $^{16}$ 

Taking these averages as steady state values, we are able to obtain the remaining eight parameters in the financial sector, that is, the auditing costs, the survival rates, the steady state cut-off values and the dispersions of idiosyncratic risk. Specifically, we use the steady state equilibrium conditions for each borrower which gives us eight equations in eight unknowns. The equations are

$$0 = \Gamma'(\bar{\omega}) - \Upsilon^{I} \left[ \Gamma'(\bar{\omega}) - \mu^{I} G'(\bar{\omega}) \right]$$
(3.46)

$$0 = \frac{R^{n}}{R^{B}} [1 - \Gamma(\bar{\omega})]$$

$$+ \alpha L \left[ \frac{R^{k}}{R^{k}} [\Pi(\bar{\omega}) - L G(\bar{\omega})] - 1 \right] = (0.47)$$

+ 
$$\Upsilon^{I}\left[\frac{R^{\kappa}}{R^{B}}\left[\Gamma\left(\bar{\omega}\right)-\mu^{I}G\left(\bar{\omega}\right)\right]-1\right]$$
 (3.47)

$$R^{B}\left(Qk-N^{I}\right) = \left[\Gamma\left(\bar{\omega}\right)-\mu^{I}G\left(\bar{\omega}\right)\right]R^{k}Qk \qquad (3.48)$$

$$N^{I} = \gamma^{I} \left\{ \left[ 1 - \Gamma\left(\bar{\omega}\right) \right] R^{k} Q k \right\} + W^{I}$$
(3.49)

$$0 = \Psi'(\bar{\chi}) - \Upsilon^B \left[ \Psi'(\bar{\chi}) - \mu^B M'(\bar{\chi}) \right] \quad (3.50)$$
$$R^B$$

$$0 = \frac{R}{R} [1 - \Psi(\bar{\chi})] + \Upsilon^B \left[ \frac{R^B}{2} [\Psi(\bar{\chi}) - \mu^B M(\bar{\chi})] - 1 \right] \quad (3.51)$$

$$\Upsilon^{B}\left[\frac{R^{B}}{R}\left[\Psi\left(\bar{\chi}\right)-\mu^{B}M\left(\bar{\chi}\right)\right]-1\right] \quad (3.51)$$

$$R\left(L-N^{B}\right) = \left[\Psi\left(\bar{\chi}\right)-\mu^{B}M\left(\bar{\chi}\right)\right]R^{B}L \qquad (3.52)$$

$$N^B = \gamma^B V^B + W^B. aga{3.53}$$

We use the function for forward in Matlab to numerically solve this system of equations and obtain a single solution.

The mean and standard deviation of the idiosyncratic risks are based on two assumptions. As in BGG, we assume idiosyncratic risk to be log-normally distributed and assume a time-invariant expected value  $E(\omega) = E(\chi) = 1$ . For the log-normal distribution, the mean becomes

<sup>&</sup>lt;sup>16</sup>Bond data are monthly yields on German sovereign bonds with a remaining time to maturity of 1 year and taken from the Bundesbank (Database ID: BBK01.WZ3400).

a function of the standard deviation, that is,  $\mu_{\omega} = -\frac{\sigma_{\omega}^2}{2}$  and  $\mu_{\chi} = -\frac{\sigma_{\chi}^2}{2}$ . The steady state bankruptcy rates  $F(\bar{\omega})$  and  $F(\bar{\chi})$  are then used to derive the value of  $\sigma_{\omega}$  and  $\sigma_{\chi}$ , respectively.

Following this approach we obtain monitoring costs in the real economy of 9% of repossessed assets, which is at the lower end of the credible range discussed by Carlstrom and Fuerst (1997). We also receive a firm survival rate of 97%, which corresponds to an average firm lifespan of 7.5 years. For the banks, monitoring costs are 16% of repossessed assets. A side-effect of this calibration strategy are short-lived banks: We obtain an average lifespan of 5 years for the banking sector. This value is lower than the ones in Gertler and Karadi (2011) and Ueda (2012) who operate with bank lifespans of 8 to 9 years. The current model and the boundaries set by the observed empirical averages do not allow for longer bank lifespans. The model implies that bank survival, among others, is a function of the leverage ratio. Our leverage ratio of 18 is considerably higher than that used e.g. by Ueda (2012) (leverage ratio of 10) and Gertler and Karadi (2011) (leverage ratio of 4). If we were to use their leverage ratios, the lifespan of the banking system would increase to 8 and 13 years, respectively.

The standard parameters of our model include household preferences, parameters associated with intermediate goods production, adjustment costs and the monetary policy rule. For these parameters, we rely on estimates from other authors that are used widely in the DSGE literature. We set the depreciation rate, the discount rate, the capital intensity of production as well as the curvature of the households disutility from labor following Smets and Wouters (2003). In a seminal contribution, these authors have estimated these parameters from euro area data using as DSGE model. The main difference between their and our model structure lies the shape of the financial sector. Howevr, the estimates for the standard parameters reflect long-run relationships in the data which should be largely invariant to the exact shape of the model economy's financial sector. As explained in the model description, we split total labor supply into three segments i.e. household, intermediate good producer and banking sector labor supply. Intermediate goods producer and banking sector labor is introduced to equip newly entering agents with start-up capital. Following BGG, we assign a miniscule value for the proportions of the labor force stemming from these two sectors to ensure that this income source does not affect the model dynamics. Furthermore, choosing small values for these proportions is also called for as there exists an inverse relationship between the average lifespan of banks and intermediate goods producers and their respective labor incomes.

For the adjustment costs, we use a common value for the price adjustment costs found in the literature of  $\varphi_P = 100$ . As shown by Keen and Wang (2007), this price adjustment cost resembles the dynamics generated by a price setting frequency of 0.25 i.e. a quarter of retail firms may reset their prices every period. Furthermore, we take the capital adjustment cost parameter  $\varphi_k = 17$  from Carrillo and Poilly (2013) since we use the same functional form as they do in their study. The elasticity of substition between retail goods,  $\theta = 11$ , is also taken from these authors.<sup>17</sup>

We specify the monetary policy rule using standard values of the literature. Monetary policy exhibits a high degree of interest rate smoothing ( $\rho_R = 0.9$ ) and is largely tailored to deviations of inflation from the steady state ( $\phi_P = 1.5$ ;  $\phi_y = 0.125$ ).

Below, we compare our model to a model in which policy rate movements transmit undisturbed into bank financing costs. Thus, the focus lies on the differences between model responses to the same shock. For the presentation below, we choose the shock size such that real output falls by one percent upon impact in our model.

<sup>&</sup>lt;sup>17</sup>Testing alternative specification for these parameters revealed that the model dynamics remain largely unaffected by their choice. For robustness, we simulated models with  $\varphi_p = 200$  or  $\theta = 6$  ( $\approx$  Calvo price adjustment frequency of 1/9) and  $\varphi_k = 10$ . The quantitative effects of these changes were miniscule and the qualitative findings unaffected by these alterations.

With two exceptions, we base the persistence of our exogenous driving forces on the estimates from Smets and Wouters (2003). As they do not feature a risk shock comparable to our specification, we use the persistence found in Christiano et al. (2013) for the shock on intermediate goods producer risk. The second exception is the persistence of the monetary policy shock which we assume to be zero. The reason for this assumption is that we only use the monetary policy shock to illustrate the transmission of monetary policy in an otherwise undisturbed state of our model economy. We choose to illustrate this with a one-off shock to the monetary policy rate which reverts to the value implied by the Taylor Rule in the next quarter.

Smets and Wouters (2003) have estimated a model that featured ten different shock sources and abstracted from a financial sector. To make a model-founded statement about monetary policy in the euro area, one needed to estimate the model on euro zone data in order to decompose the aggregate shocks into the sources we use in our model. This is beyond the scope of our paper. Rather, we base our calibration on euro area data where available in order to make a general inquiry about the mechanisms of our model. This approach is in line with those of the related models cited above, e.g. BGG, HSU, Carrillo and Poilly (2013) or Gertler and Karadi (2011).

Table 3.1 gives an overview of all parameter and relevant steady state values as well as their sources. A detailed derivation of the model's steady state can be found in appendix 3.B. The model is solved by obtaining linear approximations of the policy functions for the endogenous variables around that steady state.
Symbol	Description	Value	Source
β	Discount rate	0.99	Smets and Wouters (2003)
$\nu$	Curvature of disutility of labor	2.5	Smets and Wouters (2003)
$\psi$	Disutility weight of labor	1.11	Implied by calibration
δ	Depreciation rate	0.025	Smets and Wouters (2003)
$\alpha$	Power on capital in production function	0.33	Smets and Wouters (2003)
Ω	Proportion of labor by household	0.997	BGG
$\Omega^{I}$	Proportion of labor by IMGs	0.0015	own, based on BGG
$\Omega^B$	Proportion of labor by banks	0.0015	own, based on BGG
$\varphi_k$	Capital adjustment costs	17	Carrillo and Poilly (2013)
θ	Elasticity of substitution retail goods	11	Carrillo and Poilly (2013)
$\varphi_P$	Price adjustment costs	100	Standard
$\rho_R$	Policy smoothing parameter	0.95	Standard
$\phi_P$	Policy weight on inflation	1.5	Standard
$\phi_{y}$	Policy weight on output	0.125	Standard
$\bar{h}^{H}$	St. st. household labor supply	1	Normalized
$\bar{h}^{I}$	St. st. IMGs labor supply	1	Normalized
$\bar{h}^B$	St. st. banks' labor supply	1	Normalized
$\overline{\tau}$	St. st. exog. expend. share of output	0.2	OECD data
$\pi$	St. st. inflation rate	1	Standard
$\sigma_{\epsilon,a}$	Technology shock	0.0825	normalized, see text
$\sigma_{\epsilon, CC}$	Preference shock	0.021	normalized, see text
$\sigma_{\epsilon,q}$	Exog. spend. shock	0.0673	normalized, see text
$\sigma_{\epsilon,\omega}$	Risk shock	0.213	normalized, see text
ρa	Persistence technology	0.81	Smets and Wouters (2003)
PCC	Persistence preferences	0.83	Smets and Wouters (2003)
ρġ	Persistence exog. expend.	0.94	Smets and Wouters (2003)
Ρσω	Persistence risk	0.97	Christiano et al. (2013)
$\frac{Qk}{NL}$	St. st. leverage ratio of IMGs	4.5	Kalemli-Ozcan et al. (2012)
$F(\bar{\omega})$	St. st. bankruptcy rate of IMGs	0.0075	BGG
$EFP^{I}$	St. st. EFP of IMGs	$1.015^{0.25}$	ECB data
$\mu^{I}$	Monitoring costs for defaulting IMGs	0.0903	Implied by empirical avgs.
$\gamma^{I}$	Survival rate of IMGs	0.9672	Implied by empirical avgs.
ω	St. st. cut-off value	0.7756	Implied by empirical avgs.
$\sigma_{\omega}$	Idiosyncratic risk dispersion	0.1023	Implied by empirical avgs.
$\frac{L}{NB}$	St. st. leverage ratio of banks	18	ECB data
$F(\bar{\chi})$	St. st. bankruptcy rate of banks	0.0014	FDIC data
$EFP^B$	St. st. EFP of banks	$1.009^{0.25}$	ECB data
$\mu^B$	Monitoring costs for defaulting banks	0.1616	Implied by empirical avgs.
$\gamma^B$	Survival rate of banks	0.9518	Implied by empirical avgs.
$\dot{\bar{x}}$	St. st. cut-off value	0.9518	Implied by empirical avgs.
$\sigma_{\chi}$	Idiosyncratic risk dispersion	0.0197	Implied by empirical avgs.

 Table 3.1:
 Summary of calibration.
 Note:
 St. and IMG abbreviate steady state and intermediate goods producer, respectively.

### 3.5 Results

### 3.5.1 Transmission of monetary policy

The purpose of this paper is to examine how the presence of leveragesensitive bank financing costs affects the transmission of monetary policy. In particular, we are interested in whether the model can reproduce a scenario where, during recessions, the transmission of monetary policy is impaired in the sense that policy rate cuts are not fully passed through to bank financing costs and hence loan rates for the real economy.

In our model, the study of the bank's marginal cost of external finance  $E_t \{R_{t+1}^B\}$  suffices to infer whether the transmission of monetary policy into the real economy is impaired. To see this, recall that central bank policy aims at affecting the real economy's external financing costs conditional on borrower risk. By altering the policy rate, the central bank seeks to induce a similar move in the average lending rate for each risk category. Note that if the risk composition of the borrowers evolves during changes in the monetary policy rate, the policy rate change might not necessarily be reflected in the average lending rates. It is thus unwarranted to deduce the transmission of monetary policy from the evolution of the average marginal cost of external finance for intermediate goods producers  $E_t \{R_{t+1}^k\}$ , the expression for which we repeat here for convenience:

$$E_t \left\{ R_{t+1}^k \right\} = \Phi^I \left( \frac{Q_t k_t}{N_t^I} \right) E_t \left\{ R_{t+1}^B \right\}.$$
(3.54)

Equation (3.54) neatly disentangles the above described sources for an observed change in the average lending rate to the real economy. The term  $\Phi^I\left(\frac{Q_t k_t}{N_t^I}\right)$  can be interpreted as the markup that banks charge over their own marginal financing costs. This markup is solely a function of borrower risk, represented by the leverage ratio  $\frac{Q_t k_t}{N_t^I}$ . Any

difference between the bank's marginal financing cost and the lending rate to the real economy is due to borrower risk.

The term  $E_t \{R_{t+1}^B\}$ , banks' marginal cost of external finance, anchors the marginal cost of external finance in the real economy. It is the level from which the markup reflecting borrower risk is applied. A monetary policy that is concerned with altering the real economy's financing costs conditional on borrower risk is thus implicitly concerned with the level of  $E_t \{R_{t+1}^B\}$ . If the level of  $E_t \{R_{t+1}^B\}$  does not fully adapt to changes in monetary policy, the lending rates to the real economy cannot fully reflect the policy change either. Thus, to study the transmission of monetary policy, we analyze the evolution of banks' financing costs.

As described in section 3.2, the independence of bank financing costs is a fundamental difference from standard New Keynesian models. In the canonical models, the bank financing costs and the policy rate are identical, that is,  $E_t \{R_{t+1}^B\} = R_t$ . In these models, the banking system is a veil and monetary policy always transmits undisturbed.

For our analysis, it is important to distinguish between an exogenously and an endogenously induced change in monetary policy. Consider a decrease in the policy rate. If the decrease is induced exogenously via a shock to the policy rate, real output booms and bank financing costs unambiguously decrease due to a lower policy rate and a better aggregate state of the economy, which translates into less debt default on bank loans.

However, if the policy rate cut is endogenous, it is a reaction to a recessionary state of the economy. In this case, bank financing costs receive a decreasing impulse from monetary policy but an increasing impulse from the recessionary aggregate state of the economy. It is through these opposing signals that our model has the potential to describe a scenario in which the monetary policy transmission into bank financing costs, and hence into the financing costs of the real economy, is impaired.



Figure 3.1: Impulse responses for a monetary policy shock. Notes: Vertical axes show annual basis point deviations from the steady state. Horizontal axes show quarters.

To illustrate this, figure 3.1 displays the responses of the policy rate  $R_t$ , bank financing costs  $E_t \{R_{t+1}^B\}$  and their ratio  $efp^B$  to an exogenous policy rate decrease, that is, a monetary policy shock.<sup>18</sup> The vertical axes show basis point deviations from the steady state and horizontal axes show quarters. Quarter one corresponds to the period where the shock occurs. As the policy rate decreases, bank financing costs also decrease. The simultaneous fall in the  $efp^B$  implies that the decrease in bank financing costs is stronger than that in the policy rate. The additional decrease is due to the described expansionary effect of the exogenous rate cut.

In contrast, figures 3.2, 3.3, and 3.4 show the reactions of the policy rate, bank financing costs and the external finance premium for banks to a recessionary exogenous spending, preference, and risk shock, respectively.<sup>19</sup> The decrease in the policy rate is thus an endogenous reaction of a central bank following a monetary policy rule. Again, basis point deviations from the steady state are shown on the vertical and quarters are shown on the horizontal axes. In the cases of an exogenous spending shock (figure 3.2) and a preference shock (figure 3.3), a decrease in the policy rate is accompanied by a weaker decrease in the bank financing costs and consequently an increasing external finance premium of banks. In the case of a risk shock (figure 3.4), we see

 $<sup>^{18}</sup>$  The shock size is scaled to an initial 1% change in output.

 $<sup>^{19}\</sup>mathrm{In}$  each case, the shock size is scaled to an initial 1% change in output.



Figure 3.2: Impulse responses for an exogenous spending shock. Notes: Vertical axes show annual basis point deviations from the steady state. Horizontal axes show quarters.



Figure 3.3: Impulse responses for a preference shock. Notes: Vertical axes show annual basis point deviations from the steady state. Horizontal axes show quarters.



Figure 3.4: Impulse responses for a risk shock. Notes: Vertical axes show annual basis point deviations from the steady state. Horizontal axes show quarters.

that a decrease in the policy rate is even accompanied by an increase in bank financing costs and thus a strong increase in the external finance premium for banks. In all cases, the transmission of monetary policy is impaired in that the policy rate reductions are not passed on to bank financing costs in full or at all. Note that this scenario cannot be reproduced in traditional models featuring a financial accelerator mechanism. In these models, the bank financing costs are always equal to the risk-free rate in the economy.

### 3.5.2 Impaired transmission of monetary policy and its effects on aggregate outcomes

Having shown that the transmission of monetary policy can be impaired by the presence of leverage-sensitive bank financing costs, this section asks how this mechanism affects aggregate outcomes. To this end, we need to compare the outcomes of our model with those of a model in which policy rate movements transmit undisturbed into bank financing costs. Removing the financial friction between investors and banks by setting  $\mu^B = 0$  provides such a model.<sup>20</sup> Without monitoring costs there is no asymmetric information because investors can always observe the return on banking costlessly. Consequently, bank financing costs are not sensitive to the leverage ratio; there is no variable spread between the risk-free rate and bank financing costs.

For an exogenous spending, risk, and preference shock, the top rows of figures 3.5, 3.6 and 3.7 show the reaction of output and investment in the model with impaired and undisturbed transmission of monetary policy, respectively.<sup>21</sup> To illustrate the role of bank financing costs, the bottom row shows the external finance premia for banks and intermediate firms, respectively. For output and investment, the vertical axes

<sup>&</sup>lt;sup>20</sup>See Faia and Monacelli (2007) for a case where setting monitoring costs to zero removes a financial friction between financial intermediaries and firms.

 $<sup>^{21}\</sup>mathrm{Again},$  in each case the shock size is scaled to an initial 1% change in output.



Figure 3.5: Impulse responses for an exogenous spending shock. Notes: The solid line represents the model with an impaired transmission of the policy rate to bank financing costs while the dashed line represents the model with an undisturbed transmission mechanism. Vertical axes in the top row show percentage deviations from the steady state. Vertical axes in the bottom row show annual basis point deviations from the steady state. Horizontal axes show quarters.

show percentage deviations from the steady state, while for finance premia it shows annualized basis point deviations from the steady state. The horizontal axes show quarters.

The graphs for the external finance premium of banks visualize the difference between the two model specifications. While in the model with leverage-sensitive bank financing costs the external finance premium for banks increases with a contractionary shock, it stays constant in the model without leverage-sensitive bank financing costs. Thus, the increase in the external finance premium for intermediate goods producers is stronger in the model with a variable spread between the



Figure 3.6: Impulse responses for a risk shock. Notes: The solid line represents the model with an impaired transmission of the policy rate to bank financing costs while the dashed line represents the model with an undisturbed transmission mechanism. Vertical axes in the top row show percentage deviations from the steady state. Vertical axes in the bottom row show annual basis point deviations from the steady state. Horizontal axes show quarters.

policy rate and bank financing costs. As a consequence, the decrease in investment and output is stronger.

As is apparent in the presented impulse response functions, the additional amplification in the real sector in the model with a leveragesensitive external finance premium for banks is relatively small. This is a direct consequence of the relatively small variation in the external finance premium for banks. For the exogenous spending and the preference shock, it increases by approximately 6 basis points, while for the risk shock the increase is roughly 25 basis points.



Figure 3.7: Impulse responses for a preference shock. Notes: The solid line represents the model with an impaired transmission of the policy rate to bank financing costs while the dashed line represents the model with an undisturbed transmission mechanism. Vertical axes in the top row show percentage deviations from the steady state. Vertical axes in the bottom row show annual basis point deviations from the steady state. Horizontal axes show quarters.

The small movements in the external finance premium for banks are a consequence of the shocks' small impact on loan losses and hence bank net worth. Bank financing costs are a function of the leverage ratio, that is, the ratio of total bank assets over bank net worth. In contrast to the existing literature, our model provides an endogenous mechanism that strengthens the link between the health of bank balance sheets and aggregate economic performance. This mechanism is introduced via a non-state contingent debt contract between banks and intermediate goods producers. As explained above, banks in our model have to set interest rates at the time of loan origination and are not insured against aggregate uncertainty. In recessions, due to increased loan default,



Figure 3.8: Impulse responses of firm bankruptcy rates and loan losses for an exogenous spending, risk, and preference shock. Notes: Responses of firm bankruptcy rates are expressed in percentage point deviations from steady state. Responses to loan losses are expressed in % of steady state bank net worth.

banks suffer a revenue shortfall  $\Delta_t$  which, for convenience, is repeated here:

$$\Delta_t = \left( E_{t-1} \left\{ R_t^B \right\} - R_t^B \right) L_{t-1}.$$
 (3.55)

We interpret  $\Delta_t$  as a loss since it directly reduces bank net worth. The latter can consequently be expressed as

$$N_t^B = \gamma^B \left[ E_{t-1} \left\{ V_t \right\} - \Delta_t \right] + W_t^B.$$
 (3.56)

If banks expected a bigger payoff from their lending activities than actually materialized, losses are positive. This is the case when a negative shock hits the economy and not as many intermediate goods producers as expected can pay back their loans due to bankruptcy. The loan losses, in turn, deplete banks' net worth, increasing the leverage ratio. If these losses are small, so is the variation in the leverage ratio and thus in the external finance premium for banks.

To illustrate this point, figure 3.8 shows the reactions of firm bankruptcy rates and loan losses to an exogenous spending, risk, and preference shock, respectively. For the exogenous spending and preference shocks, the quarterly firm bankruptcy rate only increases by 0.2 percentage points from 0.75% to about 0.95%. Consequently, loan losses only amount to roughly 0.5% of bank net worth. The risk shock evokes a bigger response in the firm bankruptcy rate and in loan losses, but still only 5% of banks' net worth depletes.

### 3.5.3 Direct impact on bank net worth

The loan losses resulting from the demand shocks covered in section 3.5.2 are small compared to empirical observations for some European countries in the last couple of years. For example, the Bank of Spain reports that the Spanish banking sector's realized total losses since 2009 amount to more than a third of initial bank equity.<sup>22</sup> According to the World Bank's World Development Indicators, 16% and 19% of total bank loans in Ireland were non-performing in 2011 and 2012, respectively. Although it is clear that this is not equivalent to bank losses, it indicates substantial valuation adjustment on loan portfolios. The housing sector plays a major role in these loan losses, as noted, for example, by Lane (2011) for Ireland and the International Monetary Fund (2013) for Spain.

To assess how a depletion of bank net worth of a similar magnitude as described above affects the transmission of monetary policy, we employ

<sup>&</sup>lt;sup>22</sup>See Table 4.7 ("Equity, valuation adjustments and impairment allowances") of the Bank of Spain's Statistical Bulletin. Unfortunately, similar data from other national central banks could not be found.



Figure 3.9: Impulse responses for a bank net worth shock. Note: The shock depletes 20% of the respective steady state net worth. Responses of output and investment are expressed as percentage deviations from the steady state. The policy rate, bank financing costs, and external finance premia are expressed as annualized basis point deviations from the steady state. Horizontal axes show quarters.

an exogenous shock to bank net worth.<sup>23</sup> The net worth shock can be regarded as a proxy for loan losses incurred by a housing sector downturn and ensuing losses from mortgages and loans to real estate developers. We use a net worth shock that inflicts a loss of 1% of total assets on the banking system. We consider this magnitude as conservative in light of the reported evidence. Due to our steady state bank leverage ratio of 18 (that is, a capital-asset-ratio of 5.6%), this loss depletes 20% of bank equity in the initial period. There a no further exogenous losses in the subsequent period as the shock has zero persistence.

 $<sup>^{23}</sup>$ Ueda (2012) also uses net worth shocks in his model.

Figure 3.9 depicts the impulse responses of this exercise. A shock to bank net worth has little effect on an economy without leveragesensitive bank financing costs (dashed lines). For the model with leverage-sensitive bank financing costs (solid lines), the policy rate decreases due to the contractionary shock. However, this decrease is strongly counteracted by an increase in bank financing costs.

According to the impulse response functions, the bank financing costs increase by about 28 basis points in quarterly terms on impact. Over the first year following the shock, banks suffer an average quarterly markup of about 20 basis points. Instead of the assumed steady state mark up of 25 basis points over the risk-free rate (1% at annual rates), these banks thus face approximately 45 basis points (1.9% at annual rates). The increases in the external finance premium for the intermediate goods sector are of similar magnitude, albeit from a higher base. In the steady state, the intermediate goods sector finances itself at a markup of 60 basis points over the quarterly risk-free rate. After the net worth shock to the banking sector, the total markup averages about 90 basis points per quarter in the first year, or 3.6% at annual rates.

This strong response of the lending rate transmits into investment activity. The contraction there is considerable, being roughly 3.5% off its equilibrium value in the initial period after the shock. It also fails to recover in full over the depicted time span of 3 years. The reaction of output, 0.5% off the steady state value, is more moderate although the resulting recession grosses more than 2% of steady state output throughout the first year.

This shows that a variable spread between the policy rate and bank financing costs can have considerable consequences for aggregate variables if the effect of the macroeconomic disturbance on the bank leverage ratio is significant.

# 3.6 Monetary policy implications

## 3.6.1 Bank financing conditions and monetary policy rules

The last two sections showed that leverage-sensitive bank financing costs affect the transmission of monetary policy and aggregate outcomes. Therefore, the question arises if the central bank can achieve better welfare results if it takes into account bank financing conditions in its policy decision. When the economy is in a recession and the external finance premium for banks increases — signaling an impaired transmission from the policy rate to bank financing costs — a negative reaction of the policy rate to the external finance premium could arrange for a more undisturbed monetary policy transmission. This is because a decreasing policy rate improves conditions in the bank-ing sector and hence the leverage-dependent spread between the bank financing costs and the policy rate is reduced.

To evaluate whether a reaction to bank financing costs on behalf of the central bank can increase welfare in the economy, we employ the welfare cost concept suggested by Schmitt-Grohé and Uribe (2007). In particular, we compare the conditional welfare generated by the Ramsey optimal policy with the conditional welfare generated by different monetary policy rules which — besides the common components inflation and output — take the external finance premium for banks into account.<sup>24</sup>

<sup>&</sup>lt;sup>24</sup>We only use the household's utility function for our normative analysis. The fact that banks and intermediate goods producers also consume is not taken into account. In doing so, we rely on an argument made by Faia and Monacelli (2007). In particular, banks and intermediate goods producers are risk neutral which implies that their mean consumption is not affected by the volatility of our model. Thus, taking into account the consumption of banks and intermediate goods producers would alter the absolute level of welfare in every monetary policy regime by the same amount, but would not affect the welfare ranking among the regimes. We thus follow Faia and Monacelli (2007) and only include household utility in the welfare analysis.

The Ramsey policy is obtained by maximizing the expected sum of discounted period utility functions subject to the equilibrium conditions of the competitive economy. The alternative policies are represented by different values of the coefficients  $\phi_{\pi}$ ,  $\phi_{y}$  and  $\phi_{B}$  in the policy rule

$$\frac{R_t}{R} = \left(\frac{R_{t-1}}{R}\right)^{\rho_R} \left[ \left(\frac{\pi_t}{\pi}\right)^{\phi_p} \left(\frac{y_t}{y}\right)^{\phi_y} \left(\frac{efp_t^B}{efp^B}\right)^{\phi_B} \right]^{1-\rho_R}, (3.57)$$

where  $efp^B$  denotes the steady state value of the external finance premium for banks. The parameter capturing interest rate smoothing,  $\rho_R$ , is kept at its original calibration.

Let  $\mathcal{O}_0^r$  be the welfare of the Ramsey policy conditional on the state in period zero being the deterministic steady state,

$$\mho_0^r = E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \left( \log c_t^r - \psi \frac{(h_t^r)^{1+\nu}}{1+\nu} \right) \right\},$$
(3.58)

and let the conditional welfare of an alternative policy be

$$\mho_0^a = E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \left( \log c_t^a - \psi \frac{(h_t^a)^{1+\nu}}{1+\nu} \right) \right\}.$$
 (3.59)

The welfare costs  $\vartheta$  of a particular policy rule are defined as the share of the consumption process in the Ramsey monetary policy regime that the household is willing to forgo in order to reach the same conditional welfare in the alternative regime as in the Ramsey regime. Thus,  $\vartheta$  is implicitly defined by

$$\mathcal{O}_{0}^{a} = E_{0} \left\{ \sum_{t=0}^{\infty} \beta^{t} \left( \log \left( (1-\vartheta)c_{t}^{r} \right) - \psi \frac{(h_{t}^{r})^{1+\nu}}{1+\nu} \right) \right\}.$$
 (3.60)

Note that we can write

$$\mathcal{U}_0^s = \log(c_0^s) - \psi \frac{(h_0^s)^{1+\nu}}{1+\nu} + E_0 \left\{ \sum_{t=1}^\infty \beta^t \left( \log\left(c_t^s\right) - \psi \frac{(h_t^s)^{1+\nu}}{1+\nu} \right) \right\} (3.61)$$

for  $s \in \{r, a\}$  and hence, using (3.60),

$$\begin{aligned}
\mathcal{O}_0^a - \mathcal{O}_0^r &= \log\left(1 - \vartheta\right) + \log\left(c_0^r\right) - \log\left(c_0^r\right) + E_0 \left\{ \sum_{t=1}^{\infty} \beta^t \log(1 - \vartheta) \right\} \\
&- \vartheta \right\} + E_0 \left\{ \sum_{t=1}^{\infty} \beta^t \log(c_t^r) \right\} - E_0 \left\{ \sum_{t=1}^{\infty} \beta^t \log(c_t^r) \right\}.
\end{aligned}$$
(3.62)

Noting that  $E_0 \{\sum_{t=1}^{\infty} \beta^t \log(1-\vartheta)\} = \frac{\beta}{1-\beta}(1-\vartheta)$ , and solving (3.62) for  $\vartheta$  we get

$$\vartheta = 1 - \exp\left[(1 - \beta)(\mho_0^a - \mho_0^r)\right]^{25}$$
(3.63)

Similar to Faia and Monacelli (2007), we first evaluate some ad-hoc rules to see what role the central bank's reaction to the external finance premium for banks plays for standard values of the inflation and output coefficient. In particular, we look at (i) a rule that only reacts to

<sup>&</sup>lt;sup>25</sup>We rely on a second-order approximation of the model to compute  $\vartheta$ . As explained in Faia and Monacelli (2007) and Schmitt-Grohé and Uribe (2007), a first-order approximation of the model cannot be used to conduct welfare rankings of different policies since in this case, the expected value of an endogenous variable equals its steady state value. Comparing two variables with the same steady state thus would indicate a welfare difference of zero. When using a second-order approximation, instead, the expected value of an endogenous variable equals the steady state of that variable plus a constant "correction-term" which depends on the model's volatility. For further details see Schmitt-Grohé and Uribe (2007). Furthermore, as the welfare results depend on volatility, we cannot use the calibration of the shocks' standard deviation used in section 3.5.1. There, we chose the standard deviations to yield a 1% initial decrease in output for comparison purposes. As the implied shock sizes are big in this case, there would be implausibly large welfare gains from switching to the optimal policy. For the welfare considerations in this section we therefore employ standard deviations of the shocks that are commonly described in the literature as being empirically relevant. In particular, for the standard deviations of the productivity and the external spending shock, we follow Schmitt-Grohé and Uribe (2007) and set them to 0.0064 and 0.016, respectively. The risk shock's standard deviation is set to 0.009 as in Christiano et al. (2013). The standard deviation for the preference shock of 0.01 is taken from Smets and Wouters (2003). Note that the qualitative welfare results do not change for the shock sizes used in section 3.5.1.

inflation; (ii) a rule that reacts to inflation and positively to the external finance premium for banks; (iii) a rule that reacts to inflation and negatively to the external finance premium for banks; (iv) a standard Taylor rule; (v) a standard Taylor rule plus a positive reaction on the external finance premium for banks; (vi) a standard Taylor rule plus a negative reaction on the external finance premium for banks; (vi) a standard Taylor rule plus a negative reaction on the external finance premium for banks.

The first three data columns in table 3.2 show the specifics of these interest rate rules. Data column four in table 3.2 shows the conditional welfare as described in equation (3.59), while data column five shows the welfare loss of switching from the optimal policy to the respective policy rule as a percentage share of the optimal consumption process, as described in equation (3.63). As is apparent in table 3.2, the addition of a negative reaction to the external finance premium for banks in an otherwise unaltered policy rule increases welfare, while the addition of a positive reaction to the external finance premium for banks decreases welfare. Thus, for standard values of the inflation and output coefficient in the policy rule, the central bank can improve welfare by additionally reacting negatively to bank financing conditions. Moreover, comparing rows (i) and (iv), (ii) and (v), and (iii) and (vi), respectively, we see that a reaction to output in an otherwise unchanged policy rule improves welfare results. This is because demand shocks — causing movement of output and inflation in the same direction — play a significant role in our model.

In the following, we examine whether the welfare-improving negative reaction to the external finance premium could equally well be achieved by a higher output coefficient in the policy rule. This robustness check is called for since an increase of the external finance premium for banks coincides with a decrease in output. For a policy rule which only reacts to inflation and output, we fix the inflation coefficient at the common value of 1.5 and search for the value of the output coefficient which maximizes conditional welfare — which is 0.3. We then add a negative reaction to the external finance premium for banks to this rule and

	Rule	$\phi_{\pi}$	$\phi_y$	$\phi_B$	Welfare	$\varphi * 100$
(i)	Rule with inflation only	1.5	0	0	-8.6095	1.92
(ii)	(i) plus pos. reaction to $efp^B$	1.5	0	0.3	-10.1100	3.38
(iii)	(i) plus neg. reaction to $efp^B$	1.5	0	-0.3	-7.6727	0.99
(iv)	Standard Taylor rule	1.5	0.125	0	-7.3018	0.63
(v)	(iv) plus pos. reaction to $efp^B$	1.5	0.125	0.3	-7.6378	0.96
(vi)	(iv) plus neg. reaction to $efp^B$	1.5	0.125	-0.3	-7.1001	0.43

**Table 3.2:** Interest rate rules and associated welfare results. Notes:  $\phi_{\pi}$ ,  $\phi_{y}$  and  $\phi_{B}$  are the reaction coefficients in the policy rule 3.57.  $efp^{B}$  is the external finance premium for banks. The welfare column shows the conditional welfare obtained from 3.59 and  $\varphi * 100$  describes the percentage share of the optimal consumption process lost from switching to a non-optimal policy.

note that welfare increases. Moreover, increasing the output coefficient after the addition of the reaction to the external finance premium still results in decreasing welfare. Hence, reacting negatively to the external finance premium for banks yields welfare improvements that cannot be achieved by a stronger positive reaction to output.

To gain more general insights about the role of the external finance premium in the policy rule, we fix the output coefficient to 0.3 and search for the optimal coefficients for inflation and the external finance premium of banks jointly.<sup>26</sup> The welfare surface emerging from this exercise is displayed in figure 3.10. For a given value of the output coefficient, the optimal coefficients for inflation and the external finance premium of banks are 3 and -1.5. The main result emerging from figure 3.10 is that there is a positive effect of responding negatively to the external finance premium of banks for all levels of the inflation coefficient. Furthermore, figure 3.10 also expresses the major importance of inflation stabilization: First, the highest welfare levels are reached for an inflation coefficient on the upper bound of our search interval. Second, we see that the reaction to bank financing conditions is less welfare

<sup>&</sup>lt;sup>26</sup>For the inflation coefficient, we search over the interval [1;3]. We choose 1 as the lower bound since this value ensures the abidance of the Taylor principle. For the upper bound, a value of  $\phi_{pi} > 3$  only yields minimal welfare gains. This upper bound can also be found in the literature, for example, in Schmitt-Grohé and Uribe (2007). For the coefficient of the external finance premium for banks, we search in the interval [-5;1.5].



Figure 3.10: Welfare surface for different combinations of the central bank's reaction parameters for inflation and the external finance premium for banks. Notes: The output coefficient is fixed at 0.3.

improving for higher values of the inflation coefficient. The important role for inflation stabilization is in line with much of the literature on optimal monetary policy.

Figure 3.11 shows the welfare losses relative to the optimal policy as a percentage share of the optimal consumption process (computed as shown in equation (3.63)) for different combinations of the coefficients for inflation and the external finance premium for banks. The optimal monetary policy rule comes very close to the Ramsey-optimal policy, with welfare losses close zero. Figure 3.11 also indicates the welfare costs that arise from deviating from the optimal combination of parameters. These can be substantial for highly negative or positive values of the parameter for the external finance premium of banks. For  $\phi_B = 0$ 



Figure 3.11: Loss surface for different combinations of the central bank's reaction parameters for inflation and the external finance premium for banks. Notes: The welfare loss is expressed as percentage share of the optimal consumption process. The output coefficient is fixed at 0.3.

— that is, if the central bank does not react at all to bank financing conditions — the welfare costs from deviating from the optimal parameter amount to approximately 0.3% of the optimal consumption process if the inflation reaction parameter is fixed at its optimal value.

### 3.6.2 Discussion: Unconventional monetary policy

The previous section showed that in an economy where the monetary policy transmission is impaired by developments in the banking sector, conventional monetary policy — the central bank setting the interest rate — can improve welfare results by reacting to bank financing conditions. However, the following question arises: If the transmission of monetary policy is disturbed in the banking sector should the central bank try to circumvent this transmission channel altogether? In this sense, our model may provide a motivation for unconventional monetary policies, that is, policies where the central bank lends directly in private credit markets.

A model that can reproduce such a scenario is the one by Gertler and Karadi (2011). They rely on a model with a single financial friction in the form of moral hazard/costly enforcement: At the beginning of a period a bank can steal a fraction of its assets (and hence household deposits) at the cost of the household forcing the bank into bankruptcy. In this case the household can recover the fraction of assets that has not been embezzled but recovering the embezzled funds is too costly. This kind of friction induces tightening lending standards in downturns that depend on bank balance sheet conditions. In contrast, the central bank can intermediate funds between the household and the real economy frictionlessly because it can commit to honor its debt by assumption. The central bank is assumed to be less efficient in lending to the real economy than banks. However, in a downturn, the costs of the banks' balance sheet constraint can exceed the central bank's efficiency costs of intermediating funds. Thus, unconventional monetary policy can yield welfare gains.

The Federal Reserve and the ECB, among others, both have employed unconventional policies in the course of the recent financial and economic crisis, for example, in the form of outright purchases of asset backed securities (Federal Reserve) or government bonds (Federal Reserve and ECB) in the secondary market. With the purchase of asset backed securities, central banks can ease borrowing conditions for, say, mortgages or loans to small and medium enterprises. With the purchase of government bonds, the central bank can lower the slope of the yield curve which lowers banks' opportunity costs of loans to the producing sector and should thus result in lower borrowing rates for the real economy at large.

Said empirical relevance of unorthodox measures on behalf of the central bank and our result that deteriorated bank balance sheets decrease the effectiveness of conventional monetary policy in principle makes room for the analysis of unconventional monetary policies. Since the results in Gertler and Karadi (2011) suggest that unconventional policies might be effective even if the zero lower bound on nominal interest rates is not binding, a comparison between conventional and unconventional monetary policy might be insightful.

However, as we use a different credit market framework than Gertler and Karadi (2011), we are not able to examine a direct lending channel from the central bank to the real economy. Thus, our model currently does not allow for an examination of unconventional policies and we leave this issue for future research.

# 3.7 Bank funding costs, lending rates and bank leverage ratio: Evidence from the euro area

We complete our analysis with some empirical evidence for two of our model's main mechanisms using aggregate euro area data: First, we examine whether there is a positive relationship between bank financing costs and lending rates to the real economy. Second, we analyze if bank financing costs are positively related to the bank leverage ratio. Lacking detailed microeconomic data, it is impossible to establish causality. However, we can use aggregate data on euro area member countries from the ECB and Eurostat to provide suggestive evidence on these issues.<sup>27</sup>

 $<sup>^{27}\</sup>mathrm{See}$  appendix 3.C for a detailed description of the data.

## 3.7.1 Association between bank funding costs and lending rates to the real economy

One main implication of our model in section 3.3 is that higher bank funding costs are transmitted into higher lending rates in the real economy. That is, banks do not bear the full increase in funding costs themselves, but pass at least part of the increase on to their borrowers.

Our econometric model is guided by the distinction between three components in real sector lending rates: A component reflecting the monetary policy rate, the average borrower and business cycle risk, as well as bank financing costs.

To control for the first determinant in the lending rate toward the real sector, we include the ECB interest rate for Main Refinancing Operations (MROs hereafter). According to the ECB, these operations account for the bulk of central bank related liquidity in the euro area's banking system and the associated interest rate is the one commonly referred to as policy rate.

Controlling for the second component in the lending rate is more difficult. To account for business cycle-related risks we include annualized rates of real GDP growth at the national level. Unfortunately, we are unaware of comprehensive data that can control for the risk composition of the borrowers in any given quarter. Consequently, the estimation below is carried out under the assumption that the risk composition of the borrowers varies in proportion with the business cycle. The annualized rates of real GDP growth thus serve to capture both the average borrower and business cycle risk.

The measure of bank funding costs, in line with our theoretical model, is the average interest rate on deposits with fixed maturity, which account for roughly 15% of euro area bank liabilities.<sup>28</sup>

 $<sup>^{28}\</sup>mathrm{A}$  further 20% of bank liabilities stem from overnight deposits and deposits redeemable at notice. While interest rate data on the latter are unavailable, overnight deposits do not deviate from the central bank policy rate. This is

We also include the slope of the yield curve to account for maturity mismatch. The observed interest rates on bank lending correspond to long-term loans, while bank financing costs relate to short-term funding. We control for the expected long-term evolution of interest rates using the difference between long-term and short-term rates at the national level. The measure of national long-term rates is the yield on central government bonds with a residual maturity of 10 years. As the short-term rate, we employ the ECB policy rate.

To account for national inflation differentials, country-specific measures of the Harmonized Index of Consumer Prices are the final control variable added to our econometric model. Our dependent variable is the average lending rate to non-financial companies (NFCs hereafter). Only the interest rates on new business are used in the estimations.

The estimation is based on national, quarterly data from euro area member countries since 2003Q1 or upon entry into the currency union. To allow for an interpretation of the coefficients as changes in percentage points, the data are transformed to a common base.<sup>29</sup>

unsurprising as elusive overnight deposits yield little bargaining power to the depositor. He has thus no means to express risk concerns through the interest rate, but will rather move his funds elsewhere. In our view, the missing volatility of overnight deposit rates leaves the validity of our conjecture intact. In a homogenous agent model, the claim that bank funding costs may deviate from the central bank policy rate demands that a significant proportion of its creditors show the desired behavior. Arguably, the remaining sources of bank funding do have the necessary bargaining power to demand higher interest rates for their funds. Besides capital and reserves, a further 50% of bank liabilities stems from inter-bank lending, debt securities or money-market funds. In our view, it is a credible assumption that this class of professional investors is able to express any risk concerns through higher interest rate demands. Following this claim, about 65% of bank liabilities originate from risk-sensitive sources. We chose to leave the claim of risk-sensitive professional investors unsupported by further data. Unfortunately, representative euro area-wide data about interest rates on inter-bank lending, lending from money market funds or in the form of debt securities is either not publicly available, or its generation demands disproportionate effort. For evidence of recent price-differentiation in bank debt securities in the euro area see Gilchrist and Mojon (2013). For evidence of price-differentiation in secured and unsecured inter-bank and money market lending in Europe see e.g. Afonso et al. (2011), Heijmans et al. (2011), or Kraenzlin and von Scarpatetti (2011).

 $<sup>^{29}\</sup>mathrm{All}$  variables have been transformed to decimals. For example, 4% interest is included as 0.04 and likewise for real GDP growth and inflation.

To assess the robustness of the coefficients, we re-estimate the model with either country fixed effects, year fixed effects, or both. Moreover, the estimations are performed for the contemporaneous values of the independent variables as well as their first lag. We deem the latter specification more reliable as it avoids endogeneity concerns. Our pre-ferred specification includes the independent variables at their first lag as well as all mentioned fixed effects (see column (4) in table 3.3).<sup>30</sup>

As this paper is concerned with recession scenarios, the above estimation is repeated for a split sample. To this end, we include a binary dummy variable that is equal to one for observations more recent than 2008Q3, and equal to zero otherwise. To account for differences between the years prior and after the onset of the Great Recession, this dummy variable is interacted with all independent variables of our baseline model. Country and year fixed effects remain included without interaction.

To account for serial correlation in the error term, we employ a Prais-Winsten estimator for all specifications. The Durbin-Watson statistics before and after this transformation are reported at the bottom of the table. Recall that a statistic close to 2 implies no serial correlation in the error term. Furthermore, we generalize the estimation to account for heteroskedasticity. The asterisks mark the conventional degrees of significance.

In line with the model mechanisms in section 3.3, the results reported in table 3.3 confirm that bank financing costs are a relevant determinant of lending rates to NFCs. For the contemporaneous relationship, the coefficient for bank financing costs is positive, of considerable magnitude and strongly significant. In our preferred specification with lagged independent variables (column (4) in table 3.3), the coefficient on bank financing costs is lower, but still economically and statistically signifi-

<sup>&</sup>lt;sup>30</sup>To economize on space, the table including all tested specifications can be found in appendix 3.D. The estimation equation and further data descriptions are also relegated to appendix 3.D. For comparability, we include the specification including the contemporaneous values for the independent variables in this section.

Dependent: Lending rate to NFCs	(1)	(2)	(3)	(4)
Bank financing costs, current quarter	0.718***	0.793 * * *		
	(0.061)	(0.066)		
Interaction term	. ,	-0.126		
		(0.082)		
ECB policy rate, current quarter	0.282***	0.326***		
r,	(0.031)	(0.056)		
Interaction term	(0.001)	-0.053		
interaction term		(0.080)		
T-fl-tit	0.000	(0.035)		
innation, current quarter	-0.009	0.004		
· · · · ·	(0.018)	(0.018)		
Interaction term		-0.010		
		(0.032)		
Real GDP growth, current quarter	-0.014**	-0.008*		
	(0.006)	(0.004)		
Interaction term		-0.009		
		(0.010)		
Yield curve slope, current quarter	$0.086^{***}$	0.048		
* / *	(0.019)	(0.037)		
Interaction term	· /	0.045		
		(0.043)		
Bank financing costs previous quarter		(010 20)	0.297***	0.522***
Bunk manering cootes, previous quarter			(0.065)	(0.065)
Internation term			(0.000)	0.000)
filteraction term				(0.070)
EGD II I I			0.000***	(0.079)
ECB policy rate, previous quarter			0.383***	0.636***
			(0.066)	(0.059)
Interaction term				-0.387***
				(0.091)
Inflation, previous quarter			0.041**	0.042**
			(0.018)	(0.017)
Interaction term				0.002
				(0.026)
Real GDP growth, previous quarter			0.006	-0.010*
			(0.006)	(0.005)
Interaction term			()	0.029***
				(0.009)
Yield curve slope previous quarter			0.157***	0.173***
field carte biope, pretiode quarter			(0.021)	(0.054)
Internation term			(0.021)	0.040
interaction term				(0.060)
Crisis dummu		0.008***		0.010***
Crisis dummy		(0.002)		(0.019)
<b>a</b>	0.01.0***	(0.003)	0.010***	(0.003)
Constant	0.016***	0.013***	0.019***	0.007***
	(0.002)	(0.003)	(0.002)	(0.002)
Observations	559	559	553	553
Country fixed effects	yes	yes	yes	yes
Year fixed effects	yes	yes	yes	yes
R-squared	0.883	0.884	0.838	0.855
Adj. R-squared	0.877	0.876	0.829	0.845
Degrees of freedom	528	522	522	516
Durbin-Watson statistic, post	1.718	1.800	1.761	2.013
Durbin-Watson statistic, pre	0.348	0.439	0.510	0.493

Table 3.3: Association of lending rates with bank financing costs in the euro area. Notes: Columns (1) to (4) refer to different specifications regarding current and lagged variables as well as a dummy variable for observations before and after the onset of the Great Recession. One, two, and three asterisks signal significance on a 10%, 5%, and 1% level, respectively. Numbers in parenthesis are standard errors of coefficient estimates. All specifications include country and year fixed effects.

icant. This result carries over to the estimations including the crisis dummy and its interactions.

Note that our preferred specification points towards significant differences across the two tested periods. While ECB policy rates and bank financing costs appear to be roughly equally important prior to the Great Recession, their influence drops significantly during the crisis. That is, since the beginning of the financial crisis lending rates to NFCs were less sensitive to bank financing conditions, be it financing from the central bank or from depositors.

It is also informative to look at the evolution of the coefficient on the ECB policy rates. According to this estimate, lending rates have lost some association with ECB policy rates since 2008Q3. The sum of the individual coefficient and its interaction with the crisis dummy is only about two thirds the value of the pre-crisis estimate. Contrary to the fading influence of monetary policy rates, the significant association between bank financing costs and lending rates remains intact. The sum of the individual coefficient and the crisis dummy matches the pre-crisis estimate closely. This evidence suggests that lending rates remain sensitive to bank financing costs in the deposit market as well as the ECB policy rate throughout the studied period.

## 3.7.2 Sensitivity of bank funding costs to the leverage ratio

The second main implication of our model in section 3.3 is the sensitivity of bank funding conditions to the bank leverage ratio. Recall that in our model, banks need to pay a markup over the risk-free rate that depends on their leverage ratio, that is, the ratio of total assets over equity. To test this implication, we make use of a reduced form equation for the external finance premium (the ratio of financing costs to the risk-free rate) as in BGG. According to their model, the sensitivity of the external finance premium to the leverage ratio can be approximated by the parameter  $\nu$  in

$$log(EFP_t) = \nu [log(assets_t) - log(equity_t)].$$

Our estimation is based on this reduced form. Using the above described sources, we employ log values of quarterly aggregate bank balance sheet data and the ratio of bank funding costs over the ECB policy rate. In line with the reduced form of BGG, there are no further independent variables, nor is there a constant included in the estimation. To assess robustness, country and year fixed effects are included. As above, we interact a crisis dummy with the independent variable to identify differences before and after 2008Q3. Note, however, that only this interaction term is included. In order to preserve the reduced form, the crisis dummy is not included individually. As above, we employ a Prais-Winsten procedure and correct for heteroskedasticity in the error term.

The results in table 3.4 provide support for the a positive association of bank financing conditions and the leverage ratio.<sup>31</sup> The estimates for the sensitivity of the external finance premium for banks to the leverage ratio are positive and significant, although less pronounced when controlling for country- and year-specific effects. The estimates are of similar magnitude whether the leverage ratio is included with its contemporaneous value or at its first lag. According to our estimation, the elasticity is significantly larger since 2008Q3. Depending on the specification, the elasticity of the external finance premium for banks to the leverage ratio has tripled or even only emerged during the more recent period. The increase in the relevance of the leverage ratio for the external finance premium for banks in the recent crisis is also reflected in the adjusted  $R^2$ . While only 5% of the variation in external financing costs for banks could be explained by leverage prior to 2008Q3, the

 $<sup>^{31}\</sup>mathrm{See}$  appendix 3.C for the data description, and appendix 3.D for the estimation equation and further results.

Dependent: Bank EFP	(1)	(2)	(3)	(4)	(2)	(9)	(-)	(8)
Leverage ratio, current quarter	0.202***	0.096***	0.021	-0.019				
	(0.036)	(0.019)	(0.045)	(0.052)				
Leverage ratio, previous period					0.250 * * *	$0.119^{***}$	0.066	0.023
					(0.039)	(0.021)	(0.046)	(0.044)
Interaction term		0.199 * * *		$0.191^{***}$		0.195***		0.190 * * *
		(0.011)		(0.008)		(0.010)		(0.008)
Observations	585	568	585	568	568	568	568	568
Country fixed effects	ou	ou	yes	yes	no	no	yes	yes
Year fixed effects	ou	ou	yes	yes	no	no	yes	yes
R-squared	0.048	0.366	0.434	0.585	0.061	0.365	0.414	0.583
Adj. R-squared	0.0461	0.363	0.407	0.563	0.0593	0.363	0.385	0.562
Degrees of freedom	584	566	558	540	567	566	541	540
Durbin-Watson statistic, post	1.463	1.669	2.043	1.939	1.424	1.673	2.024	1.938
Durbin-Watson statistic, pre	0.0889	0.178	0.728	0.521	0.0877	0.178	0.734	0.521

Table 3.4: Elasticity of bank financing costs with respect to the leverage ratio in the euro area. Notes: Columns (1)-(8) refer to different specifications regarding current and lagged variables, the in- or exclusion of interactions with a dummy variable for observation before and after the onset of the Great Recession and the in- or exclusion of time and country fixed effects. One, two, and three asterisks signal significance on a 10%, 5%, and 1% level, respectively. Numbers in parenthesis are standard errors of coefficient estimates.

inclusion of the interaction term raises its explanatory power to 36% overall.

# 3.8 Conclusion

Recently, policy makers have conjectured that monetary policy is not transmitted undisturbed to bank financing costs and hence to lending rates for the real economy. To shed light on the plausibility of such a scenario, we construct a business cycle model with a financial contract between depositors and banks as well as between banks and intermediate goods producers. Unlike conventional financial accelerator models, the banking sector is independent in our model and its refinancing costs vary with its leverage ratio. Moreover, banks can incur losses due to a non-state contingent lending rate between banks and intermediate firms.

Our model can reproduce a scenario where policy rate cuts due to a recessionary state of the economy are not transmitted fully or at all to bank financing costs and hence the real economy. However, the movements in bank financing costs induced by real sector shocks such as an exogenous spending, a risk, or a preference shock — are not big enough so as to have significant effects on investment or aggregate output.

Our analysis suggests that the increases in loan default rates associated with these shocks do not result in sizable endogenous losses for the banking system. Consequently, the exposure of bank net worth to aggregate downturns is small. By contrast, a shock which depletes bank bank net worth directly by a significant amount — which could be regarded as a proxy for loan losses incurred by a housing sector downturn — shows that leverage-sensitive bank financing costs have the potential to cause a significant economic downturn. In the presence of leverage sensitive bank financing costs, it is optimal for the central bank to decrease interest rates when bank financing conditions tighten. In this way, it can mitigate the incomplete transmission of movements in the policy rates to bank financing costs. Our analyses of optimal policy rules also shows that inflation stabilization remains of high importance for welfare.

We also provide suggestive evidence for two of our main model implications. First, aggregate euro area data confirm a positive relationship between bank financing costs and lending rates to the real economy. Second, bank financing costs are positively related to the bank leverage ratio. The relevance of the latter relationship seems to be more pronounced in economic downturns.

Future research could focus on endogenous mechanisms associated with large bank net worth decreases. One possibility is to incorporate additional asset classes such as housing finance which turned out to be of particular relevance in the recent economic downturn. Another possibility is to increase the sensitivity of bank funding costs to the the state of the aggregate economy more generally. In the structure of our model, this sensitivity can be either enhanced by by increasing the cyclicality and/or magnitude of intermediate goods producer default, or by augmenting the risk consciousness of the depositors. Finally, in light of the recent crisis, it would be instructive to examine the effect of leverage-sensitive bank financing costs at the zero lower bound. If the central bank can no longer adjust its nominal interest rate downward, the impact of recessions on bank financing conditions and hence aggregate economic outcomes is likely to be stronger.

# Appendix 3.A External finance premia

 $Derivation \ of \ external \ finance \ premium \ equation \ in \ the \ investor-bank \ contract$ 

The calculations below are re-arrangements of the first-order conditions from the bank's problem. For convenience, these conditions are repeated here:

$$0 = \Psi'(\bar{\chi}_{t+1}) - \Upsilon^B_t[\Psi'(\bar{\chi}_{t+1}) - \mu^B M'(\bar{\chi}_{t+1})]$$

$$0 = E_t \left\{ \frac{R^B_{t+1}}{R} \left[ 1 - \Psi(\bar{\chi}_{t+1}) \right] \right\}$$
(3.64)

$$+ \Upsilon_{t}^{B} \left[ \frac{R_{t+1}^{B}}{R_{t}} [\Psi(\bar{\chi}_{t+1}) - \mu^{B} M(\bar{\chi}_{t+1})] - 1 \right] \right\}$$

$$(3.65)$$

$$R_t \left( L_t - N_t^B \right) = \left[ \Psi \left( \bar{\chi}_{t+1} \right) - \mu^B M \left( \bar{\chi}_t \right) \right] R_{t+1}^B L_t. \quad (3.66)$$

From equation (3.64), we receive an expression for  $\Upsilon_t^B$ :

$$\Upsilon^B_t = \frac{\Psi'(\bar{\chi}_{t+1})}{\Psi'(\bar{\chi}_{t+1}) - \mu^B M'(\bar{\chi}_{t+1})},$$
(3.67)

where

$$\Psi'(\bar{\chi}_{t+1}) = 1 - F_{\chi}(\bar{\chi}_{t+1}), \qquad (3.68)$$

$$M'(\bar{\chi}_{t+1}) = \bar{\chi}_{t+1} f_{\chi}(\bar{\chi}_{t+1}).$$
(3.69)

Furthermore, we can re-arrange (3.66) to  $\left[\Psi(\bar{\chi}_{t+1}) - \mu^B M(\bar{\chi}_t)\right] = \frac{R_t \left(L_t - N_t^B\right)}{R_{t+1}^B L_t}$ , and plug it into (3.65). We receive:

$$0 = E_t \left\{ \frac{R_{t+1}^B}{R_t} \left[ 1 - \Psi \left( \bar{\chi}_{t+1} \right) \right] + \Upsilon_t^B \left[ \frac{R_{t+1}^B}{R_t} \frac{R_t \left( L_t - N_t^B \right)}{R_{t+1}^B L_t} - 1 \right] \right\}$$
(3.70)

$$E_t \left\{ \Upsilon^B_t \right\} \frac{N^B_t}{L_t} = E_t \left\{ \frac{R^B_{t+1}}{R_t} \left[ 1 - \Psi \left( \bar{\chi}_{t+1} \right) \right] \right\}$$
(3.71)

which is commonly expressed as

$$\frac{L_t}{N_t^B} = E_t \left\{ \frac{\Upsilon_t^B}{\frac{R_{t+1}^B}{R_t} \left[1 - \Psi(\bar{\chi}_{t+1})\right]} \right\} = \phi^B \left(\frac{R_{t+1}^B}{R_t}, \, \bar{\chi}_{t+1}\right).(3.72)$$

The external finance premium equation presented above is the inverted form of equation (3.72): The function  $\Phi^B$  (.) referred to in the main text is thus defined as

$$E_t \left\{ R_{t+1}^B \right\} = \Phi^B \left( \frac{L_t}{N_t^B}, \, \bar{\chi}_{t+1} \right) R_t. \tag{3.73}$$

Derivation of external finance premium equation in the bank-intermediate goods producer contract

The calculations below are re-arrangements of the first-order conditions from the intermediate firm's problem. For convenience, these conditions

are repeated here:

$$0 = \Gamma'(\bar{\omega}_{t+1}) - \Upsilon_{t}^{I} \left[ \Gamma'(\bar{\omega}_{t+1}) - \mu^{I}G'(\bar{\omega}_{t+1}) \right] (3.74) 0 = E_{t} \left\{ \frac{R_{t+1}^{k}}{R_{t+1}^{B}} \left[ 1 - \Gamma(\bar{\omega}_{t+1}) \right] + \Upsilon_{t}^{I} \left[ \frac{R_{t+1}^{k}}{R_{t+1}^{B}} \left[ \Gamma(\bar{\omega}_{t+1}) \right] - \mu^{I}G(\bar{\omega}_{t+1}) \right] - 1 \right] \right\}$$
(3.75)  
$$\left\} \left( Q_{t}k_{t} - N_{t}^{I} \right) = E_{t} \{ \left[ \Gamma(\bar{\omega}_{t+1}) \right] \right\}$$

$$E_{t} \{ R_{t+1}^{B} \} (Q_{t}k_{t} - N_{t}^{I}) = E_{t} \{ [\Gamma (\bar{\omega}_{t+1}) - \mu^{I}G(\bar{\omega}_{t+1})] R_{t+1}^{k} \} Q_{t}k_{t}.$$
(3.76)

From equation (3.74), we get  $\Upsilon_t^I = \frac{\Gamma'(\bar{\omega}_{t+1})}{\Gamma'(\bar{\omega}_{t+1}) - \mu^I G'(\bar{\omega}_{t+1})}$ , and the definition of its components is analogous to the previous section.

Again, we re-arrange the participation constraint to

$$E_t\left\{\left[\Gamma\left(\bar{\omega}_{t+1}\right) - \mu^I G\left(\bar{\omega}_{t+1}\right)\right]\right\} = \frac{E_t\left\{R_{t+1}^B\right\}\left(Q_t k_t - N_t^I\right)}{E_t\left\{R_{t+1}^k\right\}Q_t k_t} \quad (3.77)$$

and use this in (3.75):

$$0 = E_{t} \left\{ \frac{R_{t+1}^{k}}{R_{t+1}^{B}} \left[ 1 - \Gamma \left( \bar{\omega}_{t+1} \right) \right] + \Upsilon_{t}^{I} \left[ \frac{R_{t+1}^{k}}{R_{t+1}^{B}} \frac{E_{t} \left\{ R_{t+1}^{B} \right\} \left( Q_{t} k_{t} - N_{t}^{I} \right)}{E_{t} \left\{ R_{t+1}^{k} \right\} Q_{t} k_{t}} - 1 \right] \right\}$$

$$(3.78)$$

$$E_t \left\{ \Upsilon_t^I \right\} \frac{N_t^I}{Q_t k_t} = E_t \left\{ \frac{R_{t+1}^k}{R_{t+1}^B} \left[ 1 - \Gamma \left( \bar{\omega}_{t+1} \right) \right] \right\}.$$
(3.79)

With the same manipulations as above, we can write

$$E_t \left\{ R_{t+1}^K \right\} = \Phi^I \left( \frac{Q_t k_t}{N_t^I} \right) E_t \left\{ R_{t+1}^B \right\}.$$
(3.80)

### Appendix 3.B Steady state calculations

The steady state values for the variables of the model are computed as follows, using the parameter values given in table 3.1:

- 1. Using steady state inflation  $\Pi = 1$  and the discount factor  $\beta$ , one can solve for the risk-free rate  $R = \frac{\Pi}{\beta}$ .
- 2. The steady state values of the return on capital and the return on banking are calculated using the calibrated premia, that is,  $E\{R^B\} = R^B = efp^B \cdot R$  and  $E\{R^k\} = R^k = efp^I \cdot R^B$ .
- 3. To maintain a constant capital stock in the steady state, it must be that  $i = \delta k$ . Using this in the equilibrium condition of the capital producer yields the steady state price of capital  $q = \frac{1}{[1-\varphi_k(\frac{i}{k}-\delta)]} = 1$ .
- 4. With final output as the numeraire good, the price of intermediate goods is given by the inverse of the markup, that is,  $P_x = \frac{\theta 1}{\theta}$ .
- 5. To recieve a ratio of steady state output to capital, we re-arrange the definition of the return to capital  $\frac{Y}{K} = \frac{R^k 1 + \delta}{P_r \alpha}$ .
- 6. Given the calibration assumtion of  $h^h = 1$ , total labor input is given by h = 1.
- 7. The production function can be re-arranged (first taken to the power  $\frac{1}{1-\alpha}$ , then multiply by  $Y^{\alpha}$ ) to find the value for steady state output  $Y = h\left(\frac{K}{Y}\right)^{\frac{\alpha}{1-\alpha}}$ .
- 8. Thus, the steady state value of the capital stock is  $K = \frac{K}{Y}Y$ .

- 9. The steady state value of intermediate net worth can then be calculated as  $N^{I} = \frac{N^{I}}{OK}K$ .
- 10. This allows us to calculate the steady state loan volume  $L = QK N^I$ .
- 11. This in turn gives the steady state bank net worth using the calibrated leverage ratio  $N^B = \frac{N^B}{L}L$ .
- 12. Steady state deposits are then  $D = L N^B$ .
- 13. The steady state revenues are calculated using the calibrated values for cut-off productivity, that is,  $V^{I} = [1 \Gamma(\bar{\omega})] R^{k} K$  and  $V^{B} = [1 \Psi(\bar{\chi})] R^{B} L.$
- 14. Wages are given by their marginal product  $w^{j} = (1 \alpha) \Omega^{j} p_{x} Y$ .
- 15. This yields the survival rates  $\gamma^B = \frac{N^B w^B}{V^B}$  and  $\gamma^I = \frac{N^I w^I}{V^I}$ .
- 16. With this, we can calculate steady state bank and intermediate goods producer consumption  $C^{j} = (1 \gamma^{j}) V^{j}, j = \{B, I\}.$
- 17. Investment and government consumption are given by  $I = \delta K$ and  $G = \tau Y$ .
- 18. Household consumption can then be computed via the budget constraint  $C = Y C^I C^B G I \mu^B F(\bar{\chi}) R^B L \mu^I F(\bar{\omega}) R^k Q K$ .
- 19. What remains is the Lagrange multiplier of the household  $\Lambda = C^{-\sigma}$ .
- 20. The final piece is the disutility weight of labor  $\psi = \frac{(1-\alpha)\Omega\Lambda p_x Y}{(h^h)^{1+\nu}}$ .
# Appendix 3.C Data description and summary statistics

The data used to provide empirical evidence on the association of bank financing costs with lending rates and bank financing costs with the leverage ratio consist of observations on the national level for all euro area members. The time series start in 2003Q1 as the deposit rate is not available for prior periods. For countries that have joined the monetary union after 2003Q1, only observations after their respective date of entrance were included. Data on real GDP is only available until 2012Q4 thus limiting the time span of the econometric analysis. Real GDP and inflation have been transformed into annual growth rates. No further transformations have been applied to the data. The following list gives details about the variables used:

- **Bank assets** Country-specific, quarterly values for total assets and stated composition taken from the ECB MFI balance sheet statistics.
- **Bank liabilities** Country-specific, quarterly values for total assets and stated composition taken from the ECB MFI balance sheet statistics.
- **External finance premium for banks** Country-specific, quarterly values calculated as the ratio of bank financing costs over the ECB MRO rate.
- **Bank leverage** Country-specific, quarterly values calculated as total liabilities over "capital and reserves".
- **ECB MRO rate** Quarterly values for official MRO rates taken from Eurostat (Database ID: irt\_cb\_q).
- **Bank financing costs** Country-specific, annualized agreed percentage rate on new business in deposits with agreed maturity sourced from non-financial corporations, households or non-profit institu-

tions. Deposits of all maturities included. Monthly data taken from Eurostat (Database ID: irt\_rtl\_dep\_m).

- Inflation Country-specific, simple averages of monthly values for Harmonized Index of Consumer Prices taken from Eurostat (Database ID: prc\_hicp\_midx).
- Lending rate Country-specific, annualized agreed percentage rate on new business in loans to non-financial companies. Loans of all maturities included. Monthly data taken from Eurostat (Database ID: irt\_rtl\_lnfc\_m).
- **Real GDP** Country-specific, quarterly values of seasonally adjusted data taken from Eurostat (Database ID: namq\_gdp\_c).
- **Slope of the yield curve** Country-specific, quarterly values for central government bond yields with a residual maturity of 10 years taken from Eurostat (Database ID: irt\_euryld\_q). The slope of the yield curve is constructed as the difference between the bond yields and the ECB policy rate.

Variable	Ν	Mean	Std. dev.	Min.	Max.
EFP for banks	585	1.795	1.028	0.657	6.031
Bank leverage ratio	585	15.809	5.296	4.873	29.117
Lending rate	585	0.047	0.013	0.021	0.078
Bank financing costs	585	0.028	0.009	0.006	0.054
ECB MRO rate	585	0.020	0.011	0.008	0.043
Inflation	585	0.023	0.013	-0.028	0.065
Real GDP growth	567	0.032	0.040	-0.107	0.139
Yield curve slope	576	0.024	0.026	-0.001	0.244

 Table 3.5:
 Summary statistics for selected variables.
 Notes:
 EFP abbreviates

 external finance premium.

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## Appendix 3.D Estimation equations and additional results

Association between bank financing costs and lending rates The full estimation equation is

$$\begin{split} R_{i,t}^{L} &= \beta_{0} + \beta_{1}R_{i,t(-1)}^{D} + \beta_{2}R_{t(-1)} + \beta_{3}\pi_{i,t(-1)} + \beta_{4}\Delta y_{i,t(-1)} \\ &+ \beta_{5}R_{i,t(-1)}^{GOV} + \beta_{6}\left[crisis \times R_{i,t(-1)}^{D}\right] + \beta_{7}\left[crisis \times R_{t(-1)}\right] \\ &+ \beta_{8}\left[crisis \times \pi_{i,t(-1)}\right] + \beta_{9}\left[crisis \times \Delta y_{i,t(-1)}\right] \\ &+ \beta_{10}\left[crisis \times R_{i,t(-1)}^{GOV}\right] + \beta_{11}country_{i} + \beta_{12}year_{j} + \epsilon_{i,t}. \end{split}$$

Table 3.6 displays the results for the different specifications. All models use the stated Prais-Winsten estimator and differ along the inclusion of country and year fixed effects.

Association between bank financing costs and leverage ratio The full estimation equation is

$$\begin{split} log\left(\frac{R_{i,t}^{D}}{R_{t}}\right) &= \beta_{0}log\left(\frac{liabilities_{i,t(-1)}}{capital_{i,t(-1)}}\right) \\ &+ \beta_{1}\left[crisis \times log\left(\frac{liabilities_{i,t(-1)}}{capital_{i,t(-1)}}\right)\right] \\ &+ \beta_{2}country_{i} + \beta_{3}year_{j} + \epsilon_{i,t}. \end{split}$$

Table 3.7 displays the results for different specifications. All models use the Prais-Winsten estimator and differ along the inclusion of country and year fixed effects.

Bank financing costs, c. q. $0.366^{***}$ $0.718^{***}$ $0.718^{***}$ $0.718^{***}$ ECB policy rate, c. q. $(0.06)$ $(0.06)$ $(0.06)$ $(0.031)$ Inflation, c. q. $(0.038)$ $(0.037)$ $(0.012)$ $(0.013)$ Inflation, c. q. $(0.038)$ $(0.037)$ $(0.013)$ $(0.013)$ Real GDP growth, c. q. $(0.017)$ $(0.018)$ $(0.013)$ $(0.013)$ Yield curve slope, c. q. $(0.006)$ $(0.013)$ $(0.013)$ $(0.031)$ Bank financing costs, p. q. $(0.021)$ $(0.020)$ $(0.013)$ $(0.070)$ Bank financing costs, p. q. $(0.022)$ $(0.020)$ $(0.006)$ $(0.007)$ Bank financing costs, p. q. $(0.021)$ $(0.020)$ $(0.006)$ $(0.007)$ Bank financing costs, p. q. $(0.022)$ $(0.020)$ $(0.020)$ $(0.006)$ $(0.007)$ Bank financing costs, p. q. $(0.020)$ $(0.020)$ $(0.006)$ $(0.007)$ Inflation, p. q. $(0.020)$ $(0.020)$ $(0.006)$ $(0.007)$ <	Dependent: Lending rate to NFCs	(1)	(2)	(3)	(4)	(2)	(9)	(2)	(8)
$ \begin{array}{llllllllllllllllllllllllllllllllllll$	Bank financing costs, c. q.	0.868*** (0.069)	0.836***	0.718*** (0.066)	0.718*** (0.061)				
Inflation, c. q. $(0.038)$ $(0.033)$ $(0.034)$ $(0.034)$ Real GDP growth, c. q. $(0.017)$ $(0.018)$ $(0.018)$ $(0.018)$ $(0.018)$ Yield curve slope, c. q. $0.006$ $(0.013)$ $(0.013)$ $(0.013)$ $(0.013)$ Bank financing costs, p. q. $0.006$ $(0.006)$ $(0.006)$ $(0.007)$ $(0.070)$ Bank financing costs, p. q. $(0.023)$ $(0.021)$ $(0.013)$ $(0.070)$ $(0.070)$ Bank financing costs, p. q. $(0.022)$ $(0.021)$ $(0.013)$ $(0.070)$ $(0.070)$ Bank financing costs, p. q. $(0.022)$ $(0.021)$ $(0.013)$ $(0.070)$ $(0.017)$ Bank financing costs, p. q. $(0.023)$ $(0.020)$ $(0.020)$ $(0.070)$ $(0.070)$ $(0.070)$ Bank financing costs, p. q. $(0.021)$ $(0.020)$ $(0.020)$ $(0.070)$ $(0.017)$ Real GDP growth, p. q. $(0.020)$ $(0.020)$ $(0.020)$ $(0.004)$ $(0.070)$ Yield curve slope, p. q. $(0.002)$	ECB policy rate, c. q.	0.321***	0.322***	0.295***	0.282***				
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$		(0.038) 0.012	(0.037) 0.016	(0.029)-0.011	(0.031) - 0.009				
Real GDP growth, c. q.         0.006         0.005         0.007         0.001         0.005         0.001         0.00	Inflation, c. q.	(0.017)	(0.018) 0.000	(0.018)	(0.018)				
Yield curve slope, c. q. $0.067^{***}$ $0.070^{***}$ $0.086^{***}$ $0.086^{***}$ Bank financing costs, p. q. $(0.025)$ $(0.021)$ $(0.020)$ $(0.019)$ $(0.073)$ Bank financing costs, p. q. $(0.025)$ $(0.021)$ $(0.020)$ $(0.019)$ $(0.073)$ ECB policy rate, p. q. $(0.021)$ $(0.020)$ $(0.073)$ $(0.071)$ $(0.071)$ Halton, p. q. $(0.021)$ $(0.020)$ $(0.016)$ $(0.061)$ $(0.061)$ Real GDP growth, p. q. $(0.023)$ $(0.016)$ $(0.016)$ $(0.017)$ Yield curve slope, p. q. $0.016^{***}$ $0.024^{***}$ $0.026^{***}$ $0.026^{***}$ Constant $(0.003)$ $(0.002)$ $(0.002)$ $(0.002)$ $(0.002)$ Observations $590^{***}$ $0.024^{***}$ $0.024^{***}$ $0.026^{***}$ $0.026^{***}$ Constant $(0.003)$ $(0.002)$ $(0.022)$ $(0.022)$ $(0.022)$ $(0.022)$ $(0.022)$ $(0.022)$ $(0.022)$ $(0.022)$ $(0.022)$ $(0$	Real GDP growth, c. q.	(0.006)	(0.006)	(900.0)	(900.0)				
Bank financing costs, p. q. $0.037$ Bank financing costs, p. q. $0.037$ ECB policy rate, p. q. $0.0703$ Inflation, p. q. $0.0703$ Inflation, p. q. $0.0703$ Real GDP growth, p. q. $0.0169$ Wield curve slope, p. q. $0.015$ Constant $0.003$ Constant $0.003$ Constant $0.002$ Constant $0.002$ Const	Yield curve slope, c. q.	0.067*** (0.025)	0.070*** (0.021)	0.088***	0.086***				
Constant $(0.073)$ $(0.017)$ $(0.0102)$ <	Bank financing costs, n. d.					0.048	0.087	0.275***	0.297***
ECB policy rate, p. q. $(0.06)$ $(0.06)$ $(0.06)$ $(0.06)$ $(0.06)$ $(0.06)$ $(0.06)$ $(0.016)$ $(0.016)$ $(0.016)$ $(0.016)$ $(0.016)$ $(0.017)$ Real GDP growth, p. q. $(0.016)$ $(0.016)$ $(0.017)$ $(0.016)$ $(0.017)$ Yield curve slope, p. q. $(0.015***********************************$						(0.070) 0.700***	(0.073) 0.705***	(0.072) 0.367***	(0.065) 0.383***
Inflation, P. q.         0.050***         0.044*         0.044*         0.044*         0.044*         0.044*         0.044*         0.044*         0.044*         0.044*         0.044*         0.044*         0.044*         0.044*         0.044*         0.044*         0.015         0.004         0.0015         0.0016         0.0015         0.0016         0.0017         0.0015         0.0017         0.0015         0.0017         0.0017         0.0021         0.0021         0.0021         0.0021         0.0021         0.0021         0.0021         0.0021         0.0016         0.0021         0.0016*         0.0016*         0.0016*         0.0016*         0.0016*         0.0021         0.0021         0.0021         0.0021         0.0021         0.0021         0.0021         0.0016*         0.0016*         0.0016*         0.0016*         0.0016*         0.0021         0.0021         0.0021         0.0021         0.0021         0.0021         0.0016*         0.0016*         0.0016*         0.0016*         0.0016*         0.0016*         0.0016*         0.0016*         0.0021         0.0021         0.0021         0.0016*         0.0016*         0.0016*         0.0016*         0.0016*         0.0016*         0.0016*         0.0016*         0.0016*         0.0016*	ECB policy rate, p. q.					(0.066)	(0.061)	(0.068)	(0.066)
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	Inflation a c					0.050***	0.046***	0.047**	$0.041^{**}$
Real GDP growth, p. q.         0.004         0.005         0.0064         0.005           Yield curve slope, p. q.         0.005         0.006         0.0064         0.006           Yield curve slope, p. q.         0.0167***         0.0107           Constant         0.028***         0.0107         0.028***         0.0107           Constant         0.0216***         0.016***         0.0167***         0.0167           Constant         0.0228***         0.0168**         0.0167         0.0167           Constant         0.0228***         0.0163**         0.0167         0.002           Constant         0.016**         0.016**         0.0167**         0.0167           Constant         0.002         0.0023         0.0168**         0.0167         0         0         0         0         0         0         0         0         0	Initation, p. q.					(0.016)	(0.017)	(0.019)	(0.018)
Note of the curve slope, p. q. $(0.007)$ $(0.007)$ $(0.007)$ $(0.007)$ $(0.007)$ $(0.007)$ $(0.007)$ $(0.007)$ $(0.007)$ $(0.002)$ $(0.002)$ $(0.022)$ $(0.022)$ $(0.021)$ $(0.022)$ $( 0.022)$ $( 0.022)$ $( 0.022)$ $( 0.022)$ $( 0.022)$ $( 0.022)$ $( 0.022)$ $( 0.022)$ $( 0.022)$ $( 0.022)$ $( 0.022)$ $( 0.022)$ $( 0.022)$ $( 0.022)$ $( 0.022)$ $( 0.022)$ $( 0.022)$ $( 0.022)$ <td>Real GDP growth n c</td> <td></td> <td></td> <td></td> <td></td> <td>0.004</td> <td>0.002</td> <td>0.009</td> <td>0.006</td>	Real GDP growth n c					0.004	0.002	0.009	0.006
Yield curve slope, p. q. $0.167^{***}_{-10}$ $0.167^{***}_{-10}$ $0.167^{***}_{-10}$ $0.167^{***}_{-10}$ $0.167^{***}_{-10}$ $0.167^{***}_{-10}$ $0.127^{***}_{-10}$ $0.127^{***}_{-10}$ $0.127^{***}_{-10}$ $0.016^{***}_{-10}$ $0.002^{**}_{-10}$ $0.002$	TROUT CIPT STOWITH, P. 4.					(0.006)	(0.007)	(0.006)	(0.006)
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	Yield curve slope, p. q.					0.167***	0.177***	0.144***	0.157***
$\begin{array}{c c c c c c c c c c c c c c c c c c c $	i	0.015***	0.009***	0.024***	0.016***	0.026***	0.016***	0.030***	0.019***
Observations         559         559         559         559         553 <t< td=""><td>Constant</td><td>(0.003)</td><td>(0.002)</td><td>(0.003)</td><td>(0.002)</td><td>(0.002)</td><td>(0.002)</td><td>(0.002)</td><td>(0.002)</td></t<>	Constant	(0.003)	(0.002)	(0.003)	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)
Country fixed effects         no         yes         no         yes         no         yes         no         yes         no         yes         no         yes         no         no <t< td=""><td>Observations</td><td>559</td><td>559</td><td>559</td><td>559</td><td>553</td><td>553</td><td>553</td><td>553</td></t<>	Observations	559	559	559	559	553	553	553	553
Year fixed effects         no         no         no         no           R-squared         0.734         0.864         0.789         0.883         0.672         0.825           Adi, R-squared         0.732         0.859         0.7789         0.877         0.669         0.818	Country fixed effects	no	yes	no	yes	no	yes	no	yes
R-squared 0.734 0.864 0.789 0.883 0.672 0.825 Adj. R-squared 0.732 0.859 0.783 0.877 0.669 0.818	Year fixed effects	no	ou	yes	yes	ou	no	yes	yes
Adi. R-squared 0.732 0.859 0.783 0.877 0.669 0.818	R-squared	0.734	0.864	0.789	0.883	0.672	0.825	0.700	0.838
	Adj. R-squared	0.732	0.859	0.783	0.877	0.669	0.818	0.692	0.829
Degrees of freedom         553         538         543         528         547         532	Degrees of freedom	553	538	543	528	547	532	537	522
Durbin-Watson statistic, post 1.990 1.888 1.769 1.718 1.740 1.683	Durbin-Watson statistic, post	1.990	1.888	1.769	1.718	1.740	1.683	1.788	1.761
Durbin-Watson statistic, pre 0.106 0.421 0.0934 0.348 0.115 0.369	Durbin-Watson statistic, pre	0.106	0.421	0.0934	0.348	0.115	0.369	0.117	0.510

Table 3.6: Association of lending rates with bank financing costs in the euro area, additional results. Notes: Columns (1) to (8) refer to different specifications regarding current and lagged variables as well as the in- or exclusion of country and time fixed effects. One, two, and three asterisks signal significance on a 10%, 5%, and 1% level, respectively. Numbers in parenthesis are standard errors of coefficient estimates. c. q. and p. q. abbreviate current quarter and previous quarter, respectively.

Dependent: Bank EFP	(1)	(2)	(3)	(4)	(5)	(9)	(2)	(8)
leverage ratio (log), c. q.	0.202***	0.142	$0.094^{***}$	0.021	0.096***	-0.024	0.053***	-0.019
	(0.036)	(0.107)	(0.020)	(0.045)	(0.019)	(0.061)	(0.018)	(0.052)
Interaction term					0.199***	0.206***	0.190***	$0.191^{***}$
					(0.011)	(0.011)	(0.00)	(0.008)
Observations	585	585	585	585	568	568	568	568
Country fixed effects	ou	yes	ou	yes	no	yes	no	yes
Year fixed effects	no	ou	yes	yes	no	no	yes	yes
R-squared	0.048	0.073	0.258	0.434	0.366	0.442	0.494	0.585
Adj. R-squared	0.0461	0.0450	0.243	0.407	0.363	0.423	0.483	0.563
Degrees of freedom	584	568	574	558	566	550	556	540
Durbin-Watson statistic, post	1.463	1.463	2.022	2.043	1.669	1.674	1.929	1.939
Durbin-Watson statistic, pre	0.0889	0.121	0.390	0.728	0.178	0.277	0.294	0.521

to different specifications regarding the in- or exclusion of interactions with a dummy variable for observation before and after the onset of the Great Recession and the in- or exclusion of time and country fixed effects. One, two, and three asterisks signal significance on a 10%, 5%, and 1% level, respectively. Numbers in parenthesis are standard errors of coefficient estimates. c. q. Table 3.7: Elasticity of bank financing costs with respect to the leverage ratio, additional results. Notes: Columns (1)-(8) refer abbreviates current quarter.

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#### Curriculum Vitae - Daniel Kienzler

Born April 28, 1982 in Blaubeuren (Germany); married, one child

#### Education

08/2008 - 06/2014	PhD, Economics University of St. Gallen, Switzerland
01/2013 - 05/2014	Visiting PhD student, Economics Columbia University, New York, USA
01/2009 - 01/2010	Swiss Program for Doctoral Students in Economics Study Center Gerzensee, Switzerland
10/2002 - 06/2008	Diplom, Economics University of Hohenheim, Germany
08/2005 - 06/2006	Exchange student, Economics McMaster University, Hamilton, Canada
Work experience	
08/2008 - 12/2012	Research and teaching assistant University of St. Gallen, Switzerland
05/2011 - 10/2011	Traineeship European Central Bank, Frankfurt, Germany
04/2007 - 12/2007	Research assistant Stuttgart Financial, Stuttgart, Germany
06/2006 - 08/2006	Internship ZEW, Mannheim, Germany
10/2003 - 08/2005	Student research assistant University of Hohenheim, Germany