Essays on Performance Measurement, Risk Valuation, and Regulation in the Insurance Sector

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Abstract

The European insurance industry has experienced significant changes over recent years – natural disasters, the recent financial crisis, and new regulatory and accounting frameworks. These changes make insurance an interesting and timely field of research. This thesis comprises four research papers which seek to find economics-based answers to several open questions regarding performance measurement, risk valuation, and regulation in the insurance industry.

First, participating life insurance contracts, which are very common in the European life insurance market, are examined. In addition to providing term life insurance and a minimum interest rate guarantee, these policies include bonus participation rules regarding the insurer's profit. Researchers model these bonus policies in very different ways. We present the most common participating life insurance bonus distribution mechanisms – which closely mirror the Danish, German, UK, and Italian regulatory frameworks – and perform a comparative analysis of these different models with regard to risk valuation (see Part I). We gain valuable insights into the risk associated with different bonus distribution policies. Then, as very little research has been conducted into the performance of participating life insurance contracts, we conduct a performance analysis based on contracts offered in the German market, in order to provide evidence to support decision making by policyholders (see Part II).

Second, we move from this specific insurance product to examine insurance regulation. We conduct an in-depth analysis of current regulatory developments in the European insurance industry. We focus on IFRS 4 Phase II, Solvency II, Market Consistent Embedded Value, and insurance guaranty schemes. We present these four frameworks, analyze them from different stakeholder perspectives, and compare and contrast them (see Part III). Our results suggest that the four frameworks need to be considered jointly rather than separately, due to various interrelations and interactions. Next, we turn to one particular current regulatory issue – insurance guaranty schemes. In the context of current insurance guaranty and deposit insurance schemes, we propose a capital market-based financial guaranty system for the financial service industry. Closed-form solutions for the input parameters are derived and the major advantages and disadvantages are analyzed (see Part IV). This analysis provides new insights into the possibility of the practical implementation of such a guaranty system.

Zusammenfassung

Die Europäische Versicherungsindustrie hat in den vergangenen Jahren zahlreiche Veränderungen erlebt. Zu nennen sind zum Beispiel Naturkatastrophen, die letzte Finanzmarktkrise und neue regulatorische Vorschriften. Diese Dynamik macht das Themengebiet Versicherung zu einem interessanten und wichtigen Forschungsfeld. Die vorliegende Dissertation enthält vier Forschungsarbeiten, die darauf abzielen, ökonomische Antworten auf verschiedenen offene Fragen im Bereich der Performancemessung, der Risikobewertung und der Regulierung in der Versicherungsindustrie zu geben.

Die ersten zwei Teile befassen sich mit der gemischten Kapitallebensversicherung. Die gemischte Kapitallebensversicherung ist ein Versicherungsprodukt, welches im Europäischen Lebensversicherungsmarkt weitverbreitet ist und sich aus einer Risikolebensversicherung, einer garantierten Mindestverzinsung sowie einem Überschussbeteiligungsmechanismus zusammensetzt. Wir stellen die gängigsten Überschussbeteiligungsmechanismen dar – welche die dänische, die deutsche, die britische und die italienische regulatorische Praxis widerspiegeln – und führen eine vergleichende Analyse zur Risikobewertung durch (Teil I). Wir gewinnen dabei Einblicke in die Risiken, welche die verschiedenen Überschussbeteiligungsmechanismen mit sich bringen. Da sich die Forschung bisher kaum mit der Performance von Kapitallebensversicherungsverträgen auseinandergesetzt hat, führen wir in Teil II eine Performanceanalyse durch, um Versicherungsnehmern eine Entscheidungshilfe zur Verfügung zu stellen. Wir betrachten dabei einen Vertragstypus, der in Deutschland üblich ist.

Im dritten und vierten Teil verlassen wir den Bereich der Lebensversicherung und konzentrieren uns auf Versicherungsregulierung. Wir führen eine detaillierte Analyse aktueller regulatorischer Entwicklungen in der Europäischen Versicherungsindustrie durch. Dabei fokussieren wir uns auf IFRS 4 Phase II, Solvency II, Market Consistent Embedded Value und Versicherungsgarantiefonds. Wir erklären die vier Konzepte, analysieren sie aus der Perspektive verschiedener Anspruchsgruppen und stellen sie einander gegenüber (Teil III). Unsere Ergebnisse implizieren, dass die vier Konzepte wegen zahlreicher Wechselbeziehungen gemeinsam betrachtet werden müssen. Im nächsten Schritt konzentrieren wir uns auf eines dieser regulatorischen Konzepte – Versicherungsgarantiefonds. Vor dem Hintergrund heutiger Insolvenzgarantiefonds für Versicherungsunternehmen und der gegenwärtigen Einlagensicherung bei Banken schlagen wir ein kapitalmarktorientiertes Garantiesystem für die Finanzindustrie vor. Wir leiten geschlossene Lösungen für eine Basiskalibrierung her und betrachten wesentliche Vor- und Nachteile (Teil IV). Diese Analyse liefert neue Einblicke hinsichtlich der praktischen Implementierbarkeit eines solchen kapitalmarktbasierten Garantiesystems.

Introduction

Motivation and Objective

Over recent years, the insurance industry has faced various challenges. Natural disasters have increased the frequency and size of claims. The recent financial crisis revealed major gaps in current insurance regulatory systems. Finally, new regulatory frameworks – such as Solvency II in the European Union and the Swiss Solvency Test (SST) in Switzerland, as well as changes in the International Financial Reporting Standards (IFRS) – require new levels of compliance. All these changes in the market make insurance an interesting and timely field of research.

At the same time, the valuation and risk management of insurance contracts become an increasingly important area of academic research. In particular, participating life insurance contracts, which embed various, primarily path-dependent, options, have drawn attention due to their complexity.

This leads to the the two main research areas of this thesis. Firstly, we focus on participating life insurance contracts and analyze them in two different regards. On the one hand, we perform a comparative analysis of different bonus distribution policies with regard to risk (Part I). On the other hand, we focus on participating contracts sold in Germany and conduct an extensive performance analysis (Part II). Secondly, we turn to insurance regulation. We conduct an in-depth analysis of current regulatory developments in the European insurance industry (Part III). In addition, we propose a capital market-based financial guaranty system and analyze its major advantages and disadvantages (Part IV).

Areas of Research and Major Contributions

This thesis contains four research papers which seek to find economics-based answers to several open questions regarding performance measurement, risk valuation, and regulation in the insurance sector.

Part I examines different bonus distribution mechanisms found in participating life insurance contracts. These insurance contracts usually comprise term life insurance, a minimum interest rate guarantee, and bonus participation rules regarding the insurer's profit. The embedded bonus distribution mechanisms are replicated quite differently in research. Part I categorizes and presents in formal terms the most common bonus participation rules, which closely mirror the Danish, German, UK, and Italian regulatory frameworks. Subsequently, a comparative analysis of the different bonus models is performed with regard to risk valuation. To do this, we calibrate contract parameters such that the contracts compared have a net present value of zero and the same safety level as the initial position using risk-neutral valuation. Subsequently, we analyze the effect of changes in asset volatility and in the initial reserve amount (per contract) on the value of the default put option (DPO), while holding all other parameters constant.

We contribute to the literature by introducing a new method for calibrating participating life insurance contracts in order to compare model risks of different bonus distribution models. Only Gatzert and Kling (2007) provide a framework that allows a comparison of participating contracts which embed a cliquet-style option. They analyze shortfall probability measures with regard to three bonus models. However, although they identify key risk drivers, they avoid a direct comparison across the different models. On the contrary, we propose a new method for calibrating participating life insurance contracts which allows us to directly compare the risk involved in the different bonus distribution models. In addition, by keeping our initial calibration fixed, we can isolate effects caused by changes in the underlying asset volatility and the initial reserve situation (i.e., we can directly assess model risks).

Our results show that DPO values obtained with the bonus distribution model of Bacinello (2001), which mirrors the Italian regulatory framework, are most sensitive to changes in volatility and initial reserves.

Part II also examines participating life insurance contracts. As very little research has been conducted into the performance of this kind of product, a performance analysis is conducted in order to provide evidence to support decision making of policyholders. To achieve this, we break down a participating life insurance contract into a term life insurance and a savings component and simulate the cash flow distribution of the latter. The result of the simulation is compared to the cash flows resulting from two benchmarks investing into the same portfolio but without an investment guarantee or bonus distribution scheme, in order to measure the impact of these two product features. To provide a realistic view of the two alternatives, transaction costs and wealth transfer effects between policyholders are controlled for.

The contribution is that we neither rely on a single performance measurement ratio nor do we provide an ex post analysis. Instead, the introduced framework allows a comparison of the complete payoff distribution on an ex ante basis. This general approach is subsequently not bonded to one specific subjective preference scheme. Further, we model an insurance company with various insurance collectives in order to incorporate wealth transfer effects between different groups of policyholders. Only Hansen and Miltersen (2002) analyzed participating life insurance contracts with pooled accounts before, but just for a two-policyholders case.

We show how the payoff distribution depends heavily on the initial reserve situation and managerial discretion. These results suggest that expected performance is, in general, difficult for policyholders to assess.

In Part III, we turn to insurance regulation. We perform an in-depth analysis of current regulatory and reporting developments within the European insurance industry. We focus on the four primary frameworks, namely the solvency framework Solvency II, insurance guaranty systems, the proposed IFRS 4 Phase II international accounting standards, and Market Consistent Embedded Value reporting. We present each framework and analyze it from different stakeholder perspectives. Then, we compare and contrast all four frameworks.

The contribution of Part III is twofold. On the one hand, we present a comprehensive overview of four far-reaching regulatory and reporting reforms in Europe. On the other hand, we compare and contrast these frameworks, analyze them from different stakeholder perspectives, and point out major similarities and differences. Thereby, we combine results of important publications found in industry and research as well as our own point of view. Although some authors address the relation between Solvency II and IFRS 4 Phase II (see, e.g., Duverne and Le Douit, 2009), between IFRS 4 Phase II and MCEV (see, e.g., De Mey, 2009), or between Solvency II and insurance guaranty systems (see, e.g., Rymaszewski and Schmeiser, 2011), there is no comprehensive comparison of all these frameworks. In addition, the different stakeholder perspectives on the frameworks are, in general, not taken into account.

We find that the benefits of the different regulatory frameworks need to justify the corresponding costs. If this is not the case, European insurers will be at a competitive disadvantage to less regulated markets. Coordinate introduction will be necessary to ensure that the regulatory burden is reduced and synergies can be utilized. To overcome difficulties with the planned frameworks, we propose a more holistic, comprehensive approach to insurance reporting and regulation.

In Part IV, a capital market-based financial guaranty system for the financial service industry is proposed as an alternative to current deposit insurance and insurance guaranty schemes. The proposed guaranty system secures clients' claims in the event of the default of a financial company by means of a special purpose vehicle which issues bonds to investors (similar to catastrophe bonds or other insurance-linked securities). In a first step, closed-form solutions for the input parameters are derived. Subsequently, we analyze the impact of different investment actions which might be taken by the financial companies protected by the guaranty vehicle.

We contribute to the literature by providing a detailed proposal of how a capital market-based financial guaranty system can be established. In addition, we assess the effectiveness of the proposed system by means of analyzing actions financial companies might take to lever out the guaranty system. By deriving practical implications from the numerical analysis, new insights into whether a transfer of default risk to capital markets could be feasible are delivered.

We find that if the scope of investors can be restricted, the capital market-based financial guaranty systems could be a good solution for clients in respect to the described default problem of a financial institute.

Finally, we conclude this doctoral thesis with a summary of main results.

Part I

Risk Comparison of Different Bonus Distribution Approaches in Participating Life Insurance

Abstract

The fair pricing of explicit and implicit options in life insurance products has got broad attention in the academic literature over the past years. Participating life insurance (PLI) contracts have been the focus especially. These policies are typically characterized by a term life insurance, a minimum interest rate guarantee, and bonus participation rules with regard to the insurer's asset returns or reserve situation. Researchers replicate these bonus policies quite differently. We categorize and formally present the most common PLI bonus distribution mechanisms. These bonus models closely mirror the Danish, German, UK, and Italian regulatory framework. Subsequently, we perform a comparative analysis of the different bonus models with regard to risk valuation. We calibrate contract parameters so that the compared contracts have a net present value of zero and the same safety level as the initial position, using risk-neutral valuation. Subsequently, we analyze the effect of changes in the asset volatility and in the initial reserve amount (per contract) on the value of the default put option (DPO), while keeping all other parameters constant. Our results show that DPO values obtained with the PLI bonus distribution model of Bacinello (2001), which replicates the Italian regulatory framework, are most sensitive to changes in volatility and initial reserves.[1]

[1] A. Zemp, 2011. Risk Comparison of Different Bonus Distribution Approaches in Participating Life Insurance. *Insurance: Mathematics and Economics*, 49(2): 249-264.

1 Introduction

In recent years, pricing and fair valuation of explicit and implicit options in life insurance products have got broad attention in the academic literature and by practitioners. In particular, participating life insurance (PLI hereafter) contracts have been the focus. Besides embedding a term life insurance and a minimum interest rate guarantee, these policies comprise bonus participation rules regarding the insurer's profit. Researchers replicate these bonus policies differently, depending on the regulatory framework that the model is applied to and on the research objective. A comparative analysis of these bonus models with respect to risk appears to be difficult, since designs differ greatly. Thus, earlier literature mainly focuses on risk analysis with regard to one bonus distribution model while avoiding a direct comparison across different schemes. However, changing market conditions, for instance a change in asset volatility, may affect these models quite differently. Similarly, inaccurate parameter estimations (parameter uncertainty) could have a stronger impact on one model than on the others. Thus, a direct comparison of bonus distribution models can provide important insight, in particular for regulators. Regulators can identify model risks that they impose on insurance companies which could be less pronounced with bonus distribution models found in other countries. As a consequence, regulators may reconsider the bonus distribution approach chosen and adopt one that appears to be less sensitive to parameter estimations.

In this paper, we categorize and present the most common PLI bonus distribution mechanisms. Subsequently, we perform a comparative analysis of the different bonus models with regard to risk. We calibrate contract parameters so that the compared contracts have a net present value of zero and the same safety level under the risk-neutral probability measure $\mathbb{Q}$ as initial position. The safety level is defined as the expected present value of the default put option (DPO hereafter) which corresponds to the situation in which a regulatory authority prescribes a certain safety level (e.g., Solvency II or the Swiss Solvency Test). Based on this parameterization, we derive sensitivities of the DPO value regarding the underlying asset volatility as well as the company's reserve situation.

Our results provide new insight for insurance companies, as well as regulators, on the risk involved in different bonus distribution models. Using the results of our analysis and/or applying our method of comparison will support regulatory authorities (as well as insurance companies) in selecting a bonus distribution model whose default risk is less sensitive to parameter estimations, i.e., to identify a bonus model on which parameter uncertainty has less impact. In particular, our analysis evaluates model risks associated with the misspecification of the underlying asset volatility for the different PLI bonus modeling approaches. Similarly, we compare to what extent a growing pool size – which will naturally reduce the amount of reserves per contract if no increase in equity capital takes place – influences the default risk in these bonus modeling approaches.

The field of fair valuation of PLI contracts has been researched extensively. In their basic setting, most PLI models work with single-premium contracts, whereas the policyholder is assumed to continue until maturity (i.e., does not die or surrender). As a common factor in European countries (e.g., Germany, Denmark, Switzerland) we focus on bonus models which embed a cliquet style interest rate guarantee and some kind of bonus distribution mechanism. The most fundamental and frequently applied models in this area are the ones introduced by Bacinello (2001), Haberman, Ballotta, and Wang (2003), Grosen and Jørgensen (2000), Hansen and Miltersen (2002), and Kling, Richter, and Ruß (2007). We do not analyze the bonus models of Barbarin and Devolder (2005) and of Briys and de Varenne (1997), since they involve a point-to-point guarantee, not a cliquet-style one. Furthermore, we do not focus on Albizzati and Geman (1994), Tanskanen and Lukkarinen (2003), and Kleinow (2009). Albizzati and Geman (1994) do not incorporate any bonus distribution mechanism while Tanskanen and Lukkarinen (2003) and Kleinow (2009) derive general PLI models for fair valuation which allow for implementing different bonus policies.

Bacinello (2001) introduces a model of PLI based on the Italian regulatory framework. Her basic model features a bonus distribution scheme based on the annual return on a reference portfolio. The bonus model does not incorporate any reserve building/ profit stabilization mechanism and is therefore relatively elementary. Bacinello (2003) additionally embeds a surrender option.

Unlike Bacinello (2001), Haberman et al. (2003) include a return stabilization mechanism - their bonus distribution is based on the arithmetic average return over the past years and is built upon the UK regulatory framework.[2] Additionally, Ballotta et al. (2006) incorporate the DPO. This bonus distribution model is applied by Ballotta (2005), who changed the underlying asset process to a jump-diffusion, and was adapted by Kleinow and Willder (2007).

On the contrary, Grosen and Jørgensen (2000) do not distribute bonuses based on returns, but rather based on the company's reserve situation. Their bonus model is based on the Danish regulatory environment. It has been applied by Jensen, Jørgensen, and Grosen (2001), Prieul, Putyatin, and Nassar (2001), Siu (2005), and Gerstner, Griebel, Holtz, Goschnick, and Haep (2008). An important extension of the model of Grosen and Jørgensen (2000) can be found in Hansen and Miltersen (2002). They include terminal bonus payments and annual fees to the insurance company. Similar bonus models can be found in Miltersen and Persson (2000) and Miltersen and Persson (2003).

Finally, Kling et al. (2007) developed a framework which strives to closely mirror the German insurance market. They work with a target interest rate, which leads to stable profits for policyholders as long as the reserve situation remains relatively stable. This bonus model was applied by Bauer, Kiesel, Kling, and Ruß (2006) and Zaglauer and Bauer (2008).

The purpose of this paper is to present and analyze the five basic PLI bonus distribution models under the risk-neutral measure $\mathbb{Q}$, namely Bacinello (2001), Haberman et al. (2003), Grosen and Jørgensen (2000), Hansen and Miltersen (2002), and Kling et al. (2007). In particular, we

[2] In fact, Haberman et al. (2003) introduce three different bonus distribution schemes. We apply this scheme as it is commonly used by UK insurance companies (see Ballotta, Haberman, and Wang, 2006). Thus, if we mention the bonus model of Haberman et al. (2003) throughout this paper, we always refer to the model whose smoothing mechanism is based on the arithmetic mean return over past periods.

calculate and compare the value of the DPO given by these different models.

First, we introduce and categorize the five basic bonus models and apply an integrative notation. To the best of our knowledge, only Cummins, Miltersen, and Persson (2007) compare different PLI models. However, Cummins et al. (2007) do not perform a risk comparison, but rather an analysis of the discounted payoff distribution.

Second, we calibrate contract parameters so that the compared bonus distribution mechanisms have the same (fair) contract value and the same safety level (defined as the present value of the DPO), under the risk-neutral measure. Third, we derive sensitivities of the DPO value to changes in the underlying asset volatility, as well as the company's initial reserve situation (per contract), while keeping all other contract parameters fixed. By doing so, we are able to compare the DPO value across the introduced bonus distribution models. Only Gatzert and Kling (2007) provide a framework that allows a comparison of PLI contracts which embed a cliquet-style option with regard to risk. They analyze shortfall probability measures (e.g., lower partial moments) with regard to the bonus models of Grosen and Jørgensen (2000), Hansen and Miltersen (2002), and Briys and de Varenne (1997). However, although they identify key risk drivers, they avoid a direct comparison across the different models. By contrast, we introduce a new method for calibrating PLI contracts that allows us to directly compare the risk involved in the different bonus distribution models. In addition, by keeping our initial calibration fixed, we can isolate effects caused by changes in the underlying asset volatility and the initial reserve situation (i.e., we can directly assess model risks).

The remainder of this paper is organized as follows. Section 2 introduces the basic model properties. In Section 3, the five different PLI bonus distribution models are presented. Section 4 provides the comparative analysis. We present our conclusions in Section 5.

2 Basic Model Framework

2.1 Model Overview

We assume an insurance company with various homogeneous insurance contracts (or one single large contract). The policyholder pays a single upfront premium P_0 and is assumed to continue the contract until maturity T (i.e., we exclude death or surrender). Table 1 shows the company's balance sheet at time t. $P(t)$ denotes the value of the policyholder's account at time t. $C(t)$ is the company's account and $B(t)$ is the so-called bonus reserve, which consists of asset valuation reserves as well as equity capital. The sum of all three accounts gives the company's asset base $A(t)$. The policyholder's initial proportion of assets is denoted by

$$\theta = \frac{P_0}{P_0 + B_0 + C_0} = \frac{P_0}{A_0}, \tag{1}$$

where $P_0 = P(0)$, $B_0 = B(0)$, $C_0 = C(0)$, and $A_0 = A(0)$. In the following, we describe in detail how these different balance sheet accounts develop over time.

Assets	Liabilities
$A(t)$: assets	$P(t)$: policyholder's savings account $B(t)$: bonus account $C(t)$: company's account
$A(t)$: total assets	$A(t)$: total assets

Table 1: The insurance company's balance sheet at time t. The left column shows the insurance company's assets $A(t)$ and the right column shows the corresponding liabilities $(P(t), B(t), C(t))$.

2.2 Development of Accounts

In our model framework, the insurance company invests in an asset portfolio that evolves according to a geometric Brownian motion in a complete and frictionless market setting. Given the risk-neutral valuation framework under the risk-neutral measure $\mathbb{Q}$, the drift equals the risk-free rate r,

$$dA(t) = rA(t)dt + \sigma A(t)dW^{\mathbb{Q}}(t), \tag{2}$$

with volatility σ. $W^{\mathbb{Q}}$ is a standard $\mathbb{Q}$-Brownian motion. The well-known solution of this stochastic differential equation is given by

$$A(t) = A(0) \cdot e^{\left(r-\sigma^2/2\right)t+\sigma W^{\mathbb{Q}}(t)}.^{3} \tag{3}$$

The policyholder's account $P(t)$ increases annually by the policy interest rate $r_p(t)$ which depends on the bonus distribution model applied

$$P(t) = P(t-1)\cdot(1+r_p(t)) = P_0 \cdot \prod_{i=1}^{t} \big(1+r_p(i)\big) \qquad \forall\, t \in \{1, 2, ..., T\}. \tag{4}$$

Note that we apply discrete compounding even though some models work with continuous compounding (e.g., Hansen and Miltersen, 2002).

The company's account $C(t)$ is only included in the model of Hansen and Miltersen (2002). We describe the calculation of the company's account $C(t)$ in detail when presenting their model. For all other models, the company's account does not exist, i.e., $C(t) = 0 \;\forall\, t \in \{0, 1, ..., T\}$. Finally, the bonus reserve $B(t)$ is determined residually,

$$B(t) = A(t) - P(t) - C(t),\; B(t) \in (-\infty, +\infty). \tag{5}$$

2.3 Valuation of Policyholders' Claims and the DPO

Assuming that the default does not take place, the policyholder receives the payoff $L(T)$ at maturity which consists of the policyholder's account $P(T)$ and, depending on the model, a terminal bonus payment $M(T)$,

$$L(T) = P(T) + M(T) = P_0 \cdot \prod_{t=1}^{T} (1 + r_p(t)) + M(T). \tag{6}$$

[3] Note that Equation (2) and (3) are not valid for the model of Kling et al. (2007) if dividends are distributed. We discuss this point when introducing the model.

Given the risk-neutral valuation framework, the value of a contract without default risk can be calculated by

$$V_L = e^{-rT} \cdot E^{\mathbb{Q}}(L(T)). \tag{7}$$

As in most regulatory frameworks, policyholder's have the first claim over the company's asset base. We solely analyze defaults at maturity[4], which occur if the market value of assets is lower than the book value of the policyholder's account $A(T) < P(T)$. In other words, this occurs if the bonus reserve at maturity is negative ($B(T) < 0$) and the company account is zero ($C(T) = 0$).

The value of the DPO is the premium necessary for a risk management measure that leads to a complete securitization of the policyholder's claim. The payoff of the DPO, $D(T)$, is therefore

$$D(T) = \max[P(T) - A(T), 0]. \tag{8}$$

Under the risk-neutral measure $\mathbb{Q}$, the value of the DPO is

$$V_D = e^{-rT} \cdot E^{\mathbb{Q}}(D(T)). \tag{9}$$

Combining the results obtained with regard to the contract's value V_L (Equation (7)) and the value of the DPO V_D (Equation (9)), the contract value taking default risk into account V_{L-D} can be calculated. V_{L-D} is equal to the contract value without default risk minus the DPO value, formally:

$$V_{L-D} = V_L - V_D = e^{-rT} \cdot [E^{\mathbb{Q}}(L(T)) - E^{\mathbb{Q}}(D(T))]. \tag{10}$$

This can be simplified to

$$V_{L-D} = e^{-rT} \cdot E^{\mathbb{Q}}(\min(L(T), A(T))). \tag{11}$$

Later on, we use Monte Carlo simulation to calculate the fair value of the DPO V_D. For comparability, we use the same sequence of 100'000 paths for each model applied.

[4] We neglect intermediate shortfalls during the contract's lifetime, as done, for example, by Gatzert and Kling (2007).

3 Participating Life Insurance Bonus Distribution Models

3.1 Overview and Categorization

Table 2 provides an overview of the different PLI models we analyze. As already discussed, the models are based on four different regulatory frameworks, namely Italy, UK, Denmark, and Germany.

The basic distinction between the five models is that the bonus distribution is either return-based (Bacinello (2001) and Haberman et al. (2003)) or reserve-based (Grosen and Jørgensen (2000), Hansen and Miltersen (2002), and Kling et al. (2007)).

As already discussed, only the model of Hansen and Miltersen (2002) incorporates a company's account $C(t)$. All other analyzed models embed a policyholder's account $P(t)$ and a bonus account $B(t)$. Note that the original model of Bacinello (2001) does not have any bonus account. In order to measure the value of the DPO, we add this account to her model.

The model of Bacinello (2001) is, in fact, just a special case of that of Haberman et al. (2003). However, we include both models in our analysis, since they are based on different regulatory frameworks. Similarly, the model of Hansen and Miltersen (2002) is an extension of that of Grosen and Jørgensen (2000). Again, we analyze both models, since Hansen and Miltersen (2002) additionally include terminal bonus payments, as well as annual fees to the insurance companies, which could have interesting effects on risk valuation.

In what follows, we first specify the different models separately. Note that we do not provide any analytical solution for contract and DPO values since we base all our results in the numerical analysis on the same sequence of random numbers.

3.2 Return-based Bonus Distribution

We first introduce the model of Bacinello (2001), which distributes bonuses based on the annual return on the reference portfolio $r_a(t)$. We

	(1) Bacinello (2001)	(2) Haberman et al. (2003)	(3) Grosen and Jørgensen (2000)	(4) Hansen and Miltersen (2002)	(5) Kling et al. (2007)
Regulatory Framework	Italy	UK	Denmark	Denmark	Germany
Bonus Distribution	return-based	return-based	reserve-based	reserve-based	reserve-based
Profit Stabilization and Reserve Building	none	return smoothing	reserve stabilization	reserve stabilization	reserve stabilization
Liability Accounts	$P(t), (B(t))$	$P(t), B(t)$	$P(t), B(t)$	$P(t), B(t), C(t)$	$P(t), B(t)$

Table 2: Overview of the different PLI bonus distribution models analyzed. The first row shows the regulatory framework that the model intends to mirror, the second row shows how the bonus is distributed, the third row shows how profit for policyholders is stabilized, and the last row shows the different liability accounts regarded.

apply some modifications to this model, in order to match the basic properties across all models. First, as already mentioned, we incorporate a bonus account $B(t)$, since the PLI model of Bacinello (2001) does not include any bonus account. Second, we assume that the policyholder continues the contract until maturity and exclude premature death, as done in all other models. Hansen and Miltersen (2002) actually show that the inclusion of mortality risks does not affect their results.

The PLI contract guarantees a minimum rate of interest r_g each year. A bonus is granted during years with high asset returns. Then, the policyholder receives the fraction α of the actual return on the reference portfolio. The policy interest rate $r_p(t)$ can be written as

$$r_p(t) = \max\bigl(r_g, \alpha \cdot r_a(t)\bigr), \tag{12}$$

where

$$r_a(t) = \frac{A(t) - A(t-1)}{A(t-1)}. \tag{13}$$

$\alpha \geq 0$ denotes the distribution ratio or participation rate. Subsequently, the policyholder's account develops according to

$$P(t) = P(t-1)\bigl[1 + \max\bigl(r_g, \alpha \cdot r_a(t)\bigr)\bigr]. \tag{14}$$

$B(t)$ is determined residually. Since the policyholder does not receive any terminal bonus, the payoff at maturity is

$$L(T) = P(T) = P_0 \cdot \prod_{t=1}^{T} \bigl[1 + \max\bigl(r_g, \alpha \cdot r_a(t)\bigr)\bigr]. \tag{15}$$

3.3 Average Return-based Bonus Distribution

Next, we present the model of Haberman et al. (2003), which grants bonuses based on the average return over the past years (see also Ballotta et al. (2006)). As in all analyzed models, the contract has a minimum interest rate guaranteed r_g. Additionally, Haberman et al. (2003) embed a bonus smoothing mechanism: annual surplus is credited based on the (arithmetic) average return on the reference portfolio over the past τ

years. As done by Bacinello (2001), a distribution ratio α is applied. The annual policy interest rate is

$$r_p(t) = \max\left[r_g, \frac{\alpha}{n}\left(\frac{A(t)}{A(t-1)} + ... + \frac{A(t-n+1)}{A(t-n)} - n\right)\right], \tag{16}$$

$$\text{with } n = \min(t, \tau).$$

Thus, the key difference with Bacinello (2001) is that not only current returns but also past returns are considered. For $\tau = 1$ the policy interest rate of Haberman et al. (2003) and Bacinello (2001) match exactly. Based on this interest rate $r_p(t)$, the policyholder's account evolves as follows

$$\begin{aligned} P(t) = P(t-1)\Bigg[1 + \max\Bigg[r_g, \frac{\alpha}{n}\Bigg(\frac{A(t)}{A(t-1)} + ... \\ + \frac{A(t-n+1)}{A(t-n)} - n\Bigg)\Bigg]\Bigg]. \end{aligned} \tag{17}$$

Again, $B(t)$ is determined residually. In addition, policyholders receive a terminal bonus payment $M(T)$. Since the terminal bonus is calculated based on the policyholder's initial proportion of assets $\theta = \frac{P_0}{A_0}$, only assets financed by the policyholders are actually distributed. The terminal bonus can be described by

$$M(T) = \zeta \cdot \max\big(\theta A(T) - P(T), 0\big), \tag{18}$$

where $\zeta \geq 0$ is a distribution parameter. Therefore, the payoff at maturity is

$$\begin{aligned} L(T) =& P(T) + M(T) \\ =& P_0 \cdot \prod_{t=1}^{T}\Bigg[1 + \max\Bigg[r_g, \frac{\alpha}{n}\Bigg(\frac{A(t)}{A(t-1)} + ... \\ &+ \frac{A(t-n+1)}{A(t-n)} - n\Bigg)\Bigg]\Bigg] + \zeta \cdot \max\big(\theta A(T) - P(T), 0\big). \end{aligned} \tag{19}$$

Unless stated otherwise, we fix $\tau = 3$ years as done by Ballotta et al. (2006).

3.4 Reserve-based Bonus Distribution

We now turn to the PLI model of Grosen and Jørgensen (2000). As for all analyzed models, the model of Grosen and Jørgensen (2000) includes a minimum interest rate guarantee r_g. The bonus is distributed, in contrast to the return-based bonus distribution observed in Bacinello (2001) and Haberman et al. (2003), depending on the reserve situation of the insurance company: Surplus is only credited if the buffer ratio $\frac{B(t-1)}{P(t-1)}$ exceeds a certain limit, the target buffer ratio $\gamma \geq 0$. The participation rate α defines how much of the excess reserves is to be distributed. Thus, the bonus account $B(t)$ is built up if the buffer ratio is low and partially distributed if the buffer ratio is high. Note that bonus is distributed with respect to the buffer ratio one year before (at $t-1$). The policy interest rate can be described by

$$\begin{aligned} r_p(t) &= \max\left[r_g, \alpha\left(\frac{B(t-1)}{P(t-1)} - \gamma\right)\right] \\ &= \max\left[r_g, \alpha\left(\frac{A(t-1) - P(t-1)}{P(t-1)} - \gamma\right)\right]. \end{aligned} \tag{20}$$

Hence, the policyholder's account evolves according to

$$P(t) = P(t-1)\left[1 + \max\left[r_g, \alpha\left(\frac{B(t-1)}{P(t-1)} - \gamma\right)\right]\right]. \tag{21}$$

The contract does not provide any terminal bonus payment. Thus, the payoff at expiration is simply the value of the policyholder's account:

$$L(T) = P(T) = P_0 \cdot \prod_{t=1}^{T}\left[1 + \max\left[r_g, \alpha\left(\frac{B(t-1)}{P(t-1)} - \gamma\right)\right]\right]. \tag{22}$$

3.5 Reserve-based Bonus Distribution with Annual Fees

Now, we analyze the model suggested by Hansen and Miltersen (2002). Besides embedding a minimum interest rate guaranteed and a bonus distribution mechanism similar to that introduced by Grosen and Jørgensen (2000), the insurance company collects annual fees and provides a terminal bonus payment. Note that we use discrete compounding instead of the continuous compounding applied by Hansen and Miltersen (2002), in order to match the different models.

Unlike the previous models, the insurance company has three different liability accounts: the bonus account $B(t)$, the company's account $C(t)$, and the policyholder's account $P(t)$. Again, a target buffer ratio γ and a participating rate α are applied. Due to the different accounts, the buffer ratio is defined differently than before, namely $\frac{B(t-1)}{P(t-1)+C(t-1)}$. In addition, the company's account $C(t)$ collects an annual fee. This annual fee is either a fraction ρ of the surplus (indirect method), a fraction ξ of the assets in the policyholder's account (direct method), or both (combination of indirect and direct method). The policy interest rate is

$$r_p(t) = \max\left[r_g, \alpha\left(\frac{B(t-1)}{P(t-1)+C(t-1)} - \gamma\right)\right] - \xi. \tag{23}$$

Thus, the policyholder's account evolves according to

$$P(t) = P(t-1)\left[1 + \max\left[r_g, \alpha\left(\frac{B(t-1)}{P(t-1)+C(t-1)} - \gamma\right)\right] - \xi\right]. \tag{24}$$

The sum of the policyholder's account and the company's account are modeled as

$$\begin{aligned}\big(P+C\big)(t) = \big(P+C\big)(t-1)\Bigg[1 + \max\Bigg[r_g, (\alpha+\rho)\\ \cdot\left(\frac{B(t-1)}{P(t-1)+C(t-1)} - \gamma\right)\Bigg]\Bigg],\end{aligned} \tag{25}$$

where $\alpha, \rho, \xi \in [0,1], \xi < r_g, \alpha + \rho \leq 1$.[5] From this it follows that the company's account can be calculated as

$$C(t) = \big(P + C\big)(t) - P(t). \tag{26}$$

Again, the bonus account is residually defined as

$$B(t) = A(t) - P(t) - C(t). \tag{27}$$

If the bonus account at expiration is negative, $B(T) < 0$, a transfer from the company's account $C(T)$ takes place. The policyholder receives a terminal bonus payment equal to the amount on the bonus account if positive,

$$M(T) = \max\big(B(T), 0\big) = B(T)^+. \tag{28}$$

Therefore, the policyholder's payoff at maturity is

$$\begin{aligned} L(T) = P(T) + M(T) = P_0 \cdot \prod_{t=1}^{T} \Bigg[1 + \max \Bigg[r_g, \alpha \\ \cdot \left(\frac{B(t-1)}{P(t-1) + C(t-1)} - \gamma \right) \Bigg] - \xi \Bigg] + B(T)^+. \end{aligned} \tag{29}$$

We assume that the initial bonus account in the model of Hansen and Miltersen (2002) is empty (i.e., $B_0 = 0$). Any capital paid in by shareholders is attributed to the company's account (i.e., $C_0 \geq 0$).

In what follows, we assume that the insurance company applies the indirect method to charge fees, i.e., $\xi = 0$ and $\rho \geq 0$.[6]

3.6 Reserve-based Bonus Distribution with Target Interest

Next, we introduce the last model analyzed, the model of Kling et al. (2007). The model goes much further into detail than the other models.

[5] Note that $\big(P(t) + C(t)\big) = (P + C)(t)$, as in the original paper by Hansen and Miltersen (2002).

[6] We have decided not to apply the direct method since the direct method may just reduce the guaranteed rate of interest.

Among others, Kling et al. (2007) differentiate two cases, the 'must'- and the 'is'-case. The must-case defines what is prescribed by regulation, whereas the is-case describes what German insurance companies actually do. To obtain comparability, we reduce the number of parameters as far as possible. Therefore, we just apply the is-case of Kling et al. (2007) and neglect the must-case. We have decided to focus on the is-case, because it is closer to the models previously introduced and it coincides with what actually happens in the German insurance market.

The bonus crediting mechanism depends on two key elements: the reserve situation and the target rate of interest. As seen before, the buffer ratio $\frac{B(t)}{P(t)}$ defines whether or not any bonus is granted. However, unlike the models of Grosen and Jørgensen (2000) and of Hansen and Miltersen (2002), this ratio is calculated based on accounts at time t, and not at time $t-1$. In addition to the target buffer γ, an upper boundary φ is given, with regard to the buffer ratio. As long as the buffer ratio stays within these upper and lower boundaries, $\gamma \leq \frac{B(t)}{P(t)} \leq \varphi$, the policyholder's account earns a target rate of interest $r_z > r_g$ that is set by management. A lower (higher) interest is granted only if the buffer ratio becomes too low (or too high). Of course, the policyholder always receives at least the guaranteed interest. In addition, shareholders receive dividends, calculated as a portion β of any surplus credited. $A^-(t)$ denotes assets before the distribution of dividends and $A^+(t)$ denotes assets after dividends.

With regard to the bonus distribution, four cases can be distinguished:

- If crediting the target rate of interest r_z results in a buffer ratio above its upper boundary φ, the amount leading to a buffer ratio at its upper boundary is distributed.

- If distributing the target interest leads to a buffer ratio between its upper and lower limit, the target interest r_z is granted.

- If crediting the target interest results in a buffer ratio below the target buffer γ, the amount leading to a buffer ratio at the target buffer is distributed.

- No additional bonus is distributed if the buffer ratio before the distribution of any bonus is already below the target buffer.

Formally, this can be described as follows

$$P(t) = \begin{cases} (1+r_g)\cdot P(t-1) + \dfrac{1}{1+\varphi+\beta}\left[A^-(t) - (1+r_g)(1+\varphi)\cdot P(t-1)\right] \\ \qquad\qquad \text{if } \varphi < \dfrac{B(t)}{(1+r_z)\cdot P(t-1)} \\ (1+r_z)\cdot P(t-1) \quad \text{if } \gamma \leq \dfrac{B(t)}{(1+r_z)\cdot P(t-1)} \leq \varphi \\ (1+r_g)\cdot P(t-1) + \dfrac{1}{1+\gamma+\beta}\left[A^-(t) - (1+r_g)(1+\gamma)\cdot P(t-1)\right] \\ \qquad\qquad \text{if } \dfrac{B(t)}{(1+r_z)\cdot P(t-1)} < \gamma < \dfrac{B(t)}{(1+r_g)\cdot P(t-1)} \\ (1+r_g)\cdot P(t-1) \quad \text{if } \dfrac{B(t)}{(1+r_g)\cdot P(t-1)} \leq \gamma. \end{cases} \tag{30}$$

Note that if dividends are distributed (i.e., $\beta > 0$), Equation (2) only applies in the interval $[t-1, t[$ and Equation (3) changes to $A^-(t) = A^+(t-1)e^{\left(r-\frac{\sigma^2}{2}\right)+\sigma\left(W^{\mathbb{Q}}(t)-W^{\mathbb{Q}}(t-1)\right)}$. The corresponding asset value after dividends, A^+, is calculated by

$$A^+(t) = \begin{cases} A^-(t) - \dfrac{\beta}{1+\varphi+\beta}\left[A^-(t) - (1+r_g)(1+\varphi)\cdot P(t-1)\right] \\ \qquad \text{if } \varphi < \dfrac{B(t)}{(1+r_z)\cdot P(t-1)} \\ A^-(t) - \beta(r_z - r_g)\cdot P(t-1) \\ \qquad \text{if } \gamma \leq \dfrac{B(t)}{(1+r_z)\cdot P(t-1)} \leq \varphi \\ A^-(t) - \dfrac{\beta}{1+\gamma+\beta}\left[A^-(t) - (1+r_g)(1+\gamma)\cdot P(t-1)\right] \\ \qquad \text{if } \dfrac{B(t)}{(1+r_z)\cdot P(t-1)} < \gamma < \dfrac{B(t)}{(1+r_g)\cdot P(t-1)} \\ A^-(t) \quad \text{if } \dfrac{B(t)}{(1+r_g)\cdot P(t-1)} \leq \gamma. \end{cases} \tag{31}$$

$B(t)$ is determined residually.

In what follows, we set the dividend ratio to $\beta = 0$, in order to exclude effects caused by dividend distribution. As a consequence, Equations (2) and (3) also hold for the model of Kling et al. (2007).

4 Comparative Analysis

4.1 Approach

Next, after introducing each model separately, we derive a calibration that allows us to compare the value of the DPO across the different models. In earlier literature, two methods of parameter calibration with regard to PLI contracts have been applied. Gatzert and Kling (2007) calibrate PLI contracts to be fair priced without pricing the DPO, i.e.,

$$P_0 = V_L. \tag{32}$$

On the contrary, Gatzert (2008) calibrates contracts to be fair taking default risk into account and, additionally, fixes the default-value-to-liability ratio,

$$P_0 = V_{L-D}, \tag{33}$$

$$d = \frac{V_D}{V_L} = \text{constant}. \tag{34}$$

However, the stepwise procedure introduced by Gatzert (2008, p. 842) is not applicable unless "at least one parameter that has no influence on default" exists. This is the case of all models introduced, except for the model of Haberman et al. (2003) with regard to the parameter ζ.

We introduce a new calibration method that provides a higher degree of comparability across the different bonus distribution models. Note that the methods of Gatzert and Kling (2007) and Gatzert (2008) calibrate parameters at each point separately. That is to say, if sensitivities with respect to volatility are derived, the PLI model is calibrated for each single volatility. By contrast, we derive an initial calibration that is kept fixed during our numerical analysis. This allows us to isolate effects caused by changes in the underlying asset volatility and the initial reserve situation (i.e., we can directly assess model risks).

First, we require contracts to have the same fair value taking default risk into account

$$P_0 = V_{L-D}. \tag{35}$$

Second, we fix the safety level across all models by setting the initial value of the DPO to the level S,

$$V_D = S, \tag{36}$$

which directly corresponds to current solvency regulation (e.g., Solvency II, Swiss Solvency Test) that focuses on keeping the default probability at a low level but does not prescribe reserve levels.[7]

To conclude, our calibration approach ensures that all contracts are fair and involve the same safety level, i.e., risk value. However, results cannot be obtained stepwise like in Gatzert (2008) and, thus, numerical optimization is needed.

4.2 Optimization Technique

For each single bonus distribution model, we search for parameter combinations fulfilling Equations (35) and (36) by means of numerical optimization. We apply the differential evolution technique introduced by Storn and Price (1997). Mullen, Ardia, Gil, Windover, and Cline (2009) provide a comprehensive overview on the differential evolution algorithm.[8]

In order to obtain reasonable parameter combinations, we apply the following constraints on the different parameters

[7]That is to say, Solvency II as well as the Swiss Solvency Test require that an insurance company does not exceed a certain default probability. Both do generally not prescribe how this safety level is achieved. Similarly, we require that all contracts have the same DPO value.

[8]We apply the DEoptim procedure as implemented in R. A similar implementation can be found in Price, Storn, and Lampinen (2005).

1. Bacinello (2001):
   $0 \leq \alpha \leq 1, \quad 1 \leq B_0 \leq P_0,$
2. Haberman et al. (2003):
   $0 \leq \alpha \leq 1, \quad 0 \leq \zeta \leq 1, \quad 1 \leq B_0 \leq P_0,$
3. Grosen and Jørgensen (2000):
   $0 \leq \alpha \leq 1, \quad 0 \leq \gamma \leq 1, \quad 1 \leq B_0 \leq P_0,$
4. Hansen and Miltersen (2002):
   $0 \leq \alpha \leq 1, \quad 0 \leq \gamma \leq 1, \quad 0 \leq \rho \leq 1, \quad 1 \leq C_0 \leq P_0, \quad \alpha + \rho \leq 1,$
5. Kling et al. (2007):
   $0 \leq \gamma \leq 1, \quad 0 \leq \varphi \leq 1, \quad r_g \leq r_z \leq r, \quad 1 \leq B_0 \leq P_0, \quad \gamma \leq \varphi,$

and fix other design parameters (for all models regarded), unless otherwise stated, as follows:

$$\begin{aligned} T = 10, \qquad \tau = 3, \qquad r = 0.04, \qquad \sigma = 0.1, \\ r_g \in \{0.00, 0.02\}, \qquad P_0 = 100, \quad \text{and} \quad S = V_D = 1. \end{aligned}$$

The parameter combinations calculated by optimization may not be unique. To check the results obtained for robustness, we optimize by applying three different seeds in the differential evolution algorithm. In order to differentiate results based on the three different seeds, we refer to them as Set 1, Set 2, and Set 3.

4.3 Fixing the Reserve Level

In order to increase comparability, we additionally require that the reserve level B_0 be the same across the different models, whenever possible, i.e.,

$$B_0^{(Bac)} = B_0^{(Hab)} = B_0^{(Gro)} = C_0^{(Han)} = B_0^{(Kli)}. \tag{37}$$

Note that it will not be possible to satisfy Equations (35), (36), and (37) across all models, since degrees of freedom are, in some models, too low. Thus, we decide to abandon Equation (37) for some models if no solution can be obtained otherwise because, from our point of view, fixing the safety level is more important due to its adherence to solvency regulation. In addition, if we solely fixed the reserves without considering the value of the DPO, the degrees of freedom would become too high.[9]

By applying different design parameters and optimizing according to Section 4.2, we find that reserve levels can be kept the same across the models of Haberman et al. (2003), Grosen and Jørgensen (2000), and Kling et al. (2007). The model of Bacinello (2001) usually requires a higher reserve level, in order to reach the required safety level and to be fair priced (Equations (35) and (36)). The contrary is true for the model of Hansen and Miltersen (2002).[10] As a consequence, Equation (37) reduces to

$$B_0^{(Hab)} = B_0^{(Gro)} = B_0^{(Kli)}. \tag{38}$$

We apply the optimal reserve level obtained by calibrating the model of Grosen and Jørgensen (2000) as the common initial reserve (corresponding to Equation (38)). That is to say, we optimize (calibrate) the model of Grosen and Jørgensen (2000) according to Section 4.2, obtain the reserve level $B_0^{(Gro)}$ and apply this reserve level ($B_0^{(Gro)}$) in the optimization (calibration) of the models of Kling et al. (2007) and Haberman et al. (2003).[11] For the models of Bacinello (2001) and Hansen and Miltersen (2002), we apply individual reserve levels obtained by the optimization technique.

4.4 Calibrated Parameter Combinations

We simulate the same sequence of 100'000 paths (Monte Carlo simulation) for each model applied to find the optimal parameter combinations

[9] We provide additional numerical results, which are derived from abandoning Equation (36) if no solutions can be obtained otherwise, in Appendix E (Table 11 and 12).

[10] There can be a combination of design parameters that allow to fulfill Equations (35), (36), and (37) across all models, but this would be a special case.

[11] Applying a common reserve level, obtained by optimizing the model of Haberman et al. (2003), would also be possible. To calibrate instead the common reserve level based on the model of Kling et al. (2007) does not appear to be feasible as the model has comparably high degrees of freedom.

(leading to fair contracts with the same safety level) with the differential evolution algorithm. We report the resulting parameter combinations for each bonus distribution model in Tables 3 and 4 (based on three different seeds). Table 3 shows results for a guaranteed rate of interest of $r_g = 0.00$ (money-back guarantee), Table 4 for a guaranteed rate of $r_g = 0.02$.

Given a guaranteed rate of interest of $r_g = 0.00$, Table 3 shows that the seed applied in the optimization of the model of Bacinello (2001) does not matter, since all sets yield the same values for α and B_0. In the models of Haberman et al. (2003) and Grosen and Jørgensen (2000), we obtain very similar parameter combinations in Set 2 and Set 3, whereas those in Set 1 differ. That is to say, Set 1 shows a lower focus on final

(a) Set 1 ($r_g = 0.00$)

Parameter	(1) Bacinello (2001)	(2) Haberman et al. (2003)	(3) Grosen and Jørgensen (2000)	(4) Hansen and Miltersen (2002)	(5) Kling et al. (2007)
α	0.650	0.737	0.440	0.313	-
B_0	32.677	23.063[a]	23.063	0.000	23.063[a]
C_0	-	-	-	3.739	-
ζ	-	0.375	-	-	-
γ	-	-	0.170	0.814	0.056
ρ	-	-	-	0.344	-
r_z	-	-	-	-	0.030
φ	-	-	-	-	0.398

[a] This reserve level is that obtained by optimizing the model of Grosen and Jørgensen (2000).

Table 3: Three sets (based on three different seeds) of initial parameter combinations calculated by means of optimization, which lead to a fair contract value ($V_{L-D} = P_0$) and a fixed safety level ($V_D = S = 1$) with a guaranteed rate of interest of $r_g = 0.00$ (rounded to three decimal places). (continued on next page)

(b) Set 2 ($r_g = 0.00$)

Parameter	(1) Bacinello (2001)	(2) Haberman et al. (2003)	(3) Grosen and Jørgensen (2000)	(4) Hansen and Miltersen (2002)	(5) Kling et al. (2007)
α	0.650	0.708	0.866	0.452	-
B_0	32.677	21.952[a]	21.952	0.000	21.952[a]
C_0	-	-	-	3.778	-
ζ	-	0.514	-	-	-
γ	-	-	0.241	0.706	0.013
ρ	-	-	-	0.240	-
r_z	-	-	-	-	0.020
φ	-	-	-	-	0.347

(c) Set 3 ($r_g = 0.00$)

Parameter	(1) Bacinello (2001)	(2) Haberman et al. (2003)	(3) Grosen and Jørgensen (2000)	(4) Hansen and Miltersen (2002)	(5) Kling et al. (2007)
α	0.650	0.708	0.865	0.261	-
B_0	32.677	21.959[a]	21.959	0.000	21.959[a]
C_0	-	-	-	3.806	-
ζ	-	0.513	-	-	-
γ	-	-	0.241	0.473	0.087
ρ	-	-	-	0.073	-
r_z	-	-	-	-	0.034
φ	-	-	-	-	0.386

[a] This reserve level is that obtained by optimizing the model of Grosen and Jørgensen (2000).

Table 3: Three sets (based on three different seeds) of initial parameter combinations calculated by means of optimization, which lead to a fair contract value ($V_{L-D} = P_0$) and a fixed safety level ($V_D = S = 1$) with a guaranteed rate of interest of $r_g = 0.00$ (rounded to three decimal places). (cont.)

payouts (i.e., lower ζ) in the model of Haberman et al. (2003), and earlier (i.e., lower γ) but lower bonus payouts (i.e., lower α) in the bonus model of Grosen and Jørgensen (2000). In the bonus distribution model of Hansen and Miltersen (2002) and Kling et al. (2007), results differ in all sets.

Table 4 shows optimal parameter combinations for a guaranteed rate of interest of $r_g = 0.02$. The optimization of the model of Bacinello (2001) yields the same values for α and B_0 in all sets. In addition, the resulting parameter combinations in the model of Haberman et al. (2003) are very similar across all sets. In the bonus distribution model of Kling et al. (2007), Set 1 and 2 lead to similar results. In the models of Grosen and Jørgensen (2000) and of Hansen and Miltersen (2002), the obtained parameter combinations differ in all sets.

(a) Set 1 ($r_g = 0.02$)

Parameter	(1) Bacinello (2001)	(2) Haberman et al. (2003)	(3) Grosen and Jørgensen (2000)	(4) Hansen and Miltersen (2002)	(5) Kling et al. (2007)
α	0.514	0.497	0.887	0.223	-
B_0	44.964	35.004[a]	35.004	0.000	35.004[a]
C_0	-	-	-	30.432	-
ζ	-	0.594	-	-	-
γ	-	-	0.483	0.165	0.227
ρ	-	-	-	0.304	-
r_z	-	-	-	-	0.028
φ	-	-	-	-	0.559

[a] This reserve level is that obtained by optimizing the model of Grosen and Jørgensen (2000).

Table 4: Three sets (based on three different seeds) of initial parameter combinations calculated by means of optimization, which lead to a fair contract value ($V_{L-D} = P_0$) and a fixed safety level ($V_D = S = 1$) with a guaranteed rate of interest of $r_g = 0.02$ (rounded to three decimal places). (continued on next page)

(b) Set 2 ($r_g = 0.02$)

Parameter	(1) Bacinello (2001)	(2) Haberman et al. (2003)	(3) Grosen and Jørgensen (2000)	(4) Hansen and Miltersen (2002)	(5) Kling et al. (2007)
α	0.514	0.496	0.666	0.370	-
B_0	44.964	34.995[a]	34.995	0.000	34.995[a]
C_0	-	-	-	30.552	-
ζ	-	0.594	-	-	-
γ	-	-	0.440	0.170	0.239
ρ	-	-	-	0.359	-
r_z	-	-	-	-	0.029
φ	-	-	-	-	0.565

(c) Set 3 ($r_g = 0.02$)

Parameter	(1) Bacinello (2001)	(2) Haberman et al. (2003)	(3) Grosen and Jørgensen (2000)	(4) Hansen and Miltersen (2002)	(5) Kling et al. (2007)
α	0.514	0.496	0.757	0.087	-
B_0	44.964	34.977[a]	34.977	0.000	34.977[a]
C_0	-	-	-	30.408	-
ζ	-	0.596	-	-	-
γ	-	-	0.459	0.102	0.292
ρ	-	-	-	0.204	-
r_z	-	-	-	-	0.034
φ	-	-	-	-	0.577

[a] This reserve level is that obtained by optimizing the model of Grosen and Jørgensen (2000).

Table 4: Three sets (based on three different seeds) of initial parameter combinations calculated by means of optimization, which lead to a fair contract value ($V_{L-D} = P_0$) and a fixed safety level ($V_D = S = 1$) with a guaranteed rate of interest of $r_g = 0.02$ (rounded to three decimal places). (cont.)

This discussion shows that it is useful to consider different seeds in the differential evolution algorithm, since they may lead to different parameter combinations depending on the considered model. In what follows, the obtained parameter combinations will be applied to the respective bonus distribution models in order to measure sensitivities of the value of the DPO with regard to asset volatility and the initial reserve level. Thereby, we focus on Set 1 when presenting results. However, we point out significant differences observed in other sets and provide the corresponding data in the appendix.

4.5 Numerical Analysis

Our calculated parameter combinations result in PLI contracts with the same fair value and the same safety level (i.e., same DPO value). However, asset volatility may change over time or may be misspecified (model risk). In addition, a growing pool size will reduce the amount of reserves per contract if no increase in equity capital takes place. Thus, we analyze the effects of a changing asset volatility and a changing initial reserve amount on the value of the DPO while keeping all other parameters constant.

Figure 1 plots sensitivities with respect to the underlying asset volatility for the two different guaranteed rates, $r_g \in \{0.00, 0.02\}$, based on Set 1. We provide the underlying data tables (for all sets) in Table 5 and 6 (see Appendix A).

Figure 1 clarifies that the model of Bacinello (2001) is most sensitive to changes in the underlying asset volatility regarding both guaranteed rates. On the other hand, the model of Hansen and Miltersen (2002) is least sensitive. The model of Kling et al. (2007) responds faster to changes in asset volatility compared to that of Hansen and Miltersen (2002), but slower compared to all other bonus distribution models. The models of Grosen and Jørgensen (2000) and Haberman et al. (2003) are in between the models of Bacinello (2001) and Kling et al. (2007). For a money-back guarantee (i.e., $r_g = 0.00$), the model of Grosen and Jørgensen (2000) is less sensitive than that of Haberman et al. (2003).

(a) $r_g = 0.00$

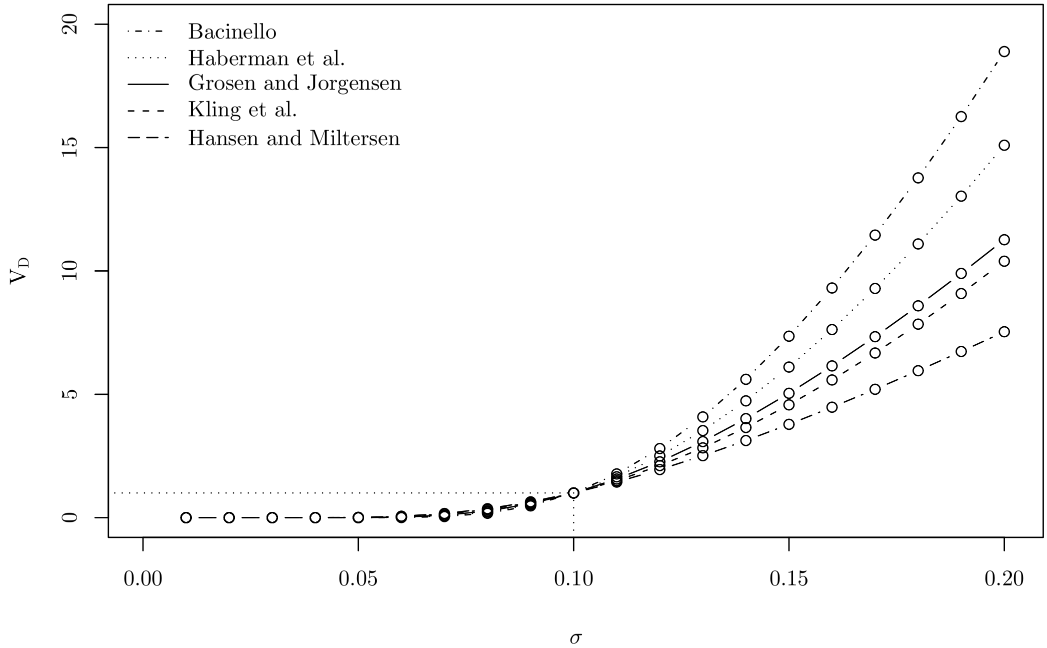

Figure 1: Value of the DPO for different volatilities σ given guaranteed rates of $r_g = 0.00$ (panel a) and $r_g = 0.02$ (panel b) (Set 1). (continued on next page)

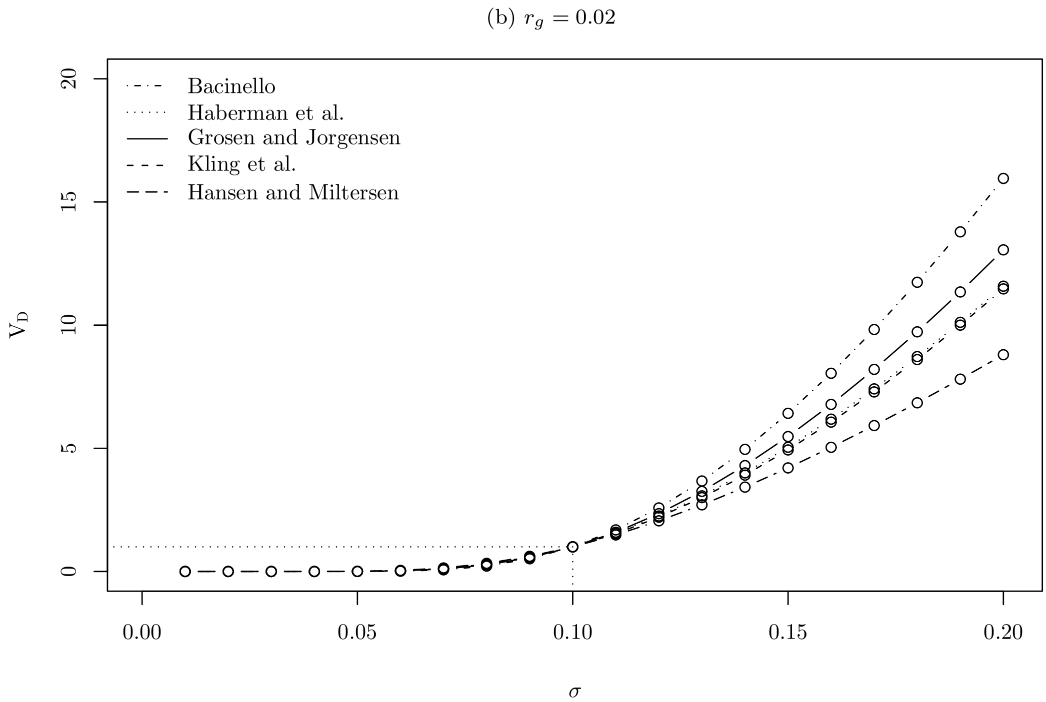

Figure 1: Value of the DPO for different volatilities σ given guaranteed rates of $r_g = 0.00$ (panel a) and $r_g = 0.02$ (panel b) (Set 1). (cont.)

The reverse is true for the higher guaranteed rate of $r_g = 0.02$: Here, the model of Grosen and Jørgensen (2000) is more sensitive than that of Haberman et al. (2003). In fact, with a guaranteed rate of $r_g = 0.02$, the model of Haberman et al. (2003) yields sensitivities that are very close to those obtained by applying the model of Kling et al. (2007).

Note that for both guaranteed rates, Set 2 and Set 3 generally lead to values that are similar to those displayed in Figure 1 (see Table 6). As a consequence, the order regarding sensitivity to changes in volatility remains the same. Thus, our general results concerning the comparison of the different bonus distribution models are robust.

Summarizing results obtained with both guaranteed rates, it becomes clear that the model of Hansen and Miltersen (2002) is the least sensitive to changes in the underlying asset volatility, followed by the model of Kling et al. (2007). With a guaranteed rate of $r_g = 0.02$, the model of Haberman et al. (2003) has a very similar sensitivity compared to that of Kling et al. (2007). On the contrary, the model of Bacinello (2001) is most sensitive to changes in asset volatility. Whether the model of Grosen and Jørgensen (2000) or that of Haberman et al. (2003) responds faster, depends on the interest rate guarantee.

In Figure 2, we show sensitivities with respect to changes in the initial reserve level, for both guaranteed rates, $r_g \in \{0.00, 0.02\}$. Note that we analyze changes in the reserve relative to the calibrated parameters because initial reserve levels are different for some bonus models (recall the discussion in Section 4.3). The 1.0 on the x-axis is the starting point of the sensitivity analysis and, for instance, 0.8 means that reserves are reduced to $B_0^{(\text{new})} = 0.80 \cdot B_0^{(\text{initial})}$. We provide the corresponding data tables for all parameter sets in Table 7 and 8 (see Appendix B).

First, notice that the DPO values are much less sensitive to changes in the initial reserve level than to changes in the underlying asset volatility given the volatility of $\sigma = 0.10$. Figure 2a clarifies that the reserve-based bonus distribution models respond little to changes in the initial reserve level in case of a money-back guarantee (i.e., $r_g = 0.00$).

Again, the model of Bacinello (2001) is the most sensitive to changes in the initial reserves, followed by that of Haberman et al. (2003). The

(a) $r_g = 0.00$

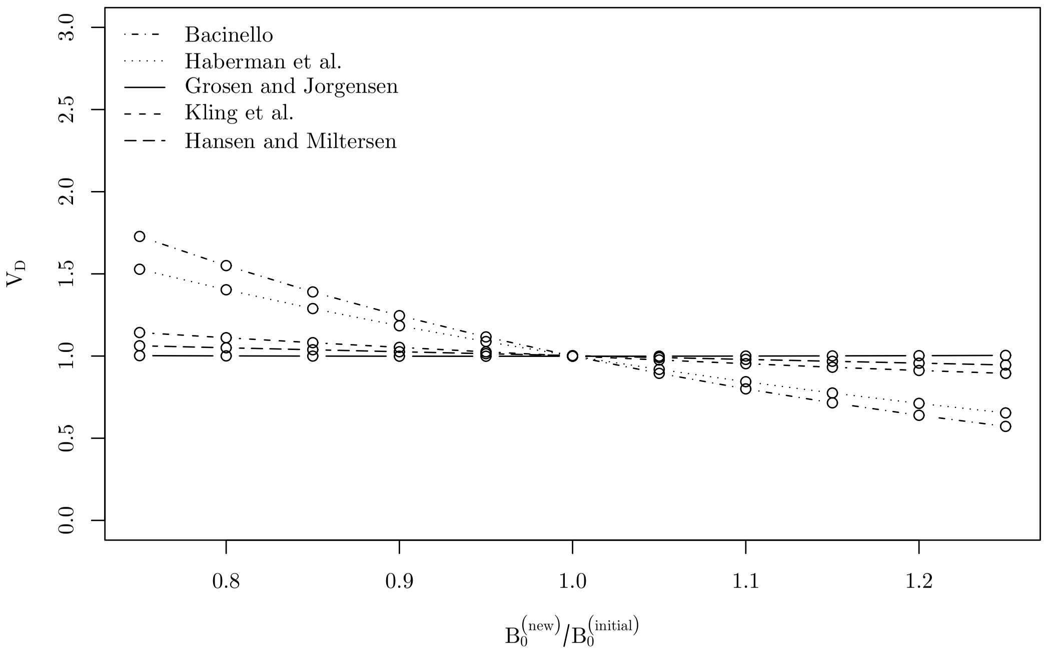

Figure 2: Value of the DPO for different reserves B_0 relative to the initial reserve level given guaranteed rates of $r_g = 0.00$ (panel a) and $r_g = 0.02$ (panel b) (Set 1). (continued on next page)

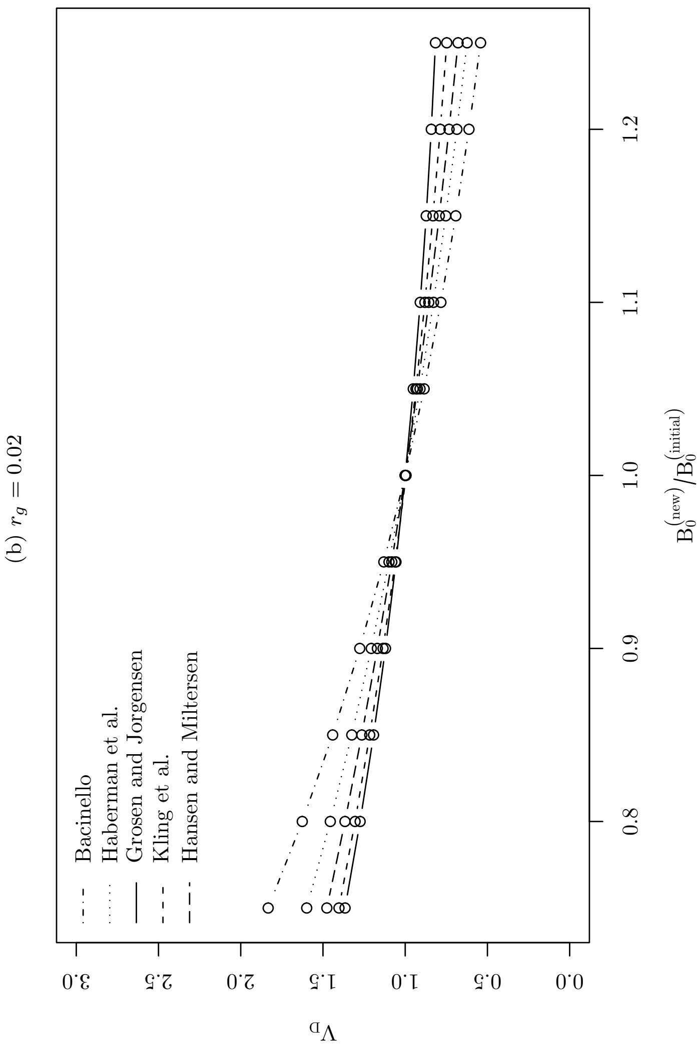

Figure 2: Value of the DPO for different reserves B_0 relative to the initial reserve level given guaranteed rates of $r_g = 0.00$ (panel a) and $r_g = 0.02$ (panel b) (Set 1). (cont.)

model of Grosen and Jørgensen (2000) is the least sensitive. The models of Hansen and Miltersen (2002) and Kling et al. (2007) are between those of Grosen and Jørgensen (2000) and Haberman et al. (2003). For a guaranteed rate of $r_g = 0.00$, the model of Kling et al. (2007) responds faster than that of Hansen and Miltersen (2002). The reverse is true for a guaranteed rate of $r_g = 0.02$: Here, the model of Hansen and Miltersen (2002) responds faster than that of Kling et al. (2007). Note that the values obtained in Set 2 and 3 are close to the ones displayed in Figure 2 (see Table 7 and 8).

To summarize results regarding sensitivities with respect to the initial reserves, the model of Bacinello (2001) is most sensitive, followed by Haberman et al. (2003); that of Grosen and Jørgensen (2000) is least sensitive, and the models of Hansen and Miltersen (2002) and Kling et al. (2007) are somewhere in between.

In addition, we have performed an analysis with regard to risk assessment under the real world measure $\mathbb{P}$. We have calculated shortfall probabilities, i.e., $\mathbb{P}(A(T) < P(T))$, for three different drift rates $\mu \in \{0.04, 0.06, 0.08\}$ and an interest rate guarantee of $r_g = 0.02$. We illustrate results in Figures 3 and 4. The corresponding data tables are reported in Table 9 (see Appendix C) and Table 10 (see Appendix D).

With a drift rate of $\mu = 0.04$ (Figures 3a and 4a), results directly correspond to our analysis with regard to risk valuation, i.e., the order regarding sensitivities is the same across the different models (e.g., the model of Bacinello (2001) is most sensitive to changes in volatility and initial reserves). For higher drift rates, $\mu \in \{0.06, 0.08\}$, results also confirm our findings with regard to risk valuation with one exception. Namely, shortfall probabilities in the model of Haberman et al. (2003) are less sensitive to changes in the asset volatility than in the model of Kling et al. (2007). Thus, the relation between these two models is just the opposite way around, compared to our results concerning risk valuation (see Figure 3b and Table 6). Here, recall that sensitivities regarding DPO values in the model of Haberman et al. (2003) have been very close to those obtained by applying the model of Kling et al. (2007). In addition, although the order does not change, with higher

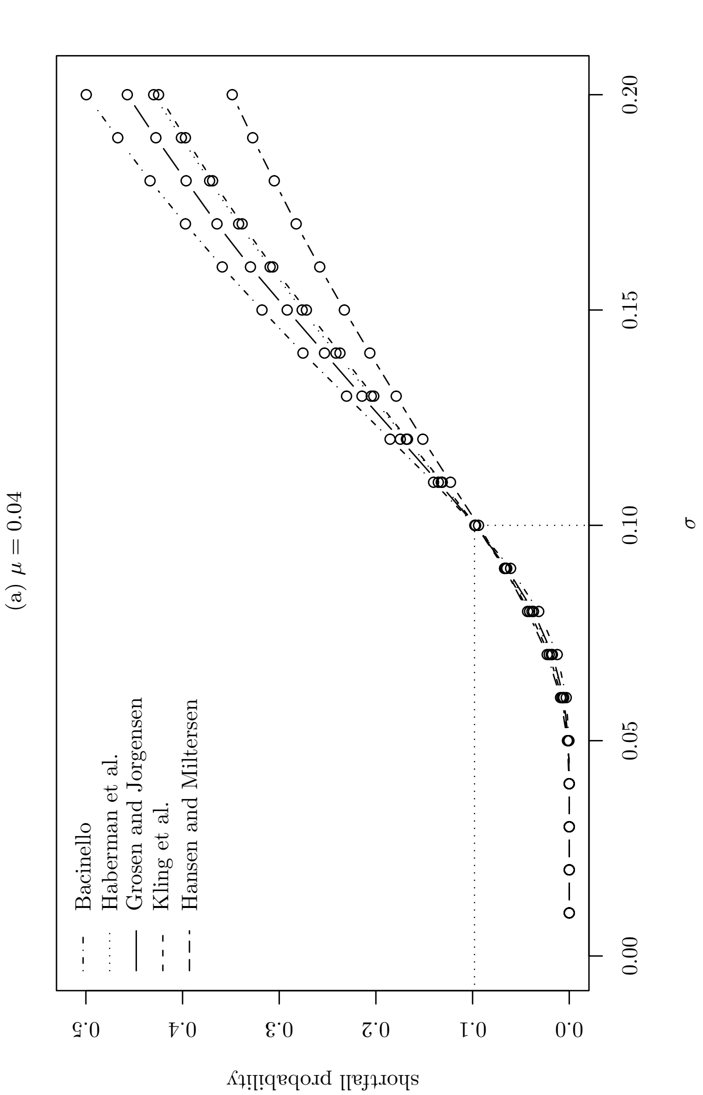

Figure 3: Shortfall probabilities for different volatilities σ given drift rates of $\mu = 0.04$ (panel a) and $\mu = 0.06$ (panel b) ($r_g = 0.02$, Set 1). (continued on next page)

(b) $\mu = 0.06$

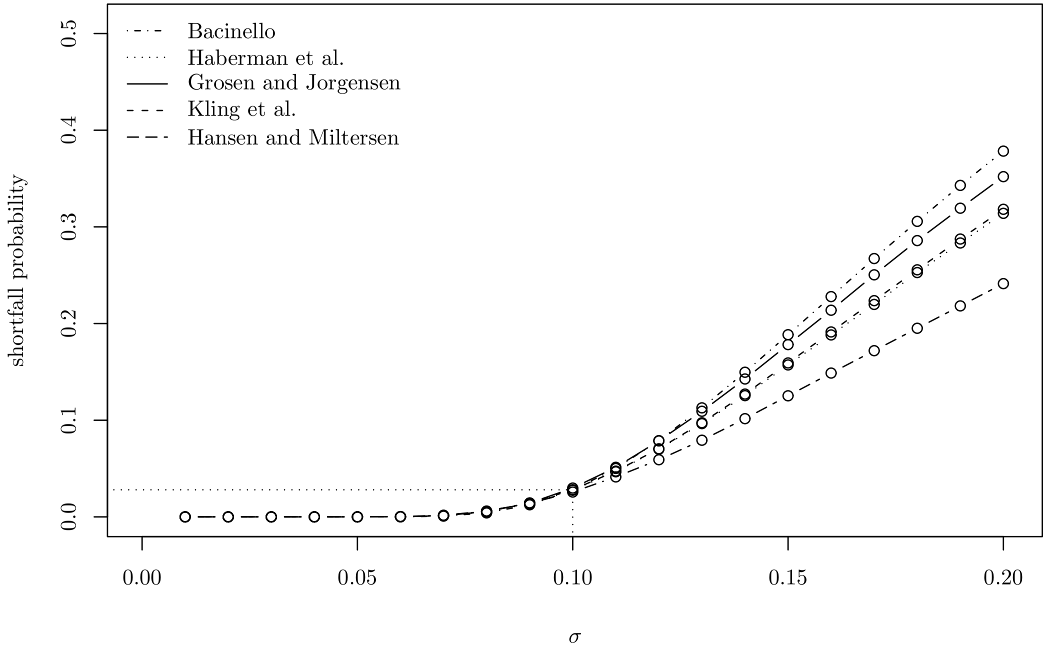

Figure 3: Shortfall probabilities for different volatilities σ given drift rates of $\mu = 0.04$ (panel a) and $\mu = 0.06$ (panel b) ($r_g = 0.02$, Set 1). (cont.)

(a) $\mu = 0.04$

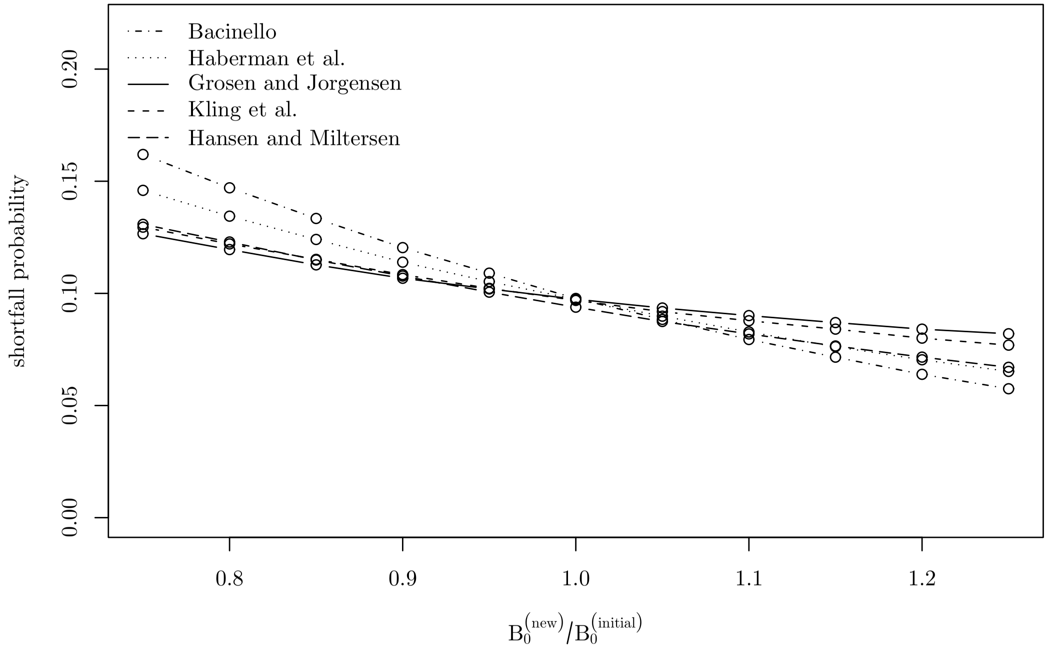

Figure 4: Shortfall probabilities for different reserves B_0 relative to the initial reserve level given drift rates of $\mu = 0.04$ (panel a) and $\mu = 0.06$ (panel b) ($r_g = 0.02$, Set 1). (continued on next page)

(b) $\mu = 0.06$

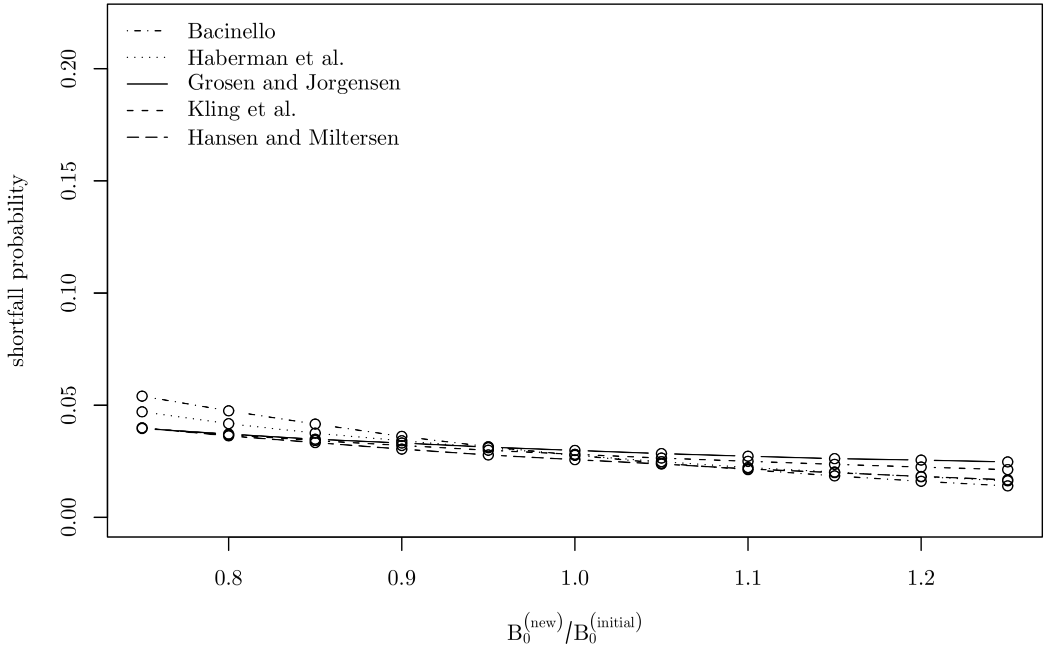

Figure 4: Shortfall probabilities for different reserves B_0 relative to the initial reserve level given drift rates of $\mu = 0.04$ (panel a) and $\mu = 0.06$ (panel b) ($r_g = 0.02$, Set 1). (cont.)

drift rates ($\mu \in \{0.06, 0.08\}$) shortfall probabilities are very similar across the reserve-based bonus distribution models (i.e., Grosen and Jørgensen (2000), Hansen and Miltersen (2002), and Kling et al. (2007)) for changing initial reserve levels (see Figure 4b).

Results confirm our findings with regard to risk valuation. The order regarding sensitivities is the same across the different models (except for the exception mentioned). In particular, shortfall probabilities in the model of Bacinello (2001) are most sensitive to changes in both parameters as we have seen in Figures 1 and 2.

Our results allow us to draw four major conclusions. First, the DPO values in the Italian based bonus framework of Bacinello (2001) are most sensitive to changes in the underlying asset volatility and the initial reserve level – which can be explained by the lack of any smoothing mechanism. Second, the reserve-based bonus distribution models (i.e.,Grosen and Jørgensen (2000), Hansen and Miltersen (2002), and Kling et al. (2007)) are generally less sensitive to changes in the initial reserve level. However, we cannot draw general conclusions in regard to whether return-based or reserve-based surplus distribution mechanisms are superior. Results regarding the model of Haberman et al. (2003) show that return-based bonus models may involve less model risks than reserve-based ones if volatility is considered (see Figures 1b and 3). Third, the models of Hansen and Miltersen (2002) and Kling et al. (2007) are the least sensitive to changes in the underlying asset volatility. And finally, the model of Grosen and Jørgensen (2000) is the least sensitive one with respect to changes in the initial reserves.

Our findings imply that return-based surplus distribution scheme of Bacinello (2001) imposes comparably high model risks on life insurance companies. If Italian insurance companies actually apply the PLI model of Bacinello (2001), regulators may reassess whether a model which is less sensitive, i.e., the one of Hansen and Miltersen (2002), could be implemented. In fact, the application of the model of Bacinello (2001) in practice appears to be problematic if a high increase in volatility takes place. Similarly, insurance companies can figure out the model that fits the required risk profile best given that they can still adhere to the regulatory framework.

5 Conclusion

We present the most common PLI bonus distribution mechanisms, namely those of Bacinello (2001), Haberman et al. (2003), Grosen and Jørgensen (2000), Hansen and Miltersen (2002), and Kling et al. (2007), and we apply an integrative notation. We develop a parameter calibration that allows us to compare the DPO values across the different models.

By applying our method of comparison, regulatory authorities can compare the model risks present in different bonus distribution schemes. In particular, we identify differences in model risks associated with the misspecification of the underlying asset volatility and for the case of a reduced initial reserve level.

Our results show that the return-based bonus distribution scheme of Bacinello (2001) generally yields the highest DPO values if the underlying asset volatility increases or the initial reserve decreases. As a consequence, given that Italian insurers actually apply the bonus distribution scheme introduced by Bacinello (2001), regulators should reassess whether a model which is less sensitive could be implemented. For instance, the models of Hansen and Miltersen (2002) and Kling et al. (2007) are the least sensitive to changes in the underlying asset volatility, whereas the model of Grosen and Jørgensen (2000) is the least sensitive with respect to changes in the initial reserves.

In summary, our analysis clarifies that the model chosen by insurance companies or prescribed by regulators should not be arbitrary. It appears to be crucial that regulatory authorities select a bonus distribution model whose default risk is less sensitive to model risks. Here, especially the model framework of Bacinello (2001) appears to be problematic – an unexpected increase in asset volatility will highly increase risk values.

Appendix

A Value of the DPO for Different Volatilities

B Value of the DPO for Different Reserve Levels

C Shortfall Probabilities for Different Volatilities

D Shortfall Probabilities for Different Reserves

E Fixed Safety Level versus a Fixed Reserve Level

(a) Set 1 ($r_g = 0.00$)

σ	Bacinello		Haberman et al.		Grosen and Jørgensen		Hansen and Miltersen		Kling et al.	
0.01	0.000	(0.000)	0.000	(0.000)	0.000	(0.000)	0.000	(0.000)	0.000	(0.000)
0.02	0.000	(0.000)	0.000	(0.000)	0.000	(0.000)	0.000	(0.000)	0.000	(0.000)
0.03	0.000	(0.000)	0.000	(0.000)	0.000	(0.000)	0.000	(0.000)	0.000	(0.000)
0.04	0.000	(0.000)	0.000	(0.000)	0.000	(0.000)	0.001	(0.000)	0.001	(0.000)
0.05	0.000	(0.000)	0.001	(0.000)	0.002	(0.000)	0.011	(0.001)	0.010	(0.001)
0.06	0.006	(0.001)	0.013	(0.001)	0.023	(0.001)	0.057	(0.002)	0.050	(0.002)
0.07	0.045	(0.002)	0.071	(0.002)	0.099	(0.003)	0.168	(0.004)	0.147	(0.003)
0.08	0.179	(0.004)	0.227	(0.005)	0.270	(0.005)	0.362	(0.006)	0.328	(0.005)
0.09	0.478	(0.007)	0.527	(0.008)	0.566	(0.008)	0.640	(0.009)	0.611	(0.008)
0.10	1.000	(0.011)	1.000	(0.011)	0.999	(0.011)	1.003	(0.012)	1.000	(0.011)
0.11	1.772	(0.016)	1.656	(0.015)	1.567	(0.015)	1.443	(0.015)	1.499	(0.014)
0.12	2.801	(0.021)	2.501	(0.020)	2.263	(0.019)	1.949	(0.018)	2.111	(0.018)
0.13	4.083	(0.027)	3.529	(0.024)	3.083	(0.023)	2.512	(0.021)	2.829	(0.021)
0.14	5.608	(0.032)	4.734	(0.029)	4.014	(0.027)	3.126	(0.024)	3.654	(0.025)
0.15	7.355	(0.038)	6.104	(0.034)	5.040	(0.031)	3.783	(0.027)	4.573	(0.029)
0.16	9.309	(0.044)	7.624	(0.039)	6.150	(0.035)	4.477	(0.030)	5.581	(0.033)
0.17	11.453	(0.050)	9.290	(0.044)	7.334	(0.039)	5.203	(0.032)	6.673	(0.037)
0.18	13.773	(0.055)	11.095	(0.049)	8.586	(0.044)	5.956	(0.035)	7.843	(0.041)
0.19	16.256	(0.061)	13.031	(0.054)	9.899	(0.048)	6.735	(0.038)	9.083	(0.045)
0.20	18.892	(0.066)	15.095	(0.059)	11.266	(0.052)	7.535	(0.041)	10.394	(0.050)

Table 5: Value of the DPO in the different PLI models for different volatilities σ given a guaranteed rate of interest $r_g = 0.00$. Values in brackets display the corresponding standard error (rounded to three decimal places). (continued on next page)

(b) Set 2 ($r_g = 0.00$)

σ	Bacinello		Haberman et al.		Grosen and Jørgensen		Hansen and Miltersen		Kling et al.	
0.01	0.000	(0.000)	0.000	(0.000)	0.000	(0.000)	0.000	(0.000)	0.000	(0.000)
0.02	0.000	(0.000)	0.000	(0.000)	0.000	(0.000)	0.000	(0.000)	0.000	(0.000)
0.03	0.000	(0.000)	0.000	(0.000)	0.000	(0.000)	0.000	(0.000)	0.000	(0.000)
0.04	0.000	(0.000)	0.000	(0.000)	0.000	(0.000)	0.001	(0.000)	0.000	(0.000)
0.05	0.000	(0.000)	0.001	(0.000)	0.001	(0.000)	0.011	(0.001)	0.007	(0.001)
0.06	0.006	(0.001)	0.014	(0.001)	0.016	(0.001)	0.057	(0.002)	0.040	(0.002)
0.07	0.045	(0.002)	0.075	(0.002)	0.079	(0.002)	0.167	(0.004)	0.130	(0.003)
0.08	0.179	(0.004)	0.233	(0.005)	0.240	(0.005)	0.360	(0.006)	0.307	(0.005)
0.09	0.478	(0.007)	0.533	(0.008)	0.539	(0.008)	0.638	(0.009)	0.593	(0.008)
0.10	1.000	(0.011)	1.000	(0.011)	1.000	(0.011)	1.000	(0.012)	1.000	(0.011)
0.11	1.772	(0.016)	1.642	(0.015)	1.624	(0.015)	1.440	(0.015)	1.530	(0.015)
0.12	2.801	(0.021)	2.463	(0.019)	2.408	(0.020)	1.947	(0.018)	2.180	(0.018)
0.13	4.083	(0.027)	3.457	(0.024)	3.344	(0.024)	2.511	(0.021)	2.949	(0.022)
0.14	5.608	(0.032)	4.618	(0.028)	4.421	(0.029)	3.128	(0.024)	3.827	(0.026)
0.15	7.355	(0.038)	5.933	(0.033)	5.620	(0.034)	3.790	(0.027)	4.810	(0.030)
0.16	9.309	(0.044)	7.388	(0.038)	6.931	(0.039)	4.494	(0.030)	5.884	(0.034)
0.17	11.453	(0.050)	8.978	(0.043)	8.342	(0.043)	5.233	(0.033)	7.047	(0.038)
0.18	13.773	(0.055)	10.697	(0.048)	9.843	(0.048)	6.004	(0.035)	8.290	(0.043)
0.19	16.256	(0.061)	12.537	(0.052)	11.427	(0.053)	6.806	(0.038)	9.608	(0.047)
0.20	18.892	(0.066)	14.492	(0.057)	13.084	(0.058)	7.636	(0.041)	10.995	(0.051)

Table 5: Value of the DPO in the different PLI models for different volatilities σ given a guaranteed rate of interest $r_g = 0.00$. Values in brackets display the corresponding standard error (rounded to three decimal places). (cont.)

(c) Set 3 ($r_g = 0.00$)

σ	Bacinello		Haberman et al.		Grosen and Jørgensen		Hansen and Miltersen		Kling et al.	
0.01	0.000	(0.000)	0.000	(0.000)	0.000	(0.000)	0.000	(0.000)	0.000	(0.000)
0.02	0.000	(0.000)	0.000	(0.000)	0.000	(0.000)	0.000	(0.000)	0.000	(0.000)
0.03	0.000	(0.000)	0.000	(0.000)	0.000	(0.000)	0.000	(0.000)	0.000	(0.000)
0.04	0.000	(0.000)	0.000	(0.000)	0.000	(0.000)	0.001	(0.000)	0.001	(0.000)
0.05	0.000	(0.000)	0.001	(0.000)	0.001	(0.000)	0.011	(0.001)	0.008	(0.001)
0.06	0.006	(0.001)	0.014	(0.001)	0.016	(0.001)	0.056	(0.002)	0.046	(0.002)
0.07	0.045	(0.002)	0.075	(0.002)	0.079	(0.002)	0.167	(0.004)	0.142	(0.003)
0.08	0.179	(0.004)	0.233	(0.005)	0.240	(0.005)	0.360	(0.006)	0.324	(0.005)
0.09	0.478	(0.007)	0.533	(0.008)	0.539	(0.008)	0.637	(0.009)	0.608	(0.008)
0.10	1.000	(0.011)	1.000	(0.011)	1.000	(0.011)	1.000	(0.012)	1.000	(0.011)
0.11	1.772	(0.016)	1.642	(0.015)	1.623	(0.015)	1.442	(0.015)	1.508	(0.014)
0.12	2.801	(0.021)	2.463	(0.019)	2.406	(0.020)	1.953	(0.018)	2.128	(0.018)
0.13	4.083	(0.027)	3.458	(0.024)	3.343	(0.024)	2.524	(0.021)	2.856	(0.022)
0.14	5.608	(0.032)	4.619	(0.028)	4.419	(0.029)	3.151	(0.024)	3.690	(0.025)
0.15	7.355	(0.038)	5.934	(0.033)	5.617	(0.034)	3.828	(0.027)	4.622	(0.029)
0.16	9.309	(0.044)	7.389	(0.038)	6.927	(0.038)	4.548	(0.030)	5.643	(0.033)
0.17	11.453	(0.050)	8.980	(0.043)	8.338	(0.043)	5.307	(0.033)	6.746	(0.037)
0.18	13.773	(0.055)	10.699	(0.048)	9.838	(0.048)	6.101	(0.035)	7.926	(0.041)
0.19	16.256	(0.061)	12.540	(0.052)	11.421	(0.053)	6.927	(0.038)	9.181	(0.046)
0.20	18.892	(0.066)	14.496	(0.057)	13.078	(0.058)	7.781	(0.041)	10.504	(0.050)

Table 5: Value of the DPO in the different PLI models for different volatilities σ given a guaranteed rate of interest $r_g = 0.00$. Values in brackets display the corresponding standard error (rounded to three decimal places). (cont.)

(a) Set 1 ($r_g = 0.02$)

σ	Bacinello		Haberman et al.		Grosen and Jørgensen		Hansen and Miltersen		Kling et al.	
0.01	0.000	(0.000)	0.000	(0.000)	0.000	(0.000)	0.000	(0.000)	0.000	(0.000)
0.02	0.000	(0.000)	0.000	(0.000)	0.000	(0.000)	0.000	(0.000)	0.000	(0.000)
0.03	0.000	(0.000)	0.000	(0.000)	0.000	(0.000)	0.000	(0.000)	0.000	(0.000)
0.04	0.000	(0.000)	0.000	(0.000)	0.000	(0.000)	0.000	(0.000)	0.000	(0.000)
0.05	0.001	(0.000)	0.003	(0.000)	0.003	(0.000)	0.007	(0.001)	0.005	(0.001)
0.06	0.012	(0.001)	0.027	(0.001)	0.026	(0.001)	0.043	(0.002)	0.034	(0.002)
0.07	0.068	(0.003)	0.108	(0.003)	0.102	(0.003)	0.142	(0.004)	0.122	(0.003)
0.08	0.219	(0.005)	0.282	(0.006)	0.269	(0.006)	0.329	(0.006)	0.298	(0.006)
0.09	0.517	(0.008)	0.575	(0.009)	0.562	(0.009)	0.616	(0.009)	0.586	(0.009)
0.10	1.000	(0.013)	1.000	(0.013)	1.000	(0.013)	1.000	(0.013)	0.995	(0.013)
0.11	1.687	(0.017)	1.558	(0.016)	1.593	(0.017)	1.484	(0.016)	1.533	(0.016)
0.12	2.580	(0.022)	2.252	(0.021)	2.347	(0.021)	2.056	(0.020)	2.204	(0.020)
0.13	3.673	(0.028)	3.069	(0.025)	3.253	(0.026)	2.706	(0.024)	2.999	(0.025)
0.14	4.957	(0.033)	4.001	(0.029)	4.299	(0.031)	3.425	(0.027)	3.911	(0.029)
0.15	6.420	(0.039)	5.043	(0.034)	5.478	(0.036)	4.205	(0.031)	4.934	(0.033)
0.16	8.045	(0.044)	6.183	(0.038)	6.782	(0.041)	5.040	(0.035)	6.063	(0.038)
0.17	9.822	(0.050)	7.413	(0.042)	8.204	(0.046)	5.924	(0.038)	7.289	(0.042)
0.18	11.740	(0.055)	8.725	(0.047)	9.728	(0.051)	6.847	(0.042)	8.603	(0.047)
0.19	13.788	(0.061)	10.115	(0.051)	11.347	(0.056)	7.807	(0.045)	10.000	(0.052)
0.20	15.956	(0.066)	11.580	(0.055)	13.056	(0.062)	8.799	(0.048)	11.471	(0.056)

Table 6: Value of the DPO in the different PLI models for different volatilities σ given a guaranteed rate of interest $r_g = 0.02$. Values in brackets display the corresponding standard error (rounded to three decimal places). (continued on next page)

(b) Set 2 ($r_g = 0.02$)

σ	Bacinello		Haberman et al.		Grosen and Jørgensen		Hansen and Miltersen		Kling et al.	
0.01	0.000	(0.000)	0.000	(0.000)	0.000	(0.000)	0.000	(0.000)	0.000	(0.000)
0.02	0.000	(0.000)	0.000	(0.000)	0.000	(0.000)	0.000	(0.000)	0.000	(0.000)
0.03	0.000	(0.000)	0.000	(0.000)	0.000	(0.000)	0.000	(0.000)	0.000	(0.000)
0.04	0.000	(0.000)	0.000	(0.000)	0.000	(0.000)	0.000	(0.000)	0.000	(0.000)
0.05	0.001	(0.000)	0.003	(0.000)	0.003	(0.000)	0.006	(0.001)	0.005	(0.001)
0.06	0.012	(0.001)	0.027	(0.001)	0.026	(0.001)	0.042	(0.002)	0.035	(0.002)
0.07	0.068	(0.003)	0.108	(0.003)	0.103	(0.003)	0.141	(0.004)	0.124	(0.004)
0.08	0.219	(0.005)	0.282	(0.006)	0.273	(0.006)	0.327	(0.006)	0.303	(0.006)
0.09	0.517	(0.008)	0.575	(0.009)	0.566	(0.009)	0.613	(0.009)	0.593	(0.009)
0.10	1.000	(0.013)	1.000	(0.013)	1.000	(0.013)	1.000	(0.013)	1.003	(0.013)
0.11	1.687	(0.017)	1.558	(0.016)	1.581	(0.017)	1.488	(0.016)	1.542	(0.016)
0.12	2.580	(0.022)	2.251	(0.021)	2.314	(0.021)	2.070	(0.020)	2.212	(0.020)
0.13	3.673	(0.028)	3.068	(0.025)	3.184	(0.025)	2.734	(0.024)	3.005	(0.025)
0.14	4.957	(0.033)	4.000	(0.029)	4.187	(0.030)	3.472	(0.027)	3.913	(0.029)
0.15	6.420	(0.039)	5.042	(0.034)	5.309	(0.035)	4.276	(0.031)	4.931	(0.033)
0.16	8.045	(0.044)	6.181	(0.038)	6.544	(0.040)	5.143	(0.035)	6.054	(0.038)
0.17	9.822	(0.050)	7.410	(0.042)	7.880	(0.044)	6.062	(0.039)	7.274	(0.042)
0.18	11.740	(0.055)	8.722	(0.047)	9.307	(0.049)	7.029	(0.042)	8.581	(0.047)
0.19	13.788	(0.061)	10.111	(0.051)	10.819	(0.054)	8.036	(0.046)	9.970	(0.051)
0.20	15.956	(0.066)	11.576	(0.055)	12.408	(0.059)	9.080	(0.049)	11.432	(0.056)

Table 6: Value of the DPO in the different PLI models for different volatilities σ given a guaranteed rate of interest $r_g = 0.02$. Values in brackets display the corresponding standard error (rounded to three decimal places). (cont.)

(c) Set 3 ($r_g = 0.02$)

σ	Bacinello		Haberman et al.		Grosen and Jørgensen		Hansen and Miltersen		Kling et al.	
0.01	0.000	(0.000)	0.000	(0.000)	0.000	(0.000)	0.000	(0.000)	0.000	(0.000)
0.02	0.000	(0.000)	0.000	(0.000)	0.000	(0.000)	0.000	(0.000)	0.000	(0.000)
0.03	0.000	(0.000)	0.000	(0.000)	0.000	(0.000)	0.000	(0.000)	0.000	(0.000)
0.04	0.000	(0.000)	0.000	(0.000)	0.000	(0.000)	0.000	(0.000)	0.000	(0.000)
0.05	0.001	(0.000)	0.003	(0.000)	0.003	(0.000)	0.007	(0.001)	0.005	(0.001)
0.06	0.012	(0.001)	0.028	(0.001)	0.026	(0.001)	0.043	(0.002)	0.034	(0.002)
0.07	0.068	(0.003)	0.108	(0.003)	0.103	(0.003)	0.142	(0.004)	0.123	(0.004)
0.08	0.219	(0.005)	0.282	(0.006)	0.272	(0.006)	0.330	(0.006)	0.301	(0.006)
0.09	0.517	(0.008)	0.575	(0.009)	0.564	(0.009)	0.616	(0.009)	0.592	(0.009)
0.10	1.000	(0.013)	1.000	(0.013)	1.000	(0.013)	0.999	(0.013)	1.001	(0.013)
0.11	1.687	(0.017)	1.558	(0.016)	1.586	(0.017)	1.479	(0.016)	1.537	(0.016)
0.12	2.580	(0.022)	2.251	(0.021)	2.328	(0.021)	2.045	(0.020)	2.203	(0.020)
0.13	3.673	(0.028)	3.067	(0.025)	3.214	(0.026)	2.685	(0.024)	2.990	(0.025)
0.14	4.957	(0.033)	3.998	(0.029)	4.235	(0.030)	3.390	(0.027)	3.889	(0.029)
0.15	6.420	(0.039)	5.039	(0.034)	5.381	(0.035)	4.152	(0.031)	4.898	(0.033)
0.16	8.045	(0.044)	6.177	(0.038)	6.644	(0.040)	4.965	(0.034)	6.011	(0.038)
0.17	9.822	(0.050)	7.405	(0.042)	8.017	(0.045)	5.820	(0.038)	7.219	(0.042)
0.18	11.740	(0.055)	8.715	(0.047)	9.485	(0.050)	6.711	(0.041)	8.511	(0.047)
0.19	13.788	(0.061)	10.103	(0.051)	11.042	(0.055)	7.633	(0.045)	9.885	(0.051)
0.20	15.956	(0.066)	11.566	(0.055)	12.682	(0.060)	8.581	(0.048)	11.332	(0.056)

Table 6: Value of the DPO in the different PLI models for different volatilities σ given a guaranteed rate of interest $r_g = 0.02$. Values in brackets display the corresponding standard error (rounded to three decimal places). (cont.)

(a) Set 1 ($r_g = 0.00$)

$B_0^{(\text{new})}/B_0^{(\text{initial})}$	Bacinello		Haberman et al.		Grosen and Jørgensen		Hansen and Miltersen		Kling et al.	
1.25	0.572	(0.008)	0.654	(0.009)	1.004	(0.012)	0.946	(0.011)	0.895	(0.011)
1.20	0.640	(0.009)	0.712	(0.009)	1.002	(0.011)	0.957	(0.011)	0.912	(0.011)
1.15	0.715	(0.010)	0.775	(0.010)	1.001	(0.011)	0.968	(0.011)	0.932	(0.011)
1.10	0.800	(0.010)	0.844	(0.010)	1.000	(0.011)	0.980	(0.012)	0.954	(0.011)
1.05	0.895	(0.011)	0.919	(0.011)	0.999	(0.011)	0.991	(0.012)	0.976	(0.011)
1.00	1.000	(0.011)	1.000	(0.011)	0.999	(0.011)	1.003	(0.012)	1.000	(0.011)
0.95	1.117	(0.012)	1.088	(0.012)	0.999	(0.011)	1.014	(0.012)	1.026	(0.011)
0.90	1.246	(0.013)	1.184	(0.012)	0.999	(0.011)	1.026	(0.012)	1.053	(0.011)
0.85	1.390	(0.014)	1.289	(0.013)	1.000	(0.011)	1.038	(0.012)	1.081	(0.012)
0.80	1.550	(0.014)	1.403	(0.013)	1.001	(0.011)	1.050	(0.012)	1.111	(0.012)
0.75	1.728	(0.015)	1.528	(0.014)	1.002	(0.011)	1.063	(0.012)	1.143	(0.012)

Table 7: Value of the DPO in the different PLI models for different reserve levels B_0 given a guaranteed rate of interest $r_g = 0.00$. Values in brackets display the corresponding standard error (rounded to three decimal places). (continued on next page)

(b) Set 2 ($r_g = 0.00$)

$B_0^{(\text{new})}/B_0^{(\text{initial})}$	Bacinello		Haberman et al.		Grosen and Jørgensen		Hansen and Miltersen		Kling et al.	
1.25	0.572	(0.008)	0.669	(0.009)	1.027	(0.012)	0.943	(0.011)	0.913	(0.011)
1.20	0.640	(0.009)	0.725	(0.009)	1.017	(0.012)	0.954	(0.011)	0.927	(0.011)
1.15	0.715	(0.010)	0.786	(0.010)	1.008	(0.012)	0.966	(0.011)	0.943	(0.011)
1.10	0.800	(0.010)	0.852	(0.010)	0.998	(0.011)	0.977	(0.011)	0.960	(0.011)
1.05	0.895	(0.011)	0.923	(0.011)	0.998	(0.011)	0.989	(0.012)	0.979	(0.011)
1.00	1.000	(0.011)	1.000	(0.011)	1.000	(0.011)	1.000	(0.012)	1.000	(0.011)
0.95	1.117	(0.012)	1.083	(0.012)	1.004	(0.011)	1.012	(0.012)	1.022	(0.011)
0.90	1.246	(0.013)	1.173	(0.012)	1.009	(0.011)	1.024	(0.012)	1.047	(0.011)
0.85	1.390	(0.014)	1.271	(0.013)	1.015	(0.011)	1.036	(0.012)	1.074	(0.012)
0.80	1.550	(0.014)	1.377	(0.013)	1.024	(0.011)	1.048	(0.012)	1.104	(0.012)
0.75	1.728	(0.015)	1.493	(0.014)	1.035	(0.011)	1.061	(0.012)	1.135	(0.012)

Table 7: Value of the DPO in the different PLI models for different reserve levels B_0 given a guaranteed rate of interest $r_g = 0.00$. Values in brackets display the corresponding standard error (rounded to three decimal places). (cont.)

(c) Set 3 ($r_g = 0.00$)

$B_0^{(\text{new})}/B_0^{(\text{initial})}$	Bacinello		Haberman et al.		Grosen and Jørgensen		Hansen and Miltersen		Kling et al.	
1.25	0.572	(0.008)	0.669	(0.009)	1.026	(0.012)	0.942	(0.011)	0.911	(0.011)
1.20	0.640	(0.009)	0.725	(0.009)	1.017	(0.012)	0.954	(0.011)	0.926	(0.011)
1.15	0.715	(0.010)	0.786	(0.010)	1.007	(0.012)	0.965	(0.011)	0.943	(0.011)
1.10	0.800	(0.010)	0.852	(0.010)	0.998	(0.011)	0.977	(0.011)	0.961	(0.011)
1.05	0.895	(0.011)	0.923	(0.011)	0.998	(0.011)	0.988	(0.012)	0.980	(0.011)
1.00	1.000	(0.011)	1.000	(0.011)	1.000	(0.011)	1.000	(0.012)	1.000	(0.011)
0.95	1.117	(0.012)	1.083	(0.012)	1.003	(0.011)	1.012	(0.012)	1.022	(0.011)
0.90	1.246	(0.013)	1.173	(0.012)	1.008	(0.011)	1.024	(0.012)	1.044	(0.011)
0.85	1.390	(0.014)	1.271	(0.013)	1.015	(0.011)	1.036	(0.012)	1.068	(0.011)
0.80	1.550	(0.014)	1.378	(0.013)	1.023	(0.011)	1.049	(0.012)	1.093	(0.012)
0.75	1.728	(0.015)	1.493	(0.014)	1.034	(0.011)	1.061	(0.012)	1.119	(0.012)

Table 7: Value of the DPO in the different PLI models for different reserve levels B_0 given a guaranteed rate of interest $r_g = 0.00$. Values in brackets display the corresponding standard error (rounded to three decimal places). (cont.)

(a) Set 1 ($r_g = 0.02$)

$B_0^{(\text{new})}/B_0^{(\text{initial})}$	Bacinello		Haberman et al.		Grosen and Jørgensen		Hansen and Miltersen		Kling et al.	
1.25	0.542	(0.009)	0.624	(0.010)	0.816	(0.011)	0.678	(0.010)	0.747	(0.011)
1.20	0.612	(0.010)	0.686	(0.010)	0.841	(0.012)	0.733	(0.011)	0.787	(0.011)
1.15	0.692	(0.010)	0.754	(0.011)	0.872	(0.012)	0.793	(0.011)	0.831	(0.011)
1.10	0.783	(0.011)	0.828	(0.011)	0.909	(0.012)	0.857	(0.012)	0.880	(0.012)
1.05	0.885	(0.012)	0.910	(0.012)	0.951	(0.012)	0.926	(0.012)	0.935	(0.012)
1.00	1.000	(0.013)	1.000	(0.013)	1.000	(0.013)	1.000	(0.013)	0.995	(0.013)
0.95	1.130	(0.013)	1.099	(0.013)	1.056	(0.013)	1.081	(0.013)	1.061	(0.013)
0.90	1.276	(0.014)	1.206	(0.014)	1.120	(0.013)	1.169	(0.014)	1.135	(0.013)
0.85	1.441	(0.015)	1.325	(0.015)	1.192	(0.014)	1.264	(0.014)	1.216	(0.014)
0.80	1.626	(0.016)	1.455	(0.015)	1.274	(0.014)	1.366	(0.015)	1.305	(0.014)
0.75	1.833	(0.017)	1.599	(0.016)	1.366	(0.015)	1.476	(0.016)	1.403	(0.015)

Table 8: Value of the DPO in the different PLI models for different reserve levels B_0 given a guaranteed rate of interest $r_g = 0.02$. Values in brackets display the corresponding standard error (rounded to three decimal places). (continued on next page)

(b) Set 2 ($r_g = 0.02$)

$B_0^{(\text{new})}/B_0^{(\text{initial})}$	Bacinello		Haberman et al.		Grosen and Jørgensen		Hansen and Miltersen		Kling et al.	
1.25	0.542	(0.009)	0.624	(0.010)	0.811	(0.011)	0.676	(0.010)	0.754	(0.011)
1.20	0.612	(0.010)	0.686	(0.010)	0.838	(0.011)	0.731	(0.011)	0.794	(0.011)
1.15	0.692	(0.010)	0.754	(0.011)	0.870	(0.012)	0.791	(0.011)	0.839	(0.011)
1.10	0.783	(0.011)	0.828	(0.011)	0.907	(0.012)	0.855	(0.012)	0.889	(0.012)
1.05	0.885	(0.012)	0.910	(0.012)	0.950	(0.012)	0.925	(0.012)	0.943	(0.012)
1.00	1.000	(0.013)	1.000	(0.013)	1.000	(0.013)	1.000	(0.013)	1.003	(0.013)
0.95	1.130	(0.013)	1.099	(0.013)	1.056	(0.013)	1.082	(0.013)	1.070	(0.013)
0.90	1.276	(0.014)	1.206	(0.014)	1.120	(0.013)	1.170	(0.014)	1.143	(0.013)
0.85	1.441	(0.015)	1.325	(0.015)	1.193	(0.014)	1.266	(0.014)	1.224	(0.014)
0.80	1.626	(0.016)	1.455	(0.015)	1.275	(0.014)	1.369	(0.015)	1.313	(0.015)
0.75	1.833	(0.017)	1.599	(0.016)	1.367	(0.015)	1.480	(0.016)	1.410	(0.015)

Table 8: Value of the DPO in the different PLI models for different reserve levels B_0 given a guaranteed rate of interest $r_g = 0.02$. Values in brackets display the corresponding standard error (rounded to three decimal places). (cont.)

(c) Set 3 ($r_g = 0.02$)

$B_0^{(\text{new})}/B_0^{(\text{initial})}$	Bacinello		Haberman et al.		Grosen and Jørgensen		Hansen and Miltersen		Kling et al.	
1.25	0.542	(0.009)	0.624	(0.010)	0.813	(0.011)	0.678	(0.010)	0.757	(0.011)
1.20	0.612	(0.010)	0.686	(0.010)	0.839	(0.011)	0.733	(0.011)	0.796	(0.011)
1.15	0.692	(0.010)	0.754	(0.011)	0.871	(0.012)	0.792	(0.011)	0.840	(0.011)
1.10	0.783	(0.011)	0.828	(0.011)	0.908	(0.012)	0.856	(0.012)	0.889	(0.012)
1.05	0.885	(0.012)	0.910	(0.012)	0.951	(0.012)	0.925	(0.012)	0.942	(0.012)
1.00	1.000	(0.013)	1.000	(0.013)	1.000	(0.013)	0.999	(0.013)	1.001	(0.013)
0.95	1.130	(0.013)	1.098	(0.013)	1.056	(0.013)	1.080	(0.013)	1.066	(0.013)
0.90	1.276	(0.014)	1.206	(0.014)	1.120	(0.013)	1.167	(0.014)	1.137	(0.013)
0.85	1.441	(0.015)	1.324	(0.015)	1.193	(0.014)	1.261	(0.014)	1.216	(0.014)
0.80	1.626	(0.016)	1.455	(0.015)	1.275	(0.014)	1.363	(0.015)	1.303	(0.014)
0.75	1.833	(0.017)	1.598	(0.016)	1.367	(0.015)	1.472	(0.016)	1.398	(0.015)

Table 8: Value of the DPO in the different PLI models for different reserve levels B_0 given a guaranteed rate of interest $r_g = 0.02$. Values in brackets display the corresponding standard error (rounded to three decimal places). (cont.)

(a) $\mu = 0.04$ ($r_g = 0.02$)

σ	Bacinello	Haberman et al.	Grosen and Jørgensen	Hansen and Miltersen	Kling et al.
0.01	0.000	0.000	0.000	0.000	0.000
0.02	0.000	0.000	0.000	0.000	0.000
0.03	0.000	0.000	0.000	0.000	0.000
0.04	0.000	0.000	0.000	0.000	0.000
0.05	0.000	0.001	0.001	0.002	0.001
0.06	0.003	0.006	0.006	0.009	0.007
0.07	0.012	0.019	0.017	0.023	0.021
0.08	0.032	0.039	0.037	0.043	0.041
0.09	0.061	0.066	0.065	0.067	0.067
0.10	0.098	0.097	0.097	0.094	0.097
0.11	0.140	0.132	0.135	0.123	0.131
0.12	0.185	0.168	0.175	0.151	0.167
0.13	0.230	0.205	0.214	0.179	0.202
0.14	0.275	0.241	0.253	0.206	0.237
0.15	0.318	0.276	0.292	0.233	0.272
0.16	0.359	0.309	0.330	0.258	0.307
0.17	0.397	0.342	0.364	0.282	0.338
0.18	0.434	0.372	0.396	0.305	0.369
0.19	0.467	0.401	0.428	0.327	0.397
0.20	0.500	0.430	0.457	0.349	0.425

Table 9: Shortfall probabilities in the different PLI models for different volatilities σ given different drift coefficients $\mu \in \{0.04, 0.06, 0.08\}$ ($r_g = 0.02$). Data are only displayed for Set 1 (rounded to three decimal places). (continued on next page)

(b) $\mu = 0.06$ ($r_g = 0.02$)

σ	Bacinello	Haberman et al.	Grosen and Jørgensen	Hansen and Miltersen	Kling et al.
0.01	0.000	0.000	0.000	0.000	0.000
0.02	0.000	0.000	0.000	0.000	0.000
0.03	0.000	0.000	0.000	0.000	0.000
0.04	0.000	0.000	0.000	0.000	0.000
0.05	0.000	0.000	0.000	0.000	0.000
0.06	0.000	0.000	0.000	0.000	0.000
0.07	0.001	0.001	0.001	0.002	0.002
0.08	0.004	0.005	0.006	0.006	0.006
0.09	0.013	0.014	0.014	0.014	0.014
0.10	0.028	0.028	0.030	0.026	0.028
0.11	0.050	0.047	0.051	0.041	0.047
0.12	0.079	0.070	0.079	0.059	0.070
0.13	0.113	0.097	0.109	0.079	0.097
0.14	0.150	0.125	0.143	0.102	0.127
0.15	0.188	0.157	0.178	0.125	0.159
0.16	0.228	0.188	0.214	0.149	0.191
0.17	0.267	0.220	0.250	0.172	0.224
0.18	0.306	0.253	0.286	0.195	0.256
0.19	0.343	0.283	0.319	0.218	0.287
0.20	0.378	0.314	0.352	0.241	0.318

Table 9: Shortfall probabilities in the different PLI models for different volatilities σ given different drift coefficients $\mu \in \{0.04, 0.06, 0.08\}$ ($r_g = 0.02$). Data are only displayed for Set 1 (rounded to three decimal places). (cont.)

(c) $\mu = 0.08$ ($r_g = 0.02$)

σ	Bacinello	Haberman et al.	Grosen and Jørgensen	Hansen and Miltersen	Kling et al.
0.01	0.000	0.000	0.000	0.000	0.000
0.02	0.000	0.000	0.000	0.000	0.000
0.03	0.000	0.000	0.000	0.000	0.000
0.04	0.000	0.000	0.000	0.000	0.000
0.05	0.000	0.000	0.000	0.000	0.000
0.06	0.000	0.000	0.000	0.000	0.000
0.07	0.000	0.000	0.000	0.000	0.000
0.08	0.000	0.000	0.001	0.000	0.000
0.09	0.002	0.002	0.002	0.002	0.002
0.10	0.005	0.005	0.007	0.005	0.006
0.11	0.014	0.012	0.016	0.010	0.013
0.12	0.027	0.023	0.030	0.018	0.025
0.13	0.046	0.038	0.049	0.029	0.041
0.14	0.069	0.056	0.073	0.043	0.060
0.15	0.098	0.078	0.101	0.058	0.083
0.16	0.129	0.102	0.130	0.075	0.109
0.17	0.163	0.128	0.160	0.093	0.136
0.18	0.198	0.156	0.193	0.113	0.166
0.19	0.234	0.184	0.227	0.134	0.195
0.20	0.269	0.213	0.260	0.155	0.226

Table 9: Shortfall probabilities in the different PLI models for different volatilities σ given different drift coefficients $\mu \in \{0.04, 0.06, 0.08\}$ ($r_g = 0.02$). Data are only displayed for Set 1 (rounded to three decimal places). (cont.)

(a) $\mu = 0.04$ ($r_g = 0.02$)

$B_0^{(new)}/B_0^{(initial)}$	Bacinello	Haberman et al.	Grosen and Jørgensen	Hansen and Miltersen	Kling et al.
1.25	0.057	0.065	0.082	0.067	0.077
1.20	0.064	0.070	0.084	0.072	0.080
1.15	0.072	0.076	0.087	0.077	0.084
1.10	0.079	0.083	0.090	0.082	0.088
1.05	0.088	0.090	0.093	0.087	0.092
1.00	0.098	0.097	0.097	0.094	0.097
0.95	0.109	0.105	0.102	0.101	0.102
0.90	0.120	0.114	0.107	0.108	0.108
0.85	0.133	0.124	0.113	0.115	0.115
0.80	0.147	0.134	0.120	0.123	0.122
0.75	0.162	0.146	0.127	0.131	0.130

Table 10: Shortfall probabilities in the different PLI models for different reserves B_0 given different drift coefficients $\mu \in \{0.04, 0.06, 0.08\}$ ($r_g = 0.02$). Data are only displayed for Set 1 (rounded to three decimal places). (continued on next page)

(b) $\mu = 0.06$ ($r_g = 0.02$)

$B_0^{(new)}/B_0^{(initial)}$	Bacinello	Haberman et al.	Grosen and Jørgensen	Hansen and Miltersen	Kling et al.
1.25	0.014	0.016	0.025	0.017	0.021
1.20	0.016	0.018	0.026	0.018	0.022
1.15	0.018	0.020	0.026	0.020	0.024
1.10	0.021	0.022	0.027	0.022	0.025
1.05	0.024	0.025	0.028	0.024	0.026
1.00	0.028	0.028	0.030	0.026	0.028
0.95	0.031	0.031	0.031	0.028	0.030
0.90	0.036	0.034	0.033	0.030	0.032
0.85	0.042	0.038	0.035	0.033	0.034
0.80	0.047	0.042	0.037	0.036	0.037
0.75	0.054	0.047	0.040	0.040	0.040

Table 10: Shortfall probabilities in the different PLI models for different reserves B_0 given different drift coefficients $\mu \in \{0.04, 0.06, 0.08\}$ ($r_g = 0.02$). Data are only displayed for Set 1 (rounded to three decimal places). (cont.)

(c) $\mu = 0.08$ ($r_g = 0.02$)

$B_0^{(new)}/B_0^{(initial)}$	Bacinello	Haberman et al.	Grosen and Jørgensen	Hansen and Miltersen	Kling et al.
1.25	0.002	0.003	0.006	0.003	0.004
1.20	0.003	0.003	0.006	0.003	0.005
1.15	0.003	0.004	0.006	0.004	0.005
1.10	0.004	0.004	0.006	0.004	0.005
1.05	0.005	0.005	0.007	0.004	0.006
1.00	0.005	0.005	0.007	0.005	0.006
0.95	0.006	0.006	0.007	0.005	0.006
0.90	0.008	0.007	0.008	0.006	0.007
0.85	0.009	0.008	0.008	0.007	0.008
0.80	0.011	0.009	0.009	0.008	0.008
0.75	0.013	0.011	0.009	0.009	0.009

Table 10: Shortfall probabilities in the different PLI models for different reserves B_0 given different drift coefficients $\mu \in \{0.04, 0.06, 0.08\}$ ($r_g = 0.02$). Data are only displayed for Set 1 (rounded to three decimal places). (cont.)

(a) **Bacinello, $\boldsymbol{r_g = 0.00}$**, $\boldsymbol{V_D = S}$, $B_0^{(Bac)} \neq B_0^{(Gro)}$
($\alpha = 0.650,\ B_0 = 32.677$)

σ	V_D		$\Delta(V_D)$	$\Delta(V_D)/V_D$
0.02	0.000	(0.000)	0.000	-
0.04	0.000	(0.000)	0.000	-
0.06	0.006	(0.001)	0.006	38.226
0.08	0.179	(0.004)	0.133	2.940
0.09	0.478	(0.007)	0.300	1.677
0.10	1.000	(0.011)	0.522	1.092
0.11	1.772	(0.016)	0.772	0.772
0.12	2.801	(0.021)	1.029	0.581
0.14	5.608	(0.032)	1.524	0.373
0.16	9.309	(0.044)	1.955	0.266
0.18	13.773	(0.055)	2.319	0.203
0.20	18.892	(0.066)	2.636	0.162

(b) **Bacinello, $\boldsymbol{r_g = 0.00}$**, $V_D \neq S$, $\boldsymbol{B_0^{(Bac)} = B_0^{(Gro)}}$
($\alpha = 0.668,\ B_0 = 23.063$)

σ	V_D		$\Delta(V_D)$	$\Delta(V_D)/V_D$
0.02	0.000	(0.000)	0.000	-
0.04	0.000	(0.000)	0.000	-
0.06	0.032	(0.001)	0.030	13.044
0.08	0.477	(0.007)	0.317	1.983
0.09	1.048	(0.011)	0.572	1.198
0.10	1.900	(0.016)	0.852	0.812
0.11	3.032	(0.021)	1.131	0.595
0.12	4.429	(0.027)	1.397	0.461
0.14	7.933	(0.038)	1.863	0.307
0.16	12.233	(0.049)	2.240	0.224
0.18	17.196	(0.060)	2.557	0.175
0.20	22.717	(0.070)	2.824	0.142

Table 11: Value of the DPO, standard error of the DPO value estimate, absolute change of the DPO value, and relative change of the DPO value for a fixed safety level or fixed reserves. Data are only displayed for Set 1 (rounded to three decimal places). (continued on next page)
Note that we obtain the optimal parameter combination for panel (b) by simulating 10'000 paths. Simulation results (i.e., DPO values) are still based on 100'000 paths.

(c) **Hansen and Miltersen, $\boldsymbol{r_g = 0.00}$, $\boldsymbol{V_D = S}$**, $B_0^{(Han)} \neq B_0^{(Gro)}$
($\alpha = 0.313$, $\gamma = 0.814$, $\rho = 0.344$, $C_0 = 3.739$)

σ	V_D		$\Delta(V_D)$	$\Delta(V_D)/V_D$
0.02	0.000	(0.000)	0.000	-
0.04	0.001	(0.000)	0.001	-
0.06	0.057	(0.002)	0.046	4.320
0.08	0.362	(0.006)	0.194	1.153
0.09	0.640	(0.009)	0.278	0.770
0.10	1.003	(0.012)	0.362	0.566
0.11	1.443	(0.015)	0.440	0.439
0.12	1.949	(0.018)	0.506	0.351
0.14	3.126	(0.024)	0.614	0.244
0.16	4.477	(0.030)	0.694	0.183
0.18	5.956	(0.035)	0.753	0.145
0.20	7.535	(0.041)	0.801	0.119

(d) **Hansen and Miltersen, $\boldsymbol{r_g = 0.00}$**, $V_D \neq S$, $\boldsymbol{B_0^{(Han)} = B_0^{(Gro)}}$
($\alpha = 0.156$, $\gamma = 0.400$, $\rho = 0.374$, $C_0 = 23.063$)

σ	V_D		$\Delta(V_D)$	$\Delta(V_D)/V_D$
0.02	0.000	(0.000)	0.000	-
0.04	0.000	(0.000)	0.000	-
0.06	0.003	(0.000)	0.003	41.578
0.08	0.061	(0.002)	0.044	2.500
0.09	0.151	(0.004)	0.090	1.472
0.10	0.299	(0.006)	0.148	0.980
0.11	0.513	(0.008)	0.214	0.716
0.12	0.792	(0.011)	0.279	0.545
0.14	1.547	(0.017)	0.410	0.361
0.16	2.536	(0.022)	0.520	0.258
0.18	3.713	(0.028)	0.611	0.197
0.20	5.048	(0.034)	0.685	0.157

Table 11: Value of the DPO, standard error of the DPO value estimate, absolute change of the DPO value, and relative change of the DPO value for a fixed safety level or fixed reserves. Data are only displayed for Set 1 (rounded to three decimal places). (cont.)
Note that we obtain the optimal parameter combination for panel (d) by simulating 10'000 paths. Simulation results (i.e., DPO values) are still based on 100'000 paths.

(e) **Bacinello, $r_g = 0.02$, $V_D = S$**, $B_0^{(Bac)} \neq B_0^{(Gro)}$
($\alpha = 0.514,\ B_0 = 44.964$)

σ	V_D		$\Delta(V_D)$	$\Delta(V_D)/V_D$
0.02	0.000	(0.000)	0.000	-
0.04	0.000	(0.000)	0.000	-
0.06	0.012	(0.001)	0.012	17.442
0.08	0.219	(0.005)	0.152	2.243
0.09	0.517	(0.008)	0.298	1.360
0.10	1.000	(0.013)	0.483	0.933
0.11	1.687	(0.017)	0.687	0.687
0.12	2.580	(0.022)	0.892	0.529
0.14	4.957	(0.033)	1.284	0.350
0.16	8.045	(0.044)	1.625	0.253
0.18	11.740	(0.055)	1.918	0.195
0.20	15.956	(0.066)	2.169	0.157

(f) **Bacinello, $r_g = 0.02$**, $V_D \neq S$, $\boldsymbol{B_0^{(Bac)} = B_0^{(Gro)}}$
($\alpha = 0.529,\ B_0 = 35.004$)

σ	V_D		$\Delta(V_D)$	$\Delta(V_D)/V_D$
0.02	0.000	(0.000)	0.000	-
0.04	0.000	(0.000)	0.000	-
0.06	0.046	(0.002)	0.041	7.597
0.08	0.481	(0.008)	0.298	1.620
0.09	0.984	(0.012)	0.503	1.045
0.10	1.712	(0.017)	0.728	0.740
0.11	2.661	(0.022)	0.949	0.554
0.12	3.821	(0.027)	1.160	0.436
0.14	6.713	(0.038)	1.534	0.296
0.16	10.253	(0.049)	1.844	0.219
0.18	14.333	(0.060)	2.101	0.172
0.20	18.878	(0.070)	2.326	0.141

Table 11: Value of the DPO, standard error of the DPO value estimate, absolute change of the DPO value, and relative change of the DPO value for a fixed safety level or fixed reserves. Data are only displayed for Set 1 (rounded to three decimal places). (cont.)
Note that we obtain the optimal parameter combination for panel (f) by simulating 10'000 paths. Simulation results (i.e., DPO values) are still based on 100'000 paths.

(g) **Hansen and Miltersen, $\boldsymbol{r_g = 0.02}$, $\boldsymbol{V_D = S}$**, $B_0^{(Han)} \neq B_0^{(Gro)}$
($\alpha = 0.223$, $\gamma = 0.165$, $\rho = 0.304$, $C_0 = 30.432$)

σ	V_D		$\Delta(V_D)$	$\Delta(V_D)/V_D$
0.02	0.000	(0.000)	0.000	-
0.04	0.000	(0.000)	0.000	-
0.06	0.043	(0.002)	0.036	5.538
0.08	0.329	(0.006)	0.187	1.318
0.09	0.616	(0.009)	0.287	0.870
0.10	1.000	(0.013)	0.385	0.625
0.11	1.484	(0.016)	0.483	0.483
0.12	2.056	(0.020)	0.573	0.386
0.14	3.425	(0.027)	0.719	0.266
0.16	5.040	(0.035)	0.836	0.199
0.18	6.847	(0.042)	0.924	0.156
0.20	8.799	(0.048)	0.992	0.127

(h) **Hansen and Miltersen, $\boldsymbol{r_g = 0.02}$**, $V_D \neq S$, $\boldsymbol{B_0^{(Han)} = B_0^{(Gro)}}$
($\alpha = 0.091$, $\gamma = 0.257$, $\rho = 0.700$, $C_0 = 35.004$)

σ	V_D		$\Delta(V_D)$	$\Delta(V_D)/V_D$
0.02	0.000	(0.000)	0.000	-
0.04	0.000	(0.000)	0.000	-
0.06	0.024	(0.001)	0.021	6.943
0.08	0.234	(0.005)	0.142	1.531
0.09	0.466	(0.008)	0.232	0.993
0.10	0.792	(0.011)	0.326	0.698
0.11	1.212	(0.015)	0.420	0.530
0.12	1.724	(0.018)	0.512	0.422
0.14	2.984	(0.025)	0.666	0.287
0.16	4.505	(0.033)	0.789	0.212
0.18	6.237	(0.040)	0.889	0.166
0.20	8.127	(0.047)	0.963	0.134

Table 11: Value of the DPO, standard error of the DPO value estimate, absolute change of the DPO value, and relative change of the DPO value for a fixed safety level or fixed reserves. Data are only displayed for Set 1 (rounded to three decimal places). (cont.)
Note that we obtain the optimal parameter combination for panel (h) by simulating 10’000 paths. Simulation results (i.e., DPO values) are still based on 100’000 paths.

(a) **Bacinello, $\boldsymbol{r_g = 0.00}$**, $\boldsymbol{V_D = S}$, $B_0^{(Bac)} \neq B_0^{(Gro)}$
($\alpha = 0.650,\ B_0 = 32.677$)

$B_0^{(\text{new})}/B_0^{(\text{initial})}$	V_D		$\Delta(V_D)$	$\Delta(V_D)/V_D$
1.25	0.572	(0.008)		
1.20	0.640	(0.009)	0.068	0.118
1.15	0.715	(0.010)	0.076	0.119
1.10	0.800	(0.010)	0.085	0.119
1.05	0.895	(0.011)	0.095	0.118
1.00	1.000	(0.011)	0.105	0.117
0.95	1.117	(0.012)	0.117	0.117
0.90	1.246	(0.013)	0.129	0.116
0.85	1.390	(0.014)	0.144	0.116
0.80	1.550	(0.014)	0.160	0.115
0.75	1.728	(0.015)	0.177	0.114

(b) **Bacinello, $\boldsymbol{r_g = 0.00}$**, $V_D \neq S$, $\boldsymbol{B_0^{(Bac)} = B_0^{(Gro)}}$
($\alpha = 0.668,\ B_0 = 23.063$)

$B_0^{(\text{new})}/B_0^{(\text{initial})}$	V_D		$\Delta(V_D)$	$\Delta(V_D)/V_D$
1.25	1.295	(0.013)		
1.20	1.399	(0.014)	0.104	0.080
1.15	1.512	(0.014)	0.112	0.080
1.10	1.632	(0.015)	0.121	0.080
1.05	1.761	(0.015)	0.129	0.079
1.00	1.900	(0.016)	0.139	0.079
0.95	2.050	(0.017)	0.149	0.079
0.90	2.210	(0.017)	0.161	0.078
0.85	2.382	(0.018)	0.172	0.078
0.80	2.567	(0.019)	0.185	0.078
0.75	2.765	(0.019)	0.198	0.077

Table 12: Value of the DPO, standard error of the DPO value estimate, absolute change of the DPO value, and relative change of the DPO value for a fixed safety level or fixed reserves. Data are only displayed for Set 1 (rounded to three decimal places). (continued on next page)
Note that we obtain the optimal parameter combination for panel (b) by simulating 10'000 paths. Simulation results (i.e., DPO values) are still based on 100'000 paths.

(c) **Hansen and Miltersen,** $\boldsymbol{r_g = 0.00}$, $\boldsymbol{V_D = S}$, $B_0^{(Han)} \neq B_0^{(Gro)}$
($\alpha = 0.313$, $\gamma = 0.814$, $\rho = 0.344$, $C_0 = 3.739$)

$B_0^{(\text{new})}/B_0^{(\text{initial})}$	V_D		$\Delta(V_D)$	$\Delta(V_D)/V_D$
1.25	0.946	(0.011)		
1.20	0.957	(0.011)	0.011	0.012
1.15	0.968	(0.011)	0.011	0.012
1.10	0.980	(0.012)	0.011	0.012
1.05	0.991	(0.012)	0.011	0.012
1.00	1.003	(0.012)	0.012	0.012
0.95	1.014	(0.012)	0.012	0.012
0.90	1.026	(0.012)	0.012	0.012
0.85	1.038	(0.012)	0.012	0.012
0.80	1.050	(0.012)	0.012	0.012
0.75	1.063	(0.012)	0.012	0.012

(d) **Hansen and Miltersen,** $\boldsymbol{r_g = 0.00}$, $V_D \neq S$, $\boldsymbol{B_0^{(Han)} = B_0^{(Gro)}}$
($\alpha = 0.156$, $\gamma = 0.400$, $\rho = 0.374$, $C_0 = 23.063$)

$B_0^{(\text{new})}/B_0^{(\text{initial})}$	V_D		$\Delta(V_D)$	$\Delta(V_D)/V_D$
1.25	0.207	(0.005)		
1.20	0.223	(0.005)	0.016	0.076
1.15	0.240	(0.005)	0.017	0.076
1.10	0.258	(0.006)	0.018	0.076
1.05	0.277	(0.006)	0.020	0.076
1.00	0.299	(0.006)	0.021	0.076
0.95	0.322	(0.006)	0.023	0.076
0.90	0.346	(0.007)	0.024	0.076
0.85	0.372	(0.007)	0.026	0.076
0.80	0.400	(0.007)	0.028	0.076
0.75	0.431	(0.007)	0.030	0.075

Table 12: Value of the DPO, standard error of the DPO value estimate, absolute change of the DPO value, and relative change of the DPO value for a fixed safety level or fixed reserves. Data are only displayed for Set 1 (rounded to three decimal places). (cont.)
Note that we obtain the optimal parameter combination for panel (d) by simulating 10'000 paths. Simulation results (i.e., DPO values) are still based on 100'000 paths.

(e) **Bacinello,** $\boldsymbol{r_g = 0.02}$, $\boldsymbol{V_D = S}$, $B_0^{(Bac)} \neq B_0^{(Gro)}$
($\alpha = 0.514,\ B_0 = 44.964$)

$B_0^{(new)}/B_0^{(initial)}$	V_D		$\Delta(V_D)$	$\Delta(V_D)/V_D$
1.25	0.542	(0.009)		
1.20	0.612	(0.010)	0.071	0.130
1.15	0.692	(0.010)	0.080	0.131
1.10	0.783	(0.011)	0.091	0.131
1.05	0.885	(0.012)	0.102	0.130
1.00	1.000	(0.013)	0.115	0.130
0.95	1.130	(0.013)	0.130	0.130
0.90	1.276	(0.014)	0.146	0.130
0.85	1.441	(0.015)	0.165	0.129
0.80	1.626	(0.016)	0.185	0.128
0.75	1.833	(0.017)	0.207	0.127

(f) **Bacinello,** $\boldsymbol{r_g = 0.02}$, $V_D \neq S$, $\boldsymbol{B_0^{(Bac)} = B_0^{(Gro)}}$
($\alpha = 0.529,\ B_0 = 35.004$)

$B_0^{(new)}/B_0^{(initial)}$	V_D		$\Delta(V_D)$	$\Delta(V_D)/V_D$
1.25	1.068	(0.013)		
1.20	1.174	(0.014)	0.107	0.100
1.15	1.291	(0.014)	0.117	0.099
1.10	1.419	(0.015)	0.128	0.099
1.05	1.559	(0.016)	0.140	0.099
1.00	1.712	(0.017)	0.153	0.098
0.95	1.880	(0.017)	0.167	0.098
0.90	2.062	(0.018)	0.183	0.097
0.85	2.262	(0.019)	0.200	0.097
0.80	2.480	(0.020)	0.218	0.096
0.75	2.718	(0.021)	0.238	0.096

Table 12: Value of the DPO, standard error of the DPO value estimate, absolute change of the DPO value, and relative change of the DPO value for a fixed safety level or fixed reserves. Data are only displayed for Set 1 (rounded to three decimal places). (cont.)
Note that we obtain the optimal parameter combination for panel (f) by simulating 10'000 paths. Simulation results (i.e., DPO values) are still based on 100'000 paths.

(g) **Hansen and Miltersen,** $\boldsymbol{r_g = 0.02}$, $\boldsymbol{V_D = S}$, $B_0^{(Han)} \neq B_0^{(Gro)}$
($\alpha = 0.223$, $\gamma = 0.165$, $\rho = 0.304$, $C_0 = 30.432$)

$B_0^{(\text{new})}/B_0^{(\text{initial})}$	V_D		$\Delta(V_D)$	$\Delta(V_D)/V_D$
1.25	0.678	(0.010)		
1.20	0.733	(0.011)	0.055	0.081
1.15	0.793	(0.011)	0.059	0.081
1.10	0.857	(0.012)	0.064	0.081
1.05	0.926	(0.012)	0.069	0.081
1.00	1.000	(0.013)	0.075	0.081
0.95	1.081	(0.013)	0.081	0.081
0.90	1.169	(0.014)	0.088	0.081
0.85	1.264	(0.014)	0.095	0.081
0.80	1.366	(0.015)	0.102	0.081
0.75	1.476	(0.016)	0.110	0.081

(h) **Hansen and Miltersen,** $\boldsymbol{r_g = 0.02}$, $V_D \neq S$, $\boldsymbol{B_0^{(Han)} = B_0^{(Gro)}}$
($\alpha = 0.091$, $\gamma = 0.257$, $\rho = 0.700$, $C_0 = 35.004$)

$B_0^{(\text{new})}/B_0^{(\text{initial})}$	V_D		$\Delta(V_D)$	$\Delta(V_D)/V_D$
1.25	0.505	(0.009)		
1.20	0.553	(0.009)	0.048	0.095
1.15	0.605	(0.010)	0.052	0.094
1.10	0.662	(0.010)	0.057	0.094
1.05	0.725	(0.011)	0.062	0.094
1.00	0.792	(0.011)	0.068	0.093
0.95	0.866	(0.012)	0.074	0.093
0.90	0.947	(0.012)	0.081	0.093
0.85	1.036	(0.013)	0.089	0.093
0.80	1.133	(0.014)	0.097	0.094
0.75	1.239	(0.014)	0.106	0.094

Table 12: Value of the DPO, standard error of the DPO value estimate, absolute change of the DPO value, and relative change of the DPO value for a fixed safety level or fixed reserves. Data are only displayed for Set 1 (rounded to three decimal places). (cont.)
Note that we obtain the optimal parameter combination for panel (h) by simulating 10'000 paths. Simulation results (i.e., DPO values) are still based on 100'000 paths.

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Part II

A Performance Analysis of Participating Life Insurance Contracts

Abstract

Participating life insurance contracts are one of the most important products in the European life insurance market. Even though these contract forms are very common, only very little research has been conducted in respect to their performance. Hence, we conduct a performance analysis to provide a decision support for policyholders. We decompose a participating life insurance contract in a term life insurance and a savings part and simulate the cash flow distribution of the latter. Simulation results are compared with cash flows resulting from two benchmarks investing in the same portfolio of assets but without investment guarantee and bonus distribution scheme in order to measure the impact of these two product features. To provide a realistic picture within the two alternatives, we take transaction costs and wealth transfers between different groups of policyholders into account. We show how the payoff distribution strongly depends on the initial reserve situation and management's discretion. Results indicate that policyholders will in general profit from a better payoff distribution of the participating life insurance compared to a mutual fund benchmark but not compared to an exchange-traded fund benchmark portfolio.[12]

[12] R. Faust, H. Schmeiser, and A. Zemp. A Performance Analysis of Participating Life Insurance Contracts. *Working Papers on Risk Management and Insurance*, 76, 2010.

1 Introduction

As a consequence of the financial crisis, private investors currently seek for safe investments with low downside risk. In this context, minimum interest rate guarantees embedded in financial products are one option for investors. Insurance companies offer investment products with such a downside protection and are often perceived as safe harbor.[13] The participating life insurance (PLI hereafter) is one of the most important products in the life insurance sector with a built in minimum interest rate guarantee. In most European countries, these contracts are typically characterized by an embedded term life insurance, a cliquet-style interest rate guarantee[14], and bonus participation rules with regard to the insurer's reserve situation (surplus fund). However, administrative costs and complex profit distribution schemes between policyholders and shareholders make it difficult to measure the performance of this product from the policyholders' point of view. We model PLI based on contract forms offered in the German market[15] and simulate the complete payoff distribution on an ex ante basis. Subsequently, we compare the cash flow distribution of the PLI with two passive portfolios which invests into the same assets. We show how the payoff distribution depends on the initial reserve situation (the surplus fund in our model) and management's discretion.

The characteristics of PLI contracts make it difficult to measure their performance. A PLI embeds various (explicit and implicit) options as well as complex bonus distribution schemes between policyholders and shareholders. In addition, an insurance company's management has a certain discretion with respect to some parameters. Furthermore, wealth transfers between different groups of policyholders take place. In order to

[13]For example, in the German life insurance market, the estimated increase in premium income in 2009 is 4.8 percent compared to 0.8% in 2008 (see GDV, 2009, Beitragseinnahmen der Versicherungswirtschaft, accessed January, 2010 at `http://www.gdv.de/Downloads/Pressemeldungen_2009/Tabellenanhang_PM_2009.pdf`). This increase might be mainly attributable to an increased risk aversion and/or risk awareness following the financial crisis.

[14]In case of a cliquet-style interest rate guarantee, the guaranteed rate of interest has to be credited to the policyholders' account on a year-to-year basis.

[15]However, the contract forms in focus are very similar to PLI contracts offered in other European insurance markets.

get over these difficulties, we measure the performance of PLI contracts from an ex ante perspective while taking embedded options, bonus distribution, and management's discretion into account. We empirically calibrate our model with market data and simulate various insurance collectives to incorporate wealth transfer effects.

In previous research on PLI, we can distinguish between two major streams of literature. The first one addresses fair pricing of participating life insurance policies based on option pricing theory. Amongst others, bonus distribution rules are often modeled and reproduced in this area of research (see, for example, Grosen and Jørgensen (2000), Bacinello (2001), Hansen and Miltersen (2002), Haberman, Ballotta, and Wang (2003), and Kling, Richter, and Ruß (2007)). For instance, Kling et al. (2007) analyze the numerical impact of interest rate guarantees found in PLI contracts on the shortfall probability of a life insurance company. Gatzert (2008) provides a general framework for pricing and risk management of participating life insurance contracts under different assumptions in respect to asset management and surplus distribution strategies. Gatzert and Schmeiser (2008) assess, in particular, the risk of different premium payment options typically offered in participating life insurance contracts. However, these fair pricing approaches generally only work under the assumption of perfect and frictionless markets.

The second stream of literature mainly analyzes performance by means of the internal rate of return, accounting ratios, and similar performance ratios based on historical cash-flows or numerical examples provided by insurance companies (see, e.g., Ferrari (1968) and Levy and Kahane (1970)). However, these approaches generally ignore embedded options and do not consider the risk-return profile of the investment. Exceptions are Waldow (2003) and Stehle, Gründl, and Waldow (2003). In these contributions, not only one single performance ratio is derived, but also historical cash flows of PLI contracts are compared with those of an alternative portfolio composed of an annual term life insurance and different investment products. Nevertheless, as most of these performance analyses are conducted from an ex post perspective, they can only indicate whether PLI contracts were advantageous in the past. Implications for the future, however, are limited.

In order to get a clear picture of the performance of PLI, we decompose PLI in a term life insurance and an investment part and simulate the cash flow distribution of the investment part under the real world measure $\mathbb{P}$. Further, we create two benchmark portfolios based on the same underlying to measure the impact of the interest rate guarantee and the bonus distribution rules on the cash flows of the portfolio. By calibrating our model with empirical market data, we are able to show in which cases the interest rate guarantee and the mechanisms applied by the insurance company can be beneficial to the policyholder. In addition, we show how the payoff distribution depends on the initial reserve situation and management's discretion. We do not benchmark the PLI using a fair (risk-neutral) pricing approach, which would mean to compare the observed market price with the calculated fair price, because we believe that the underlying assumption of perfect and frictionless markets is rather not fulfilled in this context. We doubt that instruments exist that enable policyholders to replicate the PLI's cash flows. We think that consumers will rather judge products depending on personal preferences and actually available alternatives. The contribution of this paper is that we neither rely on a single performance measurement ratio nor do we provide an ex post analysis. Instead, our framework allows a comparison of the complete payoff distribution on an ex ante basis. This general framework is subsequently not bonded to one specific subjective preference scheme. Further, we model an insurance company with various insurance collectives in order to incorporate wealth transfer effects between different groups of policyholders. Only Hansen and Miltersen (2002) analyzed PLI with pooled accounts before, but just for a two-policyholders case. In addition, the influence of the initial level of the pooled surplus fund on the performance of one single contract is analyzed. Furthermore, we examine how management's discretion, in terms of a change of the target rate of return, affect the payoff distribution.

Results indicate that all of the elements we incorporate have a strong impact on payoffs and should subsequently not be neglected. We find that if the initial level of the surplus fund is high, a PLI contract will in general yield a better payoff distribution compared to the mutual fund

(MF hereafter) benchmark but not compared to the exchange-traded fund (ETF hereafter) benchmark portfolio.

The remainder of the paper is organized as follows: In Section 2, we introduce our general framework. Results from Monte Carlo simulations are discussed in Section 3. We conclude in Section 4.

2 Model Framework

2.1 Premium Investments on a Single Contract Basis

First, we illustrate an insurance company which has only one single insurance contract. We employ a discrete time model with $t \in \{1, \ldots, T\}$ where t determines the elapsed time since inception of the contract (in years) and T denotes the contract's maturity. In section 2.5, the mechanism introduced for the single contract company is applied to an insurer with more than one contract. Our model builds on PLI contracts offered in Germany, but could be easily applied to similar regulatory frameworks (e.g., Switzerland or Austria).

The policyholder pays a constant annual premium P_{t-1} at the beginning of each year given no previous termination of the contract by death or surrender. The insurance company uses the amount $P_{c,t-1}$ of the annual premium to cover its costs. Costs are divided into annual operational costs and acquisition costs. The latter are allocated over the first five years of the contract. Another part of the premium $P_{r,t-1}$ is needed to cover the term life insurance. The remaining amount of the annual premium $P_{s,t-1}^{(\mathrm{PLI})}$ is invested in an asset portfolio. This savings part of the premium $P_{s,t-1}^{(\mathrm{PLI})}$ features an annual minimum interest rate r_g and builds up the policyholder's savings account $A_{g,t-1}$. The process can be modeled as

$$A_{g,t-1} = \sum_{i=1}^{t} P_{s,i-1}^{(\mathrm{PLI})} \exp\left(r_g(t-i)\right), \tag{39}$$

where

$$P_{s,t-1}^{(\mathrm{PLI})} = P_{t-1} - P_{c,t-1} - P_{r,t-1}. \tag{40}$$

The premium $P_{r,t-1}$ is the annual premium for a term life insurance contract. We calculate this premium using actuarial fair premiums and market loadings. To account for a decreasing sum insured I_t, the term life insurance premium is annually adjusted so that the sum insured equals the guaranteed death benefit D minus the accumulated savings account:

$$I_t = D - \exp(r_g)A_{(g,t-1)}.^{16} \tag{41}$$

Given the probability q_{x+t} of a $(x+t)$-years old individual to die within the next year and based on Equation (41), we calculate the annual risk premium as

$$P_{r,t-1} = q_{x+t-1} I_t \exp(-r_g), \tag{42}$$

under the assumption that payouts only take place at the end of each year.[17] Thereby, the guaranteed death benefit D equals the guaranteed terminal payment as common in most PLI contracts,

$$D = A_{g,T-1}\exp(r_g). \tag{43}$$

German contractual law requires that PLI product offerings explicitly report the cost components of the annual premium, i.e., $P_{c,t-1}$. However, the risk premium $P_{r,t-1}$ as well as the savings part $P_{s,t-1}^{(\mathrm{PLI})}$ are usually not shown.[18] In order to calculate these two elements, we iteratively solve for a guaranteed rate of interest r_g which fulfills Equations (41), (42), and (43).

Regarding the investment alternatives to the PLI, we denote with $P_{s,t-1}^{(\mathrm{BM})}$ the amount which is invested annually in the benchmark portfolios. $P_{s,t-1}^{(\mathrm{BM})}$ equals the annual premium P_{t-1} minus the premium for

[16] Note that the premium in t will not be paid if the policyholder dies or surrenders between $t-1$ and t. Hence, we take the savings account in $t-1$ which increases by the guaranteed rate of interest between $t-1$ and t, i.e., $\exp(r_g)A_{(g,t-1)} = A_{(g,t)} - P_{s,t}^{(\mathrm{PLI})}$.

[17] We provide more details on the calculation of the risk premium in the appendix.

[18] See Art. 2 sec. 1 of the German directive for information requirements in insurance contracts (in German: "Verordnung über Informationspflichten bei Versicherungsverträgen", VVG-InfoV).

the term life insurance contract $P_{r,t-1}$ (see Equation (42)). In addition, front-end loads Y_U as a proportion of assets invested are subtracted,

$$P_{s,t-1}^{(\mathrm{BM})} = (1 - Y_U)(P_{t-1} - P_{r,t-1}). \tag{44}$$

In order to incorporate management and administrative fees associated with these benchmark portfolios, an annual fee (defined as a percentage of the total assets in t) is deducted at the end of each year.

Because we are interested in the investment result of the PLI and not in the effect of the term life insurance, we analyze only the savings parts of both premiums, $P_{s,t-1}^{(\mathrm{PLI})}$ and $P_{s,t-1}^{(\mathrm{BM})}$. We assume in what follows that the investor wants to buy a term life insurance contract in both alternatives and hence, this part of the contract does not influence his decision. The benchmark portfolios do not include any investment guarantee and, hence, the total payout in case of death can be lower than the guaranteed death benefit D. We explicitly allow the benchmark cases to pay a lower death benefit since we intend to measure the impact of the interest rate guarantee and the bonus distribution rule on cash flows.

2.2 Portfolio Development

We illustrate a simplified balance sheet of an insurance company with market value accounting in Table 13. The liability side of this balance sheet can be divided into two different parts, the policyholders' accounts, A_g, A_{dp}, and A_{dtb} as well as the surplus fund A_f. While the policyholders' accounts are attributable to policyholders on an individual basis, the surplus fund is attributable to all policyholders as a group. Although the single contract company has only one policyholder, the surplus fund is still different from the policyholders' accounts: The surplus fund has the function of a risk buffer. That is to say it is built up in times of high returns on the asset portfolio and reduced in times of low returns. Grosen and Jørgensen (2000) work with a similar account, the so-called bonus reserve, which is determined by the difference between book and market values. Unlike the bonus reserve by Grosen and Jørgensen (2000), our surplus fund contains all assets which are attributable to policyholders

Assets (market values)	Liabilities (market values)
A_r: assets attributable to policyholders (either on an individual or collective basis)	A_f : surplus fund
	A_g : policyholders' savings accounts (subject to minimum interest rate guarantee)
	A_{dp} : policyholders' distributed profits accounts
	A_{dtb} : policyholders' distributed terminal bonus accounts

Table 13: Balance sheet of a simulated insurance company.

on a collective basis, i.e., our surplus fund consists of hidden reserves and of provisions for premium refunds.

In what follows, we describe in more detail how the different balance sheet accounts evolve. We assume that the insurance company invests in a diversified portfolio of stocks and bonds and that returns on both asset classes are independent and normally distributed.[19] The percentage of assets invested at the beginning of each year in bonds is denoted by B (with $0 \leq B \leq 1$) and the fraction invested in stocks by $1-B$. Rebalancing of the portfolio weights between bonds and stocks is performed on an annual basis. Using an annual time interval (i.e. $\Delta t = 1$), earnings $e_{a,t}$ on invested assets $A_{r,t-1}$ are given by

$$\begin{aligned} e_{a,t} =& A_{r,t-1} \left[B \left(\exp \left(\mu_B - \frac{\sigma_B^2}{2} + \sigma_B \epsilon_{1,t} \right) - 1 \right) \right. \\ & \left. + (1-B) \left(\exp \left(\mu_S - \frac{\sigma_S^2}{2} + \sigma_S \epsilon_{2,t} \right) - 1 \right) \right], \end{aligned} \tag{45}$$

where

$$A_{r,t-1} = A_{f,t-1} + A_{g,t-1} + A_{dp,t-1} + A_{dtb,t-1}. \tag{46}$$

[19]In the historical time series used later on to calibrate the model, the correlation between stock and bond returns was close to zero ($\rho = -0.0432$) and not significant on a 5% level. As a consequence, we assume independence.

σ_B (σ_S) denotes the standard deviation of bonds (stocks). The expected bond (stock) return is given by μ_B (μ_S). The random variates $\epsilon_{1,t}$ and $\epsilon_{2,t}$ are drawn from a standard normal distribution. As common in (German) PLI contracts, the minimum interest rate guarantee is granted on a year-to-year basis and only applies to the savings part of the premium $P^{(\mathrm{PLI})}_{s,t-1}$. The guaranteed minimum interest earned in period t is thus

$$e_{g,t} = (\exp(r_g) - 1)\, A_{g,t-1}, \tag{47}$$

where r_g denotes the guaranteed rate of interest. In our model, the return on the insurer's asset portfolio $e_{a,t}$ is first used to cover this interest rate guarantee. Subsequently, the achieved earnings on assets after covering the guaranteed minimal interests are

$$e_{s,t} = \max(e_{a,t} - e_{g,t}, 0). \tag{48}$$

If the achieved return is insufficient to cover the guarantee, additional capital will be required to cover the interest rate guarantee. We assume that the insurance company is always able to cover this required amount of capital by equity capital.[20]

On the contrary, the benchmark portfolios do not involve any interest rate guarantee or bonus distribution scheme. Earnings $e_{b,t}$ on invested assets $A_{b,t-1}$ for the benchmark portfolios are given by

$$\begin{aligned} e_{b,t} = {} & A_{b,t-1} B \left(\exp\left(\mu_B - \frac{\sigma_B^2}{2} + \sigma_B \epsilon_{1,t} \right) (1 - Y_B) - 1 \right) \\ & + A_{b,t-1}(1 - B) \left(\exp\left(\mu_S - \frac{\sigma_S^2}{2} + \sigma_S \epsilon_{2,t} \right) (1 - Y_S) - 1 \right), \end{aligned} \tag{49}$$

[20] By doing so, we exclude the case of insolvency. This is reasonable in the German regulatory framework since article 125 of the German law for insurance control (German: "Versicherungsaufsichtsgesetz", VAG) defines that all policyholders' claims, i.e., savings, distributed profit, and distributed terminal bonus accounts, should be secured by and transferred to a guaranty fund in case of insolvency. The safety fund continues the contracts as before.

where Y_B (Y_S) are annual fees (in percent) for the bond (stock) fraction of the portfolio. The bond fraction is given by B, the expected returns by μ_B (μ_S), and the volatility by σ_B (σ_S). The random variates $\epsilon_{1,t}$ and $\epsilon_{2,t}$ are the same as those used for the return of the PLI (see Equation (45)).

Based on Equation (49), the invested assets amount $A_{b,t-1}$ of the benchmark portfolios evolves according to

$$A_{b,t-1} = \begin{cases} P_{s,0}^{(\mathrm{BM})} & \text{if } t = 1 \\ \sum_{i=1}^{t} \left(P_{s,t-1}^{(\mathrm{BM})} + e_{b,t-1} \right) & \text{if } t > 1. \end{cases} \tag{50}$$

Considering Equations (49) and (44), it becomes clear that the benchmark portfolios have another cost structure than the PLI. While costs regarding the PLI contract are charged in absolute values in terms of annual operational costs and acquisition costs (i.e., $P_{c,t-1}$), the benchmark portfolios involve front-end loads and annual fees in percent (i.e., Y_U, Y_B, and Y_S). In order to understand why cost structures are different, the different business models of an insurance and an investment company need to be considered. An investment company which sells mutual or exchange-traded funds generates profits by means of front-end loads, back-end loads, and management fees. On the contrary, an insurance company has a different business model and, therefore, various other sources of income (i.e., risk profits). Besides, the costs of the PLI do usually not coincide with asset management costs since marketing and other operational costs are also included. Hence, even if asset management fees in the insurance and the investment company were the same, total costs would be different.

2.3 Bonus Distribution

If the PLI's earnings on assets are positive after covering the interest rate guarantee, i.e., $e_{s,t} > 0$, the remaining profit is distributed to the surplus fund A_f, to shareholders in form of dividends, and to the insurer's equity capital (retentions of earnings). The fraction F will be allocated to the surplus fund (i.e., the policyholders on a collective basis). Then, f_t is the absolute amount that is distributed to the surplus fund,

$$f_t = \begin{cases} 0 & \text{if } e_{s,t} = 0 \\ F e_{s,t} & \text{if } e_{s,t} > 0 \end{cases} \tag{51}$$

under the constraint that $0 \leq F \leq 1$. The remaining fraction $(1 - F)$ is distributed to equity capital or paid out as dividends.[21]

In participating policies, the insurance company is obligated to give policyholders a share in profits. The surplus fund A_f provides an intermediate mechanism with the goal to stabilize returns to policyholders over time. We introduce a decision rule based on the framework presented in Kling et al. (2007) in order to establish a bonus distribution mechanism in our model.[22] The insurance company defines a certain target rate of interest $r_z > r_g$ which is planned to be granted to the policyholders' accounts annually in order to maintain returns for policyholders stable. This target rate of interest is given to the policyholders as long as the surplus fund quota $Q_t = A_{f,t}/A_{g,t}$ stays within a defined range $[Q^L, Q^U]$. Let $Q_{x,t}$ be the surplus fund quota after distributing the amount f_t to the surplus fund A_f but before distributing profits to individual accounts (i.e., A_{dp} and A_{dtb}),

$$Q_{x,t} = (A_{f,t-1} + f_t)/A_{g,t}. \tag{52}$$

In addition, let $e_{z,t}$ be the additional amount which is required to achieve the target rate of interest after covering the interest rate guarantee,

$$e_{z,t} = (\exp(r_z) - 1)\left(A_{g,t-1} + A_{dp,t-1} + A_{dtb,t-1}\right) - e_{g,t}. \tag{53}$$

[21] Art. 4 sec. 3 of the German directive for minimum premium refund in life insurance (in German: "Mindestzuführungsverordung", MindZV) states that at least 90% of the creditable asset returns (less actuarial interest) need to be allocated to the provision for premium refunds. On the basis of this article, the legal quote of 90% remains in our model framework even though we do not consider any equity capital. Profits are distributed to shareholders as they provide, for instance, the interest rate guarantee and solvency capital.

[22] Kling et al. (2007) use the respective decision rule in a similar context. However, their quota is calculated by means of hidden reserves and the book value of liabilities. As our portfolio is composed differently, we calculate our quota based on the surplus fund and the policyholders' savings accounts. This quota retains the idea that reserves are built up in times of high returns and reduced in times of low returns in order to smooth the result and the contract's participation.

In order to maintain returns to policyholders stable on a year-to-year basis whenever possible, the target rate of interest r_z generally applies to all accounts which are attributable to policyholders on an individual basis even though the interest rate guarantee only applies to the savings account.

Finally, we define z_t as the bonus distributed to individual policyholders based on our decision rule. Four different cases can be distinguished which determine how much bonus z_t is distributed to individual policyholders' accounts (after distributing the amount f_t to the surplus fund):

- If crediting the target interest $e_{z,t}$ leads to a surplus fund quota above its upper limit Q^U, the amount leading to a surplus fund quota at its upper limit is distributed.

- If distributing the target interest $e_{z,t}$ leads to surplus fund quota between its upper and lower limit, the target interest is granted.

- If crediting the target interest $e_{z,t}$ leads to a surplus fund quota below its lower limit Q^L, the amount leading to a surplus fund quota at its lower limit is distributed.

- No additional bonus is distributed if the surplus fund quote is already below its lower limit Q^L before the distribution of any bonus.

Formally, this can be expressed as follows:

$$z_t = \begin{cases} (Q_{x,t} - Q_U)A_{g,t} & \text{if } Q^U < Q_{x,t} - e_{z,t}/A_{g,t} \\ e_{z,t} & \text{if } Q^L \leq Q_{x,t} - e_{z,t}/A_{g,t} \leq Q^U \\ (Q_{x,t} - Q_L)A_{g,t} & \text{if } Q^L - e_{z,t}/A_{g,t} < Q_{x,t} - e_{z,t}/A_{g,t} \\ & \text{and } Q_{x,t} - e_{z,t}/A_{g,t} < Q^L \\ 0 & \text{if } Q_{x,t} \leq Q^L, \end{cases} \tag{54}$$

given that the surplus fund after the distribution of profits is

$$A_{f,t} = A_{f,t-1} + f_t - z_t. \tag{55}$$

In this context, z_t stands for the profit distribution assigned to policyholders on an individual basis in addition to the minimum interest rate guarantee.[23]

These profits are allocated between the policyholders' terminal bonus accounts $A_{dtb,t}$ and the policyholders' distributed profits account $A_{dp,t}$. We assume that a percentage M (with $0 \leq M \leq 1$) of z_t should be distributed to $A_{dtb,t}$. Hence, the policyholders' terminal bonus accounts evolve as follows:

$$A_{dtb,t} = \begin{cases} M z_t & \text{if } t = 1 \\ A_{dtb,t-1} + M z_t & \text{if } t > 1. \end{cases} \tag{56}$$

The remaining amount of z_t is allocated to $A_{dp,t}$. In addition, $A_{dp,t}$ increases by annually distributed profits on expenses d_t since policyholders do not only participate in high asset returns but also in an improved cost situation (i.e., if actual costs are lower than those charged).[24] Thus the distributed profits account develops according to

$$A_{dp,t} = \begin{cases} (1-M) z_t + d_t & \text{if } t = 1 \\ A_{dp,t-1} + (1-M) z_t + d_t & \text{if } t > 1. \end{cases} \tag{57}$$

2.4 Cash Flows

We distinguish between three possible events which lead to a payoff to the policyholder (or his heirs respectively). Namely, surrender of the policy before maturity, death before maturity, or survival until maturity. In case of death between $t-1$ and t, policyholders (or rather their heirs)

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[23] In order to understand the intuition behind Equation (54), recall Equation (52) and the definition of the surplus fund quota ($Q_t = A_{f,t}/A_{g,t}$). The first condition in Equation (54) can be transformed to $Q^U < Q_{x,t} - e_{z,t}/A_{g,t} \Leftrightarrow Q^U < (A_{f,t-1} + f_t - e_{z,t})/A_{g,t}$. This equals the definition of the surplus fund quota with the exception that $e_{z,t}$ is used to calculate the quota instead of z_t (cf. Equation (55)). Hence, the first condition just describes the case in which crediting the target interest $e_{z,t}$ would lead to a surplus fund quota above its upper limit Q^U.

[24] See Art. 4 sec. 5 of the German directive for minimum premium refund in life insurance (in German: "Mindestzuführungsverordung", MindZV).

receive the total amount on their accounts, i.e., their savings accounts[25], their distributed profits accounts, and their distributed terminal bonus accounts,

$$\text{Payoff}_{t,\text{death}} = \exp(r_g)A_{g,t-1} + A_{dp,t} + A_{dtb,t}. \tag{58}$$

In addition, policyholders' heirs would receive the sum insured of the term life insurance contract (see Equation (41)). However, since we are interested in the investment result of the PLI and not in the effect of the term life insurance, we do not include this cash flow in our subsequent analysis.

If a policyholder cancels his policy between $t-1$ and t, he receives the amount on his savings account, on his distributed bonus account, and the fraction $0 \leq W \leq 1$ of his distributed terminal bonus account. The policyholder, in general, does not receive the total amount on his distributed terminal bonus account,

$$\text{Payoff}_{t,\text{surrender}} = \exp(r_g)A_{g,t-1} + A_{dp,t} + WA_{dtb,t}. \tag{59}$$

Finally, if a policyholder continues the contract until maturity, the insurer pays the total amount of his different accounts. As we employ a discrete time model, death and cancellation between $T-1$ and T are assumed to lead to equal payoffs at maturity,

$$\text{Payoff}_{\text{maturity}} = \exp(r_g)A_{g,T-1} + A_{dp,T} + A_{dtb,T}. \tag{60}$$

Unlike the PLI contract, the benchmarks do not differentiate between death of the policyholder, surrender, and survival until maturity. Hence, the current value of the respective benchmark portfolio is paid out in all three possible events,

$$\text{Payoff}_{t,\text{benchmark}} = A_{b,t-1} + e_{b,t}. \tag{61}$$

[25] As already mentioned, in the case of death or surrender of the insured between $t-1$ and t, no premium in t is paid by the policyholder. Hence, the policyholders' savings account subject to the minimum interest rate guarantee in t is given by $\exp(r_g)A_{g,t-1} = A_{g,t} - P_{s,t}^{(\text{PLI})}$.

Consequently, there is not any explicit surrender charge. However, there is an implicit surrender charge since front-end loads are distributed over less periods and, therefore, loads are higher in percent.

2.5 Modeling the Insurer's Portfolio

After introducing our model for a single contract insurance company, we apply it to an insurance company with more than one contract. We simulate a life insurance company's underwriting portfolio with T insurance collectives. The contract duration is the same for all collectives (T years) but the different collectives vary in their remaining time to maturity. Each insurance collective is homogeneous, i.e., contains policyholders of same age and mortality whose contracts have the same remaining time to maturity. The insurance company starts with one single insurance collective at point in time 0. Then, every year a new collective is initiated. After $T-1$ years, T collectives exist. From then on, every year one new collective is initiated with T years to maturity and one is terminated so that there will always be T insurance collectives. The basic mechanisms introduced remain the same. However, there is only one surplus fund account A_f for all contracts whereas the policyholders' accounts ($A_g^{(i)}$, $A_{dp}^{(i)}$, and $A_{dtb}^{(i)}$) remain on an individual basis. As the surplus fund is not individually attributable to the policyholders, we introduce a mechanism in order to distribute the amount z_t source-related.

Given n policyholders, each policyholder i participates in profits distributed additionally to the minimum interest with

$$z_t^{(i)} = \frac{A_{g,t-1}^{(i)} + A_{dp,t-1}^{(i)} + A_{dtb,t-1}^{(i)}}{A_{g,t-1} + A_{dp,t-1} + A_{dtb,t-1}} z_t \tag{62}$$

whereas

$$\begin{aligned} A_{g,t-1} &= \sum_{i=1}^{n} A_{g,t-1}^{(i)}, \\ A_{dp,t-1} &= \sum_{i=1}^{n} A_{dp,t-1}^{(i)}, \text{ and} \\ A_{dtb,t-1} &= \sum_{i=1}^{n} A_{dtb,t-1}^{(i)}. \end{aligned} \tag{63}$$

Based on Equation (63), Equations (52), (53), (54), and (55) do not change. However, in order to calculate distributions to individual accounts ($A_{g,t-1}^{(i)}$, $A_{dp,t-1}^{(i)}$, and $A_{dtb,t-1}^{(i)}$), Equations (39), (56), and (57) change, given the definition in Equation (62), to

$$A_{g,t-1}^{(i)} = \sum_{j=1}^{t} P_{s,j-1}^{(i,\mathrm{PLI})} \exp\left(r_g(t-j)\right), \tag{64}$$

$$A_{dtb,t}^{(i)} = \begin{cases} Mz_t^{(i)} & \text{if } t = 1 \\ A_{dtb,t-1}^{(i)} + Mz_t^{(i)} & \text{if } t > 1, \end{cases} \tag{65}$$

$$A_{dp,t}^{(i)} = \begin{cases} (1-M)z_t^{(i)} + d_t^{(i)} & \text{if } t = 1 \\ A_{dp,t-1}^{(i)} + (1-M)z_t^{(i)} + d_t^{(i)} & \text{if } t > 1. \end{cases} \tag{66}$$

One additional difference between the previously introduced single contract company and the various insurance collectives has to be noted, namely that with more than one contract cash outflows occur every year based on how many members of each collective die or cancel their policy.[26] If one policyholder i surrenders, the amount on his terminal bonus account which is not paid out $(1-W)A_{dtb,t}^{(i)}$ is distributed to the joint surplus fund A_f. Hence, policyholders profit from the cancellation of others. In our numerical analysis, we will focus on single contracts out of the T collectives given the surplus fund in order to analyze payoffs obtained by individual policyholders.

[26] In the single contract company, only one cash flow will occur after which the insurance company ceases to exist (as the single contract was paid out).

3 Numerical Analysis

3.1 Model Calibration

We apply our model to contracts with a maturity of twelve years ($T = 12$).[27] We assume that policyholders start premium payments at the beginning of age 53 so that they would receive their survival benefit at the beginning of age 65 (retirement). We use the current mortality tables, loadings of 34% (so called first order mortality), and probabilities of cancellation published by the German Actuary Association.[28] The data provided by the German Actuary Association typically serves as the basis of product calculation of German life insurance companies.

We base our contract parameters on the actual offering of a German life insurance company.[29] The policyholder pays an annual premium of $P_{t-1} = 5000€$ and has a guaranteed death benefit of $D = 61491€$. Annual profit on expenses are estimated to be $d_t = 50.74€$. Acquisition costs of 1487.70€ are allocated over the first five years. Annual administrative costs are 202.97€. Hence,

$$P_{c,t-1} = \begin{cases} 500.51€ \text{ if } t \leq 5 \\ 202.97€ \text{ if } t > 5 \end{cases} \tag{67}$$

The guaranteed death benefit and the guaranteed terminal payment are equal (i.e., $D = A_{g,T-1}\exp(r_g)$). To achieve this, the minimum interest rate needs to be set to $r_g = 2.20\%$ (cf. Section 2.1).[30] Based on this calibration, Table 14 provides an overview on the composition of the in-

[27]PLIs in Germany feature tax benefits if the duration of the policy is at least 12 years (Art. 20 sec. 6 no. 2 of the income tax law (in German: "Einkommenssteuergesetz", EStG).

[28]DAV, 2008, Raucher- und Nichtrauchersterbetafeln für Lebensversicherungen mit Todesfallcharakter and DAV, 1995, Stornoabzüge in der Lebensversicherung, DAV-Mitteilung Nr. 5. We use the DAV 2008 T mortality table.

[29]We used a contract offered by the HUK Coburg (cf. `www.huk.de`). The information used for our simulation in respect to the contract calibration are publicly available.

[30]This number is close to the current maximum permitted actuarial interest rate of 2.25% under the German law (Art. 2 sec. 1 of the German directive for the calculation of policy reserves (in German: "Deckungsrückstellungsverordnung", DeckRV)).

surance premium including the calculated risk premium and the savings part of the premium.

To obtain estimates for volatility and drift of the asset portfolio, we use monthly data from January 1990 to December 2009 of German Federal Securities with a remaining time to maturity of 10 years[31] and a Euro countries based stock index (MSCI EMU total return index), i.e., $\mu_S = 6.74\%$, $\sigma_S = 19.00\%$, $\mu_B = 3.50\%$, and $\sigma_B = 0.47\%$.[32]

We reduced the drift for bonds from 5.45% to $\mu_B = 3.50\%$ in order to account for the current low interest rate environment. The drift μ_B we apply equals the return on German Federal Securities as of December 2009. As the stock ratio in insurance companies' portfolios is approximately 8.5%[33], we apply a stock ratio of $1 - B = 8.5\%$ and a corresponding bond ratio of $B = 91.5\%$.

We assume that each insurance collective consists of $n = 10'000$ contracts and simulate $100'000$ paths. The initial surplus fund is assumed to be $A_{f,\text{initiation}} = 0$. We set the fraction distributed to the surplus fund to $F = 90\%$ which is the minimum amount that has to be credited to policyholders according to German law (legal quote).[34] We assume that a percentage $M = 10\%$ of the profits which are to be distributed to the policyholders are distributed to their terminal bonus accounts. This is close to what we observe on average in the German market.[35] As terminal bonus payments aim at motivating policyholders to continue their

[31] We use the time series WZ3409 as published by the German central bank and available at `http://www.bundesbank.de/statistik/statistik_zeitreihen.php?lang=de&open=zinsen&func=row&tr=WZ3409`.

[32] The MSCI EMU covers the European Economic and Monetary Union. We use this Euro countries based index because the German directive for investments (in German: “Anlageverordnung”, AnlV) requires that the currencies of assets and liabilities match (congruency rule).

[33] GDV, 2008, Kennzahlen zur Kapitalanlage der Versicherer. Accessed February 2010 at `http://www.gdv.de/Downloads/Veranstaltungen_2008/KAPLV_2007_Koll_2008.pdf`.

[34] cf. Art. 4 sec. 3 of the German directive for minimum premium refund in life insurance (in German: “Mindestzuführungsverordung”, MindZV)

[35] In Germany, terminal bonus payments policyholders receive are between 5.25% and 30.68% of total interest earnings with an arithmetic mean of 13.27% (see Assekurata, 2010, Marktstudie 2009: Die Überschussbeteiligung in der Lebensversicherung, accessed January, 2010 at `http://www.assekurata.de/content.php?baseID=130&dataSetID=703`). For simplicity, we assume that 10% of annual distributed profits are distributed to the terminal bonus account.

time t	annual premium P_{t-1}	annual costs $P_{c,t-1}$	risk premium $P_{r,t-1}$	savings part $P_{s,t-1}^{(\mathrm{PLI})}$
1	5000.00	500.51	296.82	4202.67
2	5000.00	500.51	302.43	4197.06
3	5000.00	500.51	305.26	4194.23
4	5000.00	500.51	304.42	4195.07
5	5000.00	500.51	299.07	4200.42
6	5000.00	202.97	285.61	4511.42
7	5000.00	202.97	265.34	4531.69
8	5000.00	202.97	237.18	4559.85
9	5000.00	202.97	199.86	4597.17
10	5000.00	202.97	150.99	4646.04
11	5000.00	202.97	86.53	4710.50
12	5000.00	202.97	0.00	4797.03

Table 14: Composition of PLI premium over time given the empirical calibration based on the actual offering of a German life insurance company.

contract until maturity, we assume that only $W = 50\%$ of the terminal bonus account is paid out in case of cancellation. For our surplus fund quota, we use the bounds $[Q^L, Q^U] = [2.5\%, 7.5\%]$. Unless otherwise stated, we apply a target rate of interest $r_z = 3.5\%$.

We consider two benchmark portfolios: A mutual fund (MF) and an exchange-traded fund (ETF). Both are very common investment alternatives for private investors but involve different transaction costs. Thereby, the ETF benchmark is a kind of extreme case due to its low fees. We calculate fees for these benchmark portfolios based on fees reported by Khorana, Servaes, and Tufano (2007) for MFs sold in Germany and based on calculations provided by the Frankfurt Stock Exchange for ETFs[36]. Thus, we apply annual fees of $Y_B \in \{0.91\%, 0.17\%\}$ for the bond fraction, $Y_S \in \{1.47\%, 0.17\%\}$ for the stock fraction, and averaged upfront fees of $Y_U \in \{3.22\%, 0.36\%\}$, whereas the first element stands for the fees associated with the MF and the second with the ETF portfolio.

3.2 Surplus Fund

Besides the function of stabilizing profits over time, the surplus fund is also an additional source of interest income for policyholders. If a policyholder enters an insurance company possessing a high amount of assets in the surplus fund, this policyholder will profit from interest earnings of a surplus fund which was built up by others. On the other hand, if the policyholder enters a contract when the surplus fund is comparably low, he will tend to build it up whereof future policyholders will profit. Hence, there is a kind of cross-subsidization between policyholders. Thus, from a policyholder's perspective, the level of the surplus fund is crucial. However, individuals who enter a PLI contract do in general not know

[36]See `http://www.boerse-frankfurt.de/DE/MediaLibrary/Document/Sonstiges/etf_handbuch.pdf`.

whether the surplus fund of the respective insurance company is rather stable or not.[37]

Figure 5 shows how the surplus fund develops on average over time in our sample case. The dashed lines provide the lower and upper bounds in each year, which are constant in our setting once the 12th insurance collective has been set up. Based on the convergence behavior observable, we analyze contracts with three different starting points. Contract 1 starts at point in time 0 when the surplus fund is empty ($A_{f,\text{initiation}} = 0$). Contract 2 is established at the end of the 12th year when 12 collectives exist and the surplus fund has partially been built up. At point in time 24, when the surplus fund is rather stable, contract 3 is initiated. Each contract (contract 1, contract 2, contract 3) refers to one single policyholder in the collective of 10'000.

Costumers benefit if they enter when the surplus fund has already been built up (contract 3). Then they will (on average) earn interest on assets others paid for and do not have to pay for assets which others will benefit of. Certainly, it is less beneficial if policyholders still have to build up the surplus fund (contract 1, contract 2). However, entering the contract when the surplus fund is greater than zero (contract 2), the policyholders might still profit from this mechanism due to earnings provided by assets already in the surplus fund.

In Table 15 to 18 we provide descriptive statistics of the payoff distribution of contract 1, contract 3, and of the two benchmark portfolios (MF and ETF). As results for contract 2 are just between those of contract 1 and 3, they are omitted and are available upon request. Reported results are for all T periods conditional upon being paid out during the respective period. The last column gives the probability of payout in each period. That is to say, Table 15 to 18 show expected payoffs a single policyholder would receive if he died or surrendered during the

[37] In order to gather an indication of the current level of the surplus fund, policyholders could analyze the balance sheet of the insurance company. However, the balance sheet might only provide information on book values but not on the required market values. Hence, a policyholder would need a high level of financial literacy in order to be able to derive implications on the actual level of the surplus fund.

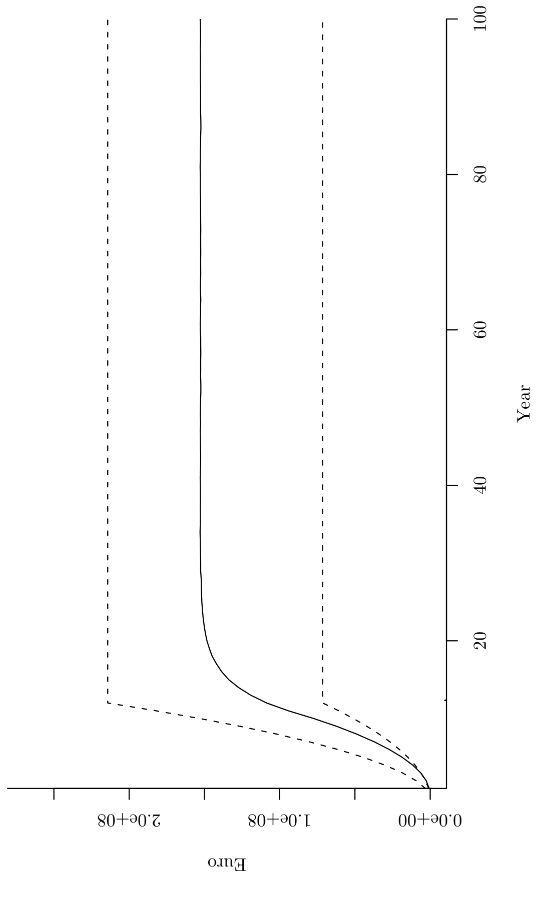


Figure 5: Development of the expected surplus fund in the sample case. The dashed lines provide the lower and upper bounds in each year, which are constant once the 12th insurance collective has been set up.

respective period whereas the corresponding probability (sum of probability of death and probability of surrender) is shown in the last column.

Regarding the contract's mean payoff, the life insurance payouts are dominated by the MF in most periods. Only in the last three periods, the mean payoff of contract 3 is higher than the one of the MF. However, as the last three periods cover 73.7% of all cases, the mean is in favor of PLI contract 3 in the most likely periods. On the other hand, the relative difference is much higher in the first periods than in the last periods. In period 1, the mean payoff of contract 3 is 8.371% lower than the one of the MF but only 2.489% higher in the last period. Comparing median payoffs yields the same structure. Concerning contract 1, mean and median are worse compared to the MF benchmark in all periods.

Although some investors might be more concerned with the mean of the payoff distribution, others may care more about the distribution's dispersion and its shape, i.e., standard deviation, skewness, and kurtosis. Concerning the standard deviation, the MF always shows higher values than the different PLI contracts. Looking at the third and fourth moment, contract 3 has a higher skewness and a higher kurtosis than the MF during all periods. Contract 1 possesses a higher skewness in periods 2 to 7 and a lower kurtosis in periods 5 to 11 compared to the MF. However, it is not possible to draw general conclusions about possible preferences solely based on these moments.

Besides considering the first four moments and the median, Table 15 to 18 also report the 5%, 25%, 75%, and the 95% quantile. All reported quantiles are higher for the MF than for contract 1. This suggests that contract 1 is – at least down to the 5% quantile – dominated by the MF benchmark for all periods. Concerning contract 3, all quantiles are lower than those of the MF in early periods (1 to 8). However, from period 9 on, the 5% and the 25% quantile of contract 3 and from period 11 on the 75% and the 95% quantile contain higher payoffs compared to the MF portfolio. This supports results reported with respect to the mean payoff, namely that contract 3 appears to be favorable in late periods.

The ETF dominates PLI contract 1 and 3 as well as the MF benchmark concerning mean payoffs and all reported quantiles. The standard deviation of the ETF portfolio is higher whereas skewness and kurtosis

Period	Mean	St. dev.	Skewness	Kurtosis	5%	25%	Median	75%	95%	Prob
1	4347	1.163	37.992	1551.293	4347	4347	4347	4347	4347	0.023
2	8787	17.301	5.356	28.648	8783	8783	8783	8783	8795	0.042
3	13339	56.713	2.354	4.760	13314	13314	13314	13314	13487	0.039
4	18028	120.75	1.379	1.009	17946	17946	17946	18111	18285	0.035
5	22876	207.049	0.880	-0.142	22685	22685	22817	22995	23264	0.031
6	28207	310.859	0.561	-0.576	27845	27900	28188	28434	28794	0.028
7	33746	431.047	0.350	-0.662	33141	33393	33707	34045	34510	0.025
8	39514	563.347	0.189	-0.623	38581	39089	39514	39940	40457	0.022
9	45521	712.180	0.113	-0.493	44326	44982	45509	46040	46690	0.019
10	51799	875.588	0.065	-0.311	50345	51166	51809	52433	53211	0.016
11	58376	1061.656	0.067	-0.037	56623	57613	58390	59141	60059	0.014
12	65424	1293.041	0.101	0.202	63275	64528	65440	66326	67442	0.706

Table 15: Descriptive statistics for the payoff distributions of PLI contract 1 conditional upon payout in the respective period. The probability of payout is given in the last column.

Period	Mean	St. dev.	Skewness	Kurtosis	5%	25%	Median	75%	95%	Prob
1	4422	36.951	2.339	9.807	4373	4411	4415	4419	4499	0.023
2	9012	95.183	1.678	5.696	8884	8978	8990	9025	9205	0.042
3	13779	178.199	1.372	3.913	13530	13709	13735	13834	14135	0.039
4	18734	283.425	1.146	2.783	18334	18609	18661	18846	19291	0.035
5	23886	410.989	1.011	2.201	23299	23688	23786	24071	24685	0.031
6	29563	564.995	0.948	2.061	28751	29268	29435	29837	30638	0.028
7	35477	742.128	0.831	1.527	34406	35064	35325	35859	36885	0.025
8	41647	942.925	0.781	1.334	40275	41101	41471	42146	43410	0.022
9	48100	1169.332	0.727	1.122	46397	47395	47900	48747	50260	0.019
10	54855	1427.806	0.681	0.950	52765	53963	54634	55658	57496	0.016
11	61956	1729.874	0.664	0.887	59424	60855	61703	62945	65138	0.014
12	69542	2067.277	0.603	0.695	66490	68201	69266	70755	73308	0.706

Table 16: Descriptive statistics for the payoff distributions of PLI contract 3 conditional upon payout in the respective period. The probability of payout is given in the last column.

Period	Mean	St. dev.	Skewness	Kurtosis	5%	25%	Median	75%	95%	Prob
1	4826	83.347	0.522	0.488	4702	4767	4819	4877	4974	0.023
2	9771	188.598	0.426	0.357	9485	9639	9758	9889	10101	0.042
3	14843	321.417	0.386	0.277	14352	14619	14823	15046	15404	0.039
4	20049	477.354	0.360	0.258	19317	19714	20023	20352	20881	0.035
5	25395	659.055	0.345	0.242	24377	24937	25359	25814	26541	0.031
6	30891	861.528	0.333	0.237	29556	30295	30842	31440	32385	0.028
7	36555	1082.281	0.314	0.214	34872	35803	36501	37254	38414	0.025
8	42397	1328.488	0.306	0.189	40332	41473	42333	43245	44684	0.022
9	48425	1596.633	0.313	0.206	45956	47312	48347	49447	51179	0.019
10	54665	1888.341	0.307	0.178	51725	53350	54568	55877	57920	0.016
11	61136	2212.389	0.311	0.205	57688	59594	61026	62552	64947	0.014
12	67853	2559.604	0.296	0.168	63870	66065	67737	69506	72259	0.706

Table 17: Descriptive statistics for the payoff distributions of MF conditional upon payout in the respective period. The probability of payout is given in the last column.

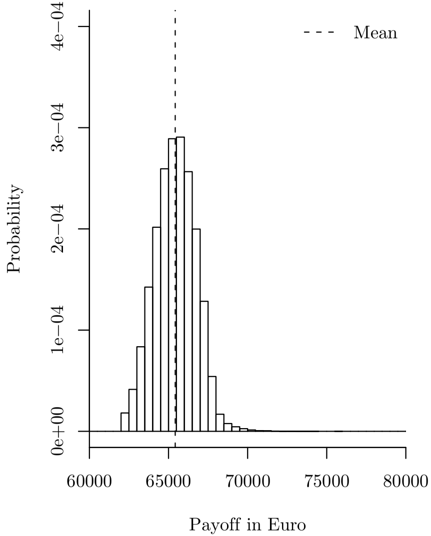
Period	Mean	St. dev.	Skewness	Kurtosis	5%	25%	Median	75%	95%	Prob
1	4858	84.458	0.522	0.478	4733	4799	4851	4910	5008	0.023
2	9890	192.310	0.425	0.356	9597	9755	9876	10010	10226	0.042
3	15102	329.901	0.388	0.276	14598	14872	15082	15310	15678	0.039
4	20510	493.151	0.355	0.266	19752	20164	20483	20824	21368	0.035
5	26120	685.900	0.344	0.249	25060	25643	26082	26556	27310	0.031
6	31948	903.316	0.333	0.232	30549	31324	31895	32523	33513	0.028
7	38013	1140.024	0.321	0.224	36240	37222	37956	38745	39972	0.025
8	44333	1408.309	0.305	0.180	42147	43352	44266	45231	46766	0.022
9	50919	1703.969	0.314	0.193	48290	49729	50836	52008	53867	0.019
10	57800	2031.720	0.315	0.214	54640	56387	57690	59098	61299	0.016
11	65001	2390.649	0.314	0.208	61284	63338	64876	66530	69127	0.014
12	72551	2789.714	0.297	0.169	68211	70601	72421	74351	77353	0.706

Table 18: Descriptive statistics for the payoff distributions of ETF conditional upon payout in the respective period. The probability of payout is given in the last column.

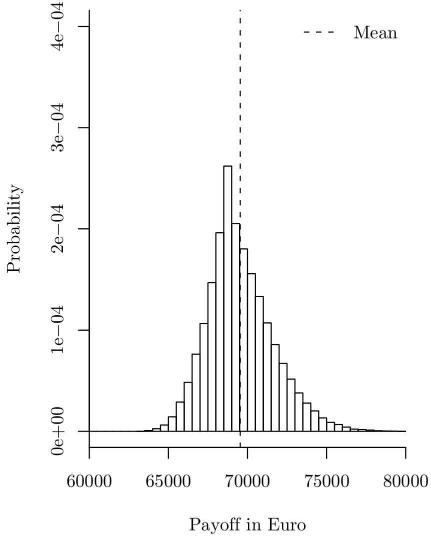
are approximately the same like those of the MF. The higher standard deviation of the ETF in comparison to the MF benchmark can be explained by the higher mean and quantile values of the payoff distribution which are caused by the comparably low transaction costs.

In order to clarify results with respect to the last period which accounts for more than 70% of all outcomes, we illustrate the payoff distributions (histograms) of the PLI contracts and the benchmark portfolios for period 12 in Figure 6. The figure shows how peaked the PLIs' payoff distributions are compared to the MF and the ETF. The payoff distribution of the ETF is very similar to the one of the MF but is shifted to the right due to the lower transaction costs. Comparing contract 1 and 3 shows that the payoff distribution of contract 1 is shifted to the left with a lower upside potential.

To summarize, the payoff distribution of the PLI depends on the level of the surplus fund at inception of the contract. If the surplus fund equals 0 when the contract is started (contract 1), the payoff distributions of both benchmark portfolios dominates the one of the PLI contract in all quantiles reported. If the surplus fund at inception is high (contract 3), the payoff distribution of the MF dominates in early periods but is dominated later on (with regard to the quantiles reported). Hence, survival until maturity without surrender appears advantageous. However, results reported suggest that the ETF portfolio might be most beneficial as it dominates all PLI contracts with regard to mean and all quantiles analyzed.

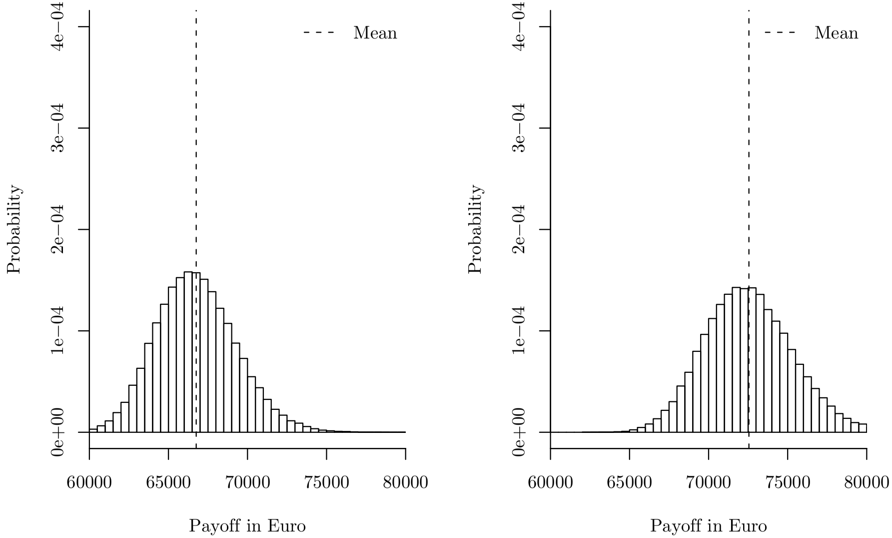


(a) PLI payoffs contract 1



(b) PLI payoffs contract 3

Figure 6: Histograms and mean of the payoff distributions conditional upon payout in the last year for each contract. (continued on next page)



(c) MF payoffs

(d) ETF payoffs

Figure 6: Histograms and mean of the payoff distributions conditional upon payout in the last year for each contract. (cont.)

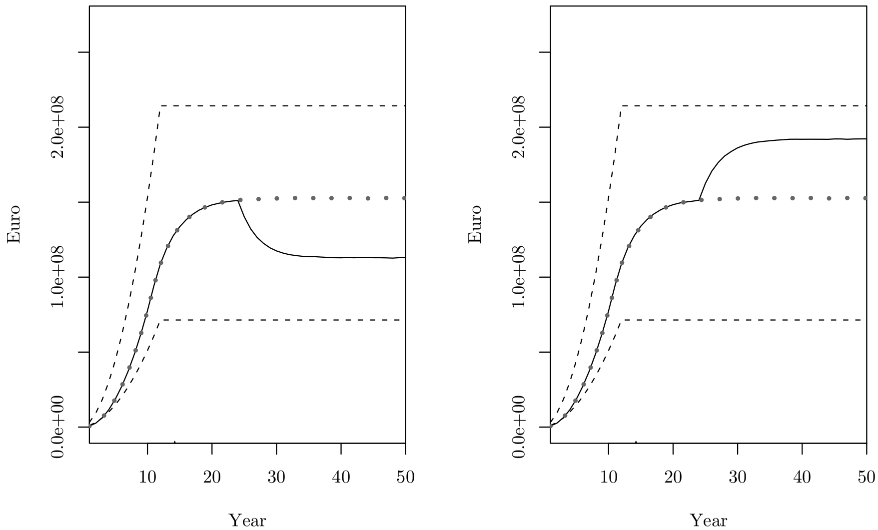
3.3 Management's Discretion

Our previous results have shown that the surplus fund has an important impact on the payoff distribution. However, we assumed parameters to be constant and differences with respect to the different contracts were caused by the initial level of the surplus fund. In what follows, we analyze the effects of management's discretion with regard to contract 3. We examine the effect on the PLI's payoff distribution if management changes the target rate of interest directly after the policyholder's first premium payment. We focus on an increase of the target rate to $r_z = 4.0\%$ and a decrease to $r_z = 3.0\%$.

Similar to Figure 5, Figure 7a and 7b show how the surplus fund develops on average over time given the change of the target rate of interest in year 24. The dashed lines provide the lower and upper bounds in each year, the dotted line displays the level of the surplus fund given no change in the target interest rate. If the target rate increases to $r_z = 4.0\%$, the surplus fund first decreases and then stabilizes at a lower level. On the contrary, with a decrease to $r_z = 3.0\%$, the surplus fund first increases and then stabilizes at a higher level. Figure 7c and 7d show the payoff distribution in the last period (similar to Figure 6). The dotted line denotes the density function given no target rate change. Both rate changes, $r_z = 3.0\%$ and $r_z = 4.0\%$, lead to a much less peaked payoff distribution compared to the contract without a change of the target rate. In addition, the rate change to $r_z = 3.0\%$ causes the payoff distribution to be more skewed than the change to $r_z = 4.0\%$.

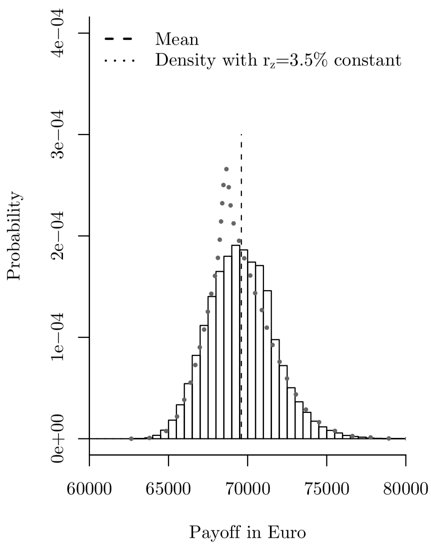
In Table 19 and 20 we provide descriptive statistics of the payoff distribution of contract 3 with the target return increase and decrease. Reported results are for all T periods conditional upon being paid out during the respective period. The probability of payout in each period is denoted in the last column.

The target rate increase to $r_z = 4.0\%$ results in a higher mean, a higher median, a lower kurtosis, and a lower skewness in all periods compared to the constant target rate. The standard deviation with the increased target rate is lower in periods 1 to 3 and higher in periods 4 to 12. The 5% and the 95% quantile are higher for the contract with the constant target rate (except for period 1). On the contrary, in most

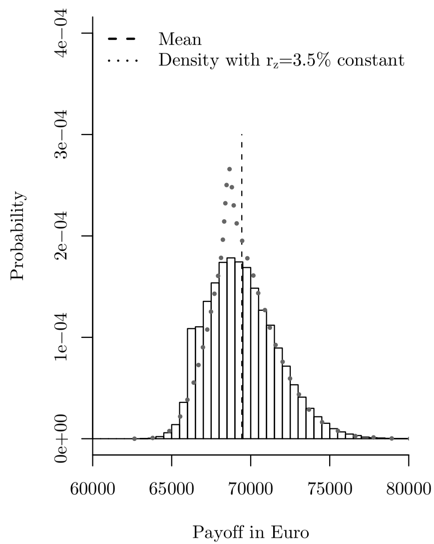


(a) Surplus fund for $r_z = 4.0\%$ (b) Surplus fund for $r_z = 3.0\%$

Figure 7: Histograms and mean of the payoff distributions of contract 3 conditional upon payout in the last year given a change in the target rate of interest at the beginning of the contract's life. Panel (a) and (b) show the corresponding expected level of the surplus fund – the dashed lines provide the lower and upper bounds in each year, the dotted line shows the case with a constant target rate of interest. (continued on next page)



(c) Payoff distributions for $r_z = 4.0\%$



(d) Payoff distributions for $r_z = 3.0\%$

Figure 7: Histograms and mean of the payoff distributions of contract 3 conditional upon payout in the last year given a change in the target rate of interest at the beginning of the contract's life. Panel (a) and (b) show the corresponding expected level of the surplus fund – the dashed lines provide the lower and upper bounds in each year, the dotted line shows the case with a constant target rate of interest. (cont.)

periods the 25% and the 75% quantile are higher for the contract with the changed target return. Hence, the target rate increase to $r_z = 4.0\%$ appears to be beneficial around the expected payoff, i.e., between the 25% and the 75% quantile. However, the higher target rate results in a lower upside potential as the equilibrium level of the surplus fund gets closer to the lower bound. Subsequently, the probability to reach the upper bound of the surplus fund and thus the probability to receive return attributions which are higher than r_z are reduced.

The decrease of the target rate of interest to $r_z = 3.0\%$ leads to a lower mean, a lower median, a higher standard deviation, and a lower kurtosis in all periods. The 5% and the 25% quantile are lower for the decreased target rate (except for period 1). On the contrary, the 75% quantile is higher from period 6 to 12 and the 95% quantile is higher for all period except for period 1. Thus, the decreased target rate of interest leads to a higher upside potential as the equilibrium level of the surplus fund gets closer to the upper bound. However, the lower target rate leads to lower expected payoffs.

These results let us draw two conclusions. First, management's discretion has an important influence in respect to the payoff distribution. Second, it depends on policyholders' (time and state) preferences if a change of the target rate is found beneficial or not. While expected payoffs increase with an increase in the target rate, a reduction leads to a higher upside potential in later periods.

Period	Mean	St. dev.	Skewness	Kurtosis	5%	25%	Median	75%	95%	Prob
1	4436	34.938	1.321	8.357	4373	4432	4436	4440	4497	0.023
2	9048	92.671	0.619	4.456	8875	9026	9054	9068	9203	0.042
3	13839	176.102	0.360	2.627	13525	13755	13861	13897	14133	0.039
4	18815	284.612	0.278	1.733	18323	18648	18862	18936	19283	0.035
5	23985	418.984	0.273	1.309	23275	23720	24049	24188	24674	0.031
6	29673	573.991	0.249	0.944	28716	29293	29728	29980	30621	0.028
7	35597	753.985	0.252	0.681	34367	35087	35639	36027	36845	0.025
8	41770	960.568	0.296	0.618	40221	41113	41790	42340	43380	0.022
9	48219	1189.749	0.309	0.557	46314	47398	48219	48955	50208	0.019
10	54963	1447.987	0.315	0.443	52661	53950	54947	55873	57399	0.016
11	62040	1739.032	0.319	0.348	59291	60821	62006	63149	64995	0.014
12	69609	2081.328	0.318	0.296	66338	68147	69549	70957	73164	0.706

Table 19: PLI contract 3, target return changing to 4%.

Period	Mean	St. dev.	Skewness	Kurtosis	5%	25%	Median	75%	95%	Prob
1	4407	40.82	2.511	8.141	4373	4390	4393	4403	4497	0.023
2	8976	108.957	1.853	4.298	8870	8915	8926	9013	9206	0.042
3	13721	206.747	1.481	2.509	13519	13584	13633	13820	14145	0.039
4	18654	328.994	1.229	1.612	18310	18406	18553	18836	19305	0.035
5	23787	475.186	1.054	1.142	23259	23397	23677	24065	24703	0.031
6	29450	646.797	0.915	0.802	28694	28903	29330	29842	30678	0.028
7	35354	840.084	0.815	0.575	34331	34666	35225	35868	36915	0.025
8	41528	1060.833	0.745	0.508	40183	40687	41390	42184	43481	0.022
9	47978	1297.929	0.673	0.380	46266	46974	47828	48787	50340	0.019
10	54741	1572.455	0.633	0.348	52596	53549	54570	55720	57593	0.016
11	61845	1868.678	0.590	0.273	59210	60448	61665	63015	65233	0.014
12	69446	2230.646	0.557	0.226	66227	67793	69235	70855	73465	0.706

Table 20: PLI contract 3, target return changing to 3%.

3.4 Performance Measurement

Next, a preference dependent valuation of the different investment opportunities is derived based on the payoff distributions shown. In order to do so, assumptions regarding the state and time preferences of the policyholder are needed. In this subsection, we assume that whenever payments take place before the end of maturity T (because of surrender or death of the investor), the corresponding cash-flows are reinvested and compounded with the annual minimum interest rate r_g. This yields one single cash flow distribution L_T at time T for each investment alternative. We provide descriptive statistics of the payoff distribution L_T of the different investment alternatives in Table 21. Regarding the mean payoff, the median, and the different quantiles shown in Table 21, the ETF benchmark leads to the highest payoffs compared to all other alternatives.

The premiums paid into the different saving products (i.e., after detaching the term life insurance) are the same for all alternatives: $P_{t-1} - P_{r,t-1}$. Compounding the premium payments $(P_{t-1} - P_{r,t-1})$ with the interest rate r_g, while taking surrender and survival probabilities of the policyholder into account, leads to a (deterministic) terminal value of premium payments of $Y_T = 55518$. As it is done in Gatzert and Schmeiser (2009), we perform a comparison of the four different cases by using modified forms of three different classical performance measures. First, an adaption of the Sharpe ratio (see Sharpe (1966)) can be defined in the following way:

$$\text{Sharpe ratio}(L_T) = \frac{E(L_T) - Y_T}{\sigma(L_T)} \tag{68}$$

For instance, in the case of the ETF benchmark portfolio, this will lead to

$$\text{Sharpe ratio}(L_T) \approx \frac{60527 - 55518}{21085} \approx 0.238 \tag{69}$$

Following Gatzert and Schmeiser (2009), a modified form of Omega and the Sortino ratio can be defined by (see Shadwick and Keating (2002), Sortino and van Der Meer (1991))

Contract type	Mean	St. dev.	Skewness	Kurtosis	5%	25%	Median	75%	95%
PLI contract 1	54441	19070	-1.397	0.355	10942	48253	64695	65950	67250
PLI contract 3	57748	20491	-1.374	0.310	11209	50802	68419	70059	72822
MF	55884	19142	-1.395	0.436	11899	49861	65327	67610	70622
ETF	60527	21085	-1.377	0.370	12398	53484	70972	73493	76816

Table 21: Descriptive statistics of the payoff distribution L_T derived under the assumption that payoffs before T had been invested to the annual minimum interest rate r_g.

Contract type	Sharpe ratio	Omega	Sortino ratio
PLI contract 1	-0.057	-0.132	-0.056
PLI contract 3	0.109	0.285	0.108
MF	0.019	0.048	0.019
ETF	0.238	0.688	0.231

Table 22: Modified performance measures for the valuation of four different investment opportunities

$$\text{Omega}(L_T) = \frac{E\left(L_T\right) - Y_T}{E\left(\max\left(Y_T - L_T, 0\right)\right)} \tag{70}$$

and

$$\text{Sortino ratio}(L_T) = \frac{E\left(L_T\right) - Y_T}{\sqrt{E\left(\max\left(Y_T - L_T, 0\right)^2\right)}}. \tag{71}$$

Table 22 provides an overview of the different performance ratios of the four investment opportunities in focus. The used performance measurements of the investment alternatives give a clear picture: The contract type ETF dominates all other investment forms analyzed. PLI contract 3 dominates MF and PLI contract 1, whereas contract 1 is dominated by all other alternatives. In addition, we further tested for first degree stochastic dominance (FSD).[38] In our simulation results, a FSD is only given for investment form ETF in comparison to PLI contract 1. More precisely, let F_1 denote the cumulative distribution function of $L_T^{(C1)}$ (PLI contract 1) and let F_2 stand for the cumulative distribution function of $L_T^{(ETF)}$ (ETF portfolio). Then $L_T^{(ETF)}$ dominates $L_T^{(C1)}$ by FSD since $F_1(x) \geq F_2(x)$ for all x and $F_1(x) > F_2(x)$ for at least some x. Performance ratios are best for the ETF portfolio and worst for PLI contract 1 as already indicated by our previous results. Further, perfor-

[38] See Bawa (1975).

mance ratios for PLI contract 3 are higher than for the MF portfolio. Hence, PLI contract 3 appears to be superior to the MF portfolio given our underlying assumptions about preferences.

To conclude, the ETF benchmark portfolio appears to be the best choice due to the low transaction costs. On the other hand, if the surplus fund is already built up, the PLI tends to perform better than the MF benchmark.

4 Conclusion

PLI contracts are popular - especially in the context of old-age provisions. This popularity might be to a large extent attributable to the downside protection. However, it is controversial if these products are actually beneficial for policyholders. More precisely, even though these contract forms are very common in insurance practice, only very little research has been conducted in respect to their performance in comparison to feasible investment alternatives. In this paper, we develop, in a first step, a framework to estimate payoffs from PLI contracts from the point of view of policyholders. We decompose PLI into an investment part and a term life insurance. Thus we are able to analyze the benefits of the minimum interest rate guarantee in combination with the profit distribution rules separately from the term life insurance. In addition, we model more than one single contract which allows us to incorporate distribution effects between policyholders. In a second step, we simulate the payoff distributions and benchmark the complete payoff distribution on an ex ante basis. We show how the payoff distribution depends on the level of the surplus fund at inception of the contract and analyze the effect of management's discretion.

We show that PLI can be beneficial to policyholders depending on the initial reserve situation. A low initial reserve situation of the insurer appears to be disadvantageous. Individuals continuing their contract until maturity without death or surrender will in general profit from a better payoff distribution compared to the MF benchmark portfolio but not compared to the ETF benchmark portfolio. Our preference dependent performance analysis shows that, in most cases, an ETF portfolio will perform better than each possible PLI contract if taxes are ignored. If taxes are accounted for, the PLI could perform better than the ETF benchmark but this will always depend on a specific investors marginal tax rate. However, if the surplus fund is already built up, the PLI tends to perform better than the MF benchmark.

Appendix

A Annual Term Life Insurance Premium

The following formulas illustrate briefly how the annual term life insurance premium can be calculated. The insured sum I_t in year t equals the guaranteed death benefit minus the accumulated savings account at the end of year t,

$$I_t = D - A_{g,t-1}\exp(r_g).$$

Recall the formulas for the savings part of the premium and the accumulated savings account:

$$P_{s,t-1}^{(\mathrm{PLI})} = P - P_{c,t-1} - P_{r,t-1}$$

and

$$A_{g,t-1} = \sum_{i=1}^{t} P_{s,i-1}^{(\mathrm{PLI})}\exp\left(r_g(t-i)\right).$$

Given the probability q_{x+t} of a $(x+t)$-years old individual to die within the next year, the term life insurance premium is (assuming that payouts only take place at the end of year t)

$$P_{r,t-1} = q_{x+t-1} I_t \exp(-r_g).$$

Insertion yields

$$\begin{aligned}
P_{r,t-1} &= q_{x+t-1} I_t \exp(-r_g) \\
&= q_{x+t-1}\left(D - A_{g,t-1}\exp(r_g)\right)\exp(-r_g) \\
&= q_{x+t-1}\left(D\exp(-r_g) - A_{g,t-1}\right) \\
&= q_{x+t-1}\left(D\exp(-r_g) - (A_{g,t-2}\exp(r_g) + P - P_{c,t-1} - P_{r,t-1})\right) \\
&\Rightarrow \frac{q_{x+t-1}}{1-q_{x+t-1}}\left(D\exp(-r_g) - (A_{g,t-2}\exp(r_g) + P - P_{c,t-1})\right).
\end{aligned}$$

Under the constraint that the guaranteed death benefit equals the guaranteed terminal payment,

$$D = A_{g,T-1}\exp(r_g).$$

Thus

$$P_{r,t-1} = \frac{q_{x+t-1}}{1-q_{x+t-1}}\left(A_{g,T-1} - \left(A_{g,t-2}\exp(r_g) + P - P_{c,t-1}\right)\right).$$

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Part III

Comparison of Stakeholder Perspectives on Current Regulatory and Reporting Reforms

Abstract

In the European insurance industry, regulatory and reporting frameworks are currently subject to far-reaching reforms. We focus on four of these frameworks, namely the Solvency II framework, insurance guaranty systems, the proposed IFRS 4 Phase II international accounting standards, and Market Consistent Embedded Value reporting. We present these frameworks, analyze them from different stakeholder perspectives, and compare and contrast them. Our analysis implies that the four frameworks need to be considered jointly rather than separately, due to various interrelations and interactions. We argue that a coordinated introduction will be necessary to ensure that the regulatory burden is reduced and synergies can be utilized in the event of all four frameworks being implemented as planned. Furthermore, we propose a more holistic, comprehensive approach to insurance reporting and regulation in order to achieve regulatory goals.[39]

[39] J. Wagner and A. Zemp. Comparison of Stakeholder Perspectives on Current Regulatory and Reporting Reforms. *Working Papers on Risk Management and Insurance*, 88, 2011.

1 Introduction

Currently, regulatory and reporting frameworks in the European insurance industry are undergoing various far-reaching reforms. In general, solvency measurement and solvency requirements appear to be the main are of focus, for instance with the European Solvency II framework. However, in addition to common capital standards, the European Union currently faces the need to harmonize other regulatory frameworks to a certain degree. The Committee of European Insurance and Occupational Pensions Supervisors (CEIOPS) (2009) specifies three other elements which need to be reconsidered as a consequence of the financial crisis: insurance guaranty schemes[40], information to policyholders, and common reporting formats.

In this paper, we present four key European insurance-related regulatory and reporting frameworks which are currently subject to important reforms. We focus on the Solvency II framework, insurance guaranty systems, the proposed IFRS 4 Phase II international accounting standards (as of July 2010), and Market Consistent Embedded Value (MCEV) reporting. We analyze these four frameworks from different stakeholder perspectives and compare and contrast them.

The first framework considered, Solvency II, is expected to be implemented by the end of 2012 (European Union, 2009). Solvency II adopts an enterprise risk management approach and takes into account the risk profile of the entire insurance company. It consists of three pillars. The first pillar prescribes capital requirements, the second pillar defines qualitative requirements, and the third pillar focuses on supervisory reporting and public disclosure. Solvency II builds on the Solvency I framework which was introduced in 2004.

Secondly, insurance guaranty systems provide last-resort protection to policyholders in the event of the default of an insurance company (see, e.g., Oxera, 2007). In a White Paper on insurance guaranty systems, the

[40] Similarly, the European Commission has announced that it would review the adequacy of existing guaranty schemes in the insurance sector as a response to the financial crisis (see the European Commission's online portal `http://ec.europa.eu/financial-crisis`).

European Commission (2010b) proposes the introduction of a directive to ensure that all Member States of the European Union have in place an insurance guaranty system meeting certain minimum requirements. Currently, only 12 out of the 30 EU-EEA countries have insurance guaranty schemes in place (European Commission, 2010b).

The third framework, IFRS 4 Phase II, is a planned set of reporting standards defining how to recognize, measure, and disclose insurance contracts (IASB, 2010b). These new reporting standards take a market value-based or risk-based approach to insurance companies. An exposure draft of IFRS 4 Phase II was published in July 2010. The standard is based on IFRS 4 Insurance Contracts, which was introduced in 2004.

Fourthly, the Market Consistent Embedded Value (MCEV) Principles (CFO Forum, 2009b) were published in June 2008 and are expected to be implemented from 31 December 2011 onwards. They are intended to harmonize embedded value reporting in Europe. The MCEV principles are based on the European Embedded Value (EEV) Principles (CFO Forum, 2004).

In the first step, the four regulatory and reporting frameworks are presented. We provide a brief overview of their current state of progress and plans for their implementation. In addition, key aspects of these frameworks are illustrated and the different underlying measurement models are explained by means of integrative, homogeneous illustrations.

In a second step, the four frameworks are analyzed from different stakeholder perspectives and compared and contrasted. First, a comparative overview of the four concepts is provided which does not consider the stakeholders' points of view. Next, we turn to the three major stakeholders, i.e., the insurance company's management, its policyholders, and its current and potential investors. In order to develop the details of the three different perspectives, we address each stakeholder in turn and analyze separately their key characteristics and interactions. Finally, in a third step, we bring together the different perspectives. In particular, we develop a proposal for a holistic, comprehensive approach to insurance reporting and regulation.

The contributions of this paper are twofold. On the one hand, we present a comprehensive overview of four far-reaching regulatory and reporting reforms within Europe. We provide an integrative illustration in order to explain the different underlying measurement models. On the other hand, we compare and contrast these frameworks, analyze them from different stakeholder perspectives, and highlight major similarities and differences. In so doing we combine results from important industry and academic publications as well as our own findings. That is to say, we discuss and analyze contributions by the insurance industry, practitioners, regulatory authorities, and the academic community. Although some authors have examined the relations between Solvency II and IFRS 4 Phase II (see, e.g., Duverne and Le Douit, 2009), between IFRS 4 Phase II and MCEV (see, e.g., De Mey, 2009), or between Solvency II and insurance guaranty systems (see, e.g., Rymaszewski and Schmeiser, 2011), there has been to date no comprehensive joint examination of these frameworks. In addition, the different stakeholder perspectives in relation to the frameworks have not, in general, been taken into account.

The remainder of the paper is structured in the following way. Section 2 presents the four regulatory and reporting frameworks, i.e., Solvency II (Section 2.1), insurance guaranty schemes (Section 2.2), IFRS 4 Phase II (Section 2.3), and MCEV (Section 2.4). In Section 3, we conduct a comparative analysis of the different frameworks from different stakeholder perspectives. Section 4 consolidates and discusses the results obtained for the different perspectives. Conclusions are provided in Section 5.

2 Regulatory and Reporting Frameworks

The recent financial crisis has revealed the need for a reconsideration of regulatory frameworks in the insurance industry. In the following, we examine major European insurance-related reporting standards and regulatory frameworks which are currently undergoing far-reaching reforms.

(a) **Regulatory frameworks**

Solvency requirements	**Insurance guaranty schemes**
- Since 1970s: Local (harmonized) capital standards - 2004: Imposition of minimum capital requirements by *Solvency I* - End 2012 (planned): Introduction of *Solvency II* comprising an enterprise risk management approach consisting of quantitative and qualitative requirements	- Since 1960s: Emergence of insurance guaranty schemes in several European countries - 2010: Heterogeneous characteristics and development in Europe (schemes in place in 12 EU Member States) - 2010: White Paper by the European Commission on harmonization of insurance guaranty systems across the European Union

Table 23: Overview of recent developments in European reporting and regulatory frameworks (see European Union, 2002a; European Union, 2002b; European Union, 2009; European Commission, 2010b; IASB, 2010c; IASB, 2010b; CFO Forum, 2004; CFO Forum, 2009b; CFO Forum, 2009a). (continued on next page)

(b) **Reporting frameworks**

Accounting standards	**Embedded value reporting**
- Before 2004: No specific IFRS reporting standard for insurance accounting	- Before 2004: heterogeneous embedded value reporting in Europe
- 2004: Introduction of *IFRS 4 Insurance Contracts*	- 2004: Publication of the *European Embedded Value* (EEV) Principles
- Planned: *IFRS 4 Phase II* adopting a market value-based perspective	- End 2011 (planned): Introduction of the *Market Consistent Embedded Value* (MCEV) Principles

Table 23: Overview of recent developments in European reporting and regulatory frameworks (see European Union, 2002a; European Union, 2002b; European Union, 2009; European Commission, 2010b; IASB, 2010c; IASB, 2010b; CFO Forum, 2004; CFO Forum, 2009b; CFO Forum, 2009a). (cont.)

The regulatory frameworks examined are *Solvency II*[41] and existing *insurance guaranty schemes*[42]. With regard to insurance reporting, we focus on the *IFRS 4 Phase II* international accounting standards and *Market Consistent Embedded Value (MCEV)* reporting. We refer to the exposure draft (IASB, 2010b, July) as "IFRS 4 Phase II". Table 23 provides an overview of recent developments in relation to these four frameworks. Unless otherwise stated, we use "regulatory frameworks" to refer to both reporting and regulatory frameworks.

2.1 Solvency Regulation

The new solvency regulation system in the European Union is adopted in a two-stage process. In 2004, Solvency I was introduced making some modifications to the capital standards which were introduced in the 1970s (European Union, 2002a; European Union, 2002b). Solvency I imposes minimum capital requirements (MCR) on life and non-life insurers. The MCR is based on liability-related, volume-based ratios. Unlike Solvency I, Solvency II takes primarily an enterprise risk management approach and considers the entire risk profile of an insurance company. It is expected to be implemented by the end of 2012 (see Table 23a). The standards are defined in a European Union directive (European Union, 2009).[43]

Solvency II consists of three pillars. The first pillar describes quantitative requirements, the second pillar focuses on qualitative requirements, and the third pillar addresses supervisory reporting and public disclosure. The main components of the second pillar are principals of internal risk management and risk control as well as the corresponding supervisory interventions (Eling, Schmeiser, and Schmit, 2007). The third pillar of Solvency II addresses market transparency and disclosure requirements,

[41]Note that Solvency II is in many aspects similar to the *Swiss Solvency Test* already in-force in Switzerland (see, e.g., Holzmüller, 2009).

[42]We refer to common mechanisms of existing schemes.

[43]A first discussion on the motivation for Solvency II can be found in European Commission (1999).

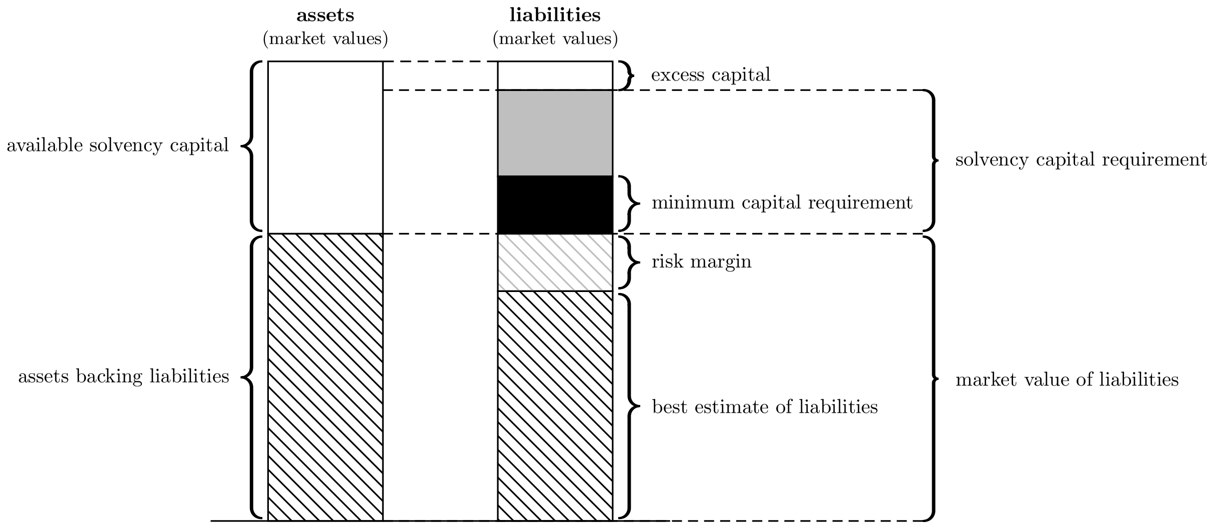

Figure 8: Economic balance sheet at market values illustrating Pillar 1 of Solvency II. The best estimate of liabilities corresponds to the "expected present value of future cash flows" (see Eling and Holzmüller, 2008; CRO Forum, 2006).

directly relating Solvency II to international reporting standards (European Union, 2009).

Figure 8 illustrates the first pillar by means of an insurer's economic balance sheet (in market values) (see Eling and Holzmüller, 2008; CRO Forum, 2006). Assets are subdivided into assets which cover liabilities and the available solvency capital (margin)[44]. The market value of liabilities corresponds to the best estimate of liabilities plus a risk margin. The best estimate of liabilities corresponds to the expected value of discounted cash flows. The risk margin is calculated using the cost of capital technique[45].

The solvency capital requirement (SCR) is determined as the value at risk of basic own funds at a 99.5% confidence level over a one-year period, where basic own funds consist of the excess of assets over liabilities and subordinated liabilities. The SCR might either be calculated by a standard model or an internal model which is subject to restrictions and approval by the supervisory authority. The minimum capital requirement is determined as the value at risk of basic own funds at a 85% confidence level over a one-year period.[46]

Solvency II requires that the available solvency capital is higher than the SCR and the MCR. Thus, it involves an escalating series of supervisory interventions: The first step corresponds to a non-compliance with the SCR, the second step to a non-compliance with the MCR (and, thus, also the SCR).

[44] There are certain restrictions regarding the composition of the available solvency capital which we do not discuss here in detail (see Art. 87 to Art. 99 of the directive).

[45] Cost of capital techniques are often applied in practice. The idea behind this technique is that the risk adjustment should reflect the costs of holding capital to cover the underlying risk (see, e.g., Rubin, Ranson, and Shi, 2009). A detailed explanation of the cost of capital approach is provided by the Swiss Federal Office of Private Insurance (FOPI) (2006) in the context of the Swiss Solvency Test.

[46] Note that the directive specifies an absolute and a relative floor as well as a relative cap for the minimum capital requirement (MCR) (see Art. 128 to Art. 131).

2.2 Insurance Guaranty Schemes

Insurance guaranty schemes provide last-resort protection to policyholders in the event of the bankruptcy of an insurance company (see, e.g., Oxera, 2007; Krogh, 1972; Yasui, 2001). Since the 1960s, insurance guaranty schemes have been introduced in various European countries (see Table 23a). Most guaranty systems resulted from the default of a single insurance company which had to be resolved. Currently, only 12 of the 30 EU-EEA countries have one or more general insurance guaranty schemes in place (European Commission, 2010b). In order to address the diversity of insurance guaranty schemes across Europe, the European Commission (2010b) proposes, in a recent White Paper, that a directive should be introduced to ensure that all Member States of the European Union have an insurance guaranty system in place which meets certain minimum requirements (see also de Larosière, Balcerowicz, Issing, Masera, Mc Carthy, Nyberg, Pérez, and Ruding, 2009).

Figure 9 shows the basic context of an insurance guaranty scheme. Policyholders take out an insurance contract with an insurance company and pay the corresponding premium. In return, the insurance company provides insurance coverage. To internalize the costs of insolvency, the insurance company pays the required contribution to the insurance guaranty fund. This contribution to the guaranty fund might either be charged on an ex post basis (based on actually incurred defaults) or an ex ante basis. In the majority of current guaranty funds, the fund contributions are levied ex ante and calculated by means of volume not of risk (Oxera, 2007; Schmeiser and Wagner, 2010).[47] Then, in case of default of the insurance company, the guaranty fund secures the interests of policyholders by a kind of compensation.[48] This compensation might either be a continuation of the insurance contract or a cash compensation. Oxera (2007) provides a detailed overview of different insurance guaranty funds in the European Union (EU).

[47]Cummins (1988) presents a theoretical approach of how risk-based premiums could be raised in an option pricing theory setting.

[48]Rymaszewski, Schmeiser, and Wagner (2011) develop conditions under which a self-supporting insurance guaranty system can be beneficial to policyholders in an imperfect market setting.

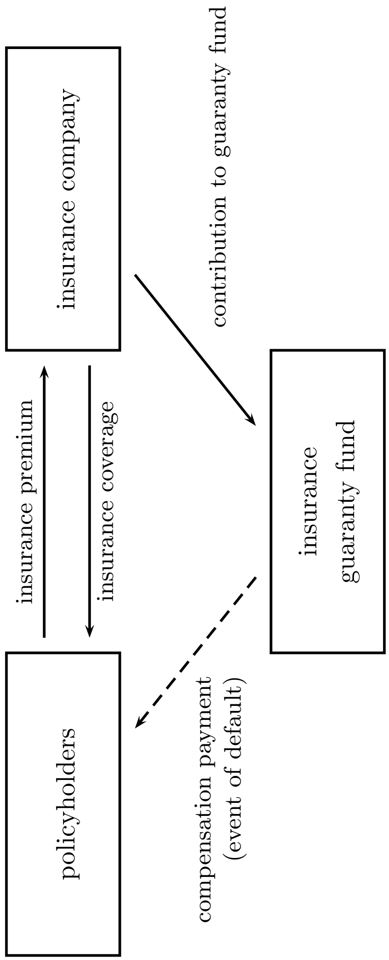

Figure 9: Illustration of basic context of an insurance guaranty fund.

2.3 International Financial Reporting Standards IFRS 4 Phase II

In July 2010, the International Accounting Standards Board (IASB) published an exposure draft proposing new standards for how to recognize, measure, and disclose insurance contracts (IASB, 2010b): IFRS 4 Phase II (see Table 23b). The IFRS 4 Phase II project aims to establish a new reporting standards providing a comprehensive and consistent basis for accounting for insurance contracts (see, e.g., IASB, 2010c). IFRS 4 Phase II builds on *IFRS 4 Insurance Contracts* which was introduced in 2004 to address some urgent issues in insurance contracts accounting (IASB, 2010c). IFRS 4 Phase II provides a market value-based or risk-based perspective on insurance companies. In contrast, the accounting approach previously taken within the insurance industry has been based on historical values rather than market values (see, e.g., Post, Gründl, Schmidl, and Dorfman, 2007).

The new insurance contracts standards apply generally to all insurance contracts. The exposure draft requires that an insurer measures the value of an insurance contract at inception as the sum of the (expected) present value of future cash flows, plus a risk adjustment and a residual margin.

We illustrate this concept in Figure 10 by means of an insurer's economic balance sheet. Note that Figure 10 aggregates the single measurements of different insurance contract liabilities. The sum of the expected present value of future cash flows and the risk adjustment provides the present value of fulfillment cash flows. Insurance companies are not allowed to realize any gains at initial recognition of an insurance contract. Thus, in the case of a negative present value of fulfillment cash flows, a residual margin is added to give an initial measurement of the insurance contract liability of zero. The risk adjustment corresponds to the maximum amount an insurer would disburse in order to be relieved of the risk of the fulfillment cash flows exceeding the ones expected. The risk adjustment can be calculated by three different methods: the confi-

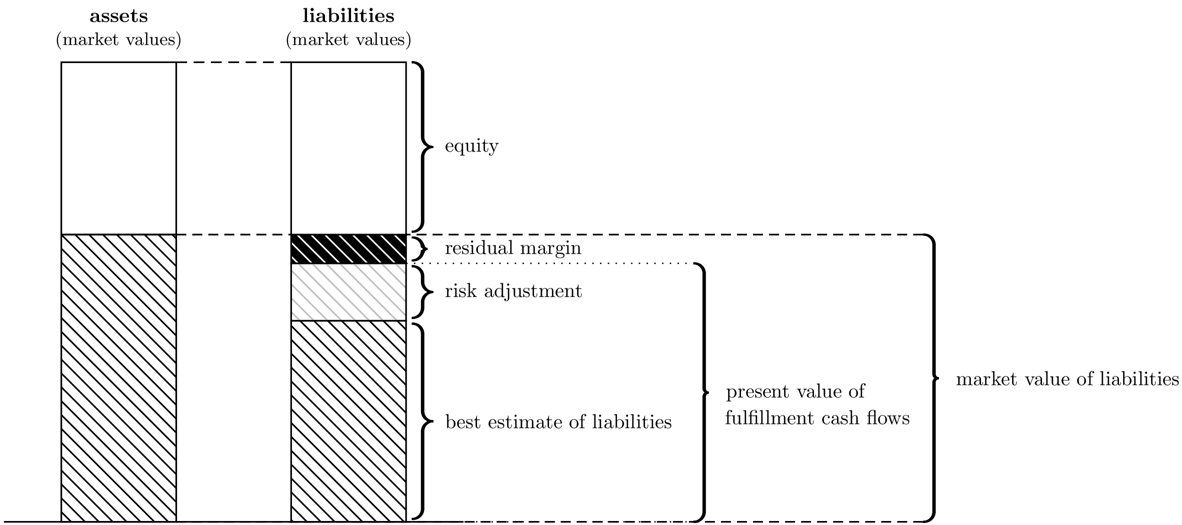

Figure 10: Economic balance sheet at market values illustrating the new standard IFRS 4 Phase II. The best estimate of liabilities corresponds to the "expected present value of future cash flows" (see the exposure draft (IASB, 2010b)). The figure aggregates the single measurements of different insurance contract liabilities.

dence level technique[49], the conditional tail expectation method[50], and the cost of capital approach.

An insurance contract (asset or liability) is recognized as soon as the insurer becomes a party to the insurance contract and is derecognized when it is extinguished. In addition, the so-called contract boundary defines which cash flows are to be included in the measurement of the liability. It separates cash flows arising from current contracts from those corresponding to future contracts. Figure 11 provides an overview of how recognition, derecognition, and contract boundaries are connected. In addition, Figure 11 shows which kind of cash flows might occur within the different time periods.

The carrying amount of the insurance contract liability is adjusted each reporting period. In addition, the exposure draft includes detailed disclosure requirements. These require both that the recognized amounts in the financial statements be stated, and that information on the nature and extent of risks resulting from insurance contracts be disclosed. We provide a figure summarizing the different disclosure requirements in Appendix A.

In the context of IFRS 4 Phase II, it is also important to consider IFRS 9. Although it does not focus on insurance companies in particular, IFRS 9 aims to create new standards for financial instruments accounting – which will have a significant effect on the asset side of insurer's financial statements. The IASB intends to replace the current financial instruments reporting standard, IAS 39, with a new set of standards, IFRS 9 (International Accounting Standards Committee Foundation (IASCF), 2009). This process of the replacement of IAS 39 is divided into three phases. The first phase involves the classification and measurement of financial instruments, the second phase focuses on impairment methodology, and the third phase addresses hedge account-

[49] The confidence level technique corresponds to a value at risk calculation (see, e.g., Rubin et al., 2009; International Actuarial Association (IAA), 2009, p.76).

[50] The conditional tail expectation (CTE) method corresponds to a tail value at risk calculation. This approach overcomes the key limitation of the value at risk technique by taking the complete tail of the distribution into account (see, e.g., Rubin et al., 2009).

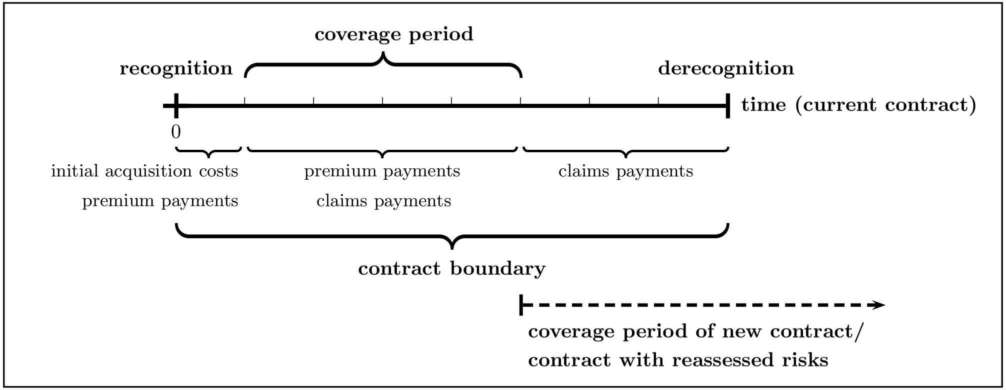

Figure 11: Illustration of time line and contract boundary. At the end of the coverage period, a new contract may start or risks may be reassessed. The contract valuation includes all cash flows (initial acquisition costs, premium and claims payments) within the contract boundary, but not those associated with new contracts.

ing (see IASCF, 2009).[51] The new standards are to enter into force in January 2013. The IASB completed the first phase, dealing with financial assets, and published *IFRS 9 Financial Instruments* in November 2009 (IASCF, 2009). Broadly speaking, this new standards require that financial assets are measured at their fair value at initial recognition (plus transaction costs, to some degree) (Art. 5.1 of IASCF, 2009). Subsequent measurements will either be at amortized costs or at fair value depending on the company's business model for managing financial assets and the contractual cash flows characteristics of the financial asset.

2.4 Market Consistent Embedded Value Principles

In May 2004, the European Insurance CFO Forum (hereafter "CFO Forum") published the European Embedded Value (EEV) Principles (CFO Forum, 2004) as the first attempt to harmonize embedded value reporting in Europe. The CFO Forum is a discussion group which was created by and is made up of the Chief Financial Officers of major European insurance companies.[52] Addressing criticism of the current EEV framework, the Market Consistent Embedded Value (MCEV) Principles (CFO Forum, 2009b) were published in June 2008 (and partially been updated in October 2009). The MCEV Principles are expected to replace the EEV Principles from 31st December 2011 onwards (CFO Forum, 2009a). At this time, the Principles will become compulsory for all members of the CFO Forum (see Table 23b).

The MCEV methodology needs to be applied to covered business. Covered business includes, as a minimum, all long-term life insurance business and may in addition include short-term life insurance as well as accident and health insurance (CFO Forum, 2009b). MCEV is the

[51] In November 2009, the IASB completed the first phase, dealing with financial assets, and published *IFRS 9 Financial Instruments* (IASCF, 2009). In addition, an exposure draft on the *Fair Value Option for Financial Liabilities* was published in May 2010 (IASB, 2010a). Regarding the second phase, an exposure draft on *Amortised Cost and Impairment* was issued in November 2009 (IASB, 2009). No exposure draft exists yet concerning hedge accounting (phase 3). Refer to the IFRS web page for current information on the project's status (http://www.ifrs.org).

[52] See http://www.cfoforum.eu.

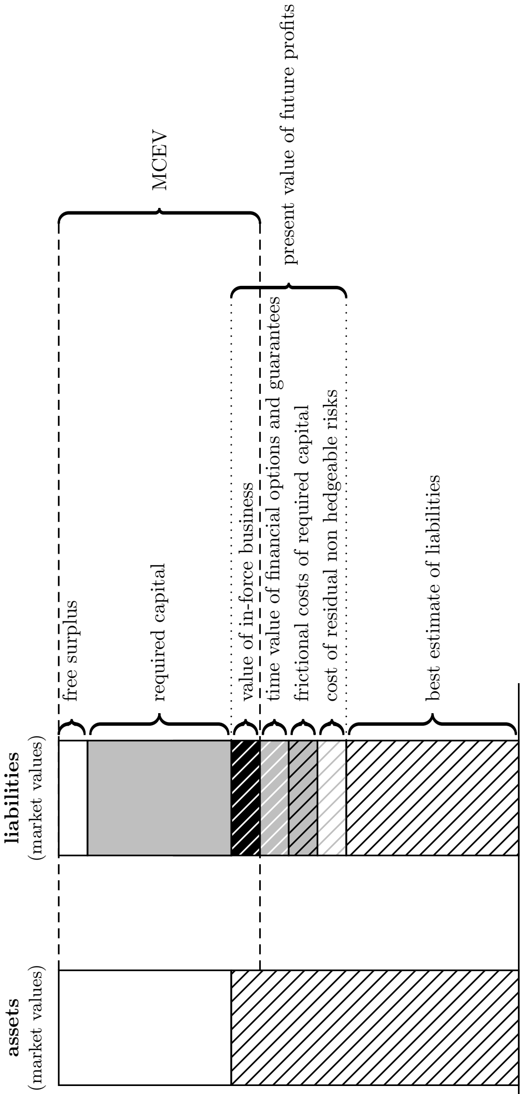

Figure 12: Economic balance sheet at market values illustrating market consistent embedded value (MCEV) (see also CFO Forum, 2009b; Diers et al., 2009).

"present value of shareholders' interests in the *earnings distributable* from assets allocated to the *covered business* after sufficient allowance for the aggregate risks in the covered business" (CFO Forum, 2009b, p.3). Figure 12 illustrates the calculation of the MCEV.

The MCEV is calculated as the sum of the free surplus allocated to the covered business, the required capital, and the value of in-force business. The required capital corresponds to the shareholders' portion of the solvency capital requirement and any additional amounts required by internal objectives. The value of in-force covered business is calculated as the present value of future profits minus the time value of financial options and guarantees, frictional costs of required capital, and costs of residual non hedgeable risks.

In order to provide a comprehensive view of both covered and non-covered business, the MCEV Principles require the calculation of a Group MCEV. The Group MCEV is calculated as the sum of the calculated MCEV of covered business and the IFRS net asset value (without any adjustments) of non-covered business.

3 Stakeholder Perspectives and Comparative Analysis

In this section, the four general frameworks are analyzed from different stakeholder perspectives, and contrasted with each other. Before turning to the different perspectives, we provide a comparative overview of the different frameworks (without considering the stakeholder points of view). We then focus on the three major stakeholders, namely insurance companies' management, policyholders, and investors. We address each stakeholder in turn and separately analyze key characteristics and interactions separately (see Sections 3.2 to 3.4).

3.1 Comparative Overview of Regulatory and Reporting Frameworks

IFRS 4 Phase II and MCEV concern insurance reporting, whereas Solvency II and insurance guaranty schemes primarily focus on insurance regulation. These different main focuses are the basis of the key differences and similarities between the four frameworks. IFRS 4 Phase II and MCEV focus on providing public information. In addition, MCEV also addresses at internal information. In contrast, both Solvency II and insurance guaranty systems have customer protection as their major goal.

Consequently, IFRS 4 Phase II targets primarily investors, MCEV targets investors and insurers (i.e., internal functions), Solvency II targets the supervisory authority and to a limited extent investors, and insurance guaranty schemes target the supervisory authority and clients. Table 24 provides a comprehensive overview of key differences between the four frameworks (see Kölschbach, 2010; PwC, 2010; Schaeffer, 2010; Schneider, 2010; Wilkins, 2008; Deloitte Research, 2008a; Deloitte Research, 2008b).

Compared to the regulatory frameworks currently in force (see Table 23), IFRS 4 Phase II, MCEV, and Solvency II take a more market- and economics-based perspective on insurance companies (see, e.g., Schneider, 2010; Farr and Wagner, 2009). Insurance business is seen as creating economic value and managing risks. The modeling in all three frameworks is based on cash flow projections and market values. However, due to their dissimilar functions, some differences in modeling persist.[53]

Regarding the relation between Solvency II and insurance guaranty schemes, the European Commission (2010b) stresses that Solvency II does not lead to a zero-failure environment and, thus, that defaulting insurance companies might still pass on losses to policyholders and taxpayers. Solvency II will merely reduce the likelihood of insolvencies and the size of losses in the event of defaults in the insurance industry (CEIPOS, 2010a). According to this reasoning, insurance guaranty schemes will not become redundant. Furthermore, an insurance guaranty scheme may be necessary in order to ensure a controlled run-off in case of an insurer's default (e.g., continuation of policies). Besides, CEIPOS (2010a) notes

[53]For instance, CEIPOS (2010b) supports the current approach of the IASB to reach a high level of conformity between Solvency II and IFRS but recognizes that differences are unavoidable due to the different goals of the two systems. Duverne and Le Douit (2009) and Schneider (2006) stress the necessity to harmonize IFRS 4 and Solvency II.

	Regulatory frameworks		Reporting frameworks	
	Solvency II	**Insurance Guaranty Funds**	**IFRS 4 Phase II**	**MCEV**
goal	customer protection	customer protection	public information	internal and public information
target group	supervisory authority, investors to some extent	supervisory authority, clients	investors	internal functions, investors
function	ensuring risk-bearing capability of insurance company	pay off policyholders in case of default of an insurance company	true and fair view of ability to generate cash flows in the future	providing information on profitability of current business
scope	insurance companies as a whole	insurance contracts covered by the respective IGS	insurance contracts based on the IASB definition	life and health insurance contracts (covered business)
valuation	expected discounted cash flows	no valuation	expected discounted cash flows	free surplus
	+ risk margin		+ risk margin	+ required capital
			+ residual margin	+ value of in-force business

Table 24: Comparison of IFRS 4 Phase II, Solvency II, MCEV, and Insurance Guaranty Schemes (see Kölschbach, 2010; PwC, 2010; Schaeffer, 2010; Schneider, 2010; Wilkins, 2008; Deloitte Research, 2008a; Deloitte Research, 2008b)

that the fifth quantitative impact study[54] on Solvency II could provide a good basis for an assessment of the required fund size for insurance guaranty schemes.

3.2 Management Perspective

With the introduction of MCEV, Solvency II, and IFRS 4 Phase II an insurance company managements face the challenge of having to adapt to three new regulatory frameworks within a comparatively short time. Clearly, this will incur costs. For instance, new data need to be captured, models need to be adapted, knowledge must be built up, and new jobs could be required. However, with the introduction of these regulatory frameworks, high costs and operational challenges are not the only effects felt by management from the introduction of these regulatory frameworks; this section will focus on the other impacts.

Regulation

Solvency II enacts new capital requirements which specify that all insurance companies must reach a certain safety level. Consequently, a unification of risk profiles of different insurance companies takes place, as a single insurer cannot have a radically different safety level to the others. Hence, management will not be able to significantly differentiate an insurance company considerably from the rest of the market by having a different safety level.[55]

However, Solvency II also provides opportunities. An insurance company's management could gain a competitive advantage by applying special risk management measures so as to comply with Solvency II; this could, for example, reduce the solvency capital requirement or involve lower regulatory costs. Thus, management needs to build up knowledge

[54] For the technical specifications of the fifth quantitative impact study (QIS5), see, e.g., European Commission (2010a).
[55] Clearly, small differences in risk profiles will still be possible after the introduction of Solvency II, but the range of feasible safety levels will be much smaller.

in their insurance company on appropriate mixes of risk management measures and how these affect the solvency capital requirement.[56]

On the other hand, Solvency II could, with excessive capital requirements, pose a threat to insurance companies. To comply with excessive capital charges, management might, for instance, be forced to move to a more conservative asset allocation, to redesign products (including repricing), to reduce capacity, or even to withdraw from certain insurance sectors. The Comité Européen des Assurances (CEA) (2010b) discusses these and other aspects in greater detail.

While Solvency II aims to keep the probability of default at a prescribed low level, insurance guaranty schemes step in when an actual default takes place. From the management perspective, the magnitude and the methodology of the calculation of the contributions charged by the guaranty fund are key. If contributions are not risk-based but rather volume-based, management is incentivized to pursue a riskier business strategy as the additional risk is not taken into account in the guaranty fund charges. As a consequence, cross-subsidization effects will occur between different insurers. Empirical studies based on US data support this idea (Lee, Mayers, and Smith, 1997; Lee and Smith, 1999).

On the other hand, if contributions are risk-based, similar arguments apply as those which as apply to Solvency II. An insurance company's management could define a particular set of risk management measures to keep the contribution to the guaranty fund at a desired level.

Reporting

The new IFRS 4 Phase II standards require that insurance companies adopt an economic perspective in their reporting. This enables managements to present their insurance business in a different way from that required by the previous reporting standards. Using IFRS 4 Phase II, management can present the business as a business which creates economic

[56] Consider, for example, the case of an insurance group. A group has to comply on two levels: group and local single entities' solvency. In order to avoid excessively high capital requirements for some entities, it is crucial that management obtains a comprehensive picture of how solvency capital requirements can be influenced and how they interact.

value and manages risks instead of being purely sales-driven. Generally, such a new accounting framework can be advantageous to an insurance company's management as soon as it is strong and credible. Once this is the case, the new standard could reduce the problem of undervaluation and attract new investors.

Furthermore, IFRS 4 Phase II provides the same risk-based, economic information on all insurance companies. This provides new opportunities for managements to benchmark their own indicators of economic performance, as well as to set themselves apart from their competitors. As the exposure draft on IFRS 4 Phase II includes broad disclosure requirements (in particular with regard to risk, as illustrated in Appendix A), insurance company managements have access to information on other market players which has not been available under previous standards.

In general, similar arguments apply to the MCEV principles. Management is able to provide information to shareholders which is based on an economic perspective on their business. The underlying principles ensure that a certain degree of comparability between the MCEV of different insurance companies is reached.

Moreover, MCEV focuses directly on shareholders and the value of their holdings. This enables an insurer's management to use MCEV as an instrument to control share prices. While reporting standards include the value of equity capital as one component of the required data, the market value of equity is the key element of the MCEV principles.

Relation of Frameworks

In order to comply with Solvency II, MCEV, and IFRS 4 Phase II, insurance company managements need to build up cash flow projection capabilities. Although the actual measurement models in the frameworks reflect in part their different objectives, the ability to project cash flows of insurance products is an essential part of all three frameworks.In addition, other synergies between the different frameworks are possible and should be considered with regard to implementation.

With regard to the relation between IFRS 4 Phase II and MCEV, it is important that the data provided in both reports are consistent and that any differences which are explained. This allows MCEV to be understood as an extension to IFRS 4 Phase II which focuses on shareholders. By complying with both, management can provide a new, comprehensive basis for decision-making to its investors. This more comprehensive information base could be used to attract new investors.

Furthermore, a single framework for economic reporting, such as IFRS 4 Phase II, could assist management into improve the governance of an insurance company. Currently, insurance companies and their subsidiaries are obliged to present IFRS data as well as MCEV data, must provide local statutory accounts, and usually have some form of internal performance accounting. IFRS 4 Phase II could – over time – allow the convergence of these different reporting efforts (Ziewer, 2010).

Solvency II and insurance guaranty funds both address the possibility of the default of an insurance company. As Solvency II aims to reduce the probability of default to a low level, and incorporates a scale of interventions which allows early detection of financial stress, the question thus arises of whether an insurance guaranty system is still needed. Both systems involve significant costs. In this context, the CEA (2010a) argues that an adequate level of policyholder protection is already offered by the current and forthcoming European insurance regulatory frameworks (e.g., investment regulation, capital requirements).

Another effect concerning both insurance guaranty systems and Solvency II relates to risk-taking incentives. If an insurance guaranty system is in place which does not charge risk-based premiums, the introduction of broad solvency requirements such as those of Solvency II could eliminate risk-taking incentives which would otherwise exist (see, e.g., European Commission, 2010b) (see also the previous discussion of insurance guaranty schemes above).

3.3 Policyholder Perspective

Insurance liabilities – and thus, the position of the policyholder as a debtholder – are among the most significant items on an insurer's balance sheet. Policyholders could use the information provided by IFRS 4 Phase II and MCEV to decide which insurance company to choose. In addition, they are to be protected by both Solvency II and insurance guaranty schemes. This paragraph considers not only private policyholders but also corporate entities seeking insurance protection.

Regulation

When policyholders become aware of a solvency regime like Solvency II, trust in the insurance industry is enhanced and the stability of the system can be improved. This is particularly important for private policyholders, who might not be able to assess the safety level of an insurance company on their own.

Conversely, when policyholders assume that the solvency system works, they do not have any incentive to gather information on the risks to which their insurance company is exposed. Policyholders then no longer act as an additional monitoring agent. This might be problematic if the solvency regulation has certain limitations.

Furthermore, excessive capital requirements may have fundamental effects on policyholders. Firstly, insurance premiums will increase if insurance companies pass the costs of the capital requirements on to their policyholders. Secondly, insurance companies could, for instance, reduce the supply of traditional life insurance products (e.g., those that involve minimum interest rate guarantees) in order to reduce their capital requirements or could even withdraw from the market. This would reduce competition (see CEA (2010) for a discussion of these and other aspects).

An insurance guaranty fund is the safety net for policyholders. In a similar way to Solvency II, guaranty systems can increase trust in insurance companies. Private policyholders in particular are often not able to secure/hedge potential losses on their own or are unaware of the fact that their insurance company could default. Besides, for long-term contracts (e.g., life insurance contracts) the counterparty risk is usually difficult to estimate.

However, insurance guaranty schemes can also generate adverse incentives. Basically, insurers pass on fund contributions to their policyholders, either in an transparent way as an additional charge on the insurance premium or simply by increasing the premium. If the contributions required from insurance companies are not risk-based, policyholders are encouraged to choose the riskiest insurer, as it can offer the lowest premiums and claims are in any case covered by the guaranty scheme.

In addition, as discussed above in relation to Solvency II, if no default risk is carried by the policyholders on their own, they do not have any incentive to monitor their insurance company (see, e.g., Cummins and Sommer, 1996; Oxera, 2007). This has the effect of an additional monitoring agent ceasing to exist.

Reporting

The target group of IFRS 4 Phase II is investors. Thus, the question arises whether the reports provided are in a format appropriate for policyholders. For instance, there may be various pieces of information which are not useful from a policyholder's point of view. For private policyholders in particular, IFRS 4 Phase II could provide grossly excessive amounts of information. Furthermore, IFRS 4 Phase II presents information from an economics-based perspective. As this perspective requires a high level of knowledge of the economics of insurance, the previous sales-based approach (premium income, claims payments, costs) may be more comprehensible and intuitive from a policyholder's point of view.

However, IFRS 4 Phase II enables policyholders to gather more information on the risk associated with different insurance companies. Thus, policyholders can assess the probability that future claims will be able to be paid. This would be of particular importance if no stringent solvency regulation and/or no insurance guaranty system were in place. This is, for instance, currently the case for corporate policyholders in some countries where only an insurance guaranty fund is only in place for private individuals.[57]

The MCEV principles are even more highly focused on shareholders. As a consequence, the information provided is likely to be of little relevance to policyholders. However, from our point of view, a policyholder could find it useful to check the MCEV of different insurance companies

[57]See, e.g., Schmeiser and Wagner (2010) for a current overview of existing insurance guaranty systems.

as a comparatively high MCEV may indicate that premiums are too high.

Relation of Frameworks

With Solvency II, IFRS 4 Phase II, and MCEV, policyholders receive broad information on the risk level of insurance companies. However, this information is not primarily addressed to policyholders and, hence, may not be presented in a way suitable for them. This applies in particular to private policyholders, who often lack financial literacy. As a result, although policyholders receive much more information under the new regimes, the question remains of whether they are able to make use of it.

With regard to Solvency II and insurance guaranty systems, a recurring question is whether the existence of both – a stringent solvency regime as well as a guaranty system – could lead to the overprotection of policyholders (i.e., whether Solvency II provides an adequate level of policyholder protection which renders insurance guaranty schemes dispensable). Both enhance trust in the insurance industry, but if one of them is redundant, the existence of both simply results in an unnecessary increase in premiums, as insurers can be expected to pass on costs associated with higher levels of regulation.

3.4 Investor Perspective

The information published by the application of financial reporting frameworks such as IFRS and MCEV is essential to current and potential investors in insurance companies. However, current insurance reporting (see Table 23) does not often provide investors with sufficient information to be able to understand an insurer's performance and risk in detail (Ziewer, 2010). This paragraph elaborates on the investor's perspective on the current regulatory and reporting reforms.

Regulation

Although Solvency II is not targeted at investors, it can provide useful information to them. Investors can use Solvency II reporting to gather additional information on the financial stability of an insurance company and, therefore, the safety of their investments. Generally, Solvency II is perceived as a more comprehensive tool than Solvency I. Accordingly, Solvency II enhances comparability among different insurance companies with regard to their safety level. Although a single solvency ratio may be based on various assumptions and the absolute figure may not be self-explanatory, the overall solvency of an insurance company can be roughly assessed.

In addition, the solvency regime reduces the probability of year-to-year changes in the safety of investments in an insurance company. In other industries without any solvency regulation, for instance, strategic changes can highly influence the riskiness of investments. The comparatively high stability of the safety level resulting from Solvency II could be of particular benefit to investors seeking for long-term investments with low risk.

However, the higher the capital charges demanded by Solvency II, the lower the profitability and the lower the returns to investors. As a result, the insurance industry could become less attractive to investors (see, e.g., CEA, 2010).

Insurance guaranty schemes, in contrast, do not appear to have any effect on investors apart from potential dividend reductions due to increased regulatory costs. In general, guaranty systems affect primarily policyholders.

Reporting

Under IFRS 4 Phase II, investors receive comprehensive, uniform information on the performance and risk profile of an insurance company based on market values. This enables investors to assess and compare the economic performance of different insurance companies. As a consequence, insurance companies could become interesting investments for investors who believe that current accounting approaches suffer from a lack of transparency and comparability.[58]

[58] De Mey (2009), for instance, identifies four key financial reporting needs of shareholders, namely comprehensiveness, comparability, timeliness, and reliability.

On the other hand, the new market-value based approach has two major drawbacks for investors. Firstly, insurance companies have a comparatively high degree of freedom, as IFRS 4 Phase II is a principle-based approach and insurance companies need to apply different model assumptions. This limits the transparency and comparability of reported results.

Secondly, a broad understanding of IFRS 4 Phase II requires a high level of knowledge of insurance economics and risks. This could discourage investors from investing in an insurance business as they may be unable to interpret the reported information.

In a similar way to IFRS 4 Phase II, the new embedded value reporting approach provides a higher level of information on the economic performance of insurance companies. In addition, MCEV gives extra information on the value of the investors' own stakes. Consequently, it is an additional source of data. However, as in IFRS 4 Phase II, the degree of freedom is relatively high and a high level of financial literacy is required to interpret the figures provided.

Relation of Frameworks

MCEV, IFRS 4 Phase II, and Solvency II make a high level of information available regarding economic value and risks. For this reason, investors are able to obtain in-depth information on the value of their investments and the potential risks their investments are exposed to. In particular, the MCEV report explicitly addresses on the value of their holdings in the insurance company.

On the other hand, all frameworks require that insurance companies make various assumptions and thus involve high degrees of freedom. This may reduce the comparability between insurers' results or with the results of companies in other industries. Nonetheless, a combination of the three frameworks could be used to obtain a holistic view, despite this problem.

Moreover, market- and risk-based values are less intuitive than sales- and volume-based figures. Hence, investors need to build up knowledge of the economic concepts applied in the three frameworks if they are to understand the different reports. Furthermore, the terminology partially differs across the frameworks (e.g., risk margin versus risk adjustment) and many vary in terms of their details.

Furthermore, the volatility of market values could deceive investors with regard to the true performance of an insurance company. As insurance companies' business is a long-term one, temporary fluctuations in value may not have any relation to the actual value of shareholders' holdings.

4 Summary and Critical Discussion

The discussions above have shown the need to consider the four regulatory frameworks jointly rather than separately. We have indicated various implications for the different stakeholders. In this section, we first consolidate the different stakeholder perspectives, under the assumption that all four frameworks will be implemented as planned. Then, we abandon this assumption and propose a holistic approach to these regulatory and reporting frameworks as a proposed improvement.

4.1 Consolidation of Stakeholder Perspectives

This section aims to bring together the different perspectives. In order to achieve this, we briefly summarize our main findings with regard to the different groups of stakeholders and, in a second step, derive one main issue for each of them.

- In order to comply with all the frameworks, the *managements* of insurance companies need to build up cash flow projection capabilities. Once this is done, both IFRS 4 Phase II and MCEV can enable managements to attract new investors. Furthermore, solvency regulation can reduce risk-taking incentives which exist under current insurance guaranty schemes.

- *Policyholders* are to be protected by these two regulatory frameworks (i.e., Solvency II and insurance guaranty systems). In addition, with the introduction of IFRS 4 Phase II, MCEV, and Solvency II reporting, policyholders gain access to comprehensive information on risk. However, the question remains of whether they want to (motivation) and can (financial literacy) make use of this information.

- Similarly, *investors* receive in-depth information on economic value and risks from the various reports (IFRS 4 Phase II, MCEV, Solvency II). However, they face challenges regarding the reliability of information (degrees of freedom) and their understanding (financial literacy). In particular, volatility of reported values could distort the realistic representation of an insurance business.

This brief synopsis of our results makes clear that the managements of insurance companies face a high regulatory burden. Capital requirements need to be fulfilled and numerous operational adjustments will be necessary. Thus, the question arises of whether the general benefits of the different regulatory frameworks can justify the associated costs. If not, European insurers will be at a competitive disadvantage to insurance companies in less regulated markets or less regulated industries (e.g., pension funds). Such a competitive disadvantage would have an impact on various stakeholders, including policyholders, investors, and the economy in general.

This question is particularly important concerning the relation between Solvency II and insurance guaranty schemes. In our view, if Solvency II is implemented appropriately and the scale of intervention is executed seriously, the probability of an insurer going bankrupt and being unable to pay off all policyholder claims is very close to zero. We agree that additional thorough analysis is necessary in order to assess the economic costs and benefits of the all frameworks for all categories of stakeholders.

In this context, the widespread information provided to investors needs to be considered. IFRS 4 Phase II and MCEV will both provide various additional risk-related, economics-based information, in comparison with current reporting. Aligning and combining these separated reports is potentially a key strength of the upcoming economic value reporting. In our view, the alignment of definitions, terminology, scope, and modeling components will be crucial in allowing IFRS 4 Phase II to replace MCEV reporting in the long term. However, if a majority of investors does not use this information, the reporting efforts may not be appropriate. Furthermore, investors need to be able to, or must learn to, understand the reported data. If this is not the case, an insurer's economic value reporting might be misinterpreted or disregarded. In the end, IFRS 4 Phase II and MCEV should enable investors to base their decision whether to invest in an insurance company or not on an improved basis of information. In order to reach this, large communication (as well as presentation) efforts and expenditures are necessary.

As already pointed out in the introduction (Section 1), CEIOPS (2009) names three elements, in addition to Solvency II, which need to be re-examined as a consequence of the financial crisis: insurance guaranty schemes, information to policyholders, as well as common reporting formats. The question of how to harmonize and how to proceed with insurance guaranty systems is discussed in the White Paper of the European Commission (2010b). Common reporting formats are a major concern of both IFRS 4 Phase II and MCEV reporting. However, the issue of the information made available to policyholders has barely been discussed. Currently, the main regulation in this area is that embedded in local contractual law. From a theoretical point of view, the new regulatory frameworks enable policyholders to gather more information on an insurance company (as discussed in Section 3.3). For policyholders, however, what would seem to be most relevant would be product-related, easily comprehensible information to enable rational choice between different insurance products and companies.

Consequently, questions arise of whether the provisions of current local contractual law (and its current reforms) are sufficient, whether European harmonization is feasible, and whether policyholders require additional information from insurance companies. It is possible that a higher level of financial literacy would suffice to allow policyholders to make informed choices between insurance products. One advantage of this would be that it would not increase the regulatory burden on European insurance companies. In addition, given the difficulty of evaluating different insurance companies and products, Solvency II and insurance guaranty systems provide a high level of protection to policyholders avoiding poor decisions with long term consequences being taken. Thus, the issues of the information provided to policyholders and financial literacy require further analysis.

In summary, we find that the four frameworks need to be considered jointly rather than separately, due to various interrelations and interactions between them. Assuming that all four frameworks are to be implemented as planned, coordination of the different timetables will be essential to exploit synergies and reduce, to some extent, the cost of implementation. In relation to IFRS in particular, it is of major importance that IFRS 4 Phase II and IFRS 9 are introduced in a synchronized way. However, our discussion showed that the planned frameworks have certain drawbacks, in particular with regard to their interrelations. Therefore, in the next paragraph we propose a more comprehensive, holistic approach.

4.2 Proposal for a Holistic Approach

The above analysis discussed various issues under the assumption that all four frameworks will be implemented as currently planned. However, the question remains of whether concrete suggestions can be made for improvements. Hence, based on our analysis in the previous sections, we now set out a proposal for a holistic approach to insurance regulation and reporting. The proposal provides the same benefits as the current regulatory plans (e.g., comprehensive information and increased customer protection) but is, in addition, internally consistent and therefore imposes much a lower regulatory burden on insurance companies in the long term.

Figure 13 illustrates the basic concept. The approach is based on an exhaustive data warehouse containing all information on book values and the distribution of discounted cash flows. These data are processed for insurance reporting in such a way as to generate two major reports:

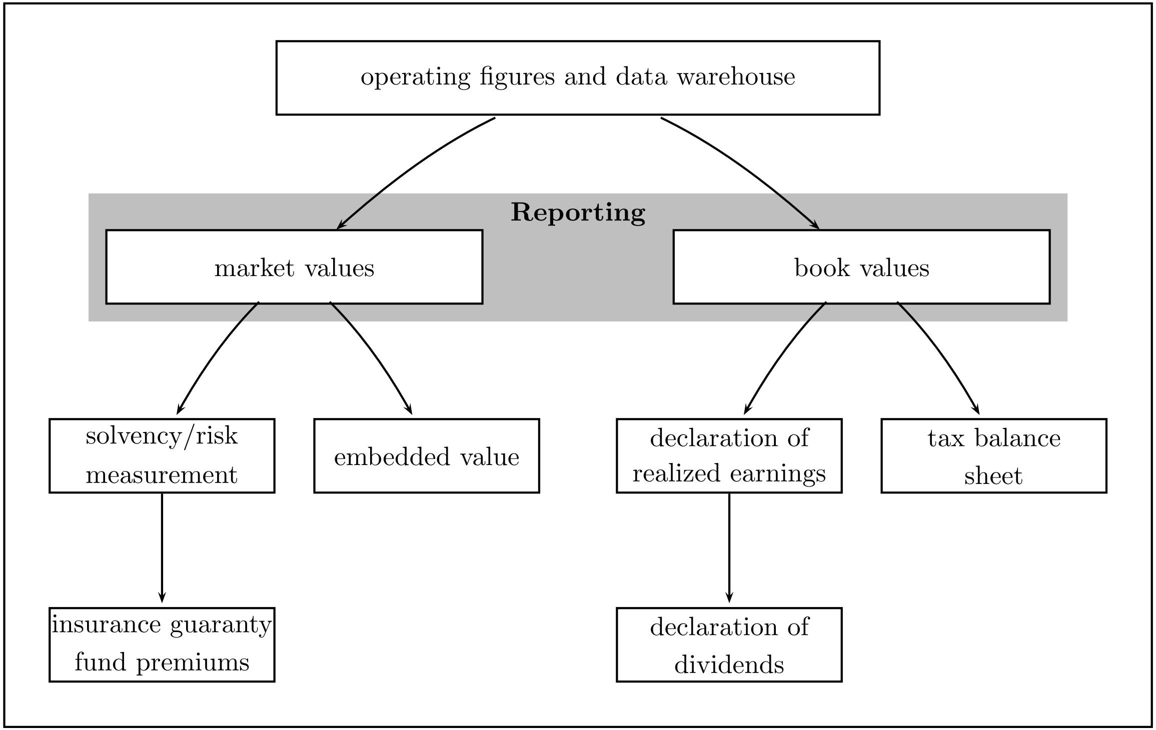

Figure 13: Proposal for a holistic approach to insurance reporting and regulation. A comprehensive reporting framework containing book and market values is the basis for coherent calculations of relevant figures in other regulatory and reporting concepts.

market value and book value reporting. We argue that both market and book values are necessary in order to provide a comprehensive view of an insurance company. While market values closely reflect the current fair value, book values provide information on historical costs and depreciation. In addition, book values are not subject to market fluctuations.

The reported market values are used for additional reporting purposes. For instance, an embedded value calculation is carried out. We argue that reporting the embedded value in addition to market value reporting is necessary in order to provide shareholders with explicit information on the value of their holdings. Furthermore, the reported market values are used for regulatory activities: the calculation of solvency capital requirements (as defined in, e.g., Solvency II) and the definition of contributions to insurance guaranty schemes. Although solvency measurements require some additional data from the data warehouse, in our view it is vital that the solvency assessment is based on the same market valuation of assets and liabilities as the reporting. The results of the solvency assessment can then be used to estimate contributions to insurance guaranty funds. Book values are required for the preparation of the tax balance sheet, to declare realized earnings, and, consequently, to determine dividends. In addition, book values can be used in order to calculate different performance ratios, e.g., the claims, expense, and combined ratios.

Thus, in our approach regulatory and reporting frameworks are based on the same data base and only one market value is derived, which is applied within other frameworks in order to measure solvency or report embedded value. In this way, the proposed approach ensures that reported data are consistent but still comprehensive. From our point of view, this holistic approach will significantly reduce regulatory costs in the long run.

5 Conclusion

Regulatory and reporting frameworks in the European insurance industry are currently subject to far-reaching reforms. In this paper, we present four of these frameworks, provide a brief overview of their current state of progress, illustrate key aspects, and explain the different underlying measurement models. Then, we analyze them from different stakeholder perspectives and compare and contrast them.

Our results are threefold. First, they show that, despite the various benefits, insurance company managements face a high regulatory burden. Thus, the question arises of whether the benefits of the different regulatory frameworks can justify the corresponding costs. If this is not the case, European insurers will be at a competitive disadvantage to less regulated markets. Second, policyholders will be well-protected from financial stress on insurance companies by the regulatory reforms. However, additional reforms may be necessary to enable policyholders to make rational choices between different insurance products and companies. In addition, the issue of financial literacy needs to be addressed. Third, the market-based perspective of the new frameworks results in a high degree of complexity. Reporting frameworks in particular should enable investors to make use of an improved basis of information. Therefore, major communication efforts and spendings will be necessary to enable investors to interpret the information reported.

In conclusion, our results make clear that the four regulatory frameworks must be considered jointly and that timetables need to be coordinated in order to reduce the regulatory burden and to exploit synergies. To overcome difficulties with the planned frameworks, we propose a more holistic, comprehensive approach to insurance reporting and regulation.

Appendix

A Disclosure Requirements in IFRS 4 Phase II

Explanation of recognized amounts
Reconciliation of contract balances from the opening to the closing balance

1. insurance contract liabilities and assets
   a) risk adjustments included
   b) residual margins included
2. reinsurance assets (as cedant)
   a) risk adjustments included
   b) residual margins included
3. impairment losses on reinsurance assets

1. carrying amounts
2. new contracts recognized
3. premiums received
4. payments
   a) claims and benefits
   b) expenses
   c) incremental acquisition costs
   ...

Explanation of recognized amounts
Methods and inputs used to develop the measures

1. methods and inputs used for estimating
   a) measurements with most material effect on recognized amounts
   b) risk adjustments (including confidence level)
   c) discount rates
   d) estimates of policyholder dividends
2. effect of changes in the inputs used (each change with material effect)
3. measurement uncertainty analysis (reasonable other inputs)

Figure 14: Overview of disclosure requirements in IFRS 4 Phase II. Disclosure requirements regarding the explanation of recognized amounts in the financial statements (see IASB, 2010b). (continued on next page)

Nature and extent of risks arising from insurance contracts

1. exposure to risk and the way they emerge
2. objectives, policies, and processes for managing risks
3. changes in 1. and 2.
4. information about the regulatory framework (e.g., minimum capital)
5. information about insurance risk (gross and net) before and after risk mitigation
   a) sensitivity analysis of insurance risk regarding its effect on profit or loss and equity
   b) concentration of insurance risk and how it is determined
   c) claims development
6. information about each other type of risk
   a) summary quantitative information (including applied risk management techniques)
   b) concentration of risks
7. information about credit risk
   a) maximum exposure at end of reporting period
   b) credit quality of reinsurance assets
8. information about liquidity risk
   a) maturity analysis or information on timing of net cash outflows
   b) how liquidity risk is managed
9. information about market risk
   a) sensitivity analysis for each kind of market risk (given exposure of insurer)
   b) explanation of methods and main inputs in sensitivity analysis
   c) explanation of objective of applied methods and limitations
   d) changes from previous periods
   e) information about exposures to market risks caused by embedded derivatives

Figure 14: Overview of disclosure requirements in IFRS 4 Phase II. Disclosure requirements regarding the nature and extent of risks regarding insurance contracts (see IASB, 2010b). (cont.)

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Part IV

A Proposal for a Capital Market-Based Guaranty Scheme for the Financial Industry

Abstract

In this paper, we introduce a capital market-based financial guaranty system as an alternative to current insurance guaranty funds and deposit insurance systems. The guaranty system secures clients' claims for the case of default of the financial company by means of a special purpose vehicle which issues bonds to investors. In a first step, we present equations in order to derive the two main input parameters of the special purpose vehicle, the premium and the principal. Subsequently, we analyze the impact of different investment actions taken by the financial companies protected by the guaranty vehicle on various shortfall measures. We find that it will be necessary to restrict the investment volume of investors from the financial industry in order to avoid systematic risk within the proposed guaranty scheme. By deriving practical implications, we show that the capital market-based solution has some key benefits compared to current deposit insurance and insurance guaranty schemes.[59]

[59] H. Schmeiser, J. Wagner, and A. Zemp. A Proposal for a Capital Market-Based Guaranty Scheme for the Financial Industry. *Working Papers on Risk Management and Insurance*, 85, 2011.

1 Introduction

High losses at numerous financial institutions and major insolvencies during the recent financial crisis have raised the questions whether current financial guaranty systems can really protect clients against defaults of financial companies in case of major economic downturns. Especially bail-outs which occurred in various countries, e.g., certain banks in Ireland and the US or the insurance company AIG, caused discussions on if and to what extent taxpayers and society should pay for economic turbulences faced by a financial company. Thus, a reconsideration of both, current deposit insurance systems found in the banking sector and insurance guaranty systems of the insurance industry, appears to be necessary. In the European Union, reviews of deposit insurance systems have led to higher coverage levels in the last year. While these coverage levels will be homogenized by end of 2011 (see, e.g., ECOFIN Council (2008)), the landscape of existing guaranty schemes in the insurance industry is still very heterogeneous with regard to practical implementation, scope and coverage (see, e.g., Oxera (2007)).

In this paper, we propose a capital market-based guaranty system as an alternative to current insurance guaranty funds and deposit insurance systems. The proposed framework has a structure which is very similar to catastrophic or other insurance-linked bonds: A special purpose vehicle for each company is established in which investors pay in a principal and financial companies a risk-adequate premium. In case of default of the financial company, the capital in the special purpose vehicle is used in order to cover the claims of the company's clients. Note, however, that the proposed market-based guaranty system exhibits one key difference to catastrophic bonds, namely that catastrophic bonds cover insurance risks whereas financial guaranty systems solely focus on credit risk.[60] The described market-based solution overcomes the problem of current guaranty systems that, generally, wealth transfers between clients of different financial companies take place (see, e.g., Rymaszewski,

[60]Nevertheless, insurance risks may have a significant impact on credit risk as insurance risks like catastrophes can cause financial companies, particularly insurers, to default.

Schmeiser, and Wagner (2010)). In addition, the capital market-based funding might allow guaranty systems to even cover shortfalls of major financial companies as it provides, at least from a conceptual point of view, access to very large amounts of capital.

In the following, we provide a brief overview on the most relevant literature regarding deposit insurance and insurance guaranty systems and point out major similarities, differences and current trends. Insurance guaranty and deposit insurance systems are similar in their basic characteristics. Both are to provide customer protection in the financial services industry and require an obligatory membership of the respective financial institutions. As banks and insurance companies are usually perceived to be system-relevant, these guaranty schemes additionally aim at enhancing financial stability. The works by Oxera (2007), Schmeiser and Wagner (2010), Feldhaus and Kazenski (1998), Demirgüç-Kunt, Kane, and Laeven (2008), Cariboni, Vanden Branden, Campolongo, and De Cesare (2008), and Frolov (2004) provide an overview of existing systems and their practical implementations in the insurance and banking industry. While Schmeiser and Wagner (2010, Sect. 2) give a worldwide outline of existing insurance guaranty funds, Oxera (2007) provides a thorough review of the existing schemes in the European Union. A detailed description of the U.S. insurance guaranty fund can be found in Feldhaus and Kazenski (1998). Demirgüç-Kunt et al. (2008) provide a comprehensive overview of different deposit insurance systems around the world as of 2003. A more recent outline of European systems, as of 2007, can be found in Cariboni et al. (2008). In addition, Frolov (2004) gives a literature review on deposit insurance designs, analyzing basic approaches and practical choices.

One major difference between deposit insurance and insurance guaranty systems can be observed with regard to compensation payments. Deposit insurance systems usually embed only one form of compensation in case of insolvency, namely cash in the amount of the current value of covered deposits up to a predefined cap. On the contrary, compensations in insurance guaranty schemes differ depending on the insurance sector and the regulatory framework. An overview of the different compensation mechanisms in European insurance guaranty funds can be found in, e.g., Oxera (2007, pp. 23-26). On the one hand, losses might be compensated with cash by covering claim events which occurred before insolvency and a certain period afterwards. That is to say, as soon as an insolvency occurs, the client can close an insurance contract with another insurance company in order to be insured without interruptions. On the other hand, a continuation of insurance contracts might be more appropriate than a cash compensation in certain insurance sectors, e.g., with regard to life or health insurance contracts which are usually of long-term nature. This is the case, for instance, in German health and life insurance guaranty funds which secure the continuation of insurance contracts in case of insolvency.

Another difference can be observed with regard to the coverage of guaranty schemes. Deposit insurance systems usually cover 100% of each deposit account up to a certain cap (maximum coverage). And, as the name implies, they only focus on deposit accounts. By contrast, insurance guaranty systems are very heterogeneous in this regard. They cover either 100% or less whereas some involve a cap and some do not (see, e.g., Schmeiser and Wagner (2010)). In addition, insurance guaranty schemes are often related to various different kinds of insurance products.

Another key difference is the current state of practice and research with regard to risk-based premium calculation. Practice in the area of deposit insurance appears to be more advanced: Whereas eight risk-based deposit insurance systems are applied in the European Union (see European Commission JRC (2008)), risk-based insurance guaranty funds, to a minimum extent, can only be found in Germany and in Japan (Oxera (2007); Schmeiser and Wagner (2010)). The situation is similar when looking at the state of current research. In the context of deposit insurance schemes, the European Commission JRC (2009) proposes three different risk-based models, the first one building on a single indicator (capital adequacy), the second one on multiple indicators (capital adequacy, asset quality, profitability, liquidity), and a market-based one. Building on these, Bernet and Walter (2009) describe the deposit insurance premium as a function of systematic risk, specific risk, and the eligible deposit amount of the respective bank. Another risk-based premium approach (by means of risk-neutral valuation) can be found in Duffie, Jarrow, Purnanandam, and Yang (2003). On the contrary, just a few risk-based models are available with regard to insurance guaranty funds. Cummins (1988) calculates risk-adequate premiums based on option pricing theory in a one period context. This model is usually extended by others, see, e.g., Duan and Yu (2005) who expand the model to multiple periods.

Regarding funding of these systems, neither detailed proposals nor models can be found which are concerned with a market-based funding of deposit insurance or insurance guaranty funds. However, there are detailed discussions whether a market-based funding of deposit insurance systems is realizable and certain models which propose reinsurance solutions. Moreover, the U.S. deposit insurer, the Federal Deposit Insurance Corporation (FDIC), is actually allowed to transfer up to 10% of their risk exposure to the market (see, e.g., Sheehan (2003)). One reinsurance solution can be found in Plaut (1991) who provides a conceptual framework under the assumption that deposit insurance is a reinsurance of different banks, i.e., the deposit insurer is only responsible for securitization and steps in in case of default. Another reinsurance framework is provided by Madan and Unal (2008) who present a framework to price excess-of-loss reinsurance contracts on deposit insurance losses.

The possibility of a market-based funding of deposit insurance based on catastrophe bonds or credit derivatives is briefly discussed by Bernet and Walter (2009) and by the International Association of Deposit Insurers (IADI) (2009). Sheehan (2003) discusses advantages and disadvantages of reinsurance or securitization of deposit insurance risks. His key argument in favor of securitization is the access to a larger pool of liquid capital which allows to cover larger losses. However, he points out that moral hazard, transaction costs, and structuring costs are problematic issues in this context. Pennacchi (2009) presents advantages and disadvantages of the application of CDS spreads. While CDS spreads are likely to incorporate systematic and firm-specific risk factors, CDS spreads can lead to an excessive volatility in deposit insurance premiums. Thus, since current literature on market-based guaranty systems is limited, this provides the starting point to propose a market-based guaranty framework.

Note that practitioners and researchers currently discuss the question whether the loss-absorbing buffer should be increased by higher solvency capital requirements. Here, mezzanine capital instruments like contractual contingent convertible bonds (CoCos) and preferred equity are in focus. By contrast, our analysis will focus on the situation where a given solvency capital exists and on the question how to secure clients' claims in the case of a financial company's default by means of a run-off system. Thus, the proposed system does not aim at preventing a financial company's default and at system protection but rather at client protection. As a consequence, it does not focus on increasing capital requirements but rather on providing a fair system which steps in whenever a financial company defaults.

In a first step, we introduce the conceptual framework of our capital market-based financial guaranty system. After describing the basic design of the scheme in detail, the key players and their interactions are identified. Next, we characterize the guaranty bonds and the positions of all relevant key players. A first analysis derives closed-form solutions for the clients' premium and the investors' principal. The latter are discussed with respect to different coverage levels. If capital markets were perfect, one would not need any guaranty system as clients could secure their claims on their own under fair conditions. However, clients may not be aware of the fact that their financial company can default or are not able to secure potential losses on their own.

Subsequently, we analyze the impact of two different actions which might be taken by the financial companies protected by the guaranty vehicle: First, the financial company might purchase guaranty bonds of its own guaranty vehicle. Second, it might purchase guaranty bonds of another financial company whereas both companies' assets have a certain positive correlation. Effects of these actions on major stakeholders, namely clients, regulator, and investors, are measured by means of four groups of measures. We consider the spread received by investors, single shortfall probabilities, expected shortfalls conditional upon default, joint shortfall probabilities, probabilities that the guaranty vehicle cannot cover all clients' claims, i.e., that the coverage of guaranty vehicle is not sufficient, and the expected amount of this insufficient coverage. Results for these different risk figures are generated by means of numerical simulation for a worst case scenario. Main conclusions are that investments in the own guaranty vehicle lever out the purpose of the guaranty system and that investments in a foreign one might lead to the same result depending on the correlation structure. In addition, contagion effects might occur. Finally, we discuss resulting practical implications for the design of such a guaranty system in detail. We find that if the scope of investors can be restricted, the capital market-based financial guaranty systems could be a good solution for clients in respect to the described default problem of a financial institute.

The contribution of our analysis is twofold. One the one hand, a detailed proposal how a capital market-based financial guaranty system can be established is introduced and closed-form solutions of the input parameters are derived. On the other hand, we assess the effectiveness of the proposed system by means of analyzing actions financial companies might take to lever out the guaranty system. By deriving practical implications out of the numerical analysis, new insights for regulators and financial companies into whether a transfer of default risk to capital markets might be feasible are provided.

The remainder of this paper is organized as follows. Section 2 introduces the model framework and the design of the proposed guaranty scheme. Key players' positions are valued under the assumption of perfect, frictionless and complete markets. The influence of financial companies' actions on the guaranty system is analyzed in Section 3. In Section 4 we derive practical implications out of our results. Section 5 concludes.

2 Conceptual Framework

Financial guaranty systems – whether focusing on banks or insurance companies – are subject to numerous discussions. Although capital market solutions are briefly discussed by some authors, a proposal on the actual design does not exist. In this section, we formally present a framework how a guaranty system which requires financial institutions to transfer their default risk to capital markets could look like. The model framework is presented in detail and considers the stakes of all involved players. This system can either be applied to insurance companies or banks whereas the application to banks is subject to some restrictions we discuss in paragraph 2.1.

The model structure is similar to that currently found in capital markets regarding catastrophic or other insurance-linked bonds (see, e.g., Cummins (2008)). Generally speaking, a special purpose vehicle is established in which financial companies pay a premium for default protection and investors provide the corresponding principal. Premium and principal are invested risk-free. If no default takes place, investors receive principal, premium and the risk-free rate earned on them as investment return. In case of default, clients receive an indemnity payment of up to the whole amount in the special purpose vehicle, i.e., principal, premium, and the risk-free rate earned on them.

2.1 Basic Design of a Capital Market-Based Guaranty Scheme

In what follows, we introduce the general framework of our proposed capital market-based financial guaranty scheme by identifying key players and their periodical interactions. Furthermore, the corresponding guaranty bonds are characterized and the positions of the involved parties are analyzed. Finally, the illustration in Figure 16 gives a synopsis of the guaranty system.

Key Players

There are six key players in our model framework, namely the financial company, its clients, the guaranty vehicle, the organizer of the guaranty vehicle, the investors into the guaranty vehicle, and the capital market. The *financial company* can be an insurance company, a bank, a pension fund, or any other financial organization which is relevant in the context of customer protection. Note, however, that a complete securitization of all claims in a bank or pension fund would not be feasible since this would correspond to a simple risk-free investment (and, thus, banks would not be needed any more). Rather, the guaranty system should focus on certain kinds of liabilities (i.e., deposit accounts) and not on the financial company as a whole. This is not necessary but clearly possible with regard to insurance companies. Hence, in what follows, the denotation financial company means either a bank that only has deposit accounts or any kind of insurance company. A refinement with regard to an application of the system to a bank with different liability classes – of which only some are to be protected – is straightforward.

The *clients* are to be protected by the financial guaranty system against the financial company's default. Clients protected by the system can be privates, small and medium-sized enterprises, companies in general, and all other potential investors. In case of default, protected clients receive a compensation payment from the *guaranty vehicle*. In order to establish the financial guaranty system, the *organizer* structures a special purpose vehicle, also called financial guaranty vehicle hereafter, and places the corresponding bonds in the capital market. This organizer may be an independent party, e.g., an investment bank, be part of the financial company itself, or be a special division of the regulatory authority.

The established guaranty vehicle receives a premium payment from the financial company for the default protection and a principal payment from investors. In return, *investors* receive a risk-adequate return for providing this capital. The investors might either be *external investors*, for instance, privates and other financial companies, or *internal investors*, i.e., the financial company itself, in which case the company provides the required capital on its own. The basic idea behind the inclusion of internal investors is that the scope of investors can hardly be limited if the guaranty bonds are publicly traded. As a consequence, the financial company will clearly be able to invest in its own guaranty vehicle.

However, the question remains whether such a self-investment leads to undesirable results. In Section 3, we analyze this aspect in detail. The guaranty vehicle invests premium and principal in the *capital market*. The capital market consists of all other potential market participants offering investments to the guaranty vehicle.

Interactions between Key Players

Since periodical funding of the system is necessary in order to adjust premium and principal to the changing risk structure of the financial company, a one-period setting is employed. The six key players introduced above mainly interact with each other at two points in time: First, when the guaranty vehicle is established and, second, when it is dissolved. Figure 15 shows these interactions. The left column displays interactions which take place at inception of the guaranty system (time $t = 0$), the right column those occurring after a one-year time horizon (time $t = 1$).

In $t = 0$, the organizer establishes the guaranty vehicle. In return, the organizer receives a fee payment. For illustrative purposes, we do not include these fee payments (transaction costs) in our subsequent model framework. However, the implementation is straightforward. Subsequently, the guaranty vehicle issues guaranty bonds which are purchased by external investors and the financial company itself. Simultaneously, the financial company pays a premium to the guaranty vehicle for the default protection and charges this premium payment back to its clients. Next, the guaranty vehicle invests all proceeds, i.e., principal and premium payment, risk-free in the capital market.

In $t = 1$, the guaranty vehicle retrieves principal and premium from the capital market, both compounded with the risk-free rate of interest. In case of default of the financial company between times $t = 0$ and $t = 1$ (dotted line), the guaranty vehicle provides an indemnity payment to the financial company's clients. If no capital remains in the guaranty vehicle after the indemnity payment, the organizer dissolves the guaranty vehicle and the investors go away empty-handed. Otherwise, the remaining capital is distributed to the investors. If the financial company does not default, all capital is transferred to the investors, after what the guaranty vehicle is dissolved.

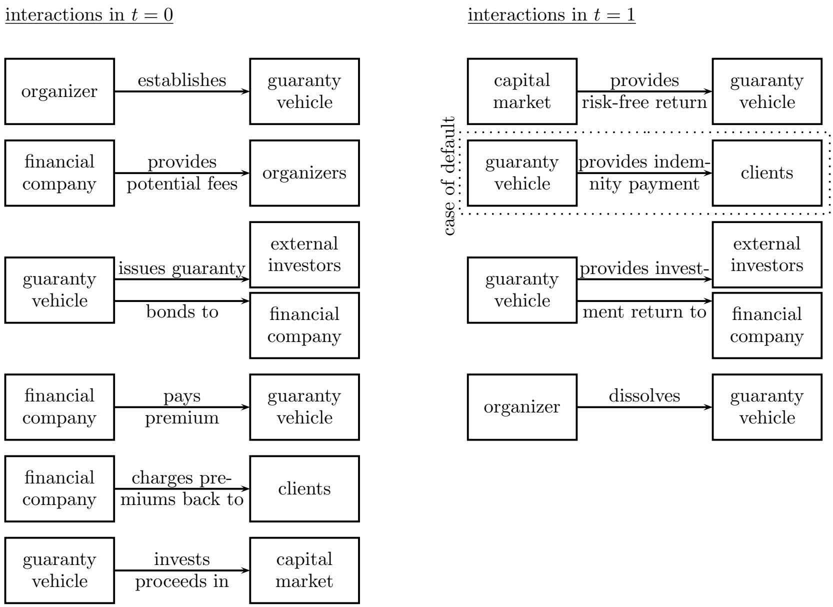


Figure 15: Illustration of the interactions between key players in a market-based financial guaranty system. The left column displays interactions which take place at inception in $t = 0$, the right column those in $t = 1$. The dotted line marks transactions which only take place in case of default of the financial company.

Characterization of Guaranty Bonds

Following the presentation of all key players and interactions relevant in the conceptual framework, a description of the guaranty bonds issue with its underlying parameters is given. At time $t = 0$, the guaranty vehicle issues bonds with a principal of M_0, whereof the amount $M_0^{(\text{ext})}$ is purchased by external investors and $M_0^{(\text{int})}$ by the financial company itself, so that

$$M_0 = M_0^{(\text{ext})} + M_0^{(\text{int})}. \tag{72}$$

For the purpose of our subsequent discussion, it is convenient to express both parts $M_0^{(\text{ext})}$ and $M_0^{(\text{int})}$ relative to the total principal M_0. Hence, we introduce the percentage α, $0 \leq \alpha \leq 1$, of the principal M_0 which is purchased by external investors, whereas the remaining part, $(1-\alpha)$, purchased by the financial company itself, i.e., we have,

$$M_0^{(\text{ext})} = \alpha M_0, \quad \text{and} \quad M_0^{(\text{int})} = (1-\alpha) M_0. \tag{73}$$

There are two general types of investments the guaranty vehicle could issue to investors which are known from the insurance linked securities literature (see, e.g., Cummins (2008)). On the one hand, an issue of the type *principal-at-risk* means that investors can lose their capital invested, i.e., M_0. On the other hand, a *coupon-at-risk* issue is principal protected and, thus, only coupon payments may be lost (corresponding to a money-back-guaranty). In what follows, we assume that investors can lose their total capital invested (principal-at-risk) as a coupon-at-risk framework would require to raise much more capital in order to cover potential compensation payments. In this sense, Cummins (2008, p. 26) argues that "principal-protected tranches have become relatively rare, primarily because they do not provide as much risk capital to the sponsor as a principal-at-risk bond".

At time $t = 0$, the financial company pays a premium P_0 to the guaranty vehicle to cover the spread between the risk-free rate of interest and the interest rate required by investors. In general, one can expect that this premium payment will be charged back to the company's clients. In return, clients receive an indemnity payment in case of the financial company's default. The financial company defaults if its assets A_1 are not sufficient to cover its liabilities L_1 at $t = 1$, i.e., if $A_1 < L_1$. Then, clients' claims occur, corresponding to the difference between assets and liabilities,

$$S_1 = (L_1 - A_1)^+, \tag{74}$$

where $(\cdot)^+$ stands for $\max(\cdot, 0)$. Here, note that liabilities L_1 at $t = 1$ will be stochastic for an insurance company but will generally be deterministic for banks. In what follows, we work with stochastic liabilities so that results can be applied to insurers and banks.

Given default of the financial company, i.e., if $S_1 > 0$, clients receive the compensation (indemnity) payment I_1. This indemnity payment has a certain cap $S_1^{(\beta)} \geq 0$, i.e., a given maximum claims amount which can be covered, as the capital hold by the guaranty vehicle is limited. That is to say, clients receive the lower of their actual claims S_1 and the cap $S_1^{(\beta)}$. Thus, the compensation payment is determined by

$$I_1 = \min\left(S_1^{(\beta)}, S_1\right). \tag{75}$$

The special purpose vehicle invests the principal M_0 as well as the premium payment P_0 at the risk-free rate of interest r_{f}. Subsequently, given that investors receive all capital available after covering the compensation payments to the company's clients, the investors' rate of return r_{s} can be expressed as

$$r_{\mathrm{s}} = \frac{(M_0 + P_0)(1 + r_{\mathrm{f}}) - I_1}{M_0} - 1. \tag{76}$$

Hence, the investors' return equals the principal and the premium payment both compounded with the risk-free rate of interest minus possible indemnity payments, the whole divided by the initial capital investment M_0. Next, the principal M_0 which has to be invested in order to exactly match the maximum claims amount covered $S_1^{(\beta)}$ in all states of the world can be calculated:

$$\begin{aligned} & S_1^{(\beta)} && = (M_0 + P_0)(1 + r_{\mathrm{f}}) \geq 0 \\ \Leftrightarrow \quad & M_0 && = \frac{S_1^{(\beta)}}{1 + r_{\mathrm{f}}} - P_0. \end{aligned} \tag{77}$$

Equation (77) shows that the principal equals the coverage cap $S_1^{(\beta)}$ discounted at the risk-free rate of interest minus the initial premium payment.

Clients', Investors', and the Financial Company's Stakes

We now turn to the stakes of clients, investors, and the financial company in order to show and interpret the formal composition of them. This provides the basis for the derivation of closed-form solutions for principal and premium in a perfect, frictionless, and complete market setting. Figure 16 provides an overview of cash flows at times $t = 0$ and $t = 1$. Based on the latter and given Equations (75)-(77), the aggregate positions of the different players in $t = 0$ can be derived.

At inception, clients pay the premium P_0 and receive the present value of the indemnity payment $\mathrm{PV}[I_1]$. Thus, the aggregate clients' position in $t = 0$ is given by

$$\begin{aligned} W_0^{(c)} &= -P_0 + \mathrm{PV}\left[I_1\right] \\ &= -P_0 + \mathrm{PV}\left[\min\left(S_1^{(\beta)}, S_1\right)\right] \\ &= -P_0 + \mathrm{PV}\left[S_1\right] - \mathrm{PV}\left[\max\left(S_1 - S_1^{(\beta)}, 0\right)\right]. \end{aligned} \tag{78}$$

In the derivation of Equation (78), the present value of the indemnity payment $\mathrm{PV}[I_1]$ can be subdivided, by applying Equation (75), into the present value of the actual claims amount $\mathrm{PV}[S_1]$ from which the present value of the claims amount exceeding the coverage cap is subtracted $\mathrm{PV}\big[\max(S_1 - S_1^{(\beta)}, 0)\big]$. The latter can be interpreted as the present value of the guaranty vehicle's default put option (DPO hereafter) which expresses the marginal or fair premium which would be required for a risk management measure to completely secure all clients' claims S_1.

External investors provide the amount αM_0 to the guaranty vehicle and receive a rate r_{s} on this investment in return (cf. Equations (73) and (76)). Subsequently, we can express the aggregate position of external investors in $t = 0$ as follows:

$$W_0^{(i)} = -\,\alpha M_0 + \mathrm{PV}\left[\alpha M_0\left(1 + r_{\mathrm{s}}\right)\right]. \tag{79}$$

Equation (79) can be decomposed by means of Equations (75) and (77) to

$$
\begin{aligned}
W_0^{(i)} &= \alpha\Bigg(- M_0 + \mathrm{PV}\Big[\underbrace{(P_0 + M_0)\,(1 + r_{\mathrm{f}})}_{S_1^{(\beta)}} - I_1\Big]\Bigg) \\
&= \alpha\Bigg(- M_0 + \mathrm{PV}\left[S_1^{(\beta)}\right] + \mathrm{PV}\left[\max\left(S_1 - S_1^{(\beta)}, 0\right)\right] \\
&\qquad - \mathrm{PV}\left[S_1\right]\Bigg). \qquad (80)
\end{aligned}
$$

Equation (80) shows that the external investors' position consists of four elements (multiplied with the coefficient α): the initial payment of the principal M_0, the present value of the coverage cap $\mathrm{PV}\big[S_1^{(\beta)}\big]$, and the present value of the guaranty vehicle's DPO $\mathrm{PV}\big[\max(S_1 - S_1^{(\beta)}, 0)\big]$, minus the present value of actual claims payments $\mathrm{PV}\left[S_1\right]$.

The financial company's position consists of two different parts. On the one hand, the financial company itself might be an investor and thus have a similar position like the external investors. On the other hand, the financial company pays the premium and charges it back to its clients. Hence, the aggregate position of the financial company in $t = 0$ is

$$
\begin{aligned}
W_0^{(f)} &= (1 - \alpha)\left(-M_0 + \mathrm{PV}\left[M_0\,(1 + r_{\mathrm{s}})\right]\right) - P_0 + P_0 \\
&= (1 - \alpha)\Bigg(- M_0 + \mathrm{PV}\left[S_1^{(\beta)}\right] + \mathrm{PV}\left[\max\left(S_1 - S_1^{(\beta)}, 0\right)\right] \\
&\qquad - \mathrm{PV}\left[S_1\right]\Bigg). \qquad (81)
\end{aligned}
$$

Comparing Equations (80) and (81), we note that the financial company's position only differs from the external investors' one due to the coefficient α as premium payments are supposed to be completely transferred to clients.

Finally, and for the sake of simplification, we do not include any fee payments and hence the organizer's position is nil as the organizer pays in and receives nothing out of the system.

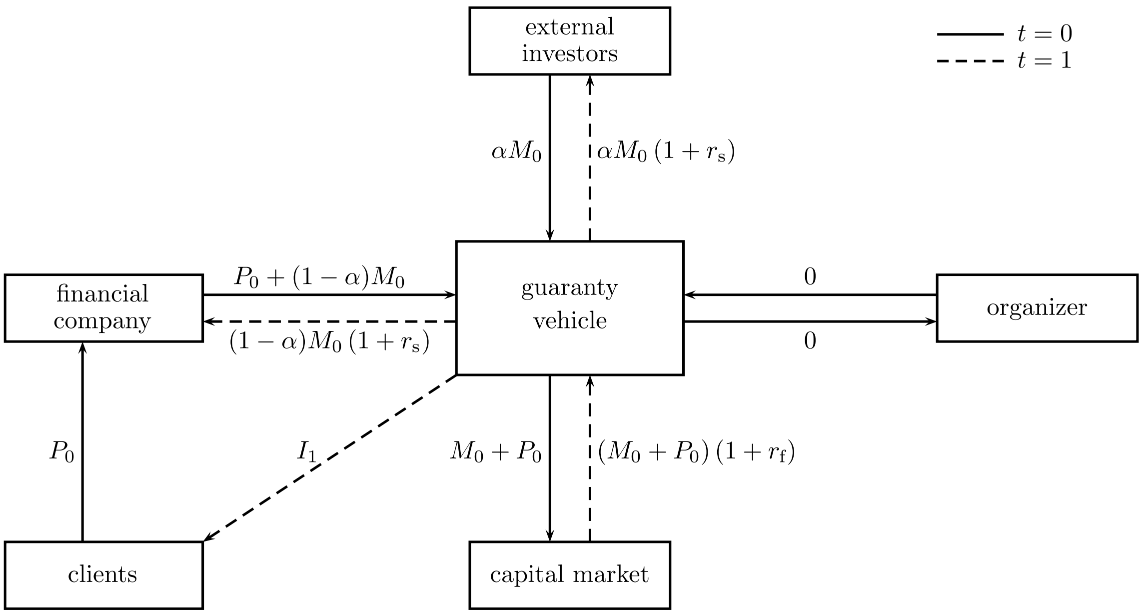


Figure 16: Illustration of cash flows in the basic model framework given no transaction costs. External investors pay the fraction α of the principal M_0 to the guaranty vehicle and earn the interest rate r_s on their investment. The financial company provides the remaining fraction $(1 - \alpha)$ bearing the same interest rate. In addition, it pays the premium P_0 which is charged back to its clients. In case of default of the financial company, clients receive an indemnity payment I_1. The guaranty vehicle invests all proceeds at the risk-free rate of interest r_f in the capital market.

2.2 Fair Valuation of Premium and Principal

In this paragraph, we derive closed-form solutions for the above-defined premium P_0 and principal M_0 for the case in which the guaranty vehicle is solely funded by external investors, i.e., $\alpha = 1$. An analysis of these values with respect to different coverage levels $S_1^{(\beta)}$ will provide an indication on the size of premium and principal and lays the basis for further analyses in different market settings.

In a perfect, frictionless, and complete market setting, the net present value of each investment should equal zero. Thus, the position of each market participant in our framework needs to be zero, i.e.,

$$W_0^{(c)} = W_0^{(i)} = W_0^{(f)} = 0. \tag{82}$$

Clients' Premium

Subsequently, we can calculate the required premium P_0 by combining Equations (78) and (82)

$$P_0 = \mathrm{PV}\left[I_1\right] = \mathrm{PV}\left[S_1\right] - \mathrm{PV}\left[\max\left(S_1 - S_1^{(\beta)}, 0\right)\right]. \tag{83}$$

Equation (83) shows that the premium equals the present value of the claims minus the present value of the guaranty vehicle's DPO. In order to derive a closed-form solution, we assume that assets A_t and liabilities L_t of the financial company follow a geometric Brownian motion with constant drifts, μ_A and μ_L, and constant volatilities, σ_A and σ_L. Thereby, we need to assume that the financial company does not invest in its own guaranty vehicle, i.e., $\alpha = 1$, since the derivation of closed-form solutions would not be possible otherwise.[61] We analyze this aspect in more detail in Section 3. Thus, asset and liability process are described by

$$\mathrm{d}A_t = \mu_A A_t \mathrm{d}t + \sigma_A A_t \mathrm{d}W_{A,t}^{\mathbb{P}}, \tag{84}$$
$$\mathrm{d}L_t = \mu_L L_t \mathrm{d}t + \sigma_L L_t \mathrm{d}W_{L,t}^{\mathbb{P}}, \tag{85}$$

[61] Later on, we will see that the assumption of a geometric Brownian motion does not hold if the financial company invests in its own guaranty vehicle.

where $W^{\mathbb{P}}_{A,t}$ and $W^{\mathbb{P}}_{L,t}$ are correlated standard $\mathbb{P}$-Brownian motions with correlation coefficient $\rho_{A,L}$, defined by $\mathrm{d}W_t^{(A)}\mathrm{d}W_t^{(L)} = \rho_{A,L}\mathrm{d}t$.

Under the risk-neutral martingale measure $\mathbb{Q}$, the drift changes to the risk-free rate of interest r_{f}. The solutions of the stochastic differential equations, Equation (84) and (85), in $t = 1$ under the risk-neutral measure are given by (see, e.g., Björk (2004))

$$A_1 = A_0 \exp\left[\left(r_{\mathrm{f}} - \sigma_A^2/2\right) + \sigma_A W^{\mathbb{Q}}_{A,1}\right], \tag{86}$$

$$L_1 = L_0 \exp\left[\left(r_{\mathrm{f}} - \sigma_L^2/2\right) + \sigma_L W^{\mathbb{Q}}_{L,1}\right]. \tag{87}$$

Next, considering the first part of Equation (83) and Equation (74), the present value of the claims $\mathrm{PV}\left[S_1\right]$ can be regarded as the value of an option to exchange one asset for another. Thus, we can derive a closed-form solution by means of the formulas provided by Fischer (1978) and Margrabe (1978). This derivation yields

$$\begin{aligned} \mathrm{PV}\left[S_1\right] &= E^{\mathbb{Q}}\left[\exp\left(-r_{\mathrm{f}}\right)\left(L_1 - A_1\right)^+\right] \\ &= L_0 N\left(d_1^{(a)}\right) - A_0 N\left(d_2^{(a)}\right), \end{aligned} \tag{88}$$

where $N(\cdot)$ denotes the value of the cumulative normal distribution and

$$d_1^{(a)} = \frac{\ln\left(L_0/A_0\right) + \hat{\sigma}^2/2}{\hat{\sigma}}, \qquad d_2^{(a)} = d_1^{(a)} - \hat{\sigma},$$
$$\hat{\sigma}^2 = \sigma_L^2 + \sigma_A^2 - 2\rho_{A,L}\sigma_L\sigma_A.$$

Next, we turn to the second part of Equation (83), namely the value of the guaranty vehicle's DPO which is formally given by $\mathrm{PV}\big[\max(S_1 - S_1^{(\beta)}, 0)\big]$. Given that the maximum claims amount covered is always positive $S_1^{(\beta)} > 0$ (see Equation (77)), the DPO value can be rewritten as follows

$$\begin{aligned} \mathrm{PV}\left[\max\left(S_1 - S_1^{(\beta)}, 0\right)\right] &= \mathrm{PV}\left[\max\left(\left(L_1 - A_1\right)^+ - S_1^{(\beta)}, 0\right)\right] \\ &= \mathrm{PV}\left[\max\left(L_1 - A_1 - S_1^{(\beta)}, 0\right)\right]. \end{aligned} \tag{89}$$

This equation cannot be solved explicitly without closer definition of the maximum coverage $S_1^{(\beta)}$. We express the coverage cap $S_1^{(\beta)}$ relative to liabilities in $t = 1$ which allows us to derive a closed-form solution,

$$S_1^{(\beta)} = \beta L_1, \tag{90}$$

where $0 \leq \beta \leq 1$ is the coverage parameter. Then, Equation (89) can be solved by applying again the formulas provided by Fischer (1978) and Margrabe (1978)

$$\begin{aligned} \mathrm{PV}\left[\max\left(L_1 - A_1 - S_1^{(\beta)}, 0\right)\right] &= \mathrm{PV}\left[\max\left((1-\beta)L_1 - A_1, 0\right)\right] \\ &= (1-\beta)L_0 N\left(d_1^{(b)}\right) \\ &\quad - A_0 N\left(d_2^{(b)}\right), \end{aligned} \tag{91}$$

with

$$\begin{aligned} d_1^{(b)} &= \frac{\ln\left((1-\beta)L_0/A_0\right) + \hat{\sigma}^2/2}{\hat{\sigma}}, \qquad d_2^{(b)} = d_1^{(b)} - \hat{\sigma}, \\ \hat{\sigma}^2 &= \sigma_L^2 + \sigma_A^2 - 2\rho_{A,L}\sigma_L\sigma_A. \end{aligned}$$

Finally, combining Equations (88) and (91), we can express the premium P_0 as

$$\begin{aligned} P_0 = &L_0\left(N\left(d_1^{(a)}\right) - (1-\beta)N\left(d_1^{(b)}\right)\right) \\ &+ A_0\left(N\left(d_2^{(b)}\right) - N\left(d_2^{(a)}\right)\right). \end{aligned} \tag{92}$$

Investors' Principal

Combining Equation (82) and Equation (80) or (81), the corresponding principal M_0 can be calculated with

$$M_0 = \mathrm{PV}\left[S_1^{(\beta)}\right] + \mathrm{PV}\left[\max\left(S_1 - S_1^{(\beta)}, 0\right)\right] - \mathrm{PV}\left[S_1\right]. \tag{93}$$

That is to say, the principal equals the sum of the present value of the coverage cap and the present value of the guaranty vehicle's DPO, minus the present value of the actual claims. Given Equations (83) and (90) the principal can be expressed as follows

$$\begin{aligned} M_0 &= \mathrm{PV}\left[S_1^{(\beta)}\right] - \mathrm{PV}\left[S_1\right] + \mathrm{PV}\left[\max\left(S_1 - S_1^{(\beta)}, 0\right)\right] \\ &= \mathrm{PV}\left[\beta L_1\right] - P_0 \\ &= \beta L_0 - P_0. \end{aligned} \tag{94}$$

Premium and Principal for Different Coverage Levels

The objective of this paragraph is to provide an indication of the magnitude of the premium P_0, the principal M_0, and their constituents. In addition, we show how and to what extent the required coverage ratio β, introduced in Equation (90), influences these elements for different asset volatilities σ_A.

For this illustration, we fix model parameters, unless stated otherwise, as follows. We consider a financial company with initial assets $A_0 = 100$ and liabilities $L_0 = 80$, whereas both quantities are expressed in million currency units. The asset volatility takes values of $\sigma_A \in \{0.05, 0.10, 0.15\}$, while the volatility of liabilities is fixed at $\sigma_L = 0.05$. We set the correlation between assets and liabilities equal to $\rho_{A,L} = 0.1$ and the risk-free rate of return to $r_{\mathrm{f}} = 0.02$.

Table 25 shows the present value of actual claims $\mathrm{PV}\left[S_1\right]$, the present value of the guaranty vehicle's DPO $\mathrm{PV}\big[(S_1 - S_1^{(\beta)})^+\big]$, the present value of the coverage cap $\mathrm{PV}\big[S_1^{(\beta)}\big]$, the principal M_0, and the premium P_0 for different coverage ratios β and the three different asset volatilities. In addition, we calculate the ratio P_0/L_0 which expresses the premium relative to liabilities in $t = 0$.

Table 25 illustrates that the value of the guaranty vehicle's DPO, $\mathrm{PV}\big[(S_1 - S_1^{(\beta)})^+\big]$, converges relatively fast to zero for increasing coverage ratios β. Naturally, the present value of claims and the default put option value are, ceteris paribus, higher the higher the asset volatility. The last column, which expresses the premium required relative to the initial liabilities P_0/L_0, clarifies that the premium which would have to be paid by the financial company – and thus by its clients – appears to be

β	$\mathrm{PV}[S_1]$	$\mathrm{PV}\big[(S_1 - S_1^{(\beta)})^+\big]$	$\mathrm{PV}\big[S_1^{(\beta)}\big]$	M_0	P_0	P_0/L_0
$\sigma_A = 0.05$						
0.00	0.00	0.000690	0	0.000	0.000	0.000%
0.05	0.00	0.000028	4	3.999	0.001	0.001%
0.10	0.00	0.000001	8	7.999	0.001	0.001%
0.15	0.00	0.000000	12	11.999	0.001	0.001%
0.20	0.00	0.000000	16	15.999	0.001	0.001%
0.25	0.00	0.000000	20	19.999	0.001	0.001%
0.30	0.00	0.000000	24	23.999	0.001	0.001%
0.40	0.00	0.000000	32	31.999	0.001	0.001%
0.50	0.00	0.000000	40	39.999	0.001	0.001%
$\sigma_A = 0.10$						
0.00	0.07	0.065332	0	0.000	0.000	0.000%
0.05	0.07	0.015557	4	3.950	0.050	0.062%
0.10	0.07	0.002772	8	7.937	0.063	0.078%
0.15	0.07	0.000351	12	11.935	0.065	0.081%
0.20	0.07	0.000030	16	15.935	0.065	0.082%
0.25	0.07	0.000002	20	19.935	0.065	0.082%
0.30	0.07	0.000000	24	23.935	0.065	0.082%
0.40	0.07	0.000000	32	31.935	0.065	0.082%
0.50	0.07	0.000000	40	39.935	0.065	0.082%
$\sigma_A = 0.15$						
0.00	0.44	0.443333	0	0.000	0.000	0.000%
0.05	0.44	0.195086	4	3.752	0.248	0.310%
0.10	0.44	0.074499	8	7.631	0.369	0.461%
0.15	0.44	0.024064	12	11.581	0.419	0.524%
0.20	0.44	0.006374	16	15.563	0.437	0.546%
0.25	0.44	0.001333	20	19.558	0.442	0.553%
0.30	0.44	0.000210	24	23.557	0.443	0.554%
0.40	0.44	0.000002	32	31.557	0.443	0.554%
0.50	0.44	0.000000	40	39.557	0.443	0.554%

Table 25: Illustration of the premium P_0, its two constituents $\mathrm{PV}[S_1]$ and $\mathrm{PV}\big[(S_1 - S_1^{(\beta)})^+\big]$, the present value of the coverage cap $\mathrm{PV}\big[S_1^{(\beta)}\big]$, and the principal M_0 for different coverage ratios β and three different asset volatilities σ_A. Values calculated are in million currency units.

relatively low for all reported volatilities. For comparison, compulsory charges in existing insurance guaranty schemes are on average around 1% of the premiums (see, e.g., Schmeiser and Wagner (2010, Table 1)).

For an insurance company, an asset volatility of $\sigma_A = 0.05$ is reasonable. Table 25 shows that, given this volatility, the present value of the guaranty vehicle's DPO, $\mathrm{PV}\big[(S_1 - S_1^{(\beta)})^+\big]$, is very low. As a consequence, a coverage ratio of $\beta = 0.05$ could already lead to a situation which is close to a full securitization. This implies that the principal M_0 which needs to be provided by the capital market will be less than 5% of the financial company's liabilities.

3 Financial Companies' Influence on the Guaranty System

The analyses presented in the previous section provide closed-form solutions for the premium P_0 and the corresponding principal M_0. Next, we turn to certain problems which might arise when implementing such a financial guaranty system in practice. In particular, we intend to analyze whether the financial companies themselves might be able to influence the effectiveness of the guaranty system by taking certain investment actions. Hereby, we focus on a worst case scenario in order to illustrate our results.

3.1 Financial Company Invests in Own Guaranty Vehicle

In Paragraph 2.2, we assume that the financial company does not invest in the guaranty vehicle covering its own defaults by setting $\alpha = 1$. Thus, the question arises what happens if the financial company invests in its own guaranty vehicle, i.e., if $(1-\alpha) > 0$. For the financial company, this investment will provide stable returns as long as its financial situation remains stable, i.e., if no shortfall occurs. However, as soon as distress arises and the guaranty vehicle is to secure clients' claims, the company's asset value will further deteriorate as the guaranty bonds will have a large loss in value.

In the extreme case, the financial company would provide the complete capital to the guaranty vehicle, i.e., $\alpha = 0$. Then, no additional capital would be provided by the guaranty vehicle. On the other hand, the asset volatility of the financial company's portfolio would decrease since the guaranty vehicle invests in the risk-free rate of interest and, thus, more capital of the financial company would ceteris paribus be invested risk-free. This direction of acting and its effects are then comparable to increasing the amount of assets held in risk-free investments, corresponding rather, e.g., to a reaction on asset allocation requirements.

To clarify these points, we formalize this discussion. The general assumption that the financial company's assets follow a geometric Brownian motion remains. However, assets invested in the company's own special purpose vehicle, i.e., for $0 \leq \alpha < 1$, do not follow a geometric Brownian motion and change the portfolio's asset process due to the dependence on the financial company's portfolio. Subsequently, and in general, assets in $t = 1$ are expressed by

$$A_1^* = \left(1 - \frac{(1-\alpha)\, M_0}{A_0}\right) A_1 + (1-\alpha)\left(S_1^{(\beta)} - I_1\right). \tag{95}$$

In the case where no investments in the own guaranty vehicle are made (case with $\alpha = 1$), Equation (95) reduces to the asset value based on the geometric Brownian motion, i.e., $A_1^* = A_1$. For $0 \leq \alpha < 1$, the fraction $(1-\alpha)\, M_0/A_0$ of all assets will be invested in the own guaranty vehicle and, thus, the fraction $(1-\alpha)$ of the guaranty vehicle's payoff in $t = 1$, $\left(S_1^{(\beta)} - I_1\right)$, is attributable to the financial company's assets.

As long as the financial company does not default ($S_1 = 0$), the indemnity payment I_1 will be zero. Simultaneously, $S_1^{(\beta)}$ is positive, given, and provides a return above the risk-free rate relative to the invested capital $(1-\alpha)M_0$ (cf. Equation (77)). As a consequence, the variance of the asset value in $t = 1$ will decrease (given no default) since

$$\sigma^2\left(A_1^*\right) = \left(1 - \frac{(1-\alpha)\, M_0}{A_0}\right)^2 \sigma_A^2, \quad \text{if } I_1 = 0, \tag{96}$$

which is smaller than σ_A^2 for all $0 \leq \alpha < 1$.

However, Equation (95) shows that losses faced by clients will become more severe in the case of default since indemnity payments will take place (i.e., $I_1 > 0$) as soon as $A_1^* < L_1$ which will additionally lower the asset value A_1^*.

3.2 Financial Company Invests in Other Guaranty Vehicle

Similarly, the question arises whether contagion effects might occur if one financial company invests in the guaranty vehicle of another financial company. Here, results will depend on the correlation structure. Generally, one can expect that the higher the correlation between assets and liabilities of the two different companies, the closer results will get to investments in the own guaranty vehicle.

In order to analyze these aspects in more detail, a second financial company is introduced. Both companies, denoted by $i = 1$ and $i = 2$ and the respective variables with superscripts (i), are supposed to be identical, meaning that their assets and liabilities follow the same process, i.e.,

$$A_0^{(1)} = A_0^{(2)}, \sigma_A^{(1)} = \sigma_A^{(1)}, \mu_A^{(1)} = \mu_A^{(2)},$$
$$L_0^{(1)} = L_0^{(2)}, \sigma_L^{(1)} = \sigma_L^{(2)}, \mu_L^{(1)} = \mu_L^{(2)},$$
$$\rho_{A^{(1)},L^{(1)}} = \rho_{A^{(2)},L^{(2)}}.$$

Assuming that both companies do not invest in their own guaranty bonds (i.e., $\alpha = 1$), assets in $t = 1$ can be described by

$$A_1^{(1)*} = \left(1 - \frac{\gamma^{(1)} M_0^{(1)}}{A_0^{(1)}}\right) A_1^{(1)} + \gamma^{(1)} \left(S_1^{(\beta)} - I_1^{(2)}\right), \quad (97)$$
$$A_1^{(2)*} = \left(1 - \frac{\gamma^{(2)} M_0^{(2)}}{A_0^{(2)}}\right) A_1^{(2)} + \gamma^{(2)} \left(S_1^{(\beta)} - I_1^{(1)}\right), \quad (98)$$

whereas the parameter $0 \leq \gamma^{(i)} \leq 1$, $i \in \{1, 2\}$,defines which percentage of the other company's guaranty bonds is purchased. Similar to Equation (95), $\gamma^{(i)} M_0^{(i)} / A_0^{(i)}$ defines the proportion of assets that is invested in the other company's guaranty vehicle.

One important observation with regard to Equations (97) and (98) is that the default of one financial company will have a negative effect on the asset value of the second one. This points out that contagion effects can occur.

To clarify the previously mentioned point that a high correlation between both companies' assets and liabilities will yield similar results like seen in the case of one company investing in its own guaranty vehicle, we consider the following: Both financial companies purchase the same stake of the other's financial guaranty bonds. That is to say, company 1 purchases a fraction $\gamma^{(1)} > 0$ of company 2's guaranty bonds and company 2 purchases $\gamma^{(1)} = \gamma^{(2)} > 0$ of company 1's guaranty bonds. Then, if the correlation between assets (and liabilities respectively) of company 1 and 2 is perfectly positive, i.e., $\rho_{A^{(1)},A^{(2)}} = 1$, results will be the same as if the companies invested in their own guaranty vehicles, and the formulas from the previous paragraph hold with $(1 - \alpha) = \gamma$.

Our discussion shows the form and direction of the influence both kinds of action have. However, to illustrate size and relevance, we provide different numerical examples. To do so, we analyze the two different actions which might be taken by the financial companies described above:

1. The financial company purchases guaranty bonds of its own guaranty vehicle.
2. The financial company purchases guaranty bonds of another financial company whereas both companies' assets have a certain positive correlation.

3.3 Stakeholders and Relevant Risk Figures

In order to measure the effects of the previously described actions, we determine relevant stakeholders and define risk figures describing how much the individual stakeholders are actually affected.

	Figure	Stakeholders	Formula
1	Expected spread received by investors	Investors	$E[r_\mathrm{s} - r_\mathrm{f}]$
2	Single shortfall (default) probabilities	Investors, Regulator	$P(S_1 > 0)$
	Expected shortfall conditional upon default	Regulator	$E[S_1 \,\vert S_1 > 0\,]$
3	Joint shortfall probabilities	Regulator	$P(S_1^{(1)} > 0 \cap S_1^{(2)} > 0)$
4	Probability that the guaranty vehicle cannot cover all clients' claims	Clients, Regulator	$P(S_1^{(\beta)} < S_1)$
	Expected amount of this insufficient coverage	Clients, Regulator	$E[(S_1 - S_1^{(\beta)}) \mid S_1^{(\beta)} < S_1]$

Table 26: Summary of the four groups of (risk) measures relevant to stakeholders (investors, clients, regulator) which might be affected by the financial company's actions. The first column describes the figure, the second column the stakeholders for which the measure might be relevant, and the last column displays the corresponding formulas.

Based on Figure 16, three major stakeholders, which could be affected by the two specific actions which the financial company can take, are identified. First, the *clients* who seek default protection who are interested in the safety of their investment. As mentioned, the possible investment actions might influence default probabilities and the extent of an actual default of the financial company. Assuming that clients are mainly interested in losses they actually face, they will be interested in whether the guaranty vehicle cannot cover all their claims given default $P(S_1^{(\beta)} < S_1)$ and the expected amount of this insufficient coverage $E[(S_1 - S_1^{(\beta)}) \mid S_1^{(\beta)} < S_1]$.

The second group of stakeholders affected are the external and internal *investors* who search for profitable investments. This group of stakeholders will not be in focus but we include them for the sake of completeness. To them, the expected spread they receive on their investment $E[r_{\mathrm{s}} - r_{\mathrm{f}}]$ and, additionally, shortfall probabilities $P(S_1 > 0)$ will be relevant.

Finally, the *regulator* whose mission it is to enhance financial stability and ensure customer protection is another major stakeholder. In the context of financial stability, shortfall probabilities $P(S_1 > 0)$, expected shortfalls $E[S_1 \,|S_1 > 0]$, and, in particular, joint shortfall probabilities $P(S_1^{(1)} > 0 \cap S_1^{(2)} > 0)$ are key risk figures. With regard to customer protection, the same figures relevant to clients appear to be important, i.e., the probability that the guaranty vehicle cannot cover all claims and the corresponding expected amount.

Summarizing, we focus on four groups of (risk) measures which are recapitulated in Table 26.

3.4 Numerical Illustration

In this paragraph, an analysis of the impact of the two different actions, which might be taken by the financial company protected by the guaranty vehicle, on the four general groups of measures defined in Table 26 is carried out.

For the numerical analysis, we fix the input parameters, unless otherwise stated, as provided in Table 27.[62] Recall that assets A_0 and liabilities L_0 are expressed in million currency units. Our calibration corresponds to a worst case scenario, i.e., the asset volatility σ_A and, thus, the coverage parameter β are higher than regular empirical data.[63] Numerical results are derived by means of Monte Carlo simulation using 10 000 000 paths. Each path solves iteratively for the asset value A_1^* along Equation (95) and Equations (97) and (98) respectively.

Parameter	Denotation	Value
Initial assets	A_0	100
Asset drift	μ_A	0.05
Volatility of assets	σ_A	0.15
Initial liabilities	L_0	80
Liability drift	μ_L	0.03
Volatility of liabilities	σ_L	0.05
Correlation between assets and liabilities	$\rho_{A,L}$	0.1
Risk-free rate of interest	r_{f}	0.02
Coverage parameter	β	0.3
Percentage purchased by external investors	α	[0;1]

Table 27: Parameter combinations applied in the numerical analysis.

[62] The analysis can be carried out using other parameter combinations. However, the effects and results are similar and yield identical practical implications.

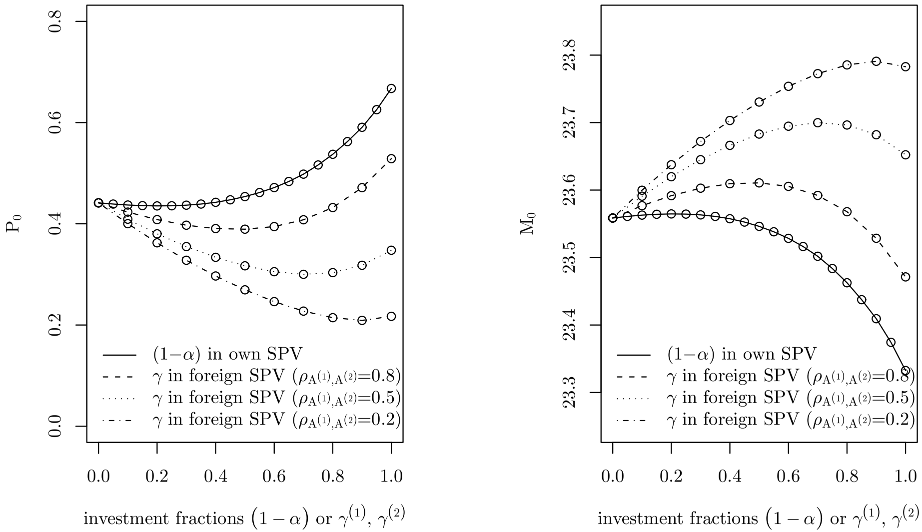
[63] We work with this worst case scenario for illustrative purposes.

Premium and Principal

In order to base our simulation results on appropriate values of premium and principal, we calculate the fair values of premium and principal for all investment fractions $(1-\alpha)$ and $\gamma^{(i)}$ regarded given the parameter combinations provided in Table 27. To do so, we numerically solve for the asset value A_1^* and for the corresponding premium P_0 and principal M_0 under the risk-neutral measure $\mathbb{Q}$ along Equations (97), (83) and (77) (for each α and $\gamma^{(i)}$ regarded). Figure 17 provides the values obtained: panel (a) shows the calculated fair premiums, panel (b) the fair principals corresponding to the different investment situations.

As seen in Table 25 (record with $\sigma_A = 0.15$ and $\beta = 0.3$), if $\alpha = 1$ and $\gamma^{(i)} = 0$, this procedure leads to a premium of $P_0 = 0.44$ and a principal of $M_0 = 23.56$. For all other α and $\gamma^{(i)}$, the premium P_0 is highest if the financial company invests in its own guaranty bonds and lowest if the financial company invests in foreign guaranty bonds with low correlation ($\rho_{A^{(1)},A^{(2)}} = 0.2$). The reverse is true for the principal since the sum of premium and principal is constant in all cases, $P_0 + M_0 = S_1^{(\beta)} = 24$.

Figure 17a shows that the premium either decreases or increases for increasing $(1-\alpha)$ and $\gamma^{(i)}$. For instance, if $\rho_{A^{(1)},A^{(2)}} = 0.5$, the premium first decreases and then increases again. This is due to two opposing effects. On the one hand, shortfall probabilities decrease with increasing $(1-\alpha)$ and $\gamma^{(i)}$ which lowers the premium P_0 – recall the already discussed decrease in asset volatility $\sigma^2(A_1^*)$ due to the higher amount that is actually invested risk-free (see Equation (96)). On the other hand, occurring defaults will yield larger losses with increasing $(1-\alpha)$ and can yield larger losses with increasing $\gamma^{(i)}$. This raises the premium P_0. Here, consider again Equations (95) and (97). As soon as a financial company defaults, indemnity payments will take place (i.e., $I_1 > 0$) which will lower the asset value A_1^* or $A_1^{(i)*}$ of the financial company investing in the guaranty bonds. The subsequent analysis of the different shortfall measures will clarify these points.



(a) Fair premiums

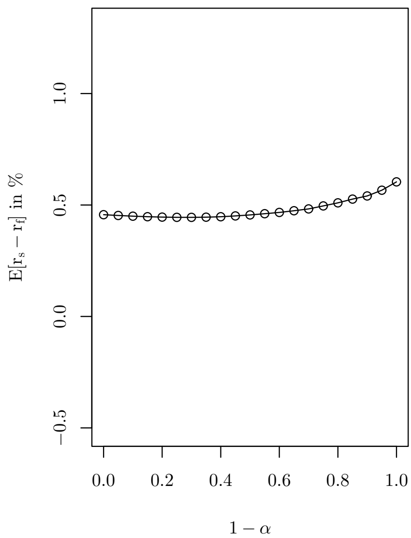
(b) Fair principals

Figure 17: Premium P_0 and principal M_0 if the financial company purchases the fraction $(1-\alpha)$ of its own financial guaranty bonds and if financial company 1 purchases fraction $\gamma^{(1)}$ of company 2's financial guaranty bonds and financial company 2 purchases fraction $\gamma^{(2)}$ of company 1's financial guaranty bonds, with $\gamma^{(1)} = \gamma^{(2)}$.

Financial Company Invests in Own Guaranty Vehicle

Now, we turn to the measures introduced in Table 26 and move to the real-world measure $\mathbb{P}$. First, we focus on the case in which the financial company invests in its own guaranty vehicle. Figure 18 shows the expected spread over the risk-free rate of return $E[r_\mathrm{s} - r_\mathrm{f}]$ investors receive for different values of α.

The expected spread appears to be relatively stable for all $(1-\alpha)$. For increasing values of $(1-\alpha)$, the expected spread marginally increases. Results directly correspond to the different underlying premiums reported in Figure 17a.



(a) Expected spread over risk-free rate

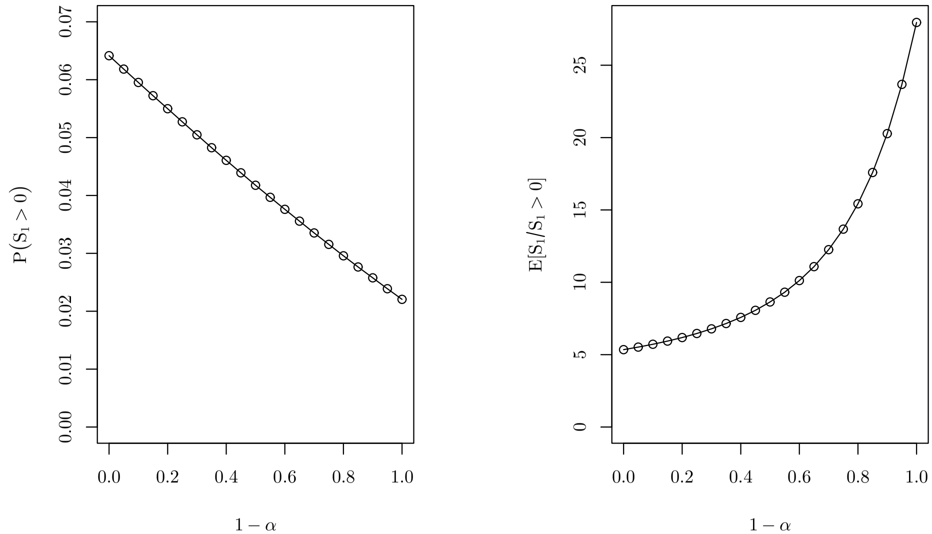
Figure 18: Expected spread over the risk-free rate $E[r_\mathrm{s} - r_\mathrm{f}]$ (in %) if the financial company purchases the fraction $(1 - \alpha)$ of its own financial guaranty bonds.

Figure 19 displays different shortfall measures for varying α. Panel (a) displays the probability that the financial company defaults $P(S_1 > 0)$ and panel (b) the expected loss conditional upon default $E[S_1 \,|S_1 > 0\,]$. Panel (c) shows the probability that the guaranty vehicle cannot cover the complete loss faced by clients $P(S_1^{(\beta)} < S_1)$ and, corresponding to these probabilities, panel (d) displays the expected amount the actual loss exceeds the maximum coverage conditional upon exceeding this maximum coverage $E[(S_1 - S_1^{(\beta)}) \,|\, S_1^{(\beta)} < S_1]$. Note that values calculated in panel (d) correspond to the probabilities reported in panel (c). As probabilities for low $(1-\alpha)$ are comparably low, we observe slight approximation errors for low $(1-\alpha)$.

The default probability decreases if the fraction of guaranty bonds purchased by the financial company itself $(1-\alpha)$ increases. This can be explained by the reduced asset volatility as long as the company does not default – recall that the higher the investment in own guaranty bonds, the higher the portion of the financial company's assets which is actually invested risk-free. However, the expected loss in case of default of the financial company increases extensively with higher participation of the financial company $(1-\alpha)$. This is due to indemnity payments to clients that will take place if the financial company defaults which will additionally lower the asset value A_1^* of the already bankrupt financial company. Thus, though a purchase of financial guaranty bonds by the financial company itself reduces the probability of default, occurring defaults will become more severe.

In line with these results, the probability that the loss exceeds the maximum coverage increases with higher participation of the financial company. The expected amount which cannot be covered by the guaranty vehicle slightly increases for increasing $(1-\alpha)$.

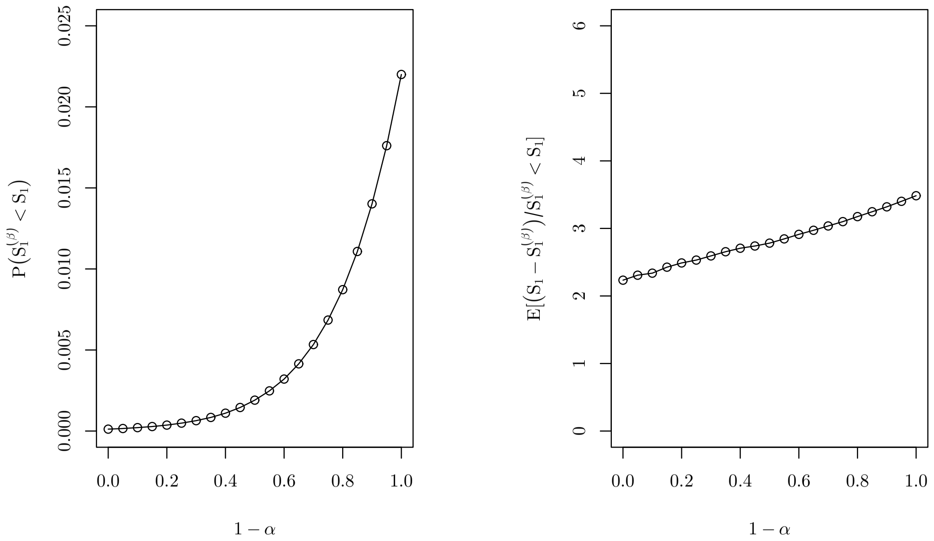
To conclude, a purchase of own guaranty bonds leads to a reduction in probabilities of default, an increase in probabilities that the guaranty vehicle cannot cover all claims, and occurring defaults yield larger losses.



(a) Shortfall probability

(b) Expected shortfall given default

Figure 19: Effects on shortfall measures when the financial company purchases the fraction $(1-\alpha)$ of its own guaranty bonds. Panel (a) shows the shortfall probability $P(S_1 > 0)$ and panel (b) the expected shortfall given default $E[S_1 \mid S_1 > 0]$. Panel (c) displays the probability that the guaranty vehicle cannot cover all losses $P(S_1^{(\beta)} < S_1)$ and panel (d) the expected amount of insufficient coverage $E[(S_1 - S_1^{(\beta)}) \mid S_1^{(\beta)} < S_1]$. (continued on next page)



(c) Probability of insufficient coverage

(d) Expected amount of insufficient coverage

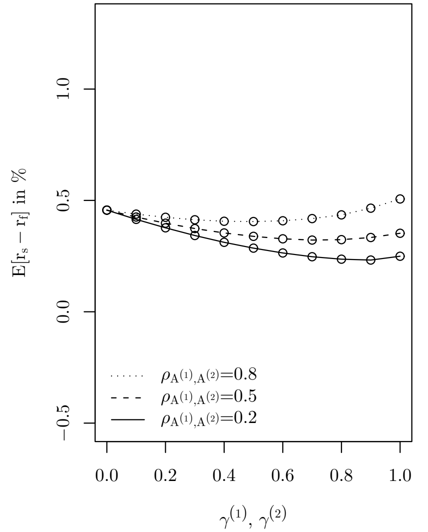
Figure 19: Effects on shortfall measures when the financial company purchases the fraction $(1-\alpha)$ of its own guaranty bonds. Panel (a) shows the shortfall probability $P(S_1 > 0)$ and panel (b) the expected shortfall given default $E[S_1 \mid S_1 > 0]$. Panel (c) displays the probability that the guaranty vehicle cannot cover all losses $P(S_1^{(\beta)} < S_1)$ and panel (d) the expected amount of insufficient coverage $E[(S_1 - S_1^{(\beta)}) \mid S_1^{(\beta)} < S_1]$. (cont.)

Financial Company Invests in other Guaranty Vehicle

Next, we allow both financial companies to purchase a stake of the other's financial guaranty bonds. That is to say, company 1 purchases a fraction $\gamma^{(1)}$ of the second company's guaranty bonds, and company 2 purchases a part $\gamma^{(2)}$ of company 1's guaranty bonds. For our numerical analysis, we always assume that $\gamma^{(1)} = \gamma^{(2)}$. We focus on three different correlations between the assets of the two companies, namely $\rho_{A^{(1)},A^{(2)}} \in \{0.20, 0.50, 0.80\}$, and assume the same correlation coefficients regarding liabilities, i.e., $\rho_{A^{(1)},A^{(2)}} = \rho_{L^{(1)},L^{(2)}}$.

Figure 20 displays the expected spread over the risk-free rate $E[r_{\mathrm{s}} - r_{\mathrm{f}}]$ investors receive for increasing $\gamma^{(i)}$, $i = 1, 2$. Note that we always show numbers for one of the two financial companies. As the companies are homogeneous, results for both are the same. The expected spread is always lowest with a low correlation coefficient (curve for $\rho_{A^{(1)},A^{(2)}} = 0.2$) and highest with a high one (curve for $\rho_{A^{(1)},A^{(2)}} = 0.8$). The calculated spreads can directly be related to the different underlying premiums reported in Figure 17a.

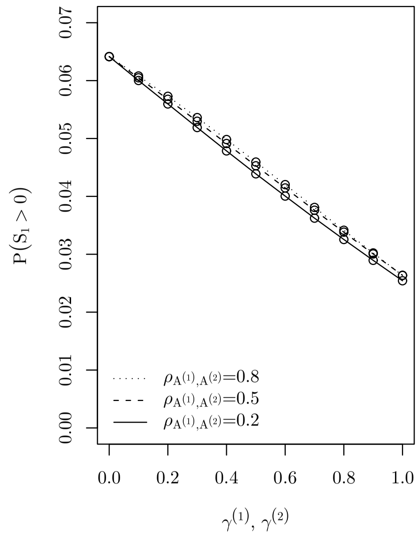
In Figure 21a we plot shortfall probabilities $P(S_1 > 0)$ and in Figure 21b the expected shortfall given default $E[S_1 \mid S_1 > 0]$ of one of the two financial companies for increasing $\gamma^{(i)}$. Shortfall probabilities decrease for increasing $\gamma^{(i)}$ whereas the decline is highest with low correlation ($\rho_{A^{(1)},A^{(2)}} = 0.2$). Expected shortfalls given default are highest with a high correlation and lowest with a low one. With $\rho_{A^{(1)},A^{(2)}} = 0.8$ and $\rho_{A^{(1)},A^{(2)}} = 0.5$, expected shortfalls increase for increasing $\gamma^{(i)}$, with $\rho_{A^{(1)},A^{(2)}} = 0.2$ the expected shortfall first decreases slightly and then increases again. As with regard to Figure 19, these results can generally be explained by a decreasing volatility of the asset portfolio and indemnity payments that will lower the asset value ($A_1^{(i)*}$) of the financial company investing in these guaranty bonds of the bankrupt one. The decrease in expected shortfalls with a low correlation ($\rho_{A^{(1)},A^{(2)}} = 0.2$) can be explained by diversification effects.



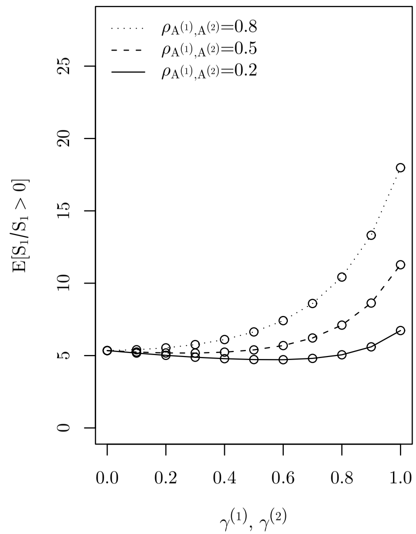
(a) Expected spread over risk-free rate

Figure 20: Expected spread over the risk-free rate $E[r_s - r_f]$ (in %) received by investors of financial company 1 (or 2) if financial company 1 purchases fraction $\gamma^{(1)}$ of company 2's financial guaranty bonds and financial company 2 purchases fraction $\gamma^{(2)}$ of company 1's financial guaranty bonds, with $\gamma^{(1)} = \gamma^{(2)}$.

Table 28 shows the corresponding joint shortfall probabilities $P(S_1^{(1)} > 0 \cap S_1^{(2)} > 0)$ and shortfall probabilities of company 1 (2) conditional on shortfall of company 2 (1) $P(S_1^{(1)} > 0 \mid S_1^{(2)} > 0)$ for different values of $\gamma^{(1)} = \gamma^{(2)}$. Naturally, joint shortfall probabilities are highest with a high correlation between the two companies' assets and lowest with a low one. For increasing $\gamma^{(i)}$, joint shortfall probabilities decrease for all reported correlation coefficients. Here, remember that single shortfall probabilities also decrease. However, the conditional shortfall probabilities show that the probability that one financial company defaults given

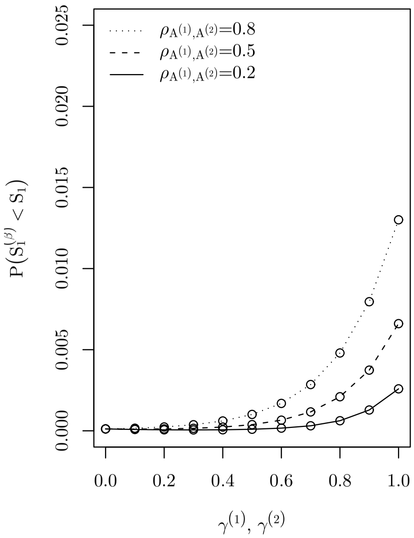


(a) Shortfall probability

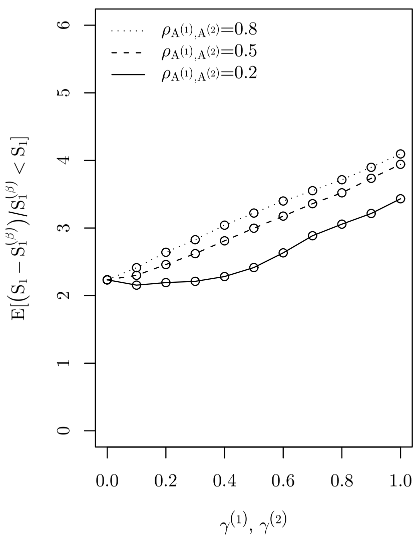


(b) Expected shortfall given default

Figure 21: Effects on shortfall measures of financial company 1 (or 2) if financial company 1 purchases fraction $\gamma^{(1)}$ of company 2's financial guaranty bonds and financial company 2 purchases fraction $\gamma^{(2)}$ of company 1's financial guaranty bonds, with $\gamma^{(1)} = \gamma^{(2)}$. Panel (a) shows the shortfall probability $P(S_1 > 0)$ and panel (b) the expected shortfall given default $E[S_1 \mid S_1 > 0]$. Panel (c) displays the probability that the guaranty vehicle cannot cover all losses $P(S_1^{(\beta)} < S_1)$ and panel (d) the expected amount of insufficient coverage $E[(S_1 - S_1^{(\beta)}) \mid S_1^{(\beta)} < S_1]$. (continued on next page)



(c) Probability of insufficient coverage



(d) Expected amount of insufficient coverage

Figure 21: Effects on shortfall measures of financial company 1 (or 2) if financial company 1 purchases fraction $\gamma^{(1)}$ of company 2's financial guaranty bonds and financial company 2 purchases fraction $\gamma^{(2)}$ of company 1's financial guaranty bonds, with $\gamma^{(1)} = \gamma^{(2)}$. Panel (a) shows the shortfall probability $P(S_1 > 0)$ and panel (b) the expected shortfall given default $E[S_1 \mid S_1 > 0]$. Panel (c) displays the probability that the guaranty vehicle cannot cover all losses $P(S_1^{(\beta)} < S_1)$ and panel (d) the expected amount of insufficient coverage $E[(S_1 - S_1^{(\beta)}) \mid S_1^{(\beta)} < S_1]$.

that one financial company has already defaulted highly increases with increasing $\gamma^{(i)}$ (contagion). Consider, for example, the extreme case in which $\gamma^{(1)} = \gamma^{(2)} = 1$. The probability that the second financial company defaults given that the other one already defaulted is 15.8% with a low correlation and even 63.6% with a high correlation.

Next, Figure 21c shows the probability that the guaranty vehicle cannot cover all losses faced by the financial company's clients $P(S_1^{(\beta)} < S_1)$ and Figure 21d the expected amount which cannot be covered given that not all claims can be covered $E[(S_1 - S_1^{(\beta)}) \mid S_1^{(\beta)} < S_1]$. Note that values calculated in Figure 21d correspond to the comparably low probabilities reported in Figure 21c. As a consequence, we observe a small approximation error in Figure 21d. The probability that not all claims can be covered generally increases with increasing $\gamma^{(i)}$ for all correlations whereas highest probabilities can be observed with high correlation and lowest with low correlation. The expected amount which cannot be covered increases with increasing $\gamma^{(i)}$ for high and medium correlation but first decreases and then increases with low correlation. Again, this decrease with a low correlation ($\rho_{A^{(1)},A^{(2)}} = 0.2$) can be explained by diversification effects. The other results can generally be explained by a decreasing volatility of the asset portfolio and indemnity payments that will lower the asset value $A_1^{(i)*}$.

To conclude this section, let us point out that, if both companies purchase financial guaranty bonds of the other financial company in the same amount, probabilities of default decrease, occurring defaults yield larger losses (except for low correlation between the two companies' assets), the probability that not all claims can be covered generally increases, and the expected amount which cannot be covered increase as well (except for low correlation between the two companies' assets). The effect on joint shortfall probabilities depends on the participation, but contagion effects appear to increase with increasing investments in the other's guaranty vehicle. Note that we also analyzed the case in which only one financial company invests in the guaranty vehicle of another financial company whereas the other financial company does not

	$P(S_1^{(1)} > 0 \cap S_1^{(2)} > 0)$			$P(S_1^{(1)} > 0 \mid S_1^{(2)} > 0)$		
$\gamma^{(1)}, \gamma^{(2)}$	$\rho_{A^{(1)},A^{(2)}} = 0.2$	$\rho_{A^{(1)},A^{(2)}} = 0.5$	$\rho_{A^{(1)},A^{(2)}} = 0.8$	$\rho_{A^{(1)},A^{(2)}} = 0.2$	$\rho_{A^{(1)},A^{(2)}} = 0.5$	$\rho_{A^{(1)},A^{(2)}} = 0.8$
0	0.007	0.016	0.032	0.107	0.250	0.503
0.1	0.007	0.016	0.032	0.116	0.270	0.526
0.2	0.007	0.016	0.031	0.125	0.287	0.547
0.3	0.007	0.016	0.030	0.132	0.303	0.566
0.4	0.007	0.016	0.029	0.139	0.317	0.582
0.5	0.006	0.015	0.027	0.145	0.329	0.596
0.6	0.006	0.014	0.026	0.150	0.339	0.607
0.7	0.006	0.013	0.023	0.154	0.348	0.617
0.8	0.005	0.012	0.021	0.156	0.355	0.625
0.9	0.005	0.011	0.019	0.158	0.359	0.631
1	0.004	0.010	0.017	0.158	0.363	0.636

Table 28: Joint shortfall probabilities of financial company 1 and 2 and conditional shortfall probabilities of company 1 (2) given default of company 2 (1) if financial company 1 purchases fraction $\gamma^{(1)}$ of company 2's financial guaranty bonds and financial company 2 purchases fraction $\gamma^{(2)}$ of company 1's financial guaranty bonds, with $\gamma^{(1)} = \gamma^{(2)}$.

change its behavior. Results, however, are very similar and do not notedly deepen the insights.[64]

4 Practical Implications and Further Comments

The numerical analysis presented in Section 3.4 shows that financial companies can significantly influence the effectiveness of our proposed capital market-based guaranty scheme. In what follows, we first discuss our numerical results with regard to their practical implications. Then, we further comment on advantages and drawbacks of the proposed system.

First, we consider the investment of a financial company in its own guaranty vehicle. At first sight, this action might even appear to be advantageous as probabilities of default decrease. However, in fact, the financial company levers the guaranty system out as the capital which should be additionally raised in the system is not raised. Instead, the financial company imposes more or less restrictions on its own capital investments as the guaranty vehicle invests all proceeds at the risk-free rate. This effect could be achieved more easily by just imposing capital allocation requirements for financial companies. As a consequence, if defaults occur, they become more severe than without any *self-investments*. From a client's perspective, one of the most important questions is whether and to what extent the guaranty vehicle with the corresponding cap might not be able to cover clients' claims. Our numerical analysis shows that the probability of such events and the extent of these events increases with an increasing amount of investment in the own guaranty vehicle. Thus, self-investments appear to be highly problematic as they counteract regulatory intentions. Nevertheless, prohibiting these self-investments is straightforward since companies already have to account for holdings of own stocks and bonds in their balance sheet.

[64]Further analysis results are available upon request.

Second, consider investments of financial companies in other financial companies' guaranty bonds. Here, results depend on the actual correlation between assets and liabilities of the different financial companies. Generally, low correlations between the financial companies' portfolios might lead to positive effects on shortfall probabilities and expected shortfalls due to diversification effects. However, the higher the correlation, the closer results get to our observations with regard to investments in the own guaranty vehicle. And, importantly, contagion effects seem to increase with increasing investments in the other's guaranty vehicle. Thus, investments in foreign guaranty vehicles can be problematic, especially if correlations between the respective financial companies are high. On the other hand, investments in the guaranty vehicle of a financial company with low correlation, e.g., in another business segment, might generate additional diversification effects. The question remains whether a supervisor can restrict the kind of investors if a product is publicly traded, especially if guaranty bonds are part of a diversified fund financial companies would usually invest in. Here, clear investment limits (caps regarding investments in guaranty vehicles) need to be established in supervisory law.

Furthermore, there are some other challenges concerning the proposed framework which need to be considered:

- *Transaction costs*: Transaction costs and organization fees – which have been put aside in the model illustration – might make the proposed guaranty system highly expensive. Especially the establishment of one SPV per financial company might appear problematic in this regard. However, instead of establishing various SPVs, one could structure one large credit-linked note per company which would be hold by a trust company. Results and implications would remain the same but structuring costs would decrease. Besides, as already mentioned, our proposed framework issues bonds focusing on credit risk and the credit market is already well established (CDS, CDOs, etc.).

- *Spreads and volume*: There might not be enough investors willing to invest in this kind of financial product. This could limit the liquidity or lead to exaggerated spreads. This problem could be solved by raising capital through a stepwise increase of the principal starting from zero. This mode of financing and establishing a guaranty fund has been chosen, for instance, in the German life insurance guaranty fund where required funds are collected over a period of five years.[65] In addition, the market for credit risks is well established and apparent risks should be comprehensible to investors. Thus, investors should, in general, be willing to invest into the guaranty bonds as long as an adequate risk premium is provided.

Besides, spreads might highly change on a year to year basis. Thus, if a financial company is already in financial distress, its premiums are expected to rise and might even worsen this distress. Similarly, as seen in the recent financial crisis, the default of one financial company might lead to increasing spreads for other ones so that contagion effects might occur. Nevertheless, as long as bankruptcy is declared on time, the capital market-based guaranty system can secure clients' claims – recall that system protection is not in focus of the proposed framework. In addition, if spreads become too high, a financial company might still take other risk management measures to reduce the spread required.

- *Impact on the market*: We assume that the guaranty vehicle invests all proceeds risk-free in the capital market. Here, the question arises whether the capital market can provide enough risk-free capital and whether this capital is actually risk-free. Again, this issue could be solved by a stepwise increase of the principal over time. Besides, current insurance guaranty and deposit insurance systems have a similar fund volume which is currently invested in the capital market (and would be dissolved if our proposal was implemented). In order to ensure that the provided capital is actu-

[65]See Protektor Lebensversicherungs-AG, `http://www.protektor-ag.de/sicherungsfonds/finanzierung/72.aspx`.

ally risk-free, a swap arrangement could be used as currently done regarding insurance-linked securities.

Although there are some challenges concerning the implementation of the proposed capital market-based financial guaranty system, the discussion above shows that most of them can be solved. Then, the capital market-based financial guaranty systems provides various advantages:

- *High liquidity*: Firstly, the proposed guaranty system offers access to the high amount of capital available in financial markets. Compared to existing guaranty systems which are mostly funded through compulsory contributions from the financial companies, the set of possible investors and, thus, the sources for funding are widespread. Hence, capital market-based guaranty systems can be structured to even cover shortfalls of major financial companies whose weight in existing schemes is often too large to be solely covered by it. Therefore, ultimate help from the state is currently needed, i.e., from taxpayers. The major bail-outs which occurred during the recent financial market crisis clarified this point. Similarly, our proposed system allows to protect all potential clients, from privates to large companies, which is opposed to current systems that often only protect private customers and, sometimes, small and medium-sized enterprises.

- *Risk-adequate premiums*: The market-based funding ensures, secondly, that market-driven, risk-adequate premiums arise. Thus, in contrast to various current financial guaranty systems, financial companies pay premiums corresponding to their risk situation – not to their volume. As briefly mentioned in the introduction and as reported in, e.g., Oxera (2007), existing guaranty schemes often target a fund volume by imposing a sourcing through volume-based contributions. These charges, as discussed in Rymaszewski et al. (2010), can imply various adverse incentives among the different market players. Market-based funding incentivizes, by definition, risk-adequate charges.

- *No cross-subsidization*: Recall that existing guaranty schemes with ex post charges can never be organized in a truly risk-based way nor avoid cross-subsidization effects due to the fact that an insolvent financial company, which may have been the riskiest one, is typically not charged at all (see, e.g., Han, Lai, and Witt (1997, pp. 1119)). Furthermore, in current systems with ex ante contributions, premiums are pooled among all participants, and, if these contributions are not risk-adequate, some financial companies are better off than others, or, at least, companies do not profit to the same extent from the guaranty scheme (see, e.g., Rymaszewski et al. (2010)). The presented capital market-based guaranty scheme does not involve any cross-subsidization effects between clients of different financial companies as each financial company has its own guaranty vehicle. As a consequence, the clients' incentive to close contracts with the worst performing financial company (offering lowest premiums) caused by current guaranty schemes, which do not involve separate accounts for each company, ceases.

To conclude, there are some challenges regarding the implementation of our capital market-based financial guaranty system – self-investments, transactions costs, spread and volume, impact on market. However, we show how these challenges can be solved and clarify key advantages of the proposed framework compared to current deposit insurance and insurance guaranty schemes.

5 Conclusion

We propose a capital market-based financial guaranty system and examine whether investment actions taken by the respective financial companies might affect the effectiveness of the system. The described market-based solution overcomes the problem of current guaranty systems that systematic wealth transfers between clients of different financial companies take place.

In the first step of our analysis, we introduce the conceptual framework of our capital market based financial guaranty scheme. We derive closed-form solutions for the clients' premium and the investors' principal under the assumption of perfect, complete, and frictionless capital markets which provides the starting point for our following analysis. In the second step, we analyze the impact of two different actions which might be taken by the financial companies protected by the guaranty vehicle: First, the financial company might purchase guaranty bonds of its own guaranty vehicle. Second, it might purchase guaranty bonds of another financial company whereas both companies' assets have a certain positive correlation. We measure effects of these actions on major stakeholders by means of various risk measures. By deriving practical implications, we provide new insights for regulators and financial companies whether a transfer of default risk to capital markets might be feasible.

We find that investments in the own guaranty vehicle lever out the purpose of the guaranty system and that investments in a foreign one might lead to the same result depending on the correlation structure. In addition, contagion effects might occur. We identify other major challenges – transaction costs, spread and volume, and impact on the market – and propose possible solution. Finally, we show that the capital market-based financial guaranty systems provides various advantages: It is highly liquid, ensures risk-adequate premiums to the guaranty scheme, and eliminates potential cross-subsidization effects.

Although there are challenges regarding the implementation of the presented proposition, we show how these can be solved. The analysis of advantages of the proposed framework clarifies that the capital market-based solution has some key benefits compared to current deposit insurance and insurance guaranty schemes.

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Summary and Outlook

In Part I, the most common participating life insurance bonus distribution mechanisms, namely the ones of Bacinello (2001), Haberman et al. (2003), Grosen and Jørgensen (2000), Hansen and Miltersen (2002), and Kling et al. (2007), are presented and an integrative notation is applied. We develop a parameter calibration which allows us to compare the default put option (DPO) values across these different models. By applying our method of comparison, regulatory authorities can compare the model risks present in different bonus distribution models.

Our results show that the return-based bonus distribution scheme of Bacinello (2001) (Italy) generally yields the highest DPO values if the underlying asset volatility increases or the initial reserves decrease. As a consequence, given that Italian insurers actually apply the bonus distribution scheme introduced by Bacinello (2001), regulators should reassess whether a model which is less sensitive could be implemented. For instance, the models of Hansen and Miltersen (2002) (Denmark) and Kling et al. (2007) (Germany) are the least sensitive to changes in the underlying asset volatility, whereas the model of Grosen and Jørgensen (2000) (Denmark) is the least sensitive with respect to changes in the initial reserves.

Our analysis in Part I clarifies that the model chosen by insurance companies or prescribed by regulators cannot be chosen arbitrarily. We suggest that regulatory authorities should select a bonus distribution model whose default risk is less sensitive to model risks. Further research may address participating life insurance contracts with periodic premium payments since the form of payment will presumably have an impact on insolvency risk. In addition, mortality and interest rate risk could provide additional interesting insights.

In Part II, we analyze the controversial question whether participating life insurance contracts are actually beneficial for policyholders. Even though this contract form is very common in insurance practice, only very little research has been conducted in respect to its performance. In a first step, a framework to estimate payoffs from participating life insurance contracts from the point of view of policyholders is developed.

In order to do so, we decompose a participating life insurance contract into an investment part and a term life insurance. Hence, we are able to analyze the benefits of the minimum interest rate guarantee in combination with the profit distribution rules independent from the term life insurance. Thereby, we model more than one single contract which allows us to incorporate distribution effects between policyholders. In a second step, we simulate and benchmark the complete payoff distribution on an ex ante basis. We show how the payoff distribution depends on the level of the surplus fund at inception of the contract and analyze the effect of management's discretion.

We show that participating life insurance can be beneficial to policyholders depending on the initial reserve situation and preferences. A low initial reserve situation of the insurer appears to be disadvantageous. Individuals continuing their contract until maturity without death or surrender will in general profit from a better payoff distribution compared to the mutual fund benchmark portfolio but not the ETF benchmark portfolio. Further, investors do not know ex ante whether and when they will die or surrender. Hence, product preferences will depend on risk aversion and the rate of intertemporal substitution. Management's discretion changes payoff distributions but it depends on preferences whether the changed payoff distribution is perceived to be better or worse.

To conclude, Part II shows that policyholders have very little chance to predetermine the cash flow distribution as long as the future behavior of management and the current level of the surplus fund are unknown or realistic assumption cannot be derived in this respect. Also, our preference dependent performance analysis shows that in most cases an ETF portfolio will assumedly perform better than each possible participating life insurance contract. In order to get a better understanding of how the underlying capital market parameters influence the performance, future research could analyze a more detailed asset model (i.e., an interest rate model for the bond fraction of the asset portfolio) or derive sensitivities with regard to drift and volatility of stocks and bonds. In addition, longer contract periods may be analyzed.

Part III addresses key regulatory and reporting reforms in the European insurance industry, namely the solvency framework Solvency II, insurance guaranty systems, the proposed IFRS 4 Phase II international accounting standards, and Market Consistent Embedded Value reporting. In a first step, we present these four frameworks. A brief overview of their current state of progress and their implementation plan is provided. Furthermore, we illustrate key elements of these four different frameworks and explain the different underlying valuation models with an integrative illustration. In a second step, we analyze the four frameworks from different stakeholder perspectives and compare and contrast them. First, we provide a comparative overview of the four concepts. Second, we proceed from stakeholder to stakeholder and analyze key characteristics and interactions separately. In a third step, we bring the different perspectives together.

Regarding the position of an insurance company's management, we show that – despite various benefits – management faces a high regulatory burden. Hence, the benefits of the different regulatory frameworks need to justify the corresponding costs. Otherwise, European insurers will be at a competitive disadvantage compared to less regulated markets. Policyholders, on the other hand, will be well-protected from an insurance company's financial distress due to the regulatory reforms. However, additional reforms may be necessary in order enable policyholders to make rational choices between different insurance products. For investors, the market-based perspective adopted in the new frameworks causes high degrees of complexity. In order to qualify investors to interpret the reported information, large communication efforts and spendings will be required.

To summarize, Part III shows that the four regulatory frameworks need to be considered jointly rather than separately, due to various interrelations and interactions. A coordinate introduction will be essential so that the regulatory burden is reduced and synergies can be utilized. Furthermore, to overcome difficulties with the planned frameworks, we propose a more holistic, comprehensive approach to insurance reporting and regulation. However, further analysis will be necessary in order to estimate economic costs and benefits of all planned frameworks and for all kinds of stakeholders. Besides, there may be local regulatory and reporting concepts which require further analysis, e.g., current reforms in contractual law.

Part IV proposes a capital market-based financial guaranty system as an alternative to current insurance guaranty funds and deposit insurance systems. The described market-based solution overcomes the problem of current guaranty systems that systematic wealth transfers between clients of different financial companies take place.

In the first step of our analysis, the conceptual framework of our capital market based financial guaranty scheme is introduced. We derive closed-form solutions for the clients' premium and the investors' principal under the assumption of perfect, complete, and frictionless capital markets which provides the starting point for our following analysis. Second, we analyze the impact of two different investment actions which may be taken by the financial companies protected by the guaranty vehicle: First, the financial company may purchase guaranty bonds of its own guaranty vehicle. Second, it may purchase guaranty bonds of another financial company whereas both companies' assets have a certain positive correlation. We measure effects of these actions on major stakeholders by means of various risk measures. By deriving practical implications out of the numerical analysis, we provide new insights for regulators and financial companies whether a transfer of default risk to capital markets may be feasible.

Our results in Part IV show that financial companies may lever out the system by investments in the own guaranty vehicle or in a foreign one. In addition, contagion effects may occur. However, we find that if the scope of investors can be restricted, the capital market-based financial guaranty systems provides various advantages: It is highly liquid, ensures risk-adequate premiums to the guaranty scheme, and eliminates potential cross-subsidization effects. As a result, if a regulatory or legislative authority is able to restrict investments of financial companies regarding their guaranty vehicles, the capital market-based solution appears to be very beneficial. Future research should estimate the dunderlying parameters (e.g., volume and premiums) with empirical data. In addition, the question of pro-cyclicality of the system needs further discussion. An alternative proposal could address only those financial institutions which are system-relevant and, hence, may need more attention.

Curriculum Vitae

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