Heterogeneity in Contests

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Contents

List of Figures

List of Tables

Summary

My dissertation consists of three chapters, all three devoted to incentives in contests and the role of heterogeneity.

Chapter 1 characterizes conditions under which introducing multiple prizes in a contest can be used to guarantee efficient incentives for the production of a public good when agents are heterogeneous. With two types of individuals, efficiency can be guaranteed if: (i) the contest designer can use at least two prizes different from zero, (ii) there is a sufficient number of individuals of each type or types are sufficiently similar and (iii) the reservation utility of the individuals resulting from non-participation is sufficiently low. For a large class of problems the optimal prize structure is not monotonic.

Chapter 2 studies situations where two parties with differing valuations or abilities vie to capture some scarce resource. While one party's characteristics are common knowledge, the other's are private information. Is the right policy to mandate the disclosure of this information? When competition occurs via a noisy all-pay auction, the answer is no. Under mild conditions, decentralizing the disclosure decision produces less wasteful competition and more efficient outcomes than mandating disclosure. These results have implications for transparency policy in lobbying, electoral competition and international relations among others.

Chapter 3 reports the results of laboratory experiments on market entry. Theory predicts that entry equalizes payoffs of inside and outside option. Our findings are at odds with this prediction. In particular, entrants earn systematically less than those who stay out of the market. The payoff gap increases as a) the inside option becomes riskier; b) the outside option becomes riskier; c) the inside option becomes more strategic; and d) the outside option becomes more strategic. We discuss possible explanations.

Introduction

What is the Aim of Contest Theory?

On a daily basis we spend money and exert effort to get ahead of our rivals. We compete for jobs and promotions, as well as in sports or other forms of contests. Similarly, firms compete in advertising or R&D battles, and politicians in political campaigns. The theory on contests and tournaments aims to understand individual behavior in these kinds of situations and how such behavior is influenced by the design of the competition. This is a necessary step before one can address questions of welfare, optimal design and policy.

The literature on contests does not follow a unified research question and has evolved across a variety of literature strands. Naturally, the type of application a researcher had in mind influenced the choice of modeling assumptions and the types of questions posed. In order to clarify the nature of contest theory, I will give a short, non-exhaustive summary of the different kinds of literatures it comprises.

A classical application of contests dating back to the likes of Tullock (1967), Krueger (1974), or Tullock (1980) is the analysis of rent-seeking competition. Here the focus is on the inefficiency of the political process in the sense that resources are wasted in a competition for the redistribution of political rents which could have otherwise been used productively. Tollison (1982), Nitzan (1994) and recently Congelton, Hillmann, and Konrad (2008) survey this strand of the literature. Typically, the aim of this literature is to analyze the degree of wastefulness of rent-seeking competition (the dissipation rate) in different types of circumstances and how it can be reduced.

A related area of contest research analyzes the properties of conflicts and war and the emergence of property rights (see for example Skaperdas (1992) and Garfinkel and Skaperdas (2007)). In contrast to the rent-seeking literature, the typical assumption is the absence of a government and hence a rule of law. This strand emerged from the recognition that, without a rule of law, property rights are established endogenously by efforts to defend or acquire resources. The focus of this literature is, on the one hand, on the emergence of property rights, and, on the other, to identify the effects of conflict on economic outcomes, as for example trade, capital accumulation, development or innovation.

Another strand of literature dating back to Lazear and Rosen (1981), Nalebuff and Stiglitz (1983), and O'Keeffe, Viscusi, and Zeckhauser (1984) analyzes worker compensation schemes which reward according to output rank order, also called "tournaments". The aim is to examine the efficiency and incentive properties of reward systems based on rank-order rather than absolute individual performance. Tournament theory argues that such systems are desirable when monitoring is either unreliable or costly. Instead of monitoring and supervising workers to elicit the optimal work effort, a firm should rely on a self-enforcing reward structure. McLaughlin (1988) surveys the literature on tournaments.

Further, there are strands of literature analyzing contests in research and development (e.g. Baye and Hoppe (2003)), the design of sports competitions (e.g. Szymanski (2003)) or promotional competition (e.g. Friedman (1958) and Schmalensee (1976)).

Heterogeneity in Contests

An underlying theme of these strands of literature and also of my dissertation is a focus on the role of heterogeneity in contests. Heterogeneity can take many forms. In the following I will give a brief overview over the research of heterogeneity in contests.

Contestants are often heterogeneous in their valuation of the prize or rent, for example when lobbying a decision maker. Or they can be of different strengths, as in a military conflict. In a competition for promotion or bonuses workers in a firm typically have heterogeneous costs of effort. Baye, Kovenock, and de Vries (1996) show that all of these problems are isomorphic and can be converted into each other by a simple transformation of the utility function. They characterize the equilibrium of an all-pay auction with heterogenous players under complete information. Nti (1999) and Stein (2002) analyze the equilibrium of a "Tullock" contest, while Lazear and Rosen (1981) are the first to consider heterogeneous workers in an all-pay setting with additive noise[1].

[1]Typically the objective function in a contest takes the following form: $\pi_i(x_1, \ldots, x_n) = p_i(x_1, \ldots, x_n)R - c_i(x_i)$, where x_i is the effort expended in the competition, R the value of the rent, $c_i(\cdot)$ the cost of effort, n the number of contestants and $p_i(\cdot)$ the probability of success. In an all-pay auction $p_i(\cdot)$ takes the value 1 if i has the highest expenditure and 0 if this is not the case. For a Tullock contest, $p_i(\cdot) = \frac{x_i^r}{\sum_{j=1}^{n} x_j^r}$. In a setting with additive noise, $p_i(\cdot)$ is defined as in the all-pay

Heterogeneity can also take the form of differences in the timing of decisions in a contest. For example one contestant could be an incumbent and hence he might have the opportunity to expend effort before his opponents. Dixit (1987) is the first to show in the context of contests that there is an advantage to being able to commit to an effort choice before one's opponents. Baik and Shogren (1992), and Leininger (1993) endogenize the order of moves. They find that contestants agree to introduce heterogeneity in terms of the timing of decisions in order to reduce the fierceness of the competition. Morgan (2003) shows that heterogeneity in timing is also good for the efficiency of the contest.

Differences between contestants often arise in the degree of information about the opponents' characteristics or the contest itself. Malueg and Yates (2004), Hurley and Shogren (1998b), and Hurley and Shogren (1998a) analyze the equilibrium of a Tullock contest when contestants are asymmetrically informed about their opponents' valuations and/or costs of effort. Contestants can either be symmetrically uninformed as in Malueg and Yates (2004) or one player possesses an informational advantage, as for example in Hurley and Shogren (1998b). Glazer and Hassin (1988), Amann and Leininger (1996), Krishna and Morgan (1997), and Moldovanu and Sela (2001) are among the first to consider asymmetric information about opponents' characteristics in an all-pay auction. O'Keeffe, Viscusi, and Zeckhauser (1984) consider optimal worker incentive schemes when workers have asymmetric abilities and these are private information. The literature above has assumed that contestants are fully aware of their own valuation of the prize. There are many examples though, when contestants are not sure about their value of the prize and valuations are correlated, as for example in a contest for the rights to drill for oil in a certain region. These situations are for example analyzed in Wärneryd (2003).

Heterogeneity can also be about risk preferences. The theoretical literature so far has mostly focussed on identical individuals and analyzed different degrees of risk aversion (e.g. Hillman and Katz (1984) and Konrad and Schlesinger (1997)); exceptions are Skaperdas and Gan (1995) and Cornes and Hartley (2001) who find that when competing in the same contest, less risk-avers individuals expend more than more risk-averse ones. Millner and Pratt (1991) have taken this question to the laboratory and have found that in a simple lottery form contest less risk averse individuals over-expend relative to risk neutrality while more risk averse individuals invest approximately the same

auction except that the ranking of contestants is determined by $x_i + \epsilon$, where ϵ is the realization of a random variable.

as the risk neutral prediction. Contests with loss-averse players are analyzed theoretically in Cornes and Hartley (2010), and experimentally in Kong (2008). They find that an increase in loss-aversion decreases expenditures in the contest and that expenditures in the laboratory are systematically higher than theoretical predictions. Relatedly, Gill and Prowse (2009) find evidence of disappointment aversion in a sequential contest. A contestant facing a first-mover who expended a high effort reacts with a reduction in effort compared to risk-neutrality because he wants to avoid disappointment of losing. Gill and Prowse (2009) estimate the heterogeneity in disappointment aversion in their subject population.

The Layout of My Thesis

This thesis contains three chapters on contests and heterogeneity. In chapter 1, I show how to incentivize individuals with heterogenous abilities to produce a public good by means of a multi-prize contest. In chapter 2, I analyze how heterogeneity influences the incentives to share information ahead of a contest and how mandatory disclosure policy affects welfare. In chapter 3, I ask whether payoffs from entering a contest and an outside option equalize, and whether heterogeneity of individuals explains entry and investment behavior.

Chapter 1

Providing Public Goods Through Contests - A Case for Multiple Prizes

Joint with Martin Kolmar

1.1 Introduction

Private provision of public goods often encounters two difficulties. On the one hand externalities lead to suboptimal provision levels when not internalized in some way. On the other hand, as soon as contributions are not of a monetary nature, individual contributions can be hard to measure and reasonably only a ranking of contributions is feasible. In these cases contests are commonly used to promote incentives. An example is an information good like basic research that is non-rival in consumption, and for political or contractual reasons the market mechanism is not applied in a substantial number of cases. Because researchers' output is typically highly specialized it is often hard to compare and especially verify outputs between researchers on a cardinal scale. Hence the researchers' chances to receive tenure positions, awards, or other forms of funding are determined by ranking the number of publications and other noisy measures of scientific output. It is the relative position of a single researcher compared to his peer group that determines his success.

Typically individuals have different productivities or opportunity costs of providing the public good. These differences have important consequences for the incentive effects of a contest because efficiency requires that (potentially) each type of individual invests a different amount of effort. Most of the literature on contest design has focused on the analysis of single-prize contests. With only a single prize it is generally impossible to set efficient incentives if individuals are heterogeneous. Abstracting from the specific problems imposed by public goods, Lazear and Rosen (1981) have shown that with heterogeneous individuals it is impossible to efficiently sort individuals into 'leagues' for the case of two types of individuals and a single prize (or, more precisely, a single prize spread). As mentioned by Lazear and Rosen, abstracting from individual participation constraints their two-prize contest is structurally equivalent to a single-prize contest because only *differences* between prizes ('prize spreads') matter for incentive reasons.[1] The second prize in their paper can be interpreted as a minimum wage as it will be received for sure. A truly two-prize contest would require at least three contestants and a prize structure of the form first, second, and others, whereby others could be interpreted as a participation fee or bonus or just be set equal to zero. To be precise, only the difference between first (second) prize and participation fee/bonus sets the incentives to invest effort in the contests. This observation is the starting point for this chapter where we explore whether it is possible to shape incentives efficiently if the contest designer can use more than one prize, and how the efficient prize structure looks, if it exists. We show that it is possible to efficiently solve the incentive problem for the case of two types of individuals, at least three individuals and two prize spreads. Given that both types are risk neutral, any spread in the prize structure that leaves its expected value unchanged leaves individual incentives unchanged. Hence, as long as the types differ in their marginal probabilities of winning the different prizes, incentives for the different types of individuals can be controlled separately if the number of prize spreads is sufficiently large. Moreover, for a large class of problems the optimal prize scheme has a rather surprising non-monotonic structure, requiring the second prize to be lower than the third prize.[2]

The methods applied in this chapter are based on the theory of contests and rent-seeking pioneered by, amongst others, Tullock (1980). Clark and Riis (1998b) extend the conventional single prize Tullock contest to multiple prizes constructing a nested contest success function. We make use of the fact that a contest with such a nested Tullock function is isomorphic to a stochastic multi-prize all-pay contest as was shown by Fu and Lu (2008). Multi-prize contests can alternatively be modeled as fully discriminating

[1] This is an application of the well-studied principle that the class of efficient mechanisms is uniquely determined up to a constant of integration. See, for example, Milgrom (2004).

[2] Malcomson (1986) finds a similar non-monotonicity in a set-up with a continuum of homogeneous agents where high performing individuals need to be penalized to implement optimal efforts.

contests. This implies that the contestant with the ith highest effort will win the ith prize with certainty and would require observability of efforts to implement in our framework. These types of contests are discussed for example in Clark and Riis (1998a) and Moldovanu and Sela (2001) who focus on the optimality of multiple prizes if the objective is to maximize aggregate effort. See Sisak (2009) for a survey on the literature on multi-prize contests.

The analysis in this chapter is closely related to Morgan (2000) who analyzes the private provision of a public good by means of a lottery. The problem analyzed in Morgan (2000) is therefore one of efficient fundraising, whereas the production of the public good is not an issue. His main result is that in the limit for a large number of individuals a lottery contest can generate efficient incentives for voluntary funding. Because Morgan's focus is on fundraising for the production of public goods, not the process of production itself, he abstracts from the problem of heterogeneity of individuals in their abilities to provide the good and the associated sorting problems. However, especially for the provision of basic research an equally important problem is the selection of the right types of individuals doing research. Zenginobuz (1996) analyzes the private provision of a public good with individuals that are concerned about status. Furthermore there are some experimental studies. Lange, List, and Price (2007) extend Morgan's analysis and compare single-prize and multi-prize contests with potentially risk-averse and heterogeneous individuals to find under which of them contributions are maximal. Morgan and Sefton (2000) show that lotteries lead to significantly higher provision levels than voluntary contributions in an experiment. Faravelli and Stanca (2009) compare a single and a three-prize all-pay auction designed for public-good provision under incomplete information regarding income levels.

As in the literature on internal labor markets (e.g. Lazear and Rosen (1981), Green and Stokey (1983)) we are concerned with incentives for effort provision and compensation schemes for individuals that can be interpreted as workers. In this context Malcomson (1984) and Malcomson (1986) stress the importance of non-observability and non-verifiability of individual efforts and the resulting appropriateness of rank-order compensation schemes. Related to our work is also the literature on team production. Prominent papers here are Groves (1973) and Holmstrom (1982). Groves (1973) characterizes the optimal compensation functions in a team-production set-up with private information. Holmstrom (1982) stresses the importance of a principal who can break the budget balance requirement and of sufficient statistics summarizing information about the individuals' efforts. Recently Gershkov, Li, and Schweinzer (2009) follow up on this paper and show how to incentivize team members through a tournament without principal. In contrast to the work in this chapter joint production is a private good and the two symmetric players share the surplus in a way agreed to beforehand. Furthermore our work is related to the literature on impure public goods, as for example analyzed in Cornes and Sandler (1994). Using a contest introduces an additional, solely private benefit to public good provision.

The chapter is organized as follows. In Section 1.2 we introduce the model where a public good can be provided by individuals with different convex cost functions. In Section 1.3 we characterize the set of efficient allocations. Section 1.4 demonstrates that voluntary provision will lead to a sub-optimal provision of the public good. Section 1.5 characterizes the conditions for the efficiency of a multi-prize contest, the concavity of the implied optimization problems, and the properties to balance the budget of the contest designer. Section 1.6 concludes.

1.2 The Model

Take an economy with $N \geq 3$ individuals ('researchers'), indexed by $i = 1, \ldots, N$. These individuals have different abilities for producing a non-rival or public (because we do not focus on mechanisms based on exclusion) good ('scientific knowledge'), whose quantity is denoted by y. The individual production process is characterized by noise so that individual i's contribution to the public good x_i, leads to an output $y_i = x_i \epsilon_i$ where ϵ_i is a random variable. Individual production shocks are assumed to be i.i.d. with distribution function $F(\epsilon)$ continuous on $[0, \infty]$. Those shocks represent the fact that even researchers with the same ability can produce a different amount of output due to exogenous factors or luck. We assume that individual contributions are unobservable. $\mathbf{x} = \{x_1, ..., x_N\} = \{x_i, x_{-i}\}$ is the vector of individual contributions, $\mathbf{y} = \{y_1, ..., y_N\} = \{y_i, y_{-i}\}$ of individual outputs. For convenience we assume a linear mapping from individual outputs y_i into public-goods production y, $y = \sum_{l=1}^{N} y_l$, which implies that individual outputs $\mathbf{y}$ are perfect substitutes. A contribution of x_i involves the investment of a private good ('time') $t_i = c_i(x_i)$ for the individual (we can regard x_i as something like efficiency hours). The mappings $c_i(.)$ from contributions to the private good ('cost functions') are assumed to be strictly convex, and we assume that $c_i(0) = c_i'(0) = 0$ and $\lim_{x_i \to \infty} c_i'(x_i) = \infty$. Each individual derives utility from the total amount of the public good, $V_i(y)$, and the consumption of a private good z_i ('leisure'). $V_i(y)$ is strictly concave in its argument with $V_i(0) = 0$. Effort as well as utility functions are twice continuously differentiable. The individuals have an endowment $\bar{z}$ of the private good which is assumed to be sufficiently large to guarantee an interior solution for all subsequent optimization problems.

We assume that there are two types of individuals, $N_H \geq 1$ H-type members and $N_L \geq 1$ L-type members, $N_H + N_L = N$ who differ in their cost function $c_i(x_i)$ as well as potentially in the utility from the public good $V_i(y)$[3]. The sets of individuals of both types are denoted by $\mathcal{N}_H, \mathcal{N}_L$, $\mathcal{N} = \mathcal{N}_H \cup \mathcal{N}_L$. We denote by x_H, x_L the contribution to the public good of a generic member of each group. We discuss an extension to N potentially different types in Appendix 1.C. Individuals decide individually and non-cooperatively about the amount of research x_i they undertake. They do so by maximizing their expected utility

$$Eu_i\left(x_i, z_i\right) = E[V_i\left(\sum_{j=1}^{N} x_j \epsilon_j\right)] + z_i \tag{1.1}$$

taking into account their budget constraint $\bar{z} = z_i + c_i(x_i)$. In the following we use the expectations operator in the context of the expected utility of public good $E[V_i\left(\sum_{j=1}^{N} x_j \epsilon_j\right)]$ as a shorthand for $\int\int\cdots\int V_i\left(\sum_{j=1}^{N} x_j \epsilon_j\right) dF\epsilon_1 dF\epsilon_2 \cdots dF\epsilon_N$. Inserting the budget constraint into the utility function yields

$$Eu_i\left(x_i\right) = E[V_i\left(\sum_{j=1}^{N} x_j \epsilon_j\right)] + \bar{z} - c_i\left(x_i\right). \tag{1.2}$$

The purpose of this chapter is to design a contest that achieves efficiency even if the contest designer is asymmetrically informed about the type of an individual. To be more specific, we assume that the contest designer knows the fractions of H- and L-type individuals in the population, $N_H/N, N_L/N$, but not the identity of a single individual. This is especially realistic in the case when N is sufficiently large. The individuals share this information and, in addition, know their own type. All this is common knowledge.

[3] Our analysis generalizes to the case where there is a third group of individuals who are not able to produce the public good but enjoy its benefits. In our example this would be the population of non-researchers.

1.3 Efficiency

Assuming an interior solution with respect to the contribution to the public good, a first-best efficient allocation is characterized by

$$\max_{\mathbf{x}} \sum_{j=1}^{N} \left[E[V_j\left(\sum_{l=1}^{N} x_l \epsilon_l\right)] + \bar{z} - c_j\left(x_j\right) \right], \tag{1.3}$$

and the first-order conditions are

$$\sum_{j=1}^{N} E[\epsilon_i V_j' \left(\sum_{l=1}^{N} x_l \epsilon_l\right)] = c_i'\left(x_i\right) \quad \forall i \in \mathcal{N}. \tag{1.4}$$

(1.4) is the standard generalized Samuelson condition that determines the optimal contribution levels of all individuals i, here for a stochastic production process. From these conditions the efficient allocation $\mathbf{x}^*$ can be derived. Note that the first-order conditions imply that $c_i'(x_i^*) = c_j'(x_j^*)$ for all j, i. Since effort decisions are made before the individual (i.i.d.-) shock ϵ_i is realized different marginal-cost functions imply that different individuals should choose different contributions but individuals of the same type should choose identical contributions $x_i^* = x_j^* \quad \forall \quad i, j \in \mathcal{N}_K, K = H, L$. The optimal allocation is unique given our assumptions. Without loss of generality assume that $x_H^* > x_L^*$.

1.4 Voluntary Decentralized Provision

A voluntary-contributions (Nash) equilibrium $\mathbf{x}^D$ is an allocation where individuals maximize utility by the choice of their contributions taking as given the contributions of all other individuals,

$$\max_{x_i} Eu_i\left(x_i, x_{-i}\right) = E[V_i\left(\sum_{l \neq i} x_l^D \epsilon_l + x_i \epsilon_i\right)] + \bar{z} - c_i\left(x_i\right) \quad \forall i \in \mathcal{N}. \tag{1.5}$$

In a Nash equilibrium the following first-order conditions hold:

$$E[\epsilon_i V_i' \left(\sum_{l=1}^{N} x_l^D \epsilon_l\right)] = c_i'\left(x_i^D\right) \quad \forall i \in \mathcal{N}. \tag{1.6}$$

Hence the individual incentives to provide the public good are misspecified compared to the efficient allocation because individuals do not internalize the externality imposed on the other individuals.

1.5 Implementation of the Efficient Allocation by Means of Contests

1.5.1 General Set-Up

As in Gershkov, Li, and Schweinzer (2009) the contest designer can only base his compensation scheme on the observed noisy ranking of efforts. Hence suppose that a contest is implemented such that individuals are awarded one of N prizes with value $w_j, j \in \mathcal{N}$ (measured in units of the private good), $\mathbf{w} = \{w_1, ..., w_N\}$ according to their output rank. $P_i^j(\mathbf{x})$ is the probability that individual i wins prize j given all efforts. We assume that each individual can only win one prize, which implies that $\sum_{j=1}^{N} P_i^j = 1 = \sum_{i=1}^{N} P_i^j$.

As will become clear later on, with two types of individuals only two prizes (or more precisely prize-spreads) are needed to induce efficient incentives. We can therefore normalize w_3 to w_N to zero without loss of generality. We need the general prize structure, however, for the case of more than two types of individuals discussed in the appendix. The prizes are financed by means of lump-sum contributions t by the individuals, $\sum_i w_i = Nt$.[4] These contributions can be either voluntary or imposed by a centralized authority (taxes). We will discuss the consequences of both interpretations later.

The individual objective function becomes

$$Eu_i(x_i) = E[V_i\left(\sum_{l=1}^{N} x_l \epsilon_l\right)] + \sum_{j=1}^{2} P_i^j(\mathbf{x}) w_j + \bar{z} - t - c_i(x_i) \quad \forall i \in \mathcal{N} \quad (1.7)$$

because of the convention $w_3 = w_4 = ... = w_N = 0$.

In order to pin down more specifically the noisy ranking process we assume that only a researcher's "best shot" determines his output rank. For example the more effort a

[4] Alternatively prizes can be financed by a distortionary income tax. Our results for this case are qualitatively the same. As will become clear from the characterization of optimal prizes the distortion caused by an income tax can always be compensated by an adequate adaption of the contest prize structure.

researcher expends, the more ideas he generates. In the end though only his best ideas turn into research projects and even fewer into publications. In this case the error term ϵ_i follows a Weibull (maximum) distribution[5]. Fu and Lu (2008) establish the strategic equivalence of this set-up with a multiple-prize Tullock contest. Hence from now on we can alternatively use a multi-prize contest of the ratio form as introduced by Clark and Riis (1998b) for our analysis as a representation of the stochastic ranking process. The respective probabilities are

$$P_i^1(x_i, x_{-i}) = \frac{x_i}{\sum_{j=1}^N x_j}, \quad P_i^2(x_i, x_{-i}) = \sum_{j \neq i} \left(\frac{x_j}{\sum_{l=1}^N x_l} \frac{x_i}{\sum_{k \neq j} x_k} \right).$$

In this extension of the standard single-prize Tullock contest, the first prize is awarded as in a single-prize Tullock contest. The ratio of individual i's contributions and aggregate contributions determines its probability of winning the first prize. The probability of winning the second prize is constructed as follows. Individual $j \neq i$ wins the first prize with probability $x_j / \sum_{k=1}^N x_k$. In this case, it is excluded from the contest, and the probability for individual i of winning the second prize in this contingency is calculated as in a Tullock contest without individual j. The total probability for individual i of winning the second prize is the sum over all other individuals j of the probability that individual j wins the first prize times the probability that individual i wins the second prize given that individual j has been excluded.[6]

Denote by $dP_i^j(x_i, x_{-i}) = \partial P_i^j(\mathbf{x}) / \partial x_i$ the derivative of the Tullock contest success function. The first-order conditions of individual i are

$$\sum_{j=1}^2 dP_i^j(x_i, x_{-i}) w_j + E[\epsilon_i V_i' \left(x_i \epsilon_i + \sum_{l \neq i} x_l \epsilon_l \right)] - c_i'(x_i) = 0 \quad \forall i \in \mathcal{N}. \tag{1.8}$$

A Nash-equilibrium is a vector $\mathbf{x}^n = \{x_1^n, ..., x_N^n\}$ such that for all i the first order conditions hold at x_i^n, given x_{-i}^n.

Combining equations 1.4 and 1.8, a necessary condition for the efficiency of a Nash equilibrium is $\mathbf{x}^n = \mathbf{x}^*$, which implies that

$$\sum_{j=1}^2 dP_i^j(x_i^*, x_{-i}^*) w_j = \sum_{k \neq i} E[\epsilon_i V_k' \left(\sum_{l=1}^N x_l^* \epsilon_l \right)] \quad \forall i \in \mathcal{N}. \tag{1.9}$$

[5] The probability density function of the Weibull (maximum) distribution is $f(\epsilon) = \frac{1}{\epsilon^2} e^{-(\frac{1}{\epsilon})}$. The cumulative distribution function is $F(\epsilon) = e^{-\frac{1}{\epsilon}}$.

[6] For more details on this multiple-prize contest-success function see Clark and Riis (1996) and Clark and Riis (1998b).

This condition simply states that the marginal benefit from the contest should equal the expected marginal externality imposed by the individuals at the efficient allocation. The expected marginal externalities at the optimum will for convenience also be denoted by $EX_i(x_i^*, x_{-i}^*)$ in the following.[7] Given that individuals of the same type behave identically in equilibrium, (1.9) can be simplified as follows:

$$\begin{aligned} dP_H^1(\mathbf{x}^*)w_1 + dP_H^2(\mathbf{x}^*)w_2 &= EX_H(\mathbf{x}^*), \qquad \forall i \in \mathcal{N}_H, \\ dP_L^1(\mathbf{x}^*)w_1 + dP_L^2(\mathbf{x}^*)w_2 &= EX_L(\mathbf{x}^*), \qquad \forall i \in \mathcal{N}_L. \end{aligned} \tag{1.10}$$

Because the first-order conditions are identical for all individuals of the same type, (1.10) effectively generates a system of 2 equations with 2 unknowns (the prizes). Written differently, the individuals' FOC form a system of linear equations of the general form

$$\begin{pmatrix} dP_H^1(\mathbf{x}^*) & dP_H^2(\mathbf{x}^*) \\ dP_L^1(\mathbf{x}^*) & dP_L^2(\mathbf{x}^*) \end{pmatrix} \begin{pmatrix} w_1 \\ w_2 \end{pmatrix} = \begin{pmatrix} EX_H(\mathbf{x}^*) \\ EX_L(\mathbf{x}^*) \end{pmatrix} \tag{1.11}$$

which for convenience is written as $\mathbf{Pw} = \mathbf{EX}$. Such a system has at least one solution $\mathbf{w}^*$ if and only if Rank $[\mathbf{P}]$ = Rank $[\mathbf{P}|\mathbf{EX}]$. For the case of an extended Tullock function the following Lemma holds.

Lemma 1. *If $x_L^* \neq x_H^*$ and $N \geq 3$, Rank $[\boldsymbol{P}]$ = Rank $[\boldsymbol{P}|\boldsymbol{EX}] = 2$ for the extended Tullock function.*

Proof: See Appendix.

In order to guarantee the implementation of the efficient allocation, it is furthermore required that the prize structure $\mathbf{w}^*$ is such that the individuals' decision problems have a (global) maximum at the efficient allocation. A sufficient condition is that the decision problem be concave. We will check later if the optimal prize structure fulfills this requirement.

The special case of identical individuals follows immediately. If all individuals are identical, the rank of $\mathbf{P}$ as well as of $\mathbf{P}|\mathbf{EX}$ is one, which implies that efficient prizes can be found. In this special case, the concavity of the single-prize Tullock function is also guaranteed. Given that $x_H^* = x_L^*$ at the optimum, all entries into $\mathbf{P}$ are identical, and efficiency can be reached using a single-prize contest $w_1, w_2 = ... = w_N = 0$. However,

[7] Note that since we do not have to restrict the quantity of the marginal externality this term can be augmented to include a distortion from taxation if prizes where financed by a distortionary income tax. Our results in this section will qualitatively carry over.

every prize scheme $w_1 + k, w_2 = ... = w_N = k, k \geq 0$ induces the same incentives, which shows that the class of efficient prize structures is potentially much larger.

On the other hand, with only one prize spread and two different types of individuals, Lazear and Rosen's (1981) finding that sorting is impossible with a Tullock CSF can be (generically) extended to the case of a public good.

Result 1. *Assume a single-prize contest with two types of individuals and a Tullock CSF. Then it is generically impossible to implement an interior optimum with* $x_H^* > 0$, $x_L^* > 0$.

Proof: In a symmetric equilibrium where each individual of the same type contributes an equal amount to the public good, the optimality conditions become

$$\frac{(N_H - 1)x_H^* + (N_L)x_L^*}{(N_H x_H^* + N_L x_L^*)^2} w_1 = EX_H \tag{1.12}$$

for the H-type, and

$$\frac{N_H x_H^* + (N_L - 1)x_L^*}{(N_H x_H^* + N_L x_L^*)^2} w_1 = EX_L \tag{1.13}$$

for the L-type. The H-type condition can only be fulfilled if

$$w_1 = \frac{(N_H x_H^* + N_L x_L^*)^2}{(N_H - 1)x_H^* + N_L x_L^*} EX_H.$$

Inserting this condition into the type-L condition yields, after some rearrangements and with a small abuse of notation,

$$\frac{x_H^*}{x_L^*} = \frac{E[\epsilon_H V_H'(\mathbf{x}^* \epsilon)]}{E[\epsilon_L V_L'(\mathbf{x}^* \epsilon)]},$$

which will not be fulfilled in general. □

For the general case of two prize spreads, the two equations in (1.10) implicitly define linear functions in the prize space for each type,

$$w_1^H(w_2, 0) = \frac{EX_H^* - dP_H^{2*} w_2}{dP_H^{1*}}, \tag{1.14}$$
$$w_1^L(w_2, 0) = \frac{EX_L^* - dP_L^{2*} w_2}{dP_L^{1*}}.$$

For each point on this function each individual has efficient incentives to provide the public good, given that all other individuals behave efficiently. It is therefore necessary to have $w_1^L(w_2, 0) = w_1^H(w_2, 0)$ for overall efficiency. This condition can always be fulfilled as long as both functions are no parallels, which would imply that

$$\frac{dP_H^{2*}}{dP_H^{1*}} = \frac{dP_L^{2*}}{dP_L^{1*}},$$

which, according to Lemma 1, can never be fulfilled with a Tullock CSF as long as $x_H^* \neq x_L^*$. Hence, the efficiency conditions intersect. Then, the efficient prize structure is given by

$$w_1^* = \frac{EX_H^* dP_L^{2*} - EX_L^* dP_H^{2*}}{dP_H^{1*} dP_L^{2*} - dP_L^{1*} dP_H^{2*}}, \quad w_2^* = \frac{EX_L^* dP_H^{1*} - EX_H^* dP_L^{1*}}{dP_H^{1*} dP_L^{2*} - dP_L^{1*} dP_H^{2*}}. \tag{1.15}$$

In order to have a lean notation we use $v_H'(\mathbf{x}^*\epsilon)$ if the individual is an H-type and $v_L'(\mathbf{x}^*\epsilon)$ if it is an L-type for $\epsilon_i V_i'\left(x_i\epsilon_i + \sum_{l\neq i} x_l\epsilon_l\right)$. We can summarize our findings as follows:

Result 2. *Assume a two-prize contest with two types of individuals and a Tullock CSF. A necessary condition for the implementation of efficient incentives is given by prize structure (1.15).*

Corollary 1. *The optimal second prize w_2^* will always be negative if $E[v_H'(\mathbf{x}^*\epsilon)] = E[v_L'(\mathbf{x}^*\epsilon)]$ ($EX_H^* = EX_L^*$). It will become positive if $\frac{EX_H^*}{dP_H^{1*}} < \frac{EX_L^*}{dP_L^{1*}} \Leftrightarrow E[v_H'(\mathbf{x}^*\epsilon)] > \frac{x_H^*}{x_L^*}E[v_L'(\mathbf{x}^*\epsilon)]$. Furthermore the optimal first prize w_1^* will be positive except if $E[v_H'(\mathbf{x}^*\epsilon)]$ is much larger than $E[v_L'(\mathbf{x}^*\epsilon)]$ or $\frac{EX_H^*}{dP_H^{2*}} < \frac{EX_L^*}{dP_L^{2*}}$ which requires $\frac{EX_H^*}{dP_H^{1*}} < \frac{EX_L^*}{dP_L^{1*}}$ or $w_2^* > 0$.*

Proof: See Appendix.

The explanation for the ability of a multi-prize contest to shape incentives efficiently and the surprising possibility of non-monotonic prize structures is the following. Given that the rank of the marginal-probability matrix is two, both types of individuals have differently-sloped iso-incentive curves in the prize space. Drawing w_2 on the abscissa and w_1 on the ordinate, it can be shown that the iso-incentive curve of the low-productivity type will always be steeper in w_2-w_1-space. Since the slope is the negative ratio of marginal probabilities of winning the second versus the first prize this means that the low-productivity type is relatively more affected by a change in the second prize [8]. This property can be exploited to shape incentives. Assume that w_2 is equal to zero and we can offer a type-specific first prize w_1^H and w_1^L such that both types provide

[8] The low productivity individual will typically be more motivated in absolute terms by a change in the first prize than the high-productivity type because of the concavity of the first prize Tullock CSF. This will always be compensated by a sufficiently higher marginal probability of winning the second prize of the low-productivity type.

the efficient quantities (full-information scenario). Typically $w_1^H \neq w_1^L$. First assume that $\frac{EX_H^*}{dP_H^{1*}} > \frac{EX_L^*}{dP_L^{1*}}$, meaning that the expected marginal utility from the public good is sufficiently high for the low-productivity type compared to the high-productivity type, and hence $w_1^H > w_1^L$. This is for example always true when both types have the same utility function for the public good. Increasing w_1^L marginally will increase incentives to provide for the low-productivity type. Hence if $w_1 = w_1^H$ a low-productivity type will c.p. overprovide given that the other individuals provide their efficient amounts. To discourage him from overproviding we need to introduce a second prize and make use of the differences in the ratio of marginal probabilities. By increasing the first prize and introducing a negative second prize we can balance incentives in such a way that both types provide the efficient quantities. We lower the second prize because this will hurt low-productivity types relatively more. On the other hand, if $\frac{EX_H^*}{dP_H^{1*}} < \frac{EX_L^*}{dP_L^{1*}} \Leftrightarrow w_1^H < w_1^L$ and we decrease w_1^L the low-productivity type underprovides. This is the case if the expected marginal benefit from the public good is sufficiently small for this type. In this case the first prize should be decreased and the second increased from zero to balance incentives and so typically a monotonic prize structure arises. Note that as the difference in expected marginal utilities between types gets even more pronounced and the low-productivity type exhibits an expected marginal utility which is considerably smaller than the high-productivity types' we will need prize structures such that the first prize is less than the second or in the extreme case even "negative". In this scenario we need to discourage the high-productivity type relative to the low-productivity type and hence the first prize is reduced because it has a relatively higher impact on him. This finding is important because it shows that in contrast to the first intuition it may be optimal to use non-monotonic prize schedules when individuals are heterogeneous. The following example illustrates these two cases:

Figure 1.1 shows the efficient incentive-indifference curves of both types for two H- and two L-type individuals with efficient provision levels $x_H^* = 2$ and $x_L^* = 1$ and identical externalities $EX_H^* = EX_L^* = 1$. This yields marginal probabilities of $dP_H^1 = 1/9, dP_L^1 = 5/36, dP_H^2 = 19/600, dP_L^2 = 71/600$, which leads to opportunity costs of w_1 in terms of w_2 of 57/200 (H) and 213/250 (L) respectively. The dashed line is the indifference curve for the H-type and the solid line is the indifference curve for the L-type as defined in (1.14). The efficient prize structure is then given at the intersection of both lines.

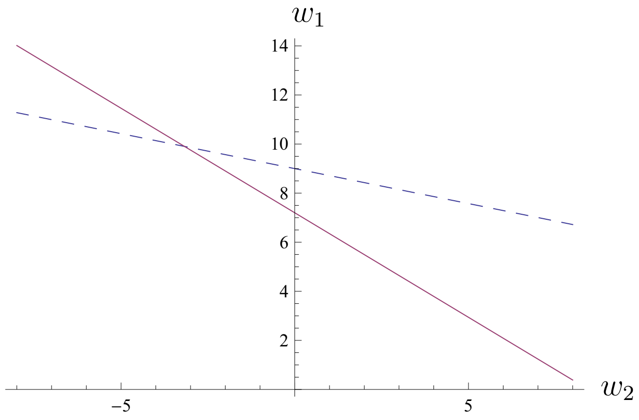


Figure 1.1: Indifference curves for H- and L- type and $EX_L^* = 1$

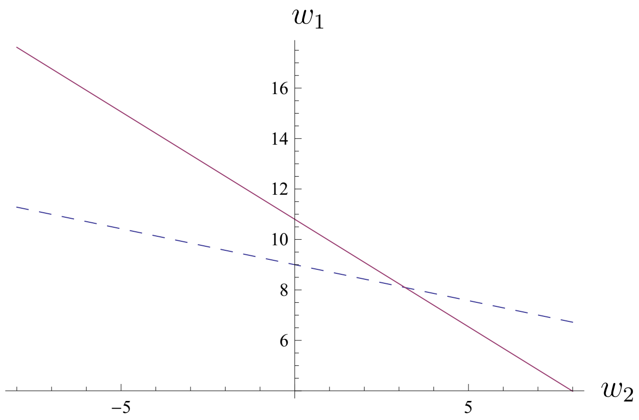


Figure 1.2: Indifference curves for H- and L- type and $EX_L^* = 1.5$

Figure 1.2 shows the efficient incentive-indifference curves for both types for the same specification as before except that EX_L increases to 1.5. In this scenario the low-productivity type has a relatively lower expected marginal benefit from the public good than the high-productivity type. The optimal second prize is positive in this case.

1.5.2 Concavity of the Objective Function

In this section we check whether the efficient prize structure (1.15) is compatible with the concavity of the individual maximization problems. A prize scheme leads to a concave individual optimization problem if

$$\underbrace{\sum_{j=1}^{2} \frac{\partial^2 P_i^j(x_i, x_{-i}^*)}{\partial x_i^2} w_j}_{\phi_i(x_i, x_{-i}^*, N_L, N_H)} \leq \left(c_i''(x_i) - E[\epsilon_i^2 V_i''\left(x_i \epsilon_i, x_{-i}^* \epsilon_{-i}\right)]\right) > 0 \; \forall i \in \mathcal{N}. \tag{1.16}$$

$N_L \diagdown x_H^*$	1	2	3	4	5	6
2	-	+	+	+	+	+
4	-	-	+	+	+	+
6	-	-	-	-	+	+
8	-	-	-	-	-	+
10	-	-	-	-	-	-

Table 1.1: Sign of $\phi_L(x_i, \mathbf{x}^*, N_L, N_H)$ for $EX_H^* = EX_L^*$ and $N_H = 1$

(1.16) defines a constraint on the prize scheme $\mathbf{w}^*$. Because we cannot define a finite and strictly positive lower bound for the right-hand side of (1.16) for general cost and utility functions, we characterize a sufficient condition for (1.16) to hold, namely that the left-hand side is negative or equal to zero.

If one denotes $\partial^2 P_i^j(x_i, x_{-i}^*)/\partial x_i^2$ by $ddP_i^j(x_i, x_{-i}^*)$, and evaluating at (1.15), the left-hand side of (1.16) becomes

$$\begin{aligned}\phi_i(x_i, \mathbf{x}^*, N_L, N_H) = & \frac{ddP_i^1(x_i, x_{-i}^*)(EX_H^* dP_L^{2*} - EX_L^* dP_H^{2*})}{dP_H^{1*} dP_L^{2*} - dP_L^{1*} dP_H^{2*}} \\ + & \frac{ddP_i^2(x_i, x_{-i}^*)(EX_L^* dP_H^{1*} - EX_H^* dP_L^{1*})}{dP_H^{1*} dP_L^{2*} - dP_L^{1*} dP_H^{2*}}, i = H, L. \quad (1.17)\end{aligned}$$

Unfortunately, (1.17) is not unambiguously negative for any combination of group-sizes and externality structures as the following simulation demonstrates. Let x_L^* equal 1. Table 1.1 displays the signs of $\phi_L(x_i, \mathbf{x}^*, N_L, N_H)$ if $EX_H^* = EX_L^*$ for different numbers of L-type individuals (rows) and different values for x_H^* (columns) if $N_H = 1$. The high-productivity type's second-order conditions are satisfied for all these parameter values.[9]

As one can see, high values of x_H^* together with low values of N_L are likely to cause the wrong sign of the second-order conditions for the L-type. The reason is that there is no competitive pressure within the H-type group, which implies that incentives for the H-type individual can only be generated by means of L-type individuals. This causes a potential conflict regarding the L-type incentives that is becoming the more severe the larger the differential $x_H^* - x_L^*$ and the lower the number of L-type individuals is: large differences in contributions create large differences in the probabilities to win the

[9]Note that as we restrain from using functional specifications the effect of a change in N_L or N_H on $\mathbf{x}^*$ cannot be included in the simulation. Hence going downwards from one cell to another cannot be interpreted as a change in N_L only but in combination with a change in the productivities such as to keep x_H^* constant. A horizontal move can be interpreted as a change in productivity only though, with N_L held constant.

$N_L \diagdown x_H^*$	1	3	5	7	9	11
2	-	-	-	+	+	+
4	-	-	-	-	+	+
6	-	-	-	-	-	-
8	-	-	-	-	-	-
10	-	-	-	-	-	-

Table 1.2: Sign of $\phi_L(x_i, \mathbf{x}^*, N_L, N_H)$ for $EX_H^* = EX_L^*$ and $N_H = 2$

prize, and low numbers of L-type individuals imply low competitive pressure within this group.

The case with only one H-type individual is a worst-case-scenario because it eliminates competition within this type-group. Increasing N_H to 2 leads to the following signs for $\phi_L(x_i, \mathbf{x}^*, N_L, N_H)$ in table 1.2 where we have increased the maximum difference $x_H^* - x_L^*$ to 10

As one can see, the general pattern that large differentials in contributions together with small numbers of L-type individuals may cause a positive sign of $\phi_L(x_i, \mathbf{x}^*, N_L, N_H)$ remains unchanged. However, the number of cases where the second-order conditions fail to hold is substantially reduced because of the stronger competition between H-type individuals.

The general intuition with respect to the sign of ϕ_i can be summarized as follows: the second-order conditions characterize a maximum if either $|x_H^* - x_L^*|$ is relatively small, the number of L-type and H-type individuals is relatively large, or both. In cases where either $|x_H^* - x_L^*|$ is sufficiently large or the number of H-type or L-type individuals is sufficiently small, the first-order conditions may characterize a minimum. To make this intuition more precise, Result 3 summarizes results for the case of symmetric externality structures whereas Result 4 summarizes results for the case of identical group sizes.

Result 3. *Assume that $EX_H^* = EX_L^*$ (individuals have the same expected marginal benefit from the public good). We then get the following results.*

(a) $\phi_H(x_i, \mathbf{x}^*, , N_L, N_H) < 0$.

(b) $\phi_L(x_i, \mathbf{x}^*, , N_L, N_H) < 0$ *for* $N_H \geq 3 \vee N_L \geq 2$.

The first part of Result 3 states that the H-type's second order condition is always fulfilled for identical marginal externalities. For the L-type we find in part 2 of result 3 that given at least 3 H-type individuals and 2 L-type individuals the second order conditions are always fulfilled. The picture portrayed by these results shows that given identical marginal externalities efficiency can be achieved by means of a multi-prize contest for a whole family of cost and utility functions if the number of individuals is only sufficiently large.

Result 4. *For general externality structures and $N_H = N_L = N$ we get the following (limit) behavior.*

(a) $\lim_{N\to\infty} \phi_H(x_i, \mathbf{x}^*, N_L, N_H) = \lim_{N\to\infty} \phi_L(x_i, \mathbf{x}^*, N_L, N_H) < 0 \Leftrightarrow \frac{E[v'_L(x^*\epsilon)]}{E[v'_H(x^*\epsilon)]} < \frac{x^*_H}{x^*_L}$.

(b) $\phi_L(x_i, \mathbf{x}^*, N_L, N_H) < 0$ *for* $E[v'_H(x^*\epsilon)] \geq E[v'_L(x^*\epsilon)] \vee N \geq 3$.

(c) $\phi_H(x_i, \mathbf{x}^*, N_L, N_H) < 0$ *for* $E[v'_H(x^*\epsilon)] \geq E[v'_L(x^*\epsilon)] \vee N \geq 2$.

The message contained in Result 4 is that the direct expected marginal benefit from an increase in public good $E[v'_i(x^*\epsilon)]$ should not be too large for the low-productivity individual compared to the high-productivity individual. Otherwise the low-productivity individual cannot be restrained from overproviding the public good. For example, if $x^*_H = 2x^*_L$, the latter's direct expected marginal benefit should not exceed twice that of the high productivity type for $N \to \infty$.

If the expected marginal benefit from the public good is higher for the H-type then under mild conditions on the minimum number of individuals efficiency can be attained.

1.5.3 Budget Balance

It follows from the direct-revelation principle that a multi-prize contest can at most be as efficient as the best direct mechanism.[10] We have seen in the prior sections that for a large class of problems it is in fact possible to shape incentives efficiently by means of contests. However, it remains to be shown whether the efficient prize structure balances the budget of the contest designer and is compatible with the participation constraints of the individuals.

Budget balance requires that $\sum w^*_i = Nt$. It is easily shown that if participation is compulsory[11] or voluntary but before individuals learn their types (*ex-ante* participation), with the reservation utility being the utility from the voluntary-contributions

[10]See Dasgupta, Hammond, and Maskin (1979), Holmstrom (1977), and Myerson (1979).

[11]See also Arrow (1979) for the case of continuous and Gradstein (1994) for the case of discrete public goods who show possibility results for the implementation of efficient mechanisms if the reservation utility of the individuals is becoming arbitrarily small. We consider this case as being the most relevant one when it comes to publicly provided contests because it is the prime objective of the state to execute compulsion. From an information-theoretic perspective the compulsory power of the state slackens the participation constraints and thereby allows to implement the optimal allocation under more general conditions.

equilibrium, implementation is possible. The more challenging case is where individuals can commit to participate in the contest after they have learned their types but before the outcome of the contest is realized, and participation is voluntary at this stage (*interim* participation). Their expected utilities are equal to

$$Eu_i(\mathbf{x}^*) = E[V_i(\mathbf{x}^*\epsilon)] + \sum_j p_i^j(\mathbf{x}^*)w_j^* + \bar{z} - t - c_i(x_i^*), \quad i = H, L. \tag{1.18}$$

We again determine the reservation utility with recourse to the voluntary-contributions equilibrium and the associated utility levels $Eu_i(\mathbf{x}^D)$. The direct mechanism is an expected-externality mechanism where the individual strategy space is their type space. The budget is balanced by an incentive-neutral transfer and hence independent of this type's decision. Individual expected utility at the efficient allocation with the direct (Bayesian) mechanism is

$$E[V_i(\mathbf{x}^*\epsilon)] - c_i(x_i^*) + \sum_{j\neq i} E[V_j(\mathbf{x}^*\epsilon)] - T + \bar{z}, \quad i = H, L. \tag{1.19}$$

T has to be chosen to balance the designer's budget and cannot depend on i's action. Since contributions x_j are not observable the transfer is of the following form:

$$T \leq \sum_{i=1}^{N} E[V_i(\mathbf{x}^*\epsilon)] - c_i(x_i^*) - Eu_i(\mathbf{x}^D) + \bar{z}, \quad i = H, L, \tag{1.20}$$

so that individual participation is guaranteed. The type for whom this equation is binding is decisive for the transfer

$$T = \min_{i=H,L}\{\sum_{j=1}^{N} E[V_j(\mathbf{x}^*\epsilon)] - c_i(x_i^*) + \bar{z} - Eu_i(\mathbf{x}^D)\}. \tag{1.21}$$

The important question for implementation is whether the deficit of this mechanism can be covered

$$\begin{aligned} NT = N\sum_{i=1}^{N} E[V_i(\mathbf{x}^*\epsilon)] - N\max_{i=H,L}\{c_i(x_i^*) + Eu_i(\mathbf{x}^D) - \bar{z}\} \geq (N-1)\sum_{i=1}^{N} E[V_i(\mathbf{x}^*\epsilon)] \\ \Leftrightarrow \qquad \sum_{i=1}^{N} E[V_i(\mathbf{x}^*\epsilon)] \geq N\max_{i=H,L}\{c_i(x_i^*) + Eu_i(\mathbf{x}^D) - \bar{z}\}. \end{aligned} \tag{1.22}$$

For $V_H(\mathbf{x}\epsilon) = V_L(\mathbf{x}\epsilon) = V(\mathbf{x}\epsilon)$ the deficit can be covered whenever the efficient allocation is preferable for each individual

$$E[V_i(\mathbf{x}^*\epsilon)] - c_i(x_i^*) + \bar{z} \geq Eu_i(\mathbf{x}^D). \tag{1.23}$$

Generally this need not be fulfilled. Rewriting the participation constraint for the contest

$$E[V_i(\mathbf{x}^*\epsilon)] - c_i(x_i^*) + \bar{z} + \sum_j p_i^j(\mathbf{x}^*)w_j^* - \frac{w_1 + w_2}{N} \geq Eu_i(\mathbf{x}^D), \quad i = H, L. \tag{1.24}$$

Here we see the additional term $\sum_j p_i^j(\mathbf{x}^*)w_j^* - \frac{w_1+w_2}{N}$ which will be positive for one type and negative for the other. Hence, the contest introduces a further distortion which might lead to a disadvantage for implementation. Note that this distortion will be larger if types are very different or if there are only a few individuals of each type. Hence we expect implementation to fail in similar cases as the second order conditions.

1.6 Conclusion

In this chapter we have analyzed whether it is possible to provide efficient incentives for the decentralized production of a public good by means of a multi-prize contest if only a noisy ranking of individual efforts is observable. Individuals are allowed to differ with respect to their costs of production as well as with respect to their utility from the consumption of the public good.

The general characterization of the problem has revealed that from an efficiency point of view only prize spreads matter. This finding generalizes the analysis of Lazear and Rosen (1981), who found a similar structure for the case of a two-player contest. Generally speaking, the dimension of the space of endogenous variables has to be equal to the dimension of the space of control variables in order to guarantee efficiency. If the relevant control variables are *prize spreads*, this implies that the absolute number of control variables has to *exceed* the number of endogenous variables. In fact we are able to show that for the case of a *two-prize contest and at least three individuals* it is possible to implement the efficient allocation in a setup similar to Lazear and Rosen's.

In addition to the general efficiency result, the two central insights of this chapter are as follows. (a) Optimal multi-prize contests are quite generally non-monotonic in prizes. We have demonstrated for the case of two different cost types that the optimal prize scheme will in fact always be $\vee$-shaped if both types have the same marginal utility function for the public good. Hence we extend the result of Malcomson (1986) who finds a $\wedge$-shaped compensation scheme for homogeneous individuals. The reason for this property is that, depending on both types' utility from the good, and given a single prize contest one type will be relatively more motivated given the efficient contributions. In a two-prize contest the difference in incentive intensity of both prizes on the two types can be used to balance incentives. Given that the low-productivity type is relatively more motivated this means increasing the spread between first and second prize. If it is the high-productivity type, a decrease in the prize spread is necessary. In the extreme it might even lead to a higher second prize. If in reality such a non-monotonicity of prizes is not feasible due to institutional reasons this implies that a multi-prize contest will typically not be able to solve this simple sorting problem. (b) Contrary to other incentive mechanisms, contests can only induce efficient incentives if there exists sufficient competition for the prizes. Hence, if there is a single high-productivity individual, it may be impossible to achieve efficiency because the only means by which this individual can be motivated is to induce the low-productivity individuals to invest more. Hence, if the cost-differential between types is becoming sufficiently large, this property may be in conflict with the concavity of the individuals' optimization problems. This finding points to an interesting implication of the analysis: if the within-group competition for the most-qualified individuals is too weak, it may be optimal to exempt them from a mechanism based on relative performance and to instead motivate them by conventional forms of incentive contracts.

1.A Proof of Lemma 1

Proof that Rank[**P**] = 2 for the case of two types of individuals and a population of at least 3 for the case of a Tullock CSF.

The following is the reduced **P**-Matrix with only one row for each type and all prizes except 1 and 2 set equal to zero. (With two types the maximal rank is two and hence we can choose $N-2$ prizes and delete all identical rows.) P_i^j stands for the multiple prize Tullock contest success function.

$$\begin{pmatrix} \frac{\partial P_H^1}{\partial x_H} & \frac{\partial P_H^2}{\partial x_H} \\ \frac{\partial P_L^1}{\partial x_L} & \frac{\partial P_L^2}{\partial x_L} \end{pmatrix} \tag{1.25}$$

The determinant of this matrix is equal to

$$\begin{aligned} &\frac{\partial P_H^1}{\partial x_H}\frac{\partial P_L^2}{\partial x_L} - \frac{\partial P_H^2}{\partial x_H}\frac{\partial P_L^1}{\partial x_L} = \\ &(x_H^* - x_L^*)((2N_H-1)x_H^* + (2N_L-1)x_L^*) \times \\ &\frac{\left(2(N_H-1)(N_L-1)x_H^*x_L^* + (N_H-1)N_H x_H^{*2} + (N_L-1)N_L x_L^{*2}\right)}{(N_H x_H^* + (N_L-1)x_L^*)^2((N_H-1)x_H^* + N_L x_L^*)^2(N_H x_H^* + N_L x_L^*)^2} \end{aligned} \tag{1.26}$$

which is always nonzero.

□

1.B Proof of Corollary 1

The optimal second prize given identical expected marginal externalities is equal to

$$\begin{aligned} w_2^* \quad = \quad & -\frac{E[v_i'(\mathbf{x}^*\epsilon)](N_H+N_L-1)(N_H x_H^* + (N_L-1)x_L^*)^2((N_H-1)x_H^* + N_L x_L^*)^2}{((N_H-1)N_H x_H^{*2} + 2(N_H-1)(N_L-1)x_L^* x_H^* + (N_L-1)N_L x_L^{*2})} \times \\ & \frac{1}{((2N_H-1)x_H^* + (2N_L-1)x_L^*)} \end{aligned} \tag{1.27}$$

which is clearly negative for $N_L, N_H \geq 1$, $N_L + N_H \geq 3$.

The optimal second prize for all possible externality structures is:

$$\begin{aligned} w_2^* = & -\frac{(N_H+N_L-1)(N_H x_H^* + (N_L-1)x_L^*)^2((N_H-1)x_H^* + N_L x_L^*)^2}{(x_H^* - x_L^*)((2N_H-1)x_H^* + (2N_L-1)x_L^*)} * \\ & \frac{(E[v_L'(\mathbf{x}^*\epsilon)]x_H^* - E[v_H'(\mathbf{x}^*\epsilon)]x_L^*)}{((N_H-1)N_H x_H^{*2} + 2(N_H-1)(N_L-1)x_L^* x_H^* + (N_L-1)N_L x_L^{*2})}. \end{aligned} \tag{1.28}$$

The sign depends on $E[v'_L]$ and $E[v'_H]$ and hence the expected marginal benefit from investing in the public good for each type. If we assume that $x^*_H > x^*_L$ the second prize will become positive if

$$\frac{E[v'_L(\mathbf{x}^*\epsilon)]}{E[v'_H(\mathbf{x}^*\epsilon)]} < \frac{x^*_L}{x^*_H}. \tag{1.29}$$

Hence if the expected marginal benefit from the public good is sufficiently small for the low-productivity type compared to the high-productivity type a positive second prize is optimal.

The condition on the first prize is a little more complicated. For identical marginal externality structures:

$$\begin{aligned}
w^*_1 =& \frac{E[v'_i(\mathbf{x}^*\epsilon)](N_H + N_L - 1)a}{((2N_H-1)x^*_H + (2N_L-1)x^*_L)\left((N_H-1)N_H x^{*2}_H + 2(N_H-1)(N_L-1)x^*_L x^*_H + (N_L-1)N_L x^{*2}_L\right)} \\
a =& (N_H-1)\,N_H^2\,(3N_H-1)\,x^{*4}_H + N_H\,(4N_L - 2 + N_H\,(8 - 7N_H + 3\,(4N_H-5)\,N_L))\,x^{*3}_H x^*_L \\
&+ \left(1 + 3\,(N_L-1)\,N_L + N_H\,(-3 + 2\,(8-9N_L)\,N_L) + 3N_H^2\,(1 + 6\,(-1+N_L)\,N_L)\right) x^{*2}_H x^{*2}_L \\
&+ N_L\,(-2 + (8-7N_L)\,N_L + N_H\,(4 + 3N_L\,(-5+4N_L)))\,x^*_H x^{*3}_L + (N_L-1)\,N_L^2\,(3N_L-1)\,x^{*4}_L.
\end{aligned} \tag{1.30}$$

This expression can be shown to be unambiguously positive if there are at least three individuals as we assume throughout the paper. For more general marginal externality structures the optimal prize becomes a complex function of all relevant variables and too unwieldy to be displayed here but can be attained upon request.

We take an indirect way of looking at the sign of this expression. Rearranging (1.14) to show the iso-incentive curves in w_1-w_2 space we get

$$\begin{aligned}
w_2^H(w_1) &= \frac{EX^*_H - dP^{1*}_H w_1}{dP^{2*}_H}, \\
w_2^L(w_1) &= \frac{EX^*_L - dP^{1*}_L w_1}{dP^{2*}_L}.
\end{aligned} \tag{1.31}$$

They are functions with slope $-\frac{dP^{1*}_i}{dP^{2*}_i}$ and axis intercept $\frac{EX^*_i}{dP^{2*}_i}$.

Fact 1. *The iso-incentive curve of the H-type is always less steep (steeper) in w_2-w_1 space (w_1-w_2 space).*

This can be shown by comparing $\frac{dP^{2*}_H}{dP^{1*}_H}$ and $\frac{dP^{2*}_L}{dP^{1*}_L}$. It is always true that $\frac{dP^{2*}_L}{dP^{1*}_L} > \frac{dP^{2*}_H}{dP^{1*}_H}$.

Since the iso-incentive curve of the H-type is steeper in w_1-w_2 space we need $\frac{EX^*_H}{dP^{2*}_H} \leq \frac{EX^*_L}{dP^{2*}_L}$ for $w^*_1 \leq 0$ - the intercept of $w_2^H(w_1)$ needs to be lower than that of $w_2^L(w_1)$.

For $w_1^* < 0$ we require that $\frac{EX_L^*}{EX_H^*} > \frac{dP_L^{2*}}{dP_H^{2*}}$. For $w_2^* > 0$ we require that $\frac{EX_L^*}{EX_H^*} > \frac{dP_L^{1*}}{dP_H^{1*}}$. By fact 1 we know that $\frac{dP_L^{1*}}{dP_H^{1*}} < \frac{dP_L^{2*}}{dP_H^{2*}}$. Hence $w_1^* < 0$ implies $w_2^* > 0$.

Fact 2. *For $x_H^* > x_L^*$ it is always true that $dP_H^{2*} < dP_L^{2*} \Leftrightarrow \frac{dP_L^{2*}}{dP_H^{2*}} > 1$.*

Together with $\frac{EX_L^*}{EX_H^*} > \frac{dP_L^{2*}}{dP_H^{2*}}$ this implies that $\frac{EX_L^*}{EX_H^*} > \frac{dP_L^{2*}}{dP_H^{2*}} > 1$ and hence $EX_L^* > EX_H^*$ which is only true for $E[v'_H] > E[v'_L]$ by definition of EX_i^*.

□

1.C Generalization to N Types of Individuals

In this appendix we discuss the generalized problem to implement the optimal allocation $\mathbf{x}^*$ by means of a multi-prize contest. Assume that there are potentially N different types and N different prizes. Then, it is a straightforward extension of the model to show that a necessary condition for a multi-prize contest to implement the optimal allocation is the following:

$$\begin{pmatrix} dP_1^1(x_1^*, x_{-1}^*) & \cdots & dP_i^N(x_1^*, x_{-1}^*) \\ \vdots & \ddots & \vdots \\ dP_N^1(x_N^*, x_{-N}^*) & \cdots & dP_N^N(x_N^*, x_{-N}^*) \end{pmatrix} \begin{pmatrix} w_1 \\ \vdots \\ w_N \end{pmatrix} = \begin{pmatrix} EX_1(x_1^*, x_{-1}^*) \\ \vdots \\ EX_N(x_N^*, x_{-N}^*) \end{pmatrix} \tag{1.32}$$

which as before is written as $\mathbf{Pw} = \mathbf{EX}$. Given that the optimization problem can be written in terms of prize spreads, it should not come as a surprise that it can be shown that the maximum rank of matrix $\mathbf{P}$ is $N-1$: Because $\sum_{j=1}^N P_i^j = 1 = \sum_{i=1}^N P_i^j$ for all $\mathbf{x}$, we know that

$$\sum_{j=1}^N dP_i^j = 0 \quad \Leftrightarrow \quad \sum_{j \neq N} dP_i^j = -dP_i^N, \tag{1.33}$$

which implies that the N-th column of $\mathbf{P}$ is a linear combination of the first $N-1$ columns. Hence, the maximum rank is $(N-1)$.

This finding implies that with N different individuals in the sense of different optimal x_i the contest mechanism cannot implement the efficient solution except if it happens that the augmented $\mathbf{P}|\mathbf{EX}$-matrix also has at most rank $N-1$.

Hence if there are $k < N$ groups of individuals that differ with respect to their cost functions, the maximum rank of $\mathbf{P}$ is k. The rank of $\mathbf{P}|\mathbf{EX}$ depends on the externality structure imposed by the individuals.

An important consequence is that it will be impossible for the contest mechanism to implement the efficient allocation if there is a subgroup of individuals which are identical in their cost functions (up to a constant of integration) but differ in their utility for the public good $V_i(x_i)$. For illustration let them be called j and k and $v'_j \neq v'_k$ (by more than a constant of integration). Then the row j and k of matrix **P** are identical as both contestants should choose the same level of research $x^*_k = x^*_j$ by condition 1.4. Hence their marginal probabilities of success will be identical. In contrast the jth and kth scalar in vector **EX**, the marginal externality on all others, will be different. Accordingly there is no vector of prizes **w** which can implement the efficient allocation. If individuals have identical cost functions their utility functions for the public good must be identical, too, up to a constant of integration. This is not too surprising though, as individuals with the same costs but different utilities can be seen as different types as well.

1.D Proof of Results 3 and 4

Proof of Result 3: All parts of the result focus on the sign of $\phi_i(.)$ and therefore constitute sufficient conditions for the concavity of the objective functions. Using the Tullock function, for $EX^*_H = EX^*_L$ (meaning $E[v'_H] = E[v'_L]$), $\phi_i(.)$ reduces to:

$$\phi_H = -\frac{2v'_H(N_H + N_L - 1)}{(N_H\,x^*_H + (N_L-1)\,x^*_L)((N_H-1)\,x^*_H + n\,x^*_L)\,(N_H\,x^*_H + N_L\,x^*_L)\,((2\,N_H-1)\,x^*_H + (2\,N_L-1)\,x^*_L)}\,\Gamma \tag{1.34}$$

where:

$$\begin{aligned}
\Gamma \quad = \quad & \frac{(N_H-1)\,N_H^2\,(1+(N_H-1)\,N_H)\,{x^*_H}^5}{((N_H-1)\,N_H\,{x^*_H}^2 + 2\,(N_H-1)\,(N_L-1)\,x^*_H\,x^*_L + (N_L-1)\,N_L\,{x^*_L}^2)} \\
+ \quad & \frac{(N_H-1)\,(N_L + N_H\,(-3+N_H\,(4-3\,N_L+N_H\,(5\,N_L-4))))\,{x^*_H}^4\,X^*_L}{((N_H-1)\,N_H\,{x^*_H}^2 + 2\,(N_H-1)\,(N_L-1)\,x^*_H\,x^*_L + (N_L-1)\,N_L\,{x^*_L}^2)} \\
+ \quad & \frac{(N_H-1)\,(2-3\,N_L-2\,N_L^2+N_H\,(-3+(11-2\,N_L)\,N_L)+2\,N_H^2\,(2+N_L\,(-8+5\,N_L)))\,{x^*_H}^3\,{x^*_L}^2}{((N_H-1)\,N_H\,{x^*_H}^2 + 2\,(N_H-1)\,(N_L-1)\,x^*_H\,x^*_L + (N_L-1)\,N_L\,{x^*_L}^2)} \\
+ \quad & \frac{(-(N_L\,(-5+N_L\,(7+N_L)))+N_H\,(1+N_L\,(-17+30\,N_L-8\,N_L^2))+N_H^2\,(-1+2\,N_L\,(6+N_L\,(-12+5\,N_L))))\,{x^*_H}^2\,{x^*_L}^3}{((N_H-1)\,N_H\,{x^*_H}^2 + 2\,(N_H-1)\,(N_L-1)\,x^*_H\,x^*_L + (N_L-1)\,N_L\,{x^*_L}^2)} \\
+ \quad & \frac{N_L\,(2+N_L\,(-10+(11-2\,N_L)\,N_L)+N_H\,(-2+(-2+N_L)\,N_L\,(-6+5\,N_L)))\,x^*_H\,{X^*_L}^4+(N_L-1)\,N_L^2\,(1+(-3+N_L)\,N_L)\,{x^*_L}^5)}{((N_H-1)\,N_H\,{x^*_H}^2 + 2\,(N_H-1)\,(N_L-1)\,x^*_H\,x^*_L + (N_L-1)\,N_L\,{x^*_L}^2)}
\end{aligned}$$

and

$$\phi_L = -\frac{2\,v'_H\,(N_H + N_L - 1)}{(N_H\,x^*_H + (N_L-1)\,x^*_N)\,((N_H-1)\,x^*_H + N_L\,x^*_N)\,(N_H\,x^*_H + N_L\,x^*_N)\,((2\,N_H-1)\,x^*_H + (2\,N_L-1)\,x^*_N)}\,\Upsilon \tag{1.35}$$

where

$$
\begin{aligned}
\Upsilon &= \frac{(N_H-1)\,N_H^2\,(1+(-3+N_H)\,N_H)\,{x_H^*}^5}{((N_H-1)\,N_H\,{x_H^*}^2+2\,(N_H-1)\,(N_L-1)\,x_H^*\,x_N^*+(N_L-1)\,N_L\,{x_N^*}^2)} \\
&+ \frac{N_H\,(2-2\,N_L+N_H\,(-10+11\,N_H-2\,N_H^2+(-2+N_H)\,(-6+5\,N_H)\,N_L))\,{x_H^*}^4\,x_N^*}{((N_H-1)\,N_H\,{x_H^*}^2+2\,(N_H-1)\,(N_L-1)\,x_H^*\,x_N^*+(N_L-1)\,N_L\,{x_N^*}^2)} \\
&+ \frac{(N_L-N_L^2+N_H\,(N_L-1)\,(-5+12\,N_L)+N_H^2\,(-7+6\,(5-4\,N_L)\,N_L)+N_H^3\,(-1+2\,N_L\,(-4+5\,N_L)))\,{x_H^*}^3\,{x_N^*}^2}{((N_H-1)\,N_H\,{x_H^*}^2+2\,(N_H-1)\,(N_L-1)\,x_H^*\,x_N^*+(N_L-1)\,N_L\,{x_N^*}^2)} \\
&+ \frac{((N_L-1)\,(2+N_L\,(-3+4\,N_L)+N_H\,(-3+(11-16\,N_L)\,N_L)+2\,N_H^2\,(-1+N_L\,(-1+5\,N_L)))\,{x_H^*}^2\,{x_N^*}^3}{((N_H-1)\,N_H\,{x_H^*}^2+2\,(N_H-1)\,(N_L-1)\,x_H^*\,x_N^*+(N_L-1)\,N_L\,{x_N^*}^2)} \\
&+ \frac{(N_L-1)\,(N_H-3\,N_H\,N_L^2+(-4+5\,N_H)\,N_L^3+N_L\,(-3+4\,N_L))\,x_H^*\,{x_N^*}^4+(N_L-1)\,N_L^2\,(1+(N_L-1)\,N_L)\,{x_N^*}^5}{((N_H-1)\,N_H\,{x_H^*}^2+2\,(N_H-1)\,(N_L-1)\,x_H^*\,x_N^*+(N_L-1)\,N_L\,{x_N^*}^2)}
\end{aligned}
$$

The denominators are unambiguously positive. The numerators can be divided by $-2v'_H(N_H+N_L-1)$, which is unambiguously negative.

$$
\begin{aligned}
\varphi_H &= (N_H-1)\,N_H^2\,(1+(N_H-1)\,N_H)\,{x_H^*}^5+(N_H-1)\,(N_L+N_H\,(-3+N_H\,(4-3\,N_L+N_H\,(-4+5\,N_L))))\,{x_H^*}^4\,x_L^* \\
&+ (N_H-1)\,(2-3\,N_L-2\,N_L^2+N_H\,(-3+(11-2\,N_L)\,N_L)+2\,N_H^2\,(2+N_L\,(-8+5\,N_L)))\,{x_H^*}^3\,{x_L^*}^2 \\
&+ (-(N_L\,(-5+N_L\,(7+N_L)))+N_H\,(1+N_L\,(-17+30\,N_L-8\,N_L^2))+N_H^2\,(-1+2\,N_L\,(6+N_L\,(-12+5\,N_L))))\,{x_H^*}^2\,{x_L^*}^3 \\
&+ N_L\,(2+N_L\,(-10+(11-2\,N_L)\,N_L)++N_H\,(-2+(-2+N_L)\,N_L\,(-6+5\,N_L)))\,x_H^*\,{x_L^*}^4 \\
&+ (N_L-1)\,N_L^2\,(1+(-3+N_L)\,N_L)\,{x_L^*}^5
\end{aligned}
$$

$$
\begin{aligned}
\varphi_L &= (N_H-1)\,N_H^2\,(1+(-3+N_H)\,N_H)\,{x_H^*}^5+N_H\,(2-2\,N_L+N_H\,(-10+11\,N_H-2\,N_H^2+(-2+N_H)\,(-6+5\,N_H)\,N_L))\,{x_H^*}^4\,x_L^* \\
&+ (N_L-N_L^2+N_H\,(N_L-1)\,(-5+12\,N_L)+N_H^2\,(-7+6\,(5-4\,N_L)\,N_L)+N_H^3\,(-1+2\,N_L\,(-4+5\,N_L)))\,{x_H^*}^3\,{x_L^*}^2 \\
&+ (N_L-1)\,(2+N_L\,(-3+4\,N_L)+N_H\,(-3+(11-16\,N_L)\,N_L)+2\,N_H^2\,(-1+N_L\,(-1+5\,N_L)))\,{x_H^*}^2\,{x_L^*}^3 \\
&+ (N_L-1)\,(N_H-3\,N_H\,N_L^2+(-4+5\,N_H)\,N_L^3+N_L\,(-3+4\,N_L))\,x_H^*\,{x_L^*}^4+(N_L-1)\,N_L^2\,(1+(N_L-1)\,N_L)\,{x_L^*}^5
\end{aligned}
$$

If this remaining part is larger or equal to zero, the optimal prize scheme constitutes an individual maximum.

Calculations reveal that φ_H is always positive for $N_H, N_L \geq 1$, $N_H + N_L \geq 3$ and $x_H^* > x_L^* > 0$, and φ_L is positive for $N_H \geq 3$, $N_L \geq 2$ and $x_H^* > x_L^* > 0$.

Proof of Result 4: The second order condition for the H-type is equal to

$$
\begin{aligned}
\phi_H &= \frac{-2\left(E[v'_L]b_1+E[v'_H]b_2\right)}{\left(-2x_H^{*2}x_L^{*2}+3Nx_H^*x_L^*\left(x_H^*+x_L^*\right)^2+N^3\left(x_H^*+x_L^*\right)^4-N^2\left(x_H^*+x_L^*\right)^2\left(x_H^{*2}+4x_H^*x_L^*+x_L^{*2}\right)\right)} \times \\
&\quad \frac{1}{N\left(x_H^*-x_L^*\right)\left(x_H^*+x_L^*\right)^2}. \qquad (1.36)
\end{aligned}
$$

with

$$
\begin{aligned}
b_1 &= 2x_H^{*3}x_L^{*3}-N^4x_L^*(x_H^*+x_L^*)^5+N^3(x_H^*+x_L^*)^6+Nx_H^{*2}x_L^*(x_H^*+x_L^*)^2\left(x_H^*+3x_L^*\right) \\
&\quad -3N^2x_H^*x_L^*(x_H^*+x_L^*)^2\left(2x_H^{*2}+x_H^*x_L^*+x_L^{*2}\right) \qquad (1.37)
\end{aligned}
$$

and

$$b_2 = 2x_H^{*4}x_L^{*2} + N^4 x_H^*(x_H^* + x_L^*)^5 - N x_H^* x_L^*(x_H^* + x_L^*)^2 \left(3x_H^{*2} + 3x_H^* x_L^* - 2x_L^{*2}\right) \\ -2N^3(x_H^* + x_L^*)^4 \left(x_H^{*2} + 2x_H^* x_L^* - x_L^{*2}\right) + N^2(x_H^* + x_L^*)^2 \left(x_H^{*4} + 7x_H^{*3}x_L^* + 9x_H^{*2}x_L^{*2} - 4x_H^* x_L^{*3} - x_L^{*4}\right). \quad (1.38)$$

The second order condition for the L-type is equal to

$$\phi_L = \frac{-2\left(-E[v'_H]b_3 - E[v'_L]b_4\right)}{\left(-2x_H^{*2}x_L^{*2} + 3N x_H^* x_L^* \left(x_H^* + x_L^*\right)^2 + N^3\left(x_H^* + x_L^*\right)^4 - N^2\left(x_H^* + x_L^*\right)^2\left(x_H^{*2} + 4x_H^* x_L^* + x_L^{*2}\right)\right)} \times \\ \frac{1}{N\left(x_H^* - x_L^*\right)\left(x_H^* + x_L^*\right)^2} \quad (1.39)$$

with

$$b_3 = -2x_H^{*3}x_L^{*3} - N^4 x_H^*(x_H^* + x_L^*)^5 + N^3(x_H^* + x_L^*)^6 + N x_H^* x_L^{*2}(x_H^* + x_L^*)^2 \left(3x_H^* + x_L^*\right) \\ -3N^2 x_H^* x_L^*(x_H^* + x_L^*)^2 \left(x_H^{*2} + x_H^* x_L^* + 2x_L^{*2}\right) \quad (1.40)$$

and

$$b_4 = 2x_H^{*2}x_L^{*4} + N^4 x_L^*(x_H^* + x_L^*)^5 + N x_H^* x_L^*(x_H^* + x_L^*)^2 \left(2x_H^{*2} - 3x_H^* x_L^* - 3x_L^{*2}\right) \\ +2N^3(x_H^* + x_L^*)^4 \left(x_H^{*2} - 2x_H^* x_L^* - x_L^{*2}\right) - N^2(x_H^* + x_L^*)^2 \left(x_H^{*4} + 4x_H^{*3}x_L^* - 9x_H^{*2}x_L^{*2} - 7x_H^* x_L^{*3} - x_L^{*4}\right) \quad (1.41)$$

The denominator of $\phi_i(.)$ is unambiguously positive also for general externality structures:

Hence, a sufficient condition for the concavity of the objective function is that the numerator of $\phi_i(.)$ is negative or zero. Call them φ_H, φ_L respectively.

Taking the limit of ϕ_H, ϕ_L for $N \to \infty$, one arrives at

sign$[\lim_{N\to\infty} \varphi_H]$ = sign$[\lim_{N\to\infty} \varphi_L]$ = sign$[E[v'_L]x_L^* - E[v'_H]x_H^*] < 0 \Leftrightarrow E[v'_L]/E[v'_H] < x_H^*/x_L^*$.

Furthermore calculations reveal that $\varphi_H < 0$ is always true given that $N \geq 2 \vee E[v'_H] \geq E[v'_L] > 0$ and $\varphi_L < 0$ is always true given that $N \geq 3 \vee E[v'_H] \geq E[v'_L] > 0$.

Chapter 2

“Where Ignorance is Bliss, ’tis Folly to be Wise”: Transparency and Welfare in Contests

Joint with Philipp Denter

2.1 Introduction

Transparency is widely seen as a remedy for agency problems. Transparency laws are easy to understand. They are very popular with politicians as well as the public. As the New York Times states “...the ideal of transparency has become as patriotic as apple pie in the post-Enron era” (The New York Times (2006)). Hence it is important to understand the implications of transparency policy. Typically, transparency works by holding the responsible actors accountable for their actions, thus making undesirable behavior less likely. Examples abound. Banking transparency and disclosure of bank activities are suggested to prevent future banking crises, money laundering, tax evasion, and other fraud. Transparency of CEO and top management wages is supposed to stop firms from making secret deals and overpaying their managers. In politics, transparency is supposed to impede selfish and corrupt behavior by politicians. But accountability is not the only implication of transparency. In this chapter we identify an aspect of transparency that is often neglected in the public debate. We show how transparency in competitive environments can have bad consequences for society – it can sharpen wasteful competition while at the same time reducing efficiency.

Consider some examples of competitive environments in which transparency policy is an issue: political campaigning, international relations, firm competition and lobbying, especially in form of rent-seeking. In the U.S., transparency in political campaigning is regulated by the Federal Election Campaign Act (FECA). It requires candidates to disclose sources of campaign contributions and campaign expenditure quarterly. The United States Supreme Court recently ruled in *Citizens United vs. Federal Election Commission* that corporate funding of political broadcasts in elections cannot be limited under the First Amendment, thus further increasing transparency. Not only is the public opinion affected by contributions disclosure but also the campaigners themselves. Disclosure of campaign contributions conveys information about the (future) financial support of a candidate and this in turn influences the outcome of the election.

Another competitive setting where transparency policy matters is international relations. Take for example transparency about nuclear armament. The amount of nuclear arms a country possesses is an indicator of its military potential, which in turn is a determinant of its bargaining power on the international stage. Recently the Obama administration formally disclosed the size of the U.S. Defense Department's stockpile of nuclear weapons: 5113 warheads as of September 30, 2009 (The Federation of American Scientists (2010)). Other countries like Israel, China or Pakistan prefer a policy of opacity.

Now consider competition between firms. In the U.S., the Securities and Exchange Commission (SEC) as well as the Federal Accounting Standards Board (FASB) regulate firms' disclosure of financial information. This information is not only accessible by stakeholders of a firm but also by its competitors, which has implications for competition between firms if private information is revealed. Our results shed light on how mandatory disclosure influences competition in winner-take-all markets, or more generally markets where competition can be represented by a contest. This is for example the case in advertising intensive markets, like the market for softdrinks.

Finally, transparency policy has also received a lot of attention in lobbying. In the U.S., lobbyists are required to disclose their client's lobbying issues and expenditures quarterly by the Lobbying Disclosure Act of 1995 and its 2007 amendment. On the other hand, lobbying disclosure in the European Union works solely on a voluntary basis. Lobbyists can choose to register with the EU register of interest representatives, follow their code of conduct and disclose their expenditures annually. Many firms and organizations actually do report their lobbying expenditures voluntarily. There is some evidence that average reported expenditures are lower in the EU than in the U.S.[1] While this can have many reasons, we offer one explanation which is consistent with these facts.

Our main results are:

- Mandating disclosure in a competitive environment can be a poor policy. We identify conditions where it leads to increased competition and less efficient outcomes.

- Decentralizing information disclosure is often beneficial. We identify conditions where competing groups will agree to transparency decisions, benefiting both the competitors and society at large.

- As the outcome of the contest becomes more sensitive to contest expenditures (e.g. luck and outside factors become less important), decentralized agreement becomes less likely. In these circumstances, a laissez-faire transparency rule is not optimal either.

Our main results may be illustrated in the following simple setting: Two groups are vying for some prize. One of these groups (the rival) has a known valuation for the prize while the valuation of the other group is (potentially) unknown, and may be either high or low. The key intuition underlying all of the results stems from the following observation: Competition is fiercest when the two rivals have similar valuations and milder when valuations diverge. Thus, if the disclosing group faces a strong opponent, competition will be fierce if it discloses a high valuation and mild when its value is revealed to be low. Since not disclosing leads to an intermediate level of competition, low valuation groups prefer to reveal while high valuation groups do not. The reverse is true when the disclosing group faces a relatively weak opponent: high valuation groups prefer disclosure while low valued groups prefer opacity. How does this translate into a group's *ex ante* disclosure policy? A group's expected payoffs are dominated by how it fares when it has a high valuation since this raises both the benefits and chances of winning the contest. As a result, the optimal policy is to disclose when the rival is relatively weak and to remain opaque when the rival is relatively strong.

[1] Friends of the Earth Europe (2010) show that 60% of the 50 largest firms disclosed voluntarily in 2008 and that they were reporting on average more lobbying expenditures in the U.S. than in their home market.

Now, let us consider the opposite situation—the decision of the rival to acquire information. While better information helps the rival to choose an optimal effort level, if the decision to acquire information is revealed, then its opponent will also respond. When the rival is relatively strong, it is better off not acquiring information since, if this information reveals that its opponent has a high valuation, competition is sharpened while if the opponent is revealed to have a low valuation, then the rival can no longer credibly commit to deter its opponent through overinvestment. Thus, information acquisition is unambiguously bad. On the other hand, when the rival is relatively weak, acquiring information reduces the efforts of the opponent regardless of valuation—in the case of high valuation, it stems from the revealed divergence of values while in the case of low valuation, it stems from discouragement.

A central insight to emerge from this analysis is that, despite the fact that the two sides have opposing interests in that both want to win, they agree that less "effort", ceteris paribus, is good. Since information sharing affects the degree of competition, there is scope for agreement. Furthermore information sharing not only influences the degree of competition but also the efficiency in allocating the prize to the party who values winning most. Surprisingly, agreement on reduced competition often also leads to greater efficiency in allocating the prize. When information sharing is optimal, it results in greater separation in the efforts of the two parties and, as a result, the prize is awarded to the higher valued group more often. Likewise, when information sharing is not optimal, it again results in greater separation of efforts. Thus, endogenous information sharing leads to *ex ante* Pareto gains. In this circumstance, mandatory disclosure policies merely serve to increase wasteful competition and distort prize allocations.

This chapter is organized as follows. Next we survey the related literature. Section 2 introduces the model. Section 3 studies information acquisition and section 4 disclosure incentives separately while section 5 puts the two decisions together. Section 6 considers a more general contest success function and section 7 draws conclusions for the desirability of mandatory disclosure policy. Section 8 studies the robustness of our findings with respect to the discriminatoryness of the competition. Section 9 concludes.

Literature Review

The nearest antecedent to our work is Kovenock, Morath, and Münster (2010), who study information disclosure between firms when the contest outcome is very sensitive to contest expenditures. Our concerns are with both information disclosure and acquisition and how they relate to the sensitivity of the contest outcome to expenditures. Baik and Shogren (1995) study the effects of spying and information acquisition in contest games. To gain tractability, they abstract away from strategic considerations in the expenditures themselves – essentially, the contest game is decision-theoretic. Our analysis, however, highlights the importance of the strategic interaction between acquisition/disclosure and contest expenditures. Indeed, our main result is that acquisition changes the behavior not just of the party gaining new information but also the party whose information was disclosed.

Information transmission from lobbies to the policy maker through lobbying has been studied for example by Potters and van Winden (1992), Lagerlöf (2007) and Grossman and Helpman (2001). The focus of this literature is on the welfare implications of lobbying when lobbyists have private information which is relevant to the policy maker and the policy maker attempts to learn by observing lobbying expenditures. In contrast we focus on information transmission between lobbyists and its implications for welfare and efficiency, and highlight consequences for disclosure policy.

The incentives for information sharing and the effect of mandatory disclosure law have been studies in the context of Cournot and Bertrand competition, e.g. Li (1985), Shapiro (1986) and Darrough (1993). We complement this literature by analyzing the incentives to disclosure as well as acquire information and the effect of mandatory disclosure policy in situations where competition can be represented by a contest. In contrast to this literature we have situations in mind where expenditures are at least partially wasteful to society, as for example in military conflict, advertising competition or lobbying.

One of our main results is to show that it can be optimal for a lobbying group or firm to remain ignorant about the valuation its rival places on “winning” the contest. The strategic value of ignorance has also been shown in the context of agency theory. A principal may benefit from ignorance as it alters the agent’s incentives to exert effort. The agent may benefit as well, as ignorance may make it harder for the principal to extract rents. Papers highlighting these effects are for example Dewatripont and Maskin (1995), Barros (1997) and Kessler (1998). While this literature focusses on vertical relationships between two distinct parties, in our model the focus is on competing parties who are essentially identical.

Information disclosure has also been studied in the context of goods markets, e.g. Jovanovic (1982), Milgrom (2008) and Daughety and Reinganum (2008), where the focus is on whether markets lead to optimal incentives for firms to disclose information about the quality of their goods. This literature revolves around the trade-off that disclosure is beneficial for the consumer but costly to the seller. In contrast, we show that mandatory disclosure can be harmful even without direct monetary costs, purely through its strategic effect.

Asymmetric information in contests has also been much studied (e.g. Hurley and Shogren (1998b), Katsenos (2009) and Moldovanu and Sela (2001)), although this literature has mainly ignored voluntary information disclosure and acquisition and the consequences for mandatory disclosure policy. Also the role of commitment in contests has received ample attention, see for example Dixit (1987), Baik and Shogren (1992), Morgan (2003), Morgan and Várdy (2007), Yildirim (2005) and Fu (2006), though the form of commitment typically consists of committing to a sequence of moves. In contrast we study contests where players are able to commit to certain informational regimes.

2.2 The Model

While we couch the model in the context of lobbying, it is easily translated into other competitive situations. See footnote 3 for an example. Consider two lobbying groups $i = A, B$ who vie for favorable legislation to be passed. Success yields lobby i a value v_i while failure yields zero. To affect the chances of success, each group chooses lobbying effort x_i. The chance that i is successful depends on the contest success function (CSF):

$$p_i\left(x_i, x_j\right) = \frac{x_i}{x_i + x_j}. \tag{2.1}$$

If both groups choose zero lobbying effort ($x_i = 0$) a coin toss determines success. Lobbyists are risk-neutral with a constant marginal cost of effort normalized to one. While each lobbying group knows its own valuation for success, information about the other party differs. In particular, the valuation of group A is commonly known while group B has private information about its value. One can think of this situation arising when group A is an "incumbent" who has engaged in many past fights over related issues while group B is a newcomer or, alternatively, where publicly available information makes it easy to estimate A's value while B's value, perhaps being more subjective, is harder for outsiders to estimate. For simplicity, we assume that B's value is binary—it is either low, $v_B = v_L$, with probability q or high, $v_B = v_H$, with the complementary probability.[2] The payoff functions are equal to

$$\begin{aligned}\pi_B &= \frac{x_B}{x_B + x_A} v_B - x_B \\ \pi_A &= \left(q \frac{x_A}{x_{BL} + x_A} + (1-q) \frac{x_A}{x_{BH} + x_A} \right) v_A - x_A.\end{aligned}$$

We focus on the case where there is uncertainty as to the identity of the higher valued lobbying group, i.e., when $v_A \in [v_L, v_H]$. Otherwise the efficient policy is obvious. Furthermore we assume that the policy is valuable enough for all lobbying groups to choose strictly positive lobbying effort.[3]

2.3 Information Disclosure

In the European Union, 60% of the top 50 European companies voluntarily disclosed their lobbying issues and expenditures in the EU register of interest representatives in the year 2008[4]. This information enables other lobbyists to infer something about their opponent's valuation for the legislation at stake. Instead of making a decision about lobbying expenditures under uncertainty about the opponent's valuation, a lobbying group can then, in the extreme case, decide on expenditures knowing the valuation of its opponent. Hence it can potentially make a better decision as to its optimal lobbying strategy. Since lobbying is a competitive activity giving one's opponent an advantage is not desirable. So why do lobbyists disclose information voluntarily?

In reality disclosure can take many forms. It can range from merely disclosing information about ones mutable actions (e.g. expenditures) to revealing information about inherent characteristics (e.g. costs or valuations). The latter is a stronger form of disclosure as disclosing information about an action can transmit information about characteristics, but not necessarily so. To keep the analysis simple we assume that disclosure is directly related to inherent characteristics. In the extreme case when

[2] In the appendix, we show that qualitatively similar results are obtained when B's distribution of values occurs on a continuum.

[3] We can easily reframe our model in terms of another introductory example – political campaigns. Two politicians $i = A, B$ are campaigning for a political office. The political office yields i a value v_i while failure yields a value normalized to zero. To affect the chances of success, each politician chooses some amount of campaign expenditures x_i. The chance that i is successful depends on the contest success function (CSF) defined in equation 2.2. The talent of the incumbent politician is more or less common knowledge and hence his value for office v_A is known. For the newcomer we assume the value is low with probability q and high else.

[4] The EU register of interest representatives can be found online at: https://webgate.ec.europa.eu/transparency/regrin/welcome.do?locale=en.

disclosure need not reveal anything about characteristics, mandatory disclosure policy has at best no effect on competition in our model. In the appendix we extend our analysis to a richer dynamic model, where actions / expenditures can be disclosed in an ongoing competition.

Let us assume that lobbying group B can choose to credibly disclose its valuation to lobbying group A before the contest. We consider two possibilities. Either group B commits to a disclosure policy before learning its value, and disclosure is costless and truthful. Alternatively, we consider a situation where group B decides whether to disclose its value after it learns about it, but disclosure is costly. Lobbying group B can signal its value by submitting a costly signal to lobbying group A. We do not make any judgement which is the more realistic scenario — in fact we show shortly that both situations lead to a qualitatively similar outcome.

Even though disclosure enables the opponent to make a more informed decision, this does not necessarily mean that the disclosing group is hurt by revealing information. For example if the opponent learns that the group has a very high valuation it will optimally react by lowering its expenditures, as its chances of success are so slim, and this is beneficial for both groups. On the other hand, if the opponent learns the lobbying group has a very low valuation, it might also find it beneficial to lower its expenditures, as not much is needed for success.

We find that whether lobbying group B knows its valuation or not, information is only disclosed when B faces a relatively weak group A. Formally,

Proposition 1. *a) Assume lobbying group B does not know its value yet. If lobbying group B expects to be relatively weak compared to lobbying group A ($\sqrt{v_L v_H} < v_A$) it strictly prefers not to disclose its valuation and votes against mandatory disclosure. On the other hand, a lobbying group B with a high expected valuation ($\sqrt{v_L v_H} > v_A$) always votes in favor of mandatory disclosure.*

b) After lobbying group B learns its valuation and given the chance to send a costly signal before the contest to group A, only a high-value lobbying group credibly reveals its valuation. This is only profitable in a situation where group A is relatively weak ($\sqrt{v_L v_H} > v_A$). Otherwise no information is disclosed.

Proof. See appendix. □

To make the intuition behind Proposition 1 clearer let us first look at the incentives of a high- and a low-value lobbying group B separately. A high-value lobbying group

B will prefer disclosure if it can discourage lobbying group A from expending lobbying effort. This is the case whenever it is relatively strong, or $v_A < \sqrt{v_L v_H}$. For $v_A \geq \sqrt{v_L v_H}$ disclosing makes A more aggressive, as it learns that its opponent is quite similar. The opposite is true for a weak lobbying group B. When facing a strong group A it prefers to disclose its valuation, as A will react with lower lobbying effort. If A is weak on the other hand, revealing its valuation makes competition stronger, as A learns that it is facing a similarly strong opponent. The weak and the strong lobbying group B's incentives are never aligned. If disclosing is beneficial for one, it is harmful to the other. From an ex-ante point of view, before learning its valuation, the strong lobbying groups' interests always dominate though. The reason is that an increase in success probability in case the value is high is worth more than in case the value turns out to be low. This intuition carries over to the second part of Proposition 1 as well. A lobbying group with a high valuation stands to gain more from a decrease in A's lobbying effort. This means that it is willing to expend more signaling effort than a low-value group. If it is in its interest, it will always be able to imitate a low-value group's signal so that no information can be credibly disclosed. Hence against a strong group A information will never be disclosed because it is detrimental to the high-value group, while against a weak group A the high-value group is willing to credibly disclose its valuation through the costly signal.

2.4 Information Acquisition

We learned in the previous section that lobbying group B uses disclosure to reduce competition by lobbying group A. But when lobbying group B chooses not disclose, it might well be in A's interest to go acquire information about the valuation of lobbying group B itself. Also, it might choose to look away when group B decides to disclose. In terms of our model, suppose that it were costless for group A to acquire a credible report as to B's valuation and consider lobbying group A's incentives for information acquisition.

One might be tempted to draw an analogy with a bargaining situation. In effect, A and B are negotiating (through their efforts) on who will receive the valuable legislative prize. The usual advice in such situations is to "know thy enemy." That is, group A should gather as much information as possible about group B, including its valuation. This information will enable it to make the best possible decision regarding its negotiation strategy, which can now be type-specific. Since information gathering is costless, it seems obvious that the optimal strategy is complete information gathering.

Where the analogy breaks down is in the form of the "negotiation" between the two parties. Here, success will be determined by performance in an imperfectly discriminating contest; thus, there is an integrative as well as distributive aspect to the "negotiation." In particular, both lobbying groups benefit if lobbying efforts are more muted and, since only relative lobbying efforts determine the outcome, equilibrium success probabilities would be unaffected if both sides could agree to scale down their efforts.

But how can ignorance enable the lobbying groups to scale down effort? Consider a lobbying group A which has a valuation above the average of lobbying group B. If it knew for sure it faces a strong group B, competition between the similarly strong groups would be very intense. But the chance to encounter a much weaker group B diminishes A's investment incentive, and hence also the strong group B's reaction. On the other hand, A overinvests against a weak group B to increase its chances in case its opponent turns out to be strong. The weak group B will react to this discouragement by lowering its investment. By optimally choosing to remain ignorant about lobbying group B's valuation, A can on the one hand discourage a weaker rival and on the other hand appease a stronger rival, thereby softening the competition between the two lobbies. Thus, unlike a decision-theoretic or negotiation context, rent-seeking competition between the two parties creates a value to ignorance.

A sharp illustration of this intuition may be seen for the case where group A has diffuse priors (i.e. $q = 1/2$). Here we show that, when group A is strong compared to B, it prefers to remain ignorant while when it is weak, it seeks information to mitigate this disadvantage. Formally,

Proposition 2. *If lobbying group A is relatively strong compared to group B ($v_A > \sqrt{v_L v_H}$) it strictly prefers not to acquire any information about B's value while a relatively weak lobbying group A ($v_A < \sqrt{v_L v_H}$) always acquires costless information about group B.*

Proof. See appendix. □

Notice that the conditions for information disclosure/withholding in Proposition 1 are identical to those in Proposition 2 when group A is determining whether to pursue this information. That is, despite competing with one another, both groups agree on information revelation. We formalize this observation in Proposition 3 below. Figure 2.1 illustrates the intuition behind the value to ignorance graphically. It shows

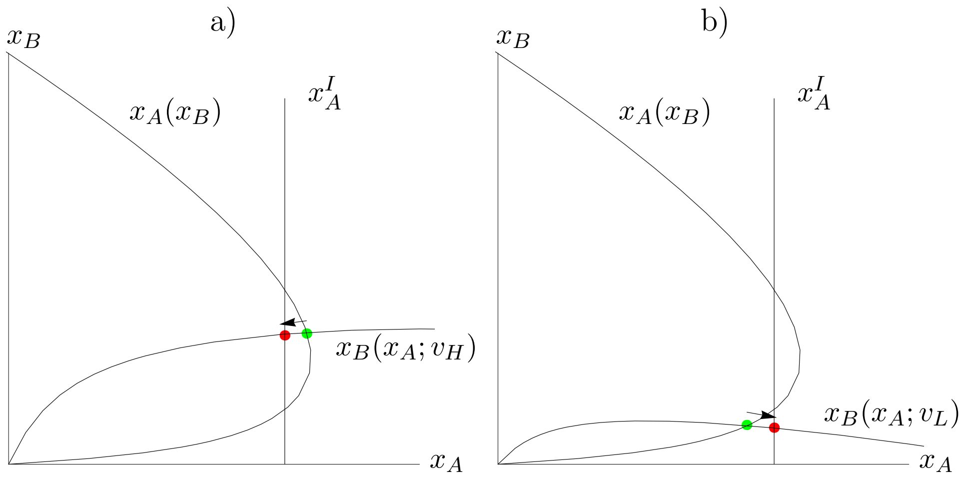


Figure 2.1: Full-information best response functions

the best response functions of both groups when A knows the valuation of group B. Optimal lobbying expenditures under full information are given where the best response functions intersect. Panel a) shows the full-information best response functions when lobbying group A faces a strong opponent, panel b) when it faces a weak opponent. x_A^I denotes the lobbying effort of A under ignorance. If group A's value is above average, its lobbying effort under ignorance (vertical line) is lower than under full information in case it faces the high value opponent (panel a)), while the opposite is true against the low value opponent (panel b)). We can directly see that this benefits A by decreasing both its opponents' lobbying efforts.[5]

Softening competition through ignorance does not always work. If group A's valuation is below average, ignorance worsens competition. A weak group A invests very little when facing a much stronger group B while it fights hard against the just slightly weaker group B, where competition is more equal. By staying ignorant A finds itself overinvesting in case it faces the stronger group B, which reacts to this threat with an increase in investment. At the same time it underinvests in case it faces the weak group B, which also reacts with an increase in investment, sensing a good opportunity. A weak lobbying group A always acquires costless information.

[5]Technically speaking, our results are due to the non-monotonicity of reaction functions. This implies that efforts are strategic complements for the favorite while they are strategic substitutes for the underdog, where in our set-up the favorite is the group with the higher valuation.

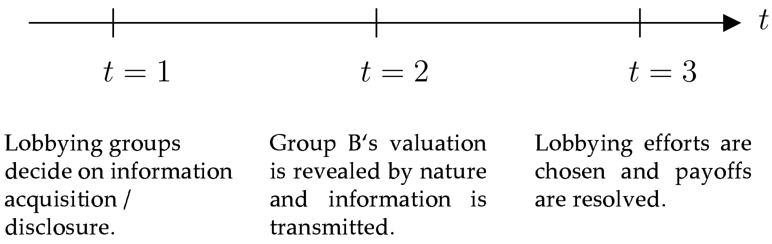


Figure 2.2: Sequence of moves

2.5 Information Transmission

So far we have analyzed the lobbying groups' disclosure and acquisition decisions separately. Now we combine these analyses to find out, how lobbying groups exchange information voluntarily. In a later section we then compare our findings to lobbying under mandatory disclosure policy. The game proceeds as follows: Prior to the start of lobbying, each lobbying group engages in information disclosure/acquisition decisions; that is, group A decides whether to pursue credible information about B's valuation while group B decides its disclosure policy. Following information acquisition/disclosure, both lobbying groups simultaneously choose lobbying efforts and payoffs are resolved. Figure 2.2 illustrates the flow of the game.

From now on, we assume that lobbying group B has not learned its valuation when deciding on the issue of information disclosure.[6] Then if both lobbying groups agree that information should be exchanged (B prefers disclosure and A acquisition) A will learn the value of group B. If on the other hand both lobbying groups agree not to disclose (B prefers non-disclosure and A ignorance), no information is transmitted. What is not so clear is what happens if A and B do not agree. For example A might want to acquire information about B's value, but B might not be willing to disclose it. Or B might want to disclose its value while A does not want to acquire it. We assume that in both cases A does not learn the value of B, even though most of our results do not depend on this assumption. We will discuss the implications of this assumption in the relevant places.

First consider the case with group A having diffuse priors (i.e. $q = 1/2$). Then the lobbying groups always agree on information transmission between them. Formally,

Proposition 3. *If lobbying group B expects to be relatively weak compared to lobbying group A ($\sqrt{v_L v_H} < v_A$) both lobbying groups agree not to transfer any information while*

[6] In Proposition 1 we showed that our results extend to the case where B has learned its valuation and has the possibility to send a costly signal to group A.

if lobbying group B expects to have a high valuation compared to A ($\sqrt{v_L v_H} > v_A$) both agree on disclosure.

Proof. Follows from the proof of Propositions 1 and 2. □

Surprisingly, we find the lobbying groups' incentives to be always aligned.[7] The reason for this is that there exist gains from coordination in the form of reduced competition. By coordinating, both parties can save on lobbying expenditures.[8] This is in a way similar to a finding in Baik and Shogren (1992) and Leininger (1993), who analyze the choice of the order of moves in sequential rent-seeking contests. Lobbying groups try to coordinate on the equilibrium where the least lobbying efforts are expended, which is possible if it is profitable for both. They find that both groups always prefer the weak group to go first. It chooses a low lobbying effort and the strong group reacts with lower lobbying effort as well. Even though the weak group ends up winning less often, it is compensated by lower lobbying costs. When choosing whether to disclose a similar logic applies. Staying ignorant can have a similar effect as moving first, if it enables A to move closer to its Stackelberg point. As we have shown, this is the case for a relatively strong lobbying group A. By staying ignorant it can credibly reduce its investment against the high-valuation lobbying group B who will react by reducing its expenditures as well. In the appendix we show that we can get a analogous result to Proposition 3 in a dynamic model of expenditure disclosure.

2.6 More General Contest Success Function

So far we have assumed that the lobbying process can be represented by a simple lottery contest. In order to show the robustness of our results, in this section we assume the political process can be represented by a more general CSF of the following form:

$$p_i(x_i, x_j) = \frac{f(x_i)}{f(x_i) + f(x_j)} \tag{2.2}$$

[7] In technical terms we assume that in the first stage group A and B decide simultaneously, A whether it wants to commit to acquire information, B whether to commit to disclose its value. There exist multiple equilibria in this set-up depending on the distribution of valuations, with the one described in proposition 3 being the pareto preferred one. Lobbying groups can solve the coordination problem for example by taking sequential decisions.

[8] Another example where voluntary exchange of information can be found, is armed conflict. As Thomas Schelling (1960) pointed out in his seminal work, "*[t]he ancients exchanged hostages, drank wine from the same glass to demonstrate the absence of poison, met in public places to inhibit the massacre of one by the other, and even deliberately exchanged spies to facilitate transmittal of authentic information*". Our analysis provides a rationale for this: exchanging authentic information decreases fierceness of conflict, something that is good for both parties.

where $f' > 0$ and $f'' \leq 0$.[9]

As we have seen in the previous section, whether ignorance is bliss for lobbying group A is determined by whether or not its value is above the average of group B's valuations. Proposition 2 shows though, that it is not the arithmetic average; rather the decision to acquire information turns on the geometric mean of B's value. Next we show that such a critical value of lobbying group A, let us denote it by $\widehat{v}_A$, exists more generally.

Lemma 2. *For every q, there exists a value $\widehat{v}_A$ such that, if $v_A = \widehat{v}_A$, lobbying group A is indifferent between acquiring information or not, and lobbying group B is indifferent between disclosing information or not.*

Proof. See appendix. □

To illustrate the intuition for the proof of this lemma, assume A knows its opponent. When A faces a weak opponent B, a small lobbying effort will basically guarantee success for A. With an increase in B's value, A increases its optimal lobbying effort until both groups have an equal value. Here competition is at its fiercest. Now an increase in B's value will start to discourage A from investing, until at one point B becomes so strong that A invests barely anything. This logic implies that there will always be two possible values of group B, one larger than A's, one smaller, such that A expends exactly the same lobbying effort. If group B has exactly these values, v_L and v_H, A's behavior will be unchanged whether it knows B's value or not.

It is tempting to reason from Lemma 2 that Propositions 1 and 2 hold for more general prior probabilities of B's values v_L and v_H and more general lobbying technologies. Indeed, we can generalize Propositions 1 and 2 locally around the critical value $\widehat{v}_A$. For information disclosure we get:

Proposition 4. *In a neighborhood of the critical value $\widehat{v}_A$, if lobbying group B expects to be relatively weak compared to lobbying group A ($v_A > \widehat{v}_A$) it strictly prefers not to disclose its valuation and votes against mandatory disclosure. On the other hand, a lobbying group B with a high expected valuation ($v_A < \widehat{v}_A$) always votes in favor of mandatory disclosure.*

Proof. See appendix. □

Exactly as with a lottery contest, information will be disclosed if lobbying group B is relatively strong. For information acquisition, we find:

[9] This is a standard contest success function, see Skaperdas (1996) for an axiomatization.

Proposition 5. *In a neighborhood of the critical value $\widehat{v}_A$, if lobbying group A is relatively strong compared to group B ($v_A > \widehat{v}_A$) it strictly prefers to stay ignorant about lobbying group B's value, while a relatively weak lobbying group A ($v_A < \widehat{v}_A$) always acquires this information. Furthermore, when there is a unique $\widehat{v}_A$ satisfying Lemma 2, then the result holds globally.*

Proof. See appendix. □

But what is the reason that Proposition 5 only holds locally? The critical value $\widehat{v}_A$ for Lemma 2 is not necessarily the only critical value for A. Take for example a very strong lobbying group A with a value close to v_H and assume that the probability of facing a strong group B is small. Then group A's lobbying effort under ignorance is similar to the lobbying effort knowing it is facing a weak group B. But if B happens to be strong and A were ignorant, it would underinvest by a large amount. Even though this leads the strong group to reduce its effort, this is not optimal for group A. In fact, there is an optimal degree of underinvestment against a stronger opponent, the so-called "Stackelberg point". If A had the opportunity to precommit lobbying effort, this would be the effort it would optimally choose. Ignorance enables lobbying group A to move closer to this optimal point in certain situations. In other situations A will surpass the Stackelberg point, like in the example above, or move away from it as with a below-average valuation. In these situations acquiring information is the optimal strategy.

Putting Propositions 4 and 5 together, we get the generalized results on information transmission:

Proposition 6. *In a neighborhood of $\widehat{v}_A$, if lobbying group B expects to be relatively weak compared to lobbying group A ($\widehat{v}_A < v_A$) both lobbying groups agree not to transfer any information while if lobbying group B expects to have a high valuation compared to A ($\widehat{v}_A > v_A$) both agree on disclosure.*

Proof. Follows from the proof of Propositions 4 and 5. □

2.7 Mandatory Disclosure Policy

Typically the lobbying groups agree on whether to disclose information between themselves. But how do we evaluate their decision from a societal point of view? Society is interested in keeping (at least partially wasteful)[10] lobbying expenditures low as well as improving the probability that the most beneficial policy is chosen. Given the lobbyists' joint decision we now ask, whether there is need for a policy intervention to achieve these goals. Can the government increase welfare by making disclosure of lobbying expenditures mandatory? Let us first assume that the policy maker is concerned with keeping the expected wastefulness of the lobbying process low and it is irrelevant for society which lobbying group is successful. This is typically the case in rent-seeking contests. We then get the following result.

Proposition 7. *Aggregate effort is lower under*

- *information disclosure if lobbying group A is relatively weak ($v_A \leq \sqrt{v_H v_L}$),*
- *asymmetric information if lobbying group A is relatively strong ($v_A > \sqrt{v_H v_L}$).*

Proof. See appendix. □

As foreshadowed in section 2.5 we find that if the uninformed lobbying group is relatively strong, mandatory information disclosure makes the lobbying process more wasteful; in other words, transparency is detrimental to society. In addition, in a majority of situations the lobbying groups would voluntarily agree not to transfer any information, as we have shown in Propositions 3 and 6. In these cases decentralization is an optimal policy. If we assume that information can only be transferred when at least lobbying group B agrees to disclose her information, we can conclude the following.

Corollary 2. *If the policy maker is interested in keeping lobbying expenditures low a laissez-faire policy is always preferable to a policy of mandatory disclosure.*

Many policies are not purely of a redistributive nature and it is desirable that the policy with the highest cost-benefit ratio is chosen. Hence a policy maker should also be concerned about the allocative efficiency of mandatory disclosure policy. We define efficiency as the probability that the lobbying group with the highest valuation wins the lobbying contest.[11] Without the noisiness of the political process, transparency is clearly beneficial for efficiency. Only if it is known which policy is the best, can it be

[10] Of course lobbying expenditures are only a transfer from lobbyists to the politician and hence not wasted in a narrow sense. On the other hand, lobbying draws financial and human resources which would otherwise have been used productively, for example for R&D. This misallocation of resources is a loss to society. For a discussion see for example Congleton (1988).

[11] Our results are robust to using an efficiency measure which weighs the probabilities with the respective valuation of the winner.

chosen. When the political process is noisy though, transparency will influence the balance of power of the lobbying groups, sometimes favoring the weaker, sometimes the stronger one. This can lead to undesirable side-effects of transparency policy.

Proposition 8. *Efficiency is greater under*

- *information disclosure if lobbying group A is relatively weak ($v_A \leq \sqrt{v_H v_L}$),*
- *asymmetric information if lobbying group A is relatively strong ($v_A > \sqrt{v_H v_L}$).*

Proof. See appendix. □

What is the intuition for this finding? As we already discussed in section 2.4, asymmetric information enables the uninformed lobbying group to act similar to a Stackelberg leader when it is sufficiently strong relative to the informed lobbying group. Morgan (2003) finds that sequential rent-seeking contests dominate simultaneous ones in terms of efficiency. Hence if asymmetric information enables A to get closer to its Stackelberg point, which is true for $v_A > \sqrt{v_H v_L}$, it will also improve efficiency. Together with the results in Propositions 3, 6 and 7 we find the following.

Corollary 3. *Assume the policy maker is interested in increasing efficiency and keeping wastefulness of the lobbying competition low. Then a laissez-faire policy is always weakly superior, independent of the relative weights the policy maker places on the two goals. Mandatory disclosure policy is in many cases strictly dominated from a welfare perspective.*

Bringing transparency to lobbying is advertised as an important goal of many governments around the world, as for example the U.S. and the EU. We show how transparency, in the form of mandatory disclosure policy, can affect lobbying competition, making it more wasteful and less efficient. Even though it would seem at first glance that the lobbyists' and society's goals are very different it turns out that, in terms of information structure, their interests are in fact aligned. At the same time, our result has the potential to explain the emergence of mandatory disclosure policies, even though shown to be inefficient. A politician interested in maximizing his rent-seeking revenues always weakly prefers mandatory disclosure to voluntary disclosure.

2.8 Uncertainty about the Decision Maker and the Scope for Agreement

So far we have implicitly assumed that policy makers are not basing their decision solely on lobbying expenditures. By spending more in the contest a lobbying group can increase its chances to succeed, but there always remains some uncertainty. Put differently, the lobbying group with the lower expenditures still has a non-zero chance of success – the lobbying process is at least somewhat noisy. For example, policy makers may have preferences over political outcomes unknown to the lobbying groups. Also, policy makers may face imperfectly observable constraints, for example they might have to toe the party line. A member of a green party is unlikely to pass a bill prolonging the use of nuclear power plants. Another reason for a noisy lobbying process from the lobbying groups' perspective is that lobbying efforts are only imperfectly observable by the policy maker. This could be due to the complexity of the subject so that it is difficult for lobbyists to communicate their concerns properly, or because it is not clear ex-ante what the best strategy to approach a political decision maker is and which consequences of the favored bill to highlight.

We have captured this uncertainty by using a non-deterministic CSF of the ratio form, as defined in equation (2.2). We now consider a CSF which can be interpreted as the limiting case when noise vanishes completely and therefore the contest is perfectly discriminatory, the all-pay auction. It represents a situation where the political process is very sensitive to lobbying effort and where the lobbying group with the highest expenditure wins with certainty.[12] This higher sensitivity implies higher marginal returns of lobbying effort and therefore increases the fierceness of the competition. It is interesting to consider this situation as a limiting case, because it is implicitly assumed that politicians do not have any private preferences about the political outcomes, do not face any constraints and the process of communication between the lobbying groups and the policy maker is free of misunderstandings and noise. We show now how noisiness influences the incentives to coordinate on information transmission.

Proposition 9. *As the the noisiness of the political process vanishes and competition takes the form of an all-pay auction*

(a) disclosing information is weakly dominated for lobbying group B,

[12]The standard references analyzing the all-pay auctions are Hillman and Riley (1989), Baye, Kovenock, and de Vries (1993, 1996), and Krishna and Morgan (1997).

(b) staying ignorant is weakly dominated for lobbying group A,

(c) the lobbying groups' incentives are never aligned and therefore they will never agree on transferring information voluntarily.

Proof. See appendix.[13] □

This result reveals that the contest's degree of sensitivity to rent-seeking efforts influences when the lobbying groups agree on information transmission. In contrast to ratio form contests, in a fully discriminating contest the lobbying groups' incentives are never aligned. The informed group never discloses its information while the uninformed group always takes the opportunity to acquire information. Because of the extreme fierceness of competition there is no scope for agreement left.

Let us consider the lobbying groups' incentives separately. Why does lobbying group B never benefit from disclosing its valuation? Under a noisy political process, by disclosing its value, a strong group B discourages a weak group A from investing. This does not work when the political process is fully discriminating. By disclosing information, a strong lobbying group will only secure itself a payoff equal to the difference in valuations between itself and its opponent. All other rents are dissipated through competition. With asymmetric information competition is less fierce and it can in addition earn informational rents. In fact, it can secure itself the exact same payoff with one-sided asymmetric information (by marginally overbidding group A's valuation) and might even do better. Technically speaking, in all-pay auctions both reaction functions are monotonically increasing until the valuation of the weakest lobbying group so there will be no discouragement effect in the relevant range.

Why is there no value to ignorance? Lobbying group A never benefits from ignorance because, as politicians become perfectly responsive to lobbying expenditures, there is no advantage to moving first[14]. In fact, the low-valuation lobbying group is indifferent with respect to timing and the group with the higher value prefers to follow. In short, when lobbying groups know their opponents' value, payoffs are exactly the same, whether groups move sequentially or simultaneously. Hence the advantage from ignorance highlighted under an imperfectly discriminating political process does not apply in a setting where policy makers are perfectly responsive to lobbyists' influence. The disadvantage of making a suboptimal decision - in form of an only "on average"

[13] A proof for part 2 of the Proposition has first been given in Kovenock, Morath, and Münster (2010) for two-sided asymmetric information and a continuous distribution of types.

[14] This was shown for example in Konrad and Leininger (2007).

best response - does still apply. Since there are only costs to ignorance, lobbying group A always acquires information.

What are the consequences for disclosure policy? First of all, Proposition 9 shows that lobbying groups don't agree on disclosure and hence it is no longer clear what happens under a laissez-faire transparency rule. Furthermore, a reduction in aggregate effort and an increase in efficiency, the policy maker's two objectives, are no longer necessarily compatible. Aggregate effort is typically smaller under full information when A's value is not too close to either v_H or v_L and under asymmetric information else. Efficiency is typically greater under asymmetric information except if v_A is relatively small and q is relatively large. Figure 2.3 illustrates this for $v_L = 1$ and $v_H = 2$. In darkgray regions full information is optimal while in lightgray regions asymmetric information is preferred. So decreasing aggregate effort often implies decreasing efficiency.

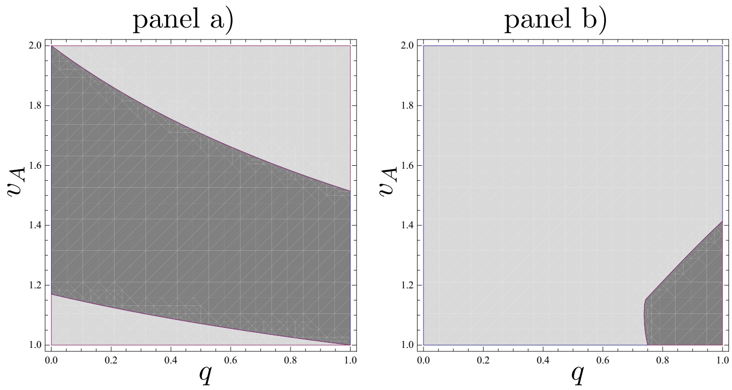


Figure 2.3: Aggregate effort (panel a)) and efficiency (panel b)).

We can draw the following conclusions regarding mandatory and voluntary disclosure policy.

Corollary 4. *Policy makers who are very responsive to the influence of lobbyists make decentralized agreements unlikely. In these circumstances, neither a laissez-faire transparency rule nor mandated disclosure is optimal. Furthermore, achieving an increase in efficiency and a decrease in aggregate effort through disclosure policy becomes unlikely as these two goals are typically in conflict.*

Asymmetric information has two effects on efficiency when the policy maker is perfectly responsive to lobbying expenditures. On the one hand it stratifies the range of efforts of lobbying group B. A low-valuation group chooses its investment from an interval of the form $[0, \underline{x}]$ while the high-valuation group chooses from $[\underline{x}, \overline{x}]$. In contrast, under full information they choose from the interval $[0, \overline{x_i}]$, $i = H, L$. This is beneficial for efficiency. On the other hand we showed that lobbying group B benefits from informational rents. Especially when A is very likely to face a low-valuation opponent and v_A is close to v_L, this becomes important for efficiency. B's informational advantage will lead to a low-valuation type winning too often, decreasing efficiency. In theses cases the detrimental effect of asymmetric information dominates and efficiency is higher under full information.

Summarizing our results, we find, as the contest becomes perfectly discriminating, the possibility to discourage an opposing lobbying group vanishes and hence the possibility to coordinate on information disclosure to reduce competition. As the strategic effect of information highlighted in the imperfectly discriminating contest becomes irrelevant, information only serves to make a better decision. As the contest becomes perfectly discriminating, the value to ignorance is lost.

2.9 Conclusion

A central insight to emerge from our analysis is that, despite the fact that parties are competing, there is broad agreement on disclosure/acquisition of private information. We find that sharing information is favored by both sides when the rival is relatively weak and favored by neither when the rival is relatively strong. The reason is that information sharing affects the degree of competition, and because both parties dislike fierce competition there is scope for agreement. We show that when the parties agree on information sharing, it also leads to greater efficiency in allocating the prize. Thus, the possibility of endogenous information sharing leads to ex ante Pareto gains.

Our results have important implications for disclosure policy. We identify how in competitive environments, as for example lobbying or political campaigning, mandatory disclosure policies can increase wasteful competition and distort prize allocations. In terms of information disclosure the competing groups' and societies' interests are often aligned and voluntary disclosure reduces wastefulness and increases efficiency. Our results may help explain why the European Commission has resisted calls to adopt mandatory disclosure laws for EU lobbying.

We have highlighted an important mechanisms underlying information transmission in contests. Since we mostly abstract away from dynamics to focus on the role of information, future research should further explore the implications of transparency in more dynamic settings. For example if information is revealed through expenditures, a trade-off between taking early leadership by investing heavily and investing cautiously to be able to react to new information becomes important. Another interesting extension of our analysis would be to allow for common values. This can be relevant in many settings. In our lobbying example the lobbyists might posses relevant information about the value of the policy at stake, as for example when lobbying for a monopoly position and each firm has done market research. Lobbying groups learn not only about their opponent's interest, but also about their own. Furthermore sabotage might be a concern in these kinds of environments and this will influence the incentives for voluntary disclosure.

2.A Proof of Propositions 1 and 2

2.A.1 Equilibrium under Full and Asymmetric Information

Equilibrium efforts, probability of success and utility under full information are equal to (see Nti (1999))

$$\begin{aligned} x_i^{FI}(v_i,v_j) &= \frac{v_i^2 v_j}{(v_i+v_j)^2} \\ p_i^{FI}(v_i,v_j) &= \frac{v_i}{v_i+v_j} \\ u_i^{FI}(v_i,v_j) &= \frac{v_i^3}{(v_i+v_j)^2}. \end{aligned} \tag{2.3}$$

It is easily verified that A will invest more against a high-value opponent than against a low-value one iff $v_A > \sqrt{v_H v_L}$. Under one-sided asymmetric information effort, probability of success and utility in an interior solution are

$$\begin{aligned} x_A^{AI}(v_A,v_L,v_H) &= \frac{v_L v_H v_A^2\left((1-q)\sqrt{v_L}+q\sqrt{v_H}\right)^2}{(v_H v_L + v_A((1-q)v_L+qv_H))^2} \\ x_H^{AI}(v_A,v_L,v_H) &= \frac{\left((1-q)\sqrt{v_L}+q\sqrt{v_H}\right)v_A v_H\sqrt{v_L v_H}}{(v_H v_L + v_A((1-q)v_L+qv_H))^2}\left(\sqrt{v_H}v_L + qv_A\left(\sqrt{v_H}-\sqrt{v_L}\right)\right) \\ x_L^{AI}(v_A,v_L,v_H) &= \frac{\left((1-q)\sqrt{v_L}+q\sqrt{v_H}\right)v_A v_L\sqrt{v_L v_H}}{(v_H v_L + v_A((1-q)v_L+qv_H))^2}\left(\sqrt{v_L}v_H - (1-q)v_A\left(\sqrt{v_H}-\sqrt{v_L}\right)\right) \\ p_A^{AI}(v_A,v_L,v_H) &= \frac{v_A\left((1-q)\sqrt{v_L}+q\sqrt{v_H}\right)^2}{(v_H v_L + v_A((1-q)v_L+qv_H))} \\ p_H^{AI}(v_A,v_L,v_H) &= 1-\frac{v_A\left((1-q)\sqrt{v_L}+q\sqrt{v_H}\right)}{(v_H v_L + v_A((1-q)v_L+qv_H))}\sqrt{v_L} \\ p_L^{AI}(v_A,v_L,v_H) &= 1-\frac{v_A\left((1-q)\sqrt{v_L}+q\sqrt{v_H}\right)}{(v_H v_L + v_A((1-q)v_L+qv_H))}\sqrt{v_H} \\ u_A^{AI}(v_A,v_L,v_H) &= \frac{v_A^3\left(q\sqrt{v_H}+(1-q)\sqrt{v_L}\right)^2(qv_H+(1-q)v_L)}{(v_A(qv_H+(1-q)v_L)+v_L v_H)^2} \\ u_L^{AI}(v_A,v_L,v_H) &= \frac{\left(v_L^{3/2}(v_A(1-q)+v_H)-(1-q)v_A\sqrt{v_H}v_L\right)^2}{(qv_A(v_H-v_L)+v_L(v_A+v_H))^2} \\ u_H^{AI}(v_A,v_L,v_H) &= \frac{\left(qv_A v_H\left(\sqrt{v_H}-\sqrt{v_L}\right)+v_H^{3/2}v_L\right)^2}{(qv_A(v_H-v_L)+v_L(v_A+v_H))^2}. \end{aligned} \tag{2.4}$$

2.A.2 Disclosing Information

To see whether group B prefers to disclose or not it is sufficient to look at group A's effort difference between full and asymmetric information. If A invests more under full information against B, B will clearly prefer asymmetric information. The difference in A's effort is equal to

$$\begin{aligned}
\Delta x_{AH} &= \frac{v_A^2 v_H}{(v_A+v_H)^2} - \frac{v_L v_H v_A^2\left((1-q)\sqrt{v_L}+q\sqrt{v_H}\right)^2}{\left(v_H v_L + v_A\left((1-q)v_L+qv_H\right)\right)^2}\\
&= \frac{q v_A^2 v_H^{3/2}\left(\sqrt{v_H}-\sqrt{v_L}\right)\left(v_A-\sqrt{v_H}\sqrt{v_L}\right)}{(v_A+v_H)^2\left(v_H v_L + v_A\left((1-q)v_L+qv_H\right)\right)^2}\\
&\quad\times\Big(q v_A\sqrt{v_H}\sqrt{v_L}+qv_Av_H+2(1-q)v_Av_L+qv_H^{3/2}\sqrt{v_L}+(2-q)v_Hv_L\Big)\\
\Delta x_{AL} &= \frac{v_A^2 v_L}{(v_A+v_L)^2} - \frac{v_L v_H v_A^2\left((1-q)\sqrt{v_L}+q\sqrt{v_H}\right)^2}{\left(v_H v_L + v_A\left((1-q)v_L+qv_H\right)\right)^2}\\
&= -\frac{(1-q) v_A^2 v_L^{3/2}\left(\sqrt{v_H}-\sqrt{v_L}\right)\left(v_A-\sqrt{v_H}\sqrt{v_L}\right)}{(v_A+v_L)^2\left(v_H v_L + v_A\left((1-q)v_L+qv_H\right)\right)^2}\\
&\quad\times\Big((1-q)\Big(v_A\sqrt{v_H}\sqrt{v_L}+v_Av_L+\sqrt{v_H}v_L^{3/2}\Big)+2qv_Av_H+qv_Hv_L+v_Hv_L\Big).
\end{aligned}$$

At $v_A = \sqrt{v_L v_H}$ A's effort is identical, while for $v_A > \sqrt{v_L v_H}$ A underinvests against a high-value opponent and overinvests against a low-value one under asymmetric information. The opposite holds true for $v_A < \sqrt{v_L v_H}$. Hence it follows that for $v_A > \sqrt{v_L v_H}$ a high-value B prefers not to disclose, while a low-value one prefers disclosure and vice versa for $v_A < \sqrt{v_L v_H}$. Now let us consider the ex-ante expected utility of group B when it has not yet learned its value.

$$\begin{aligned}
&E[\Delta u_B] = q\Delta u_L + (1-q)\Delta u_H = \frac{-(1-q)qv_A\left(\sqrt{v_H}-\sqrt{v_L}\right)^2\left(v_A-\sqrt{v_H}\sqrt{v_L}\right)}{(v_A+v_H)^2(v_A+v_L)^2(qv_A(v_H-v_L)+v_Av_L+v_Hv_L)^2}\\
&\times\Big(\left(v_H^2-v_L^2\right)qv_A^2\left(v_A^2+v_A\sqrt{v_Hv_L}+4v_Hv_L\right)+qv_A\Big(2v_A^2v_Hv_L+2v_Av_H^{3/2}v_L^{3/2}+2v_H^2v_L^2\Big)(v_H-v_L)\\
&+v_A^4v_L^2+v_A^3\Big(2v_H^{3/2}v_L^{3/2}+2v_H^2v_L+\sqrt{v_H}v_L^{5/2}+4v_Hv_L^2+2v_L^3\Big)+v_A\Big(4v_H^3v_L^2+6v_H^2v_L^3+3v_H^{5/2}v_L^{5/2}\Big)\\
&+v_A^2\Big(2v_H^{5/2}v_L^{3/2}+4v_H^{3/2}v_L^{5/2}+2v_H^3v_L+7v_H^2v_L^2+6v_Hv_L^3\Big)+2v_H^3v_L^3+2qv_A^3\left(v_H^3-v_L^3\right)\Big)
\end{aligned}$$

Hence for $v_A = \sqrt{v_L v_H}$ group B is also indifferent in expectation whether to disclose or not, while for $v_A > \sqrt{v_L v_H}$ it prefers not to disclose and for $v_A < \sqrt{v_L v_H}$ disclosure is optimal.

2.A.3 Signaling of Valuation

Now lobbying group B has the possibility to expend money before the contest in order to signal its valuation. Katsenos (2009) is the first to analyze costly signaling in a lottery contest with two-sided asymmetric information and two possible types of valuations, v_H and v_L. He finds that separating equilibria only exist, when the probability to face a strong opponent is sufficiently low. Instead, we analyze signaling with one-sided asymmetric information.

First we show that there cannot be a separating equilibrium when $v_A > \sqrt{v_H v_L}$. We have shown above that in this case H prefers non-disclosure or even being mistaken for a low-value group while L prefers disclosure. So let us assume L signals its type by expending some amount of costly signaling effort s_L while H spends s_H. s_H can only be zero, as for H it is the worst possible case that A believes him to be strong with certainty. Incentive compatibility requires that L's utility from signaling its type is larger than its utility from imitating H and vice versa. The respective utility differences are

$$\begin{aligned}
\Delta u_L &= -s_L - \frac{v_A^2 v_H}{(v_A+v_H)^2} + 2v_A\sqrt{\frac{v_H v_L}{(v_A+v_H)^2}} - \frac{v_L v_A^2 + 2v_A v_L^2}{(v_A+v_L)^2} \geq 0 \\
\Delta u_H &= s_L - \frac{v_A^2 v_L}{(v_A+v_L)^2} + 2v_A\sqrt{\frac{v_H v_L}{(v_A+v_L)^2}} + \frac{v_H^3}{(v_A+v_H)^2} - v_H \geq 0
\end{aligned}$$

and hence we require

$$\begin{aligned}
-\frac{v_A^2 v_H}{(v_A+v_H)^2} + 2v_A\sqrt{\frac{v_H v_L}{(v_A+v_H)^2}} + \frac{v_L^3}{(v_A+v_L)^2} - v_L &\geq s_L \\
\frac{v_A^2 v_L}{(v_A+v_L)^2} - 2v_A\sqrt{\frac{v_H v_L}{(v_A+v_L)^2}} - \frac{v_H^3}{(v_A+v_H)^2} + v_H &\leq s_L
\end{aligned}$$

which, is easily shown, can never be fulfilled at the same time. Hence there does not exist a separating equilibrium for $v_A > \sqrt{v_H v_L}$ and no information is credibly disclosed. On the other hand for $v_A < \sqrt{v_H v_L}$ H prefers disclosure while L prefers non-disclosure. Because now for L full-information is the worst case, it will never spend a positive amount of money to signal its type and $s_L = 0$, $s_H > 0$. The incentive compatibility constraint becomes

$$\begin{aligned}
\Delta u_L &= s_H - \frac{v_A^2 v_H}{(v_A+v_H)^2} + 2v_A\sqrt{\frac{v_H v_L}{(v_A+v_H)^2}} - \frac{v_L v_A^2 + 2v_A v_L^2}{(v_A+v_L)^2} \geq 0 \\
\Delta u_H &= -s_H - \frac{v_A^2 v_L}{(v_A+v_L)^2} + 2v_A\sqrt{\frac{v_H v_L}{(v_A+v_L)^2}} + \frac{v_H^3}{(v_A+v_H)^2} - v_H \geq 0.
\end{aligned}$$

It is easily shown that there always exists a $s_H > 0$ such that both incentive compatibility constraints are satisfied. There always exists a separating equilibrium for $v_A < \sqrt{v_H v_L}$ and information is disclosed.

2.A.4 Acquiring Information

Let us consider lobbying group A's incentives to acquire information. The difference in expected utility is equal to

$$\begin{aligned}
D_A = & \frac{(1-q)qv_A^3\left(\sqrt{v_H}-\sqrt{v_L}\right)^2\left(v_A-\sqrt{v_H}\sqrt{v_L}\right)}{(v_A+v_H)^2(v_A+v_L)^2(qv_A\left(v_H-v_L\right)+v_Av_L+v_Hv_L)^2} \\
& \times\Big(\left(v_H-v_L\right)q\left(v_A^3-3v_A^2\sqrt{v_Lv_H}-v_Av_Hv_L-v_H^{3/2}v_L^{3/2}\right) \\
& -2qv_A\sqrt{v_Lv_H}\left(v_H^2-v_L^2\right)+v_A^3v_L-3v_A^2\sqrt{v_H}v_L^{3/2}-4v_Av_H^{3/2}v_L^{3/2} \\
& -v_Av_H^2v_L-2v_A\sqrt{v_H}v_L^{5/2}-2v_Av_Hv_L^2-v_H^{5/2}v_L^{3/2}-2v_H^{3/2}v_L^{5/2}-2v_H^2v_L^2\Big)
\end{aligned}$$

For $v_A < \sqrt{v_H v_L}$ A clearly prefers to acquire information, while for $v_A = \sqrt{v_H v_L}$ it is indifferent. For v_A slightly larger than $\sqrt{v_H v_L}$ it prefers ignorance while for v_A approaching v_H it might prefer to acquire information again. This implies we have to be careful about staying in an interior solution, in other words we need $v_L \geq \frac{(1-q)^2 v_A^2 v_H}{((1-q)v_A+v_H)^2}$ or $v_A \leq \frac{v_H\sqrt{v_L}}{(1-q)\left(\sqrt{v_H}-\sqrt{v_L}\right)}$.

Let $q = \frac{1}{2}$. Then the difference in utility for group A between full-information and asymmetric information is equal to

$$\begin{aligned}
D_A = & \frac{v_A^3\left(\sqrt{v_H}-\sqrt{v_L}\right)^2\left(v_A-\sqrt{v_H}\sqrt{v_L}\right)}{2(v_A+v_H)^2(v_A+v_L)^2(v_Av_H+v_Av_L+2v_Hv_L)^2} \\
& \times\Big(\left(v_H+v_L\right)\left(v_A^3-3v_A^2\sqrt{v_H}\sqrt{v_L}-3v_Av_Hv_L-3v_H^{3/2}v_L^{3/2}\right) \\
& -2v_A\sqrt{v_H}\sqrt{v_L}\left(v_H^2+4v_Hv_L+v_L^2\right)-4v_H^2v_L^2\Big)
\end{aligned}$$

We can show that this is unambiguously positive for $v_A < \sqrt{v_L v_H}$ and negative for $v_A > \sqrt{v_L v_H}$ given that we are in an interior solution. For $v_H > 9v_L$ the condition for an interior solution is binding. So for $v_H < 9v_L$ v_A can be as high as v_H. Let us plug this into the expression in brackets: $v_H^4 - 5v_H^{7/2}\sqrt{v_L} - 14v_H^{5/2}v_L^{3/2} - 5v_H^{3/2}v_L^{5/2} - 2v_H^3 v_L - 7v_H^2 v_L^2$. This is clearly strictly negative for all $v_H < 9v_L$. For $v_H > 9v_L$ we insert the highest possible v_A into the expression in brackets carries the sign of: $-\left(4v_H^{3/2} - 7v_H\sqrt{v_L} + v_L^{3/2}\right)$ which is always negative for $v_H > 9v_L$.

2.B Proof of Lemma 2

Proof. To see this, first note that (i) reaction functions are hump-shaped and (ii) reach a maximum where $x_A = x_B$, i.e. where the reaction function crosses the 45 degree line (for a proof see Yildirim (2005)). Moreover, we find an equilibrium on this line exactly when $v_A = v_B$, i.e. when the game is symmetric. Let us denote full-information symmetric efforts for $v_A = v_L$ by x_L and for $v_A = v_H$ by x_H. Keeping the valuation of the opponent fixed, a group's effort is strictly increasing in its own valuation. So let v_A increase from v_L to v_H. Then the effort of the L-value type is strictly decreasing (strategic substitute) and the effort of the H-value type is strictly increasing (strategic complement). If the opponent is of the L-value type, x_A increases from x_L to some $x_{HL} > x_L$. To the contrary, if the opponent is of the H type x_A increases from some $x_{LH} < x_L$ to x_H. Note that $x_H > x_{HL} > x_L > x_{LH}$, i.e. if the opponent is of the H-value type A's effort is at the beginning lower and at the end higher compared to the L-value type. Accordingly, by continuity there has to be some $\widehat{v}_A \in (v_L, v_H)$ for which efforts against both types of the other group are identical and equal to $\widehat{x}_A$.

If $v_A = \widehat{v}_A$ group A will spend the same lobbying effort in the full information games and in the asymmetric information game in equilibrium. Accordingly, both types of group B will choose the same effort independent of the informational environment, implying A's costs and winning probabilities are identical and thus A is indifferent between both information regimes. □

2.C Proof of Proposition 4

At $v_A = \widehat{v}_A$ group B is exactly indifferent whether it discloses its information or not, ex-ante as well as interim, as group A always chooses the same lobbying effort. Let us increase v_A marginally from there. The derivative of the difference in the expected utility of player B between full information and asymmetric information (which we denote by D_B) with respect to v_A at $\widehat{v}_A$ can also be written as

$$\begin{aligned}
\frac{\partial D_B}{\partial v_A}|_{v_A=\widehat{v}_A} &= (1-q)\left(v_H\left(-\frac{\partial p_H}{\partial x_A}\left(\frac{\partial x_{AH}^{FI}}{\partial v_A}-\frac{\partial x_A^{AI}}{\partial v_A}\right)-\frac{\partial p_H}{\partial x_H}\left(\frac{\partial x_H^{FI}}{\partial v_A}-\frac{\partial x_H^{AI}}{\partial v_A}\right)\right)-\left(\frac{\partial x_H^{FI}}{\partial v_A}-\frac{\partial x_H^{AI}}{\partial v_A}\right)\right)\\
&+ q\left(v_L\left(-\frac{\partial p_L}{\partial x_A}\left(\frac{\partial x_{AL}^{FI}}{\partial v_A}-\frac{\partial x_A^{AI}}{\partial v_A}\right)-\frac{\partial p_L}{\partial x_L}\left(\frac{\partial x_L^{FI}}{\partial v_A}-\frac{\partial x_L^{AI}}{\partial v_A}\right)\right)-\left(\frac{\partial x_L^{FI}}{\partial v_A}-\frac{\partial x_L^{AI}}{\partial v_A}\right)\right)\\
&= \left((1-q)v_H\left(\frac{\partial x_A^{AI}}{\partial v_A}-\frac{\partial x_{AH}^{FI}}{\partial v_A}\right)+q\,v_L\left(\frac{\partial x_A^{AI}}{\partial v_A}-\frac{\partial x_{AL}^{FI}}{\partial v_A}\right)\right)\frac{1}{\widehat{v}_A},
\end{aligned}$$

using $p_i=\frac{f(x_A)}{f(x_A)+f(x_i)}$ and $x_i=x_B^i$, $i=H,L$ to shorten the exposition. We know that $v_A>0$, $0<q<1$. $\frac{\partial p_H}{\partial x_A}=\frac{\partial p_L}{\partial x_A}=\frac{1}{v_A}$ and $\frac{\partial p_L}{\partial x_L}=-\frac{1}{v_L}<\frac{\partial p_H}{\partial x_H}=-\frac{1}{v_H}<0$ follow from the first order conditions of the two groups.

The relevant equilibrium comparative statics are

$$\begin{aligned}
\frac{\partial x_{AH}^{FI}}{\partial v_A}|_{v_A=\widehat{v}_A} &= \frac{-\frac{\partial^2 p_H}{\partial x_H^2}}{\left(\frac{\partial^2 p_H}{\partial x_A^2}\frac{\partial^2 p_H}{\partial x_H^2}-\left(\frac{\partial^2 p_H}{\partial x_A\partial x_H}\right)^2\right)v_A^2}>0\\
\frac{\partial x_A^{AI}}{\partial v_A}|_{v_A=\widehat{v}_A} &= \frac{-\frac{\partial^2 p_H}{\partial x_H^2}\frac{\partial^2 p_L}{\partial x_L^2}}{\left(\frac{\partial^2 p_L}{\partial x_L^2}(1-q)\left(\frac{\partial^2 p_H}{\partial x_H^2}\frac{\partial^2 p_H}{\partial x_A^2}-\left(\frac{\partial^2 p_H}{\partial x_A\partial x_H}\right)^2\right)+\frac{\partial^2 p_H}{\partial x_H^2}q\left(\frac{\partial^2 p_L}{\partial x_L^2}\frac{\partial^2 p_L}{\partial x_A^2}-\left(\frac{\partial^2 p_L}{\partial x_A\partial x_L}\right)^2\right)\right)v_A^2}>0\\
\frac{\partial x_{AL}^{FI}}{\partial v_A}|_{v_A=\widehat{v}_A} &= \frac{-\frac{\partial^2 p_L}{\partial x_L^2}}{\left(\frac{\partial^2 p_L}{\partial x_A^2}\frac{\partial^2 p_L}{\partial x_L^2}-\left(\frac{\partial^2 p_L}{\partial x_A\partial x_L}\right)^2\right)v_A^2}>0.
\end{aligned}$$

Using these in our derivative

$$\begin{aligned}
\frac{\partial D_B}{\partial v_A}|_{v_A=\widehat{v}_A} = &\frac{\left(\frac{\partial^2 p_H}{\partial x_H^2}\left(\left(\frac{\partial^2 p_L}{\partial x_A\partial x_L}\right)^2-\frac{\partial^2 p_L}{\partial x_L^2}\frac{\partial^2 p_L}{\partial x_A^2}\right)+\frac{\partial^2 p_L}{\partial x_L^2}\left(\frac{\partial^2 p_H}{\partial x_H^2}\frac{\partial^2 p_H}{\partial x_A^2}-\left(\frac{\partial^2 p_H}{\partial x_A\partial x_H}\right)^2\right)\right)}{\left(\frac{\partial^2 p_L}{\partial x_A^2}\frac{\partial^2 p_L}{\partial x_L^2}-\left(\frac{\partial^2 p_L}{\partial x_A\partial x_L}\right)^2\right)\left(\frac{\partial^2 p_H}{\partial x_A^2}\frac{\partial^2 p_H}{\partial x_H^2}-\left(\frac{\partial^2 p_H}{\partial x_A\partial x_H}\right)^2\right)v_A^3}\times\\
&\frac{\left(\frac{\partial^2 p_H}{\partial x_H^2}v_H\left(\left(\frac{\partial^2 p_L}{\partial x_A\partial x_L}\right)^2-\frac{\partial^2 p_L}{\partial x_A^2}\frac{\partial^2 p_L}{\partial x_L^2}\right)+\frac{\partial^2 p_L}{\partial x_L^2}v_L\left(\frac{\partial^2 p_H}{\partial x_A^2}\frac{\partial^2 p_H}{\partial x_H^2}-\left(\frac{\partial^2 p_H}{\partial x_A\partial x_H}\right)^2\right)\right)q\,(1-q)}{\left(\frac{\partial^2 p_L}{\partial x_L^2}(1-q)\left(\frac{\partial^2 p_H}{\partial x_H^2}\frac{\partial^2 p_H}{\partial x_A^2}-\left(\frac{\partial^2 p_H}{\partial x_A\partial x_H}\right)^2\right)+\frac{\partial^2 p_H}{\partial x_H^2}q\left(\frac{\partial^2 p_L}{\partial x_L^2}\frac{\partial^2 p_L}{\partial x_A^2}-\left(\frac{\partial^2 p_L}{\partial x_A\partial x_L}\right)^2\right)\right)}<0,
\end{aligned}$$

where we use $\frac{\partial^2 p_L}{\partial x_A^2}<0$, $\frac{\partial^2 p_H}{\partial x_A^2}<0$, $\frac{\partial^2 p_H}{\partial x_H^2}>0$ and $\frac{\partial^2 p_L}{\partial x_L^2}>0$ which follow from the shape of the CSF. $\frac{\partial^2 p_L}{\partial x_A x_L}>0$ and $\frac{\partial^2 p_H}{\partial x_A x_H}<0$ come from the fact that at $v_A=\widehat{v}_A$ A is an underdog against an opponent with valuation v_H but a favorite against an opponent with valuation v_L and

$$\frac{\partial^2 p_H}{\partial x_H^2}\left(\left(\frac{\partial^2 p_L}{\partial x_A\partial x_L}\right)^2-\frac{\partial^2 p_L}{\partial x_L^2}\frac{\partial^2 p_L}{\partial x_A^2}\right)+\frac{\partial^2 p_L}{\partial x_L^2}\left(\frac{\partial^2 p_H}{\partial x_H^2}\frac{\partial^2 p_H}{\partial x_A^2}-\left(\frac{\partial^2 p_H}{\partial x_A\partial x_H}\right)^2\right)>0.$$

Intuitively this term relates $\frac{\partial x_{AH}^{FI}}{\partial v_A}|_{v_A=\widehat{v}_A}$ to $\frac{\partial x_{AL}^{FI}}{\partial v_A}|_{v_A=\widehat{v}_A}$. For $\frac{\partial x_{AH}^{FI}}{\partial v_A}|_{v_A=\widehat{v}_A} > \frac{\partial x_{AL}^{FI}}{\partial v_A}|_{v_A=\widehat{v}_A}$ it will be positive and for $\frac{\partial x_{AH}^{FI}}{\partial v_A}|_{v_A=\widehat{v}_A} < \frac{\partial x_{AL}^{FI}}{\partial v_A}|_{v_A=\widehat{v}_A}$ it will be negative. For our CSF given in equation 2.2 it will always be positive. This means that starting at $x_A^L = x_A^H$ a slight increase in v_A will lead to a relatively higher increase in effort on the part of group A against the high-type opponent. Hence we find that at $v_A = \widehat{v}_A$ the derivative of D_B is strictly negative. □

2.D Proof of Proposition 5

We showed in Lemma 2 that if $v_A = \widehat{v}_A$ group A will be indifferent between ignorance and full-information. To prove the proposition we show that the derivative of the difference of utilities of A (which we denote by D_A) with respect to v_A is non-zero at $v_A = \widehat{v}_A$, which implies that for some valuations v_A slightly below (above) $\widehat{v}_A$ group A prefers to stay ignorant (acquire information) or the other way around. The derivative of D_A at $\widehat{v}_A$ is equal to

$$\begin{aligned}\frac{\partial D_A}{\partial v_A}|_{v_A=\widehat{v}_A} &= \left((1-q)\left(\frac{\partial p_H}{\partial x_A}\left(\frac{\partial x_{AH}^{FI}}{\partial v_A} - \frac{\partial x_A^{AI}}{\partial v_A}\right) + \frac{\partial p_H}{\partial x_H}\left(\frac{\partial x_H^{FI}}{\partial v_A} - \frac{\partial x_H^{AI}}{\partial v_A}\right)\right)\right. \\ &+ \left. q\left(\frac{\partial p_L}{\partial x_A}\left(\frac{\partial x_{AL}^{FI}}{\partial v_A} - \frac{\partial x_A^{AI}}{\partial v_A}\right) + \frac{\partial p_L}{\partial x_L}\left(\frac{\partial x_L^{FI}}{\partial v_A} - \frac{\partial x_L^{AI}}{\partial v_A}\right)\right)\right)\widehat{v}_A - \left((1-q)\frac{\partial x_{AH}^{FI}}{\partial v_A} + q\frac{\partial x_{AL}^{FI}}{\partial v_A}\right) + \frac{\partial x_A^{AI}}{\partial v_A}.\end{aligned}$$

We use $p_i = \frac{f(x_A)}{f(x_A)+f(x_i)}$ and $x_i = x_B^i$, $i = H, L$ to shorten the exposition. We know that $v_A > 0$, $0 < q < 1$. $\frac{\partial p_H}{\partial x_A} = \frac{\partial p_L}{\partial x_A} = \frac{1}{v_A}$ and $\frac{\partial p_L}{\partial x_L} = -\frac{1}{v_L} < \frac{\partial p_H}{\partial x_H} = -\frac{1}{v_H} < 0$ follow from the first order conditions of the two groups. The derivative simplifies to

$$\frac{\partial D_A}{\partial v_A}|_{v_A=\widehat{v}_A} = -\left(\frac{(1-q)}{v_H}\left(\frac{\partial x_H^{FI}}{\partial v_A} - \frac{\partial x_H^{AI}}{\partial v_A}\right) + \frac{q}{v_L}\left(\frac{\partial x_L^{FI}}{\partial v_A} - \frac{\partial x_L^{AI}}{\partial v_A}\right)\right)\widehat{v}_A.$$

This derivative will only be zero, if a change in v_A induces the same effect on B's full-information effort as on its asymmetric information effort, or if they just offset each other for the two types weighted by the probability q and their valuation.

To find out we totally differentiate the system of first order conditions for full information and asymmetric information and use Cramer's rule to get equilibrium comparative statics regarding v_A, taking into account the previously mentioned first order conditions at $v_A = \widehat{v}_A$.

$$\frac{\partial x_H^{FI}}{\partial v_A}|_{v_A=\widehat{v}_A} = \frac{\frac{\partial^2 p_H}{\partial x_A \partial x_H}}{\left(\frac{\partial^2 p_H}{\partial x_A^2}\frac{\partial^2 p_H}{\partial x_H^2} - \left(\frac{\partial^2 p_H}{\partial x_A \partial x_H}\right)^2\right) v_A^2} > 0$$

$$\frac{\partial x_H^{AI}}{\partial v_A}|_{v_A=\widehat{v}_A} = \frac{\frac{\partial^2 p_H}{\partial x_A \partial x_H}\frac{\partial^2 p_L}{\partial x_L^2}}{\left(\frac{\partial^2 p_L}{\partial x_L^2}(1-q)\left(\frac{\partial^2 p_H}{\partial x_H^2}\frac{\partial^2 p_H}{\partial x_A^2} - \left(\frac{\partial^2 p_H}{\partial x_A \partial x_H}\right)^2\right) + \frac{\partial^2 p_H}{\partial x_H^2} q \left(\frac{\partial^2 p_L}{\partial x_L^2}\frac{\partial^2 p_L}{\partial x_A^2} - \left(\frac{\partial^2 p_L}{\partial x_A \partial x_L}\right)^2\right)\right) v_A^2} > 0$$

$$\frac{\partial x_L^{FI}}{\partial v_A}|_{v_A=\widehat{v}_A} = \frac{\frac{\partial^2 p_L}{\partial x_A x_L}}{\left(\frac{\partial^2 p_L}{\partial x_A^2}\frac{\partial^2 p_L}{\partial x_L^2} - \left(\frac{\partial^2 p_L}{\partial x_A \partial x_L}\right)^2\right) v_A^2} < 0$$

$$\frac{\partial x_L^{AI}}{\partial v_A}|_{v_A=\widehat{v}_A} = \frac{\frac{\partial^2 p_L}{\partial x_A \partial x_L}\frac{\partial^2 p_H}{\partial x_H^2}}{\left(\frac{\partial^2 p_L}{\partial x_L^2}(1-q)\left(\frac{\partial^2 p_H}{\partial x_H^2}\frac{\partial^2 p_H}{\partial x_A^2} - \left(\frac{\partial^2 p_H}{\partial x_A \partial x_H}\right)^2\right) + \frac{\partial^2 p_H}{\partial x_H^2} q \left(\frac{\partial^2 p_L}{\partial x_L^2}\frac{\partial^2 p_L}{\partial x_A^2} - \left(\frac{\partial^2 p_L}{\partial x_A \partial x_L}\right)^2\right)\right) v_A^2} < 0.$$

$\frac{\partial^2 p_L}{\partial x_A^2} < 0$, $\frac{\partial^2 p_H}{\partial x_A^2} < 0$, $\frac{\partial^2 p_H}{\partial x_H^2} > 0$ and $\frac{\partial^2 p_L}{\partial x_L^2} > 0$ follow from the shape of the CSF. $\frac{\partial^2 p_L}{\partial x_A x_L} > 0$ and $\frac{\partial^2 p_H}{\partial x_A x_H} < 0$ come from the fact that at $v_A = \widehat{v}_A$ A is an underdog against an opponent with valuation v_H but a favorite against an opponent with valuation v_L. Using this, the derivative of the difference in utilities equals

$$\frac{\partial D_A}{\partial v_A}|_{v_A=\widehat{v}_A} = -\frac{\left(\frac{\partial^2 p_L}{\partial x_L^2}\left(\frac{\partial^2 p_H}{\partial x_H^2}\frac{\partial^2 p_H}{\partial x_A^2} - \left(\frac{\partial^2 p_H}{\partial x_A \partial x_H}\right)^2\right) + \frac{\partial^2 p_H}{\partial x_H^2}\left(\left(\frac{\partial^2 p_L}{\partial x_A \partial x_L}\right)^2 - \frac{\partial^2 p_L}{\partial x_L^2}\frac{\partial^2 p_L}{\partial x_A^2}\right)\right)}{\left(\frac{\partial^2 p_L}{\partial x_A^2}\frac{\partial^2 p_L}{\partial x_L^2} - \left(\frac{\partial^2 p_L}{\partial x_A \partial x_L}\right)^2\right)\left(\frac{\partial^2 p_H}{\partial x_A^2}\frac{\partial^2 p_H}{\partial x_H^2} - \left(\frac{\partial^2 p_H}{\partial x_A \partial x_H}\right)^2\right) v_A\, v_H\, v_L} \times$$
$$\frac{\left(\frac{\partial^2 p_L}{\partial x_A \partial x_L} v_H \left(\frac{\partial^2 p_H}{\partial x_A^2}\frac{\partial^2 p_H}{\partial x_H^2} - \left(\frac{\partial^2 p_H}{\partial x_A \partial x_H}\right)^2\right) + \frac{\partial^2 p_H}{\partial x_A \partial x_H} v_L \left(\left(\frac{\partial^2 p_L}{\partial x_A \partial x_L}\right)^2 - \frac{\partial^2 p_L}{\partial x_A^2}\frac{\partial^2 p_L}{\partial x_L^2}\right)\right) q(1-q)}{\left(\frac{\partial^2 p_L}{\partial x_L^2}(1-q)\left(\frac{\partial^2 p_H}{\partial x_H^2}\frac{\partial^2 p_H}{\partial x_A^2} - \left(\frac{\partial^2 p_H}{\partial x_A \partial x_H}\right)^2\right) + \frac{\partial^2 p_H}{\partial x_H^2} q \left(\frac{\partial^2 p_L}{\partial x_L^2}\frac{\partial^2 p_L}{\partial x_A^2} - \left(\frac{\partial^2 p_L}{\partial x_A \partial x_L}\right)^2\right)\right)}$$

which has the sign of

$$\text{Sign}\left[\frac{\partial D_A}{\partial v_A}|_{v_A=\widehat{v}_A}\right] = \text{Sign}\left[-\left(\frac{\partial^2 p_L}{\partial x_L^2}\left(\frac{\partial^2 p_H}{\partial x_H^2}\frac{\partial^2 p_H}{\partial x_A^2} - \left(\frac{\partial^2 p_H}{\partial x_A \partial x_H}\right)^2\right) - \frac{\partial^2 p_H}{\partial x_H^2}\left(\frac{\partial^2 p_L}{\partial x_L^2}\frac{\partial^2 p_L}{\partial x_A^2} - \left(\frac{\partial^2 p_L}{\partial x_A \partial x_L}\right)^2\right)\right)\right].$$

Intuitively this term relates $\frac{\partial x_{AH}^{FI}}{\partial v_A}|_{v_A=\widehat{v}_A}$ to $\frac{\partial x_{AL}^{FI}}{\partial v_A}|_{v_A=\widehat{v}_A}$. For $\frac{\partial x_{AH}^{FI}}{\partial v_A}|_{v_A=\widehat{v}_A} > \frac{\partial x_{AL}^{FI}}{\partial v_A}|_{v_A=\widehat{v}_A}$ it will be negative and for $\frac{\partial x_{AH}^{FI}}{\partial v_A}|_{v_A=\widehat{v}_A} < \frac{\partial x_{AL}^{FI}}{\partial v_A}|_{v_A=\widehat{v}_A}$ it will be positive. For our CSF given in equation 2.2 it will always be negative. This means that starting at $x_A^L = x_A^H$ a slight increase in v_A will lead to a relatively higher increase in effort on the part of group A against the high-type opponent.[15] Hence we find that at $v_A = \widehat{v}_A$ the derivative of D_A is strictly negative. Thus there exist some valuations $v_A > \widehat{v}_A$ where ignorance is bliss. □

[15] Note that for more general CSF the opposite case can arise and A increases its effort more against the low-type opponent. Then there will be a value of ignorance for $v_A < \widehat{v}_A$.

2.E Proof of Propositions 7 and 8

Expected aggregate effort with contest success function $p_i = \frac{x_i}{x_i + x_j}$ under full information is equal to

$$E\left[\sum_{i=\{A,B\}} x_i^{FI}\right] = \frac{v_A\left(\left((1-q)\,v_H + qv_L\right)v_A + v_Lv_H\right)}{\left(v_A+v_H\right)\left(v_A+v_L\right)},$$

while expected aggregate effort under one-sided asymmetric information is equal to

$$E\left[\sum_{i=\{A,B\}} x_i^{AI}\right] = \left((1-q)\sqrt{v_H} + q\sqrt{v_L}\right)\frac{\left((1-q)\frac{1}{\sqrt{v_H}} + q\frac{1}{\sqrt{v_L}}\right)}{\left(\frac{1}{v_A} + \left(\frac{(1-q)}{v_H} + \frac{q}{v_L}\right)\right)}.$$

Their difference is equal to

$$\begin{aligned} E\left[\sum_{i=\{A,B\}} \Delta x\right] &= \frac{v_A\left(\left((1-q)\,v_H + qv_L\right)v_A + v_Lv_H\right)}{\left(v_A+v_H\right)\left(v_A+v_L\right)} - \frac{\left((1-q)\sqrt{v_H} + q\sqrt{v_L}\right)\left(\frac{(1-q)}{\sqrt{v_H}} + \frac{q}{\sqrt{v_L}}\right)}{\left(\frac{1}{v_A} + \left(\frac{(1-q)}{v_H} + \frac{q}{v_L}\right)\right)} \\ &= \frac{(1-q)\,qv_A\left(\sqrt{v_H}-\sqrt{v_L}\right)^2\left(v_A - \sqrt{v_Hv_L}\right)\left(v_A\left(\sqrt{v_Hv_L} + v_H + v_L\right) + v_Hv_L\right)}{(v_A+v_H)(v_A+v_L)(qv_A(v_H-v_L) + v_L(v_A+v_H))}. \end{aligned}$$

It is easily observed that this is positive for $v_A > \sqrt{v_Hv_L}$ and negative otherwise hence proving Proposition 7.

Efficiency implies that the informational regime should be chosen to maximize $q\frac{x_A}{x_A+x_L} + (1-q)\frac{x_H}{x_A+x_H}$ as we assume $v_L \le v_A \le v_H$. We get

$$\Delta\left(q\frac{x_A}{x_A+x_L} + (1-q)\frac{x_H}{x_A+x_H}\right) = -\frac{(1-q)qv_A(v_H-v_L)\left(v_A^2 - v_Hv_L\right)}{(v_A+v_H)(v_A+v_L)(qv_A(v_H-v_L)+v_L(v_A+v_H))},$$

which is positive for $v_A < \sqrt{v_Hv_L}$ and negative else. □

2.F Proof of Proposition 9

Full information strategies for a match with valuations $v_i > v_j$ are given by the bidding distribution functions

$$\begin{aligned}
F_j(x; v_j, v_i) &= \frac{v_i - v_j}{v_i} + \frac{x}{v_i} \\
F_i(x; v_i, v_j) &= \frac{x}{v_j},
\end{aligned}$$

for $x \in [0, v_j]$. In the following we stick to the notation that $F_i(x; v_j)$ indicates the bidding distribution of group i facing another group j and we will denote the corresponding density function by $f_i(x; v_j)$. The ex-ante expected full information payoffs are

$$\begin{aligned}
\pi_H^{FI} &= v_H - v_A \\
\pi_L^{FI} &= 0 \\
\pi_A^{FI} &= q\,(v_A - v_L)\,.
\end{aligned}$$

Those results are standard and the proofs can be found for example in Hillman and Riley (1989) or Baye, Kovenock, and de Vries (1996). Using the equilibrium strategies it is easily verified that expected aggregate effort is equal to

$$\begin{aligned}
X^{FI} &= q \int_0^{v_L} \left(f_A(x; v_L) + f_L(x; v_A)\right) x\,dx + (1-q) \int_0^{v_A} \left(f_A(x; v_H) + f_H(x; v_A)\right) x\,dx \\
&= q \int_0^{v_L} \left(\frac{x}{v_L} + \frac{x}{v_A}\right) dx + (1-q) \int_0^{v_A} \left(\frac{x}{v_A} + \frac{x}{v_H}\right) dx \\
&= \frac{q}{2} \left(\frac{v_L^2}{v_A} + v_L\right) + \frac{(1-q)}{2} \left(v_A + \frac{v_A^2}{v_H}\right)
\end{aligned}$$

and that efficiency (the ex-ante probability that the player with higher valuation wins) equals

$$\begin{aligned}
EF^{FI} &= q \int_0^{v_L} F_L^{FI}(x; v_A)\, f_A^{FI}(x; v_L)\, dx + (1-q) \int_0^{v_A} F_A^{FI}(x; v_H)\, f_H^{FI}(x; v_A)\, dx \\
&= q \int_0^{v_L} \left(\frac{v_A - v_L}{v_A} + \frac{x}{v_A}\right) \frac{1}{v_L}\, dx + (1-q) \int_0^{v_A} \left(\frac{v_H - v_A}{v_H} + \frac{x}{v_H}\right) \frac{1}{v_A}\, dx \\
&= (1-q) \left(1 - \frac{v_A}{2v_H}\right) + q \left(1 - \frac{v_L}{2v_A}\right).
\end{aligned}$$

Under one-sided asymmetric information consider first the case where v_A is relatively small, $v_A \leq \tilde{v}_A \equiv \frac{v_L}{q+\frac{v_L}{v_H}(1-q)}$. We then find that A's bidding/effort distribution function has a mass point at zero. The groups' equilibrium strategies are given by the distribution functions

$$\begin{aligned}
F_A^{AI}(x;v_L,v_H) &= \begin{cases} \frac{v_H-(1-q)v_A}{v_H} - \frac{qv_A}{v_L} + \frac{x}{v_L} & \text{for} \quad x \in [0, qv_A] \\ \frac{v_H - v_A}{v_H} + \frac{x}{v_H} & \text{for} \quad x \in [qv_A, v_A] \end{cases} \\
F_L^{AI}(x;v_A) &= \frac{x}{qv_A} \text{ for } x \in [0, qv_A] \\
F_H^{AI}(x;v_A) &= \frac{x - qv_A}{(1-q)\, v_A} \text{ for } x \in [qv_A, v_A].
\end{aligned}$$

That those distribution functions indeed characterize an equilibrium is easily verified and we leave this to the reader (a proof is available upon request). Equilibrium payoffs in this case are

$$\begin{aligned}
\pi_A^{AI} &= 0 < \pi_A^{FI} = q\,(v_A - v_L) \\
\pi_H^{AI} &= v_H - v_A = \pi_H^{FI} \\
\pi_L^{AI} &= v_L \frac{v_H - (1-q)\, v_A}{v_H} - qv_A > \pi_L^{FI} = 0.
\end{aligned}$$

A prefers full information while B ex-ante prefers asymmetric information, which is the case because the L-type is better off while the H-type is indifferent.

Expected aggregate effort is equal to

$$\begin{aligned}
X_{v_A \leq \tilde{v}_A}^{AI} &= q \int_0^{qv_A} \left(f_A^{AI}(x;v_L,v_H) + f_L^{AI}\right) x\, dx + (1-q) \int_{qv_A}^{v_A} \left(f_A^{AI}(x;v_L,v_H) + f_H^{AI}\right) x\, dx \\
&= \int_0^{qv_A} \left(\frac{x}{v_A} + \frac{x}{v_L}\right) dx + \int_{qv_A}^{v_A} \left(\frac{x}{v_A} + \frac{x}{v_H}\right) dx \\
&= \frac{v_A\ \left(q^2 v_A (v_H - v_L) + v_L (v_A + v_H)\right)}{2 v_H v_L}
\end{aligned}$$

and efficiency is equal to

$$
\begin{aligned}
EF^{AI}_{v_A \leq \tilde{v}_A} &= q\int_0^{qv_A} F_L^{AI}(x;v_A) f_A(x;v_L)\,dx + (1-q)\int_{qv_A}^{v_A} F_A^{AI}(x;v_H) f_H(x;v_A)\,dx \\
&= q\int_0^{q\,v_A} \frac{x}{q\,v_A}\frac{1}{v_L}\,dx + (1-q)\int_{qv_A}^{v_A}\left(\frac{v_H-(1-q)v_A}{v_H} - \frac{qv_A}{v_L} + \frac{x}{v_L}\right)\frac{1}{(1-q)v_A}\,dx \\
&= \frac{q^2 v_A v_H - (q-1)v_L[(q-1)v_A + 2v_H]}{2v_H v_L}.
\end{aligned}
$$

Now consider $v_A > \tilde{v}_A = \frac{v_L}{q+\frac{v_L}{v_H}(1-q)}$. Here only L's effort distribution has a mass point, which is at zero.

$$
\begin{aligned}
F_A^{AI}(x;v_L,v_L) &= \begin{cases} \frac{x}{v_L} & \text{for} \quad x\in[0,\underline{x}] \\ \frac{x}{v_H} + \left(1-\frac{(1-q)v_A}{v_H}\right)\left(1-\frac{v_L}{v_H}\right) & \text{for} \quad x\in[\underline{x},\overline{x}] \end{cases} \\
F_L^{AI}(x;v_A) &= \frac{x}{qv_A} + 1 - \frac{v_L}{qv_A} + \frac{v_L\,(1-q)}{qv_H} \text{ for } x\in[0,\underline{x}] \\
F_H^{AI}(x;v_A) &= \frac{x}{(1-q)\,v_A} + \frac{v_L}{v_H} - \frac{v_L}{(1-q)\,v_A} \text{ for } x\in[\underline{x},\overline{x}],
\end{aligned}
$$

where $\underline{x} = v_L - (1-q)\,v_A\frac{v_L}{v_H}$ and $\overline{x} = v_L + (1-q)\,v_A\left(1-\frac{v_L}{v_H}\right)$. The corresponding expected equilibrium payoffs are

$$
\begin{aligned}
\pi_A^{AI} &= qv_A - v_L + \frac{(1-q)\,v_A v_L}{v_H} < \pi_A^{FI} = q\,(v_A - v_L) \\
\pi_H^{AI} &= v_H - v_L - v_A\,(1-q)\left(1-\frac{v_L}{v_H}\right) > v_H - v_A = \pi_H^{FI} \\
\pi_L^{AI} &= 0 = \pi_L^{FI}.
\end{aligned}
$$

B prefers asymmetric information, since the H-type is better off while the L-type is indifferent, whereas A prefers full information. Ex-ante expected aggregate effort is equal to

$$
\begin{aligned}
X^{AI}_{v_A > \tilde{v}_A} &= \int_0^{\underline{x}} \left(f_A^{AI}(x;v_L) + f_L^{AI}(x;v_A)\right)\,x\,dx + \int_{\underline{x}}^{\overline{x}} \left(f_A^{AI}(x;v_L) + f_L^{AI}(x;v_A)\right)\,x\,dx \\
&= \frac{\frac{v_L(v_A+v_L)((q-1)v_A+v_H)^2}{v_A} + (q-1)(v_A+v_H)((q-1)v_A(v_H-2v_L) - 2v_H v_L)}{2v_H^2}
\end{aligned}
$$

and efficiency equals

$$
\begin{aligned}
EF^{AI}_{v_A > \tilde{v}_A} &= q \int_0^{\underline{x}} F^{AI}_L(x; v_A)\, f_A(x; v_L)\, dx + (1-q) \int_{\underline{x}}^{\overline{x}} F^{AI}_A(x; v_L)\, f_H(x; v_A)\, dx \\
&= \frac{v_A v_H \left((q^2-1)\, v_A + 2v_H\right) - v_L((q-1)v_A + v_H)^2}{2v_A {v_H}^2}.
\end{aligned}
$$

□

2.G A Dynamic Model of Expenditure Disclosure

Assume lobbying is dynamic and takes place over two periods. Lobbyists decide whether to voluntarily disclose their first stage lobbying expenditures x_i^1 before the second stage of lobbying begins. After period two, aggregate lobbying expenditures, $x_i^1 + x_i^2 = X_i$, determine the chance to enact the preferred legislation. The CSF is now given by

$$
p_i\left(x_i^t, x_j^t\right) = \frac{x_i^1 + x_i^2}{x_i^1 + x_i^2 + x_j^1 + x_j^2} = \frac{X_i}{X_i + X_j}. \tag{2.5}
$$

Payoffs are

$$
\begin{aligned}
\pi_B &= \frac{X_B}{X_B + X_A} v_B - X_B \\
\pi_A &= \left(\sigma \frac{X_A}{X_L + X_A} + (1-\sigma)\frac{X_A}{X_H + X_A}\right) v_A - X_A,
\end{aligned}
$$

where σ stands for the belief of lobbying group A that B is of a low valuation. We focus on the existence of two kinds of equilibria: one in which aggregate expenditures for each group correspond to the full information expenditures and one where they correspond to the asymmetric information expenditures in the valuation disclosure game. In the first case lobbying group B is sending a signal in period 1 regarding its valuation while in the latter case both types of group B expend the same lobbying effort and group A does not learn anything about its opponent's value. We show that there exists an equilibrium of this dynamic expenditure disclosure game in which aggregate lobbying expenditures for each lobbying group correspond to those in the valuation disclosure game. In addition, if lobbying groups can decide on voluntary expenditure disclosure we show that there exists an equilibrium where expenditures are disclosed if group A is relatively weak and they are not disclosed when group A is relatively strong. [16] In this sense the valuation disclosure model can be seen as a refinement or reduced form of the dynamic expenditure disclosure model, giving us a unique equilibrium prediction.

Proposition 10. *a) If lobbying group A is relatively weak ($v_A < \sqrt{v_L v_H}$) there exists a separating equilibrium in the expenditure disclosure game where both lobbying groups choose to disclose their expenditures. Aggregate expenditures of each lobbying group correspond to those in the valuation disclosure game.*

b) If lobbying group A is relatively strong ($v_A \geq \sqrt{v_L v_H}$) there exists a pooling equilibrium where both lobbying groups abstain from disclosure and hence no information is revealed. Aggregate expenditures of each lobbying group correspond to those in the valuation disclosure game.

Proof. First consider the game with observable expenditures. Let us prove the existence of a separating equilibrium for $v_A \leq \sqrt{v_L v_H}$. Start by conjecturing equilibrium expenditures of this game to be $x_A^1 = x_A^{FI}(v_H)$, $x_H^1 = x_H^{FI}$, $x_L^1 = x_L^{FI}$, $x_A^2(v_L) = x_A^{FI}(v_L) - x_A^{FI}(v_H)$ and $x_A^2(v_H) = x_H^2 = x_L^2 = 0$. The superscript FI stands for one-shot full-information equilibrium efforts while AI for one-shot asymmetric information equilibrium efforts. These equilibrium efforts are given in equations 2.4 and 2.5. We need to check if there is any profitable deviation in order to verify our assumption of equilibrium. Let us start in the second stage. The unique equilibrium strategies after separation in the second stage are given in Yildirim (2005).

Lemma 3. *(Yildirim (2005) Lemma 2) Given (x_i^1, x_j^1), the following strategy profiles constitute the unique equilibrium in the second period:*

$$\widehat{x}_i^2(x_i^1, x_j^1) = \begin{cases} 0, & \text{if } x_i^1 \geq R_i(x_j^1) \text{ and } x_j^1 \geq R_j(x_i^1), \\ x_i^{FI} - x_i^1, & \text{if } x_i^1 \leq x_i^{FI} \text{ and } x_j^1 \leq x_j^{FI}, \\ 0, & \text{if } x_i^1 \geq x_i^{FI} \text{ and } x_j^1 \leq R_j(x_i^1), \\ R_i(x_j^1) - x_i^1, & \text{if } x_i^1 \leq R_i(x_j^1) \text{ and } x_j^1 \geq x_j^{FI}. \end{cases}$$

Given that A invested $x_A^{FI}(v_H)$ and since $x_A^{FI}(v_H) < x_A^{FI}(v_L)$ (see appendix 2.A) we are in case 2 and it is optimal for both H and L to respond with $x_i^2 = x_i^{FI} - x_i^1$, which is

[16] Typically multiple equilibria will exists, reflecting "leadership" by one or the other lobbying group. The characterization of all equilibria of this game is work in progress.

$x_H^2 = x_L^2 = 0$. Given that $x_H^1 = x_H^{FI}$ and $x_L^1 = x_L^{FI}$ also A's optimal reaction is to expend $x_A^2 = x_A^{FI}(v_L) - x_A^1 = x_A^{FI}(v_L) - x_A^{FI}(v_H)$ against an L-type and $x_A^2 = x_A^{FI}(v_H) - x_A^1 = 0$ against an H-type.

Consider now the first stage expenditures. Is there a profitable deviation? Let us consider each lobbyist in turn. When group A decreases its expenditures in stage 1 it only substitutes them with higher expenditures in period 2, aggregate expenditures stay the same by Lemma 3. An increase in expenditures on the other hand decreases its utility, as it cannot induce lower expenditures from its opponent and it will be above its optimal reaction. What about group B? The exact same arguments imply that lowering first period expenditures is not profitable for either group L or H as they will be exactly offset by higher second period expenditures. Increasing expenditures is detrimental for group H as they will not induce a decrease in the opponent's expenditures, who already invests zero in period 2. It is also detrimental for group L. It is the underdog against A and hence an increase in expenditures will make A even more aggressive. The last possible deviation is imitation of the other groups expenditures. We have two possible deviations. Either H imitates and L and expends x_L^{FI} or L imitates H and expends x_H^{FI}. Since $x_A^1 = x_A^{FI}(v_H)$, H never benefits from imitation. It can only increase A's expenditures. We only need to check L's incentive constraint. In the potential signaling equilibrium its payoff is $u_L = \frac{v_L^3}{(v_L+v_A)^2}$ while a deviation brings a payoff of $u_L^D = \frac{v_H}{v_H+v_A}v_L - \frac{v_H^2 v_A}{(v_H+v_A)^2}$. It is easily verified that the latter is always smaller and hence deviating does not pay off. Note that the equivalent separating equilibrium with $x_A^1 = x_A^{FI}(v_L)$ does not exist for $v_A < \sqrt{v_H v_L}$. In this case $x_A^{FI}(v_H) > x_A^{FI}(v_L)$ and H has an incentive to imitate L. This is not costly to H because it can, in the second period, sill optimally react to A and increase its expenditures after deviating to x_L^{FI} to imitate L. Hence this separating equilibrium does not exist.

Next we prove existence of the pooling equilibrium for $v_A > \sqrt{v_H v_L}$. We conjecture that an equilibrium exists with expenditures $x_A^1 = x_A^{AI}$, $x_H^1 = x_L^1 = x_L^{AI}$, $x_A^2 = x_L^2 = 0$ and $x_H^2 = x_H^{AI} - x_L^{AI}$. If B's pool, A does not learn anything about their value and in the second period A plays an average best response $\widetilde{R}_A(X_L, X_H)$

$$\widehat{x}_A^2(x_A^1, x_B^1) = \begin{cases} x_A^{AI} - x_A^1, & \text{if } x_A^1 \leq x_A^{AI} \text{ and } x_B^1 \leq x_L^{AI}, \\ 0, & \text{if } x_A^1 \geq R_A(x_B^1) \text{ and } x_B^1 \geq R_H(x_A^1), \\ 0, & \text{if } x_A^1 \geq x_A^{AI} \text{ and } x_B^1 \leq R_L(x_A^1), \\ R_A(x_B^1) - x_A^1, & \text{if } x_A^1 \leq R_A(x_B^1) \text{ and } x_B^1 \geq x_H^{FI}, \\ x_A^{NS} - x_A^1, & \text{if } x_A^1 \leq x_A^{NS} \text{ and } x_L^{AI} \leq x_B^1 \leq x_H^{FI}, \\ 0, & \text{if } x_A^1 \geq x_A^{NS} \text{ and } R_L(x_A^1) \leq x_B^1 \leq R_H(x_A^1), \end{cases}$$

and

$$(\widehat{x}_H^2, \widehat{x}_L^2) = \begin{cases} \left(x_H^{AI} - x_H^1, x_L^{AI} - x_L^1\right), & \text{if } x_A^1 \leq x_A^{AI} \text{ and } x_B^1 \leq x_L^{AI}, \\ (0,0), & \text{if } x_A^1 \geq R_A(x_B^1) \text{ and } x_B^1 \geq R_H(x_A^1), \\ \left(R_H(x_A) - x_H^1, R_L(x_A) - x_L^1\right), & \text{if } x_A^1 \geq x_A^{AI} \text{ and } x_B^1 \leq R_L(x_A^1), \\ (0,0), & \text{if } x_A^1 \leq R_A(x_B^1) \text{ and } x_B^1 \geq x_H^{FI}, \\ \left(x_H^{NS}(x_B^1) - x_H^1, 0\right), & \text{if } x_A^1 \leq x_A^{NS} \text{ and } x_L^{AI} \leq x_B^1 \leq x_H^{FI}, \\ \left(x_H^{NS}(x_B^1) - x_H^1, 0\right), & \text{if } x_A^1 \geq x_A^{NS} \text{ and } R_L(x_A^1) \leq x_B^1 \leq R_H(x_A^1), \end{cases}$$

where $x_A^{NS}(x_B^1)$, $x_H^{NS}(x_B^1)$ are defined as the expenditure pair that solves $x_H^{NS} = R_H(x_A^{NS})$ and $x_A^{NS} = \widetilde{R}_A(x_B^1, x_H^{NS})$ The proof of the optimality of these second period strategy profiles follows the proof in Yildirim (2005) for the full-information case and can be received from the authors upon request. Given the first stage strategies we find that in fact it is optimal for A and L to invest zero and for H to invest $x_L^{AI} - x_H^{AI}$. Let us consider possible deviations in the first stage. Group A does not have an incentive to deviate. A decrease in expenditure will again be directly compensated by an increase in expenditure in stage 2 while an increase in first period expenditures leads to an increase in expenditure by group H and no decrease by L. To consider deviations by group B we need to specify out of equilibrium beliefs. In this case we need not restrict them, A can believe anything after a deviation by B. The reason is that a deviation can never lead to a decrease in expenditures, as A only expends in the first period. Hence B does not deviate and we have established the existence of a pooling equilibrium.

Now assume that lobbying groups can decide on expenditure disclosure after the first period. Since no information is revealed before this decision this is equivalent to lobbying groups choosing expenditures and disclosure at the same time. Let us start with the pooling equilibrium for $v_A > \sqrt{v_H v_L}$. We show that all groups deciding not to disclose and expend $x_i^1 = x_i^{AI}$, $x_i^2 = 0$, $i = L, A$ and $x_H^1 = x_L^{AI}$, $x_H^2 = x_H^{AI} - x_L^{AI}$ is an equilibrium of this game. Given the first period decisions all groups can do no better than to reach their best response functions and hence $x_L^2 = x_A^2 = 0$ and $x_H^2 = x_H^{AI} - x_L^{AI}$. Consider the first stage decisions. Given the expenditures of group B, A has no incentive to deviate from expending $x_A^1 = x_A^{AI}$ and not disclosing. Disclosing its expenditures does not change anything as they are anticipated in equilibrium. Choosing a different expenditure does not increase its payoff either, no matter whether it discloses or not. Disclosing a higher expenditure would only be beneficial to discourage L, but L already invests x_L^{AI} in the first stage. Disclosing a lower expenditure results in exactly the same payoff as our proposed equilibrium. Not disclosing and expending less is exactly the same, while not disclosing and expending more yields a lower payoff. Consider L's first stage incentives. Expending more in the first period, no matter whether it discloses yields a lower payoff. Expending less on the other hand yields exactly the same payoff as it cannot induce A to reduce its expenditures which are already sunk in the first period. Lastly consider H. H cannot induce lower expenditures from A than x_A^{AI} which is already sunk and hence deviating to disclosure is not profitable. Not disclosing and investing something else is also weakly not profitable.

Consider now $v_A < \sqrt{v_H v_L}$. We want to show that in the proposed separating equilibrium lobbying groups have an incentive to disclose their expenditures. The equilibrium expenditures were $x_A^1 = x_A^{FI}(v_H)$, $x_H^1 = x_H^{FI}$, $x_L^1 = x_L^{FI}$, $x_A^2(v_L) = x_A^{FI}(v_L) - x_A^{FI}(v_H)$ and $x_A^2(v_H) = x_H^2 = x_L^2 = 0$. By the analysis above we only need to check whether there is a deviation to non-disclosure. As A can only make B expend more in the second period and it ends up on its reaction function, there is no profitable deviation. The same is true for H. We have already established that it is too costly for L to imitate H. Since L is the underdog it does not want to increase its expenditures either. Decreasing them does not change anything. $\square$

2.H Continuous Uniform Distribution

Let us assume that B's value is distributed uniformly on $[a, b]$. The expected utility of lobbying group A if it does not know the value of group B is equal to

$$E[u_A] = \frac{1}{b-a} \int_a^b \frac{x_A}{x_A + x_B(v_B)} dv_B v_A - x_A.$$

Taking the derivative and setting it equal to zero

$$\frac{\partial E[u_A]}{\partial x_A} = \frac{1}{b-a}\int_a^b \frac{x_B(v_B)}{(x_A + x_B(v_B))^2} dv_B v_A - 1$$

we get A's first order condition. Plugging this into group B's reaction function $x_B(x_A) = \max\left\{\sqrt{x_A v_B} - x_A, 0\right\}$ we can solve for the equilibrium efforts. Focussing on interior solutions we get the following equilibrium efforts.

$$\begin{aligned}
\frac{\partial E[u_A]}{\partial x_A} &= \frac{1}{b-a}\int_a^b \frac{\sqrt{x_A v_B} - x_A}{\left(x_A + \sqrt{x_A v_B} - x_A\right)^2} dv_B v_A - 1 \\
&= \frac{2v_A}{(b-a)\sqrt{x_A}}\left(\sqrt{b} - \sqrt{a}\right) - \frac{v_A}{b-a}\left(\ln[b] - \ln[a]\right) - 1 \overset{!}{=} 0 \\
\Leftrightarrow x_A^{AI} &= \left(\frac{2v_A\left(\sqrt{b} - \sqrt{a}\right)}{v_A\left(\ln[b] - \ln[a]\right) + (b-a)}\right)^2 \\
x_B^{AI} &= \sqrt{v_B \frac{2v_A\left(\sqrt{b} - \sqrt{a}\right)}{v_A\left(\ln[b] - \ln[a]\right) + (b-a)}} - \left(\frac{2v_A\left(\sqrt{b} - \sqrt{a}\right)}{v_A\left(\ln[b] - \ln[a]\right) + (b-a)}\right)^2
\end{aligned}$$

A and B's equilibrium utility under one-sided asymmetric information is equal to

$$\begin{aligned}
E[u_A^{AI}] &= \frac{\frac{2v_A\left(\sqrt{b}-\sqrt{a}\right)}{v_A(\ln[b]-\ln[a])+(b-a)}}{b-a}\int_a^b \frac{1}{\sqrt{v_B}} dv_B v_A - \left(\frac{2v_A\left(\sqrt{b} - \sqrt{a}\right)}{v_A\left(\ln[b] - \ln[a]\right) + (b-a)}\right)^2 \\
&= \frac{\frac{2v_A\left(\sqrt{b}-\sqrt{a}\right)}{v_A(\ln[b]-\ln[a])+(b-a)}}{b-a} 2\left(\sqrt{b} - \sqrt{a}\right) v_A - \left(\frac{2v_A\left(\sqrt{b} - \sqrt{a}\right)}{v_A\left(\ln[b] - \ln[a]\right) + (b-a)}\right)^2
\end{aligned}$$

$$\begin{aligned}
u_B^{AI} &= \frac{\sqrt{v_B x_A} - x_A}{\sqrt{v_B x_A}} v_B - \sqrt{v_B x_A} + x_A = v_B - 2\sqrt{x_A v_B} + x_A \\
&= v_B - 2\sqrt{\frac{2v_A\left(\sqrt{b} - \sqrt{a}\right)}{v_A\left(\ln[b] - \ln[a]\right) + (b-a)} v_B} + \left(\frac{2v_A\left(\sqrt{b} - \sqrt{a}\right)}{v_A\left(\ln[b] - \ln[a]\right) + (b-a)}\right)^2
\end{aligned}$$

and B's expected utility before it learns its type

$$E[u_B^{AI}] = \frac{b-a}{2} - \frac{4}{3}\left(b^{\frac{3}{2}} - a^{\frac{3}{2}}\right)\sqrt{\frac{2v_A\left(\sqrt{b}-\sqrt{a}\right)}{v_A\left(\ln[b]-\ln[a]\right)+(b-a)}} + \left(\frac{2v_A\left(\sqrt{b}-\sqrt{a}\right)}{v_A\left(\ln[b]-\ln[a]\right)+(b-a)}\right)^2$$

If both lobbying groups know their respective valuations equilibrium efforts are

$$x_i^{FI}(v_i, v_j) = \frac{v_i^2 v_j}{(v_i+v_j)^2},$$

and utilities

$$\begin{aligned}
E[u_A^{FI}] &= \int_a^b \frac{v_A^3}{(v_A+v_B)^2} dF(v_B) = \frac{1}{b-a}\left(\frac{v_A^3}{v_A+a} - \frac{v_A^3}{v_A+b}\right) \\
u_B^{FI} &= \frac{v_B^3}{(v_B+v_A)^2} \\
E[u_B^{FI}] &= \frac{\frac{v_A^3}{v_A+b} + 3v_A^2\ln[v_A+b] - 2v_A b + \frac{b^2}{2} - \left(\frac{v_A^3}{v_A+a} + 3v_A^2\ln[v_A+a] - 2v_A a + \frac{a^2}{2}\right)}{b-a}.
\end{aligned}$$

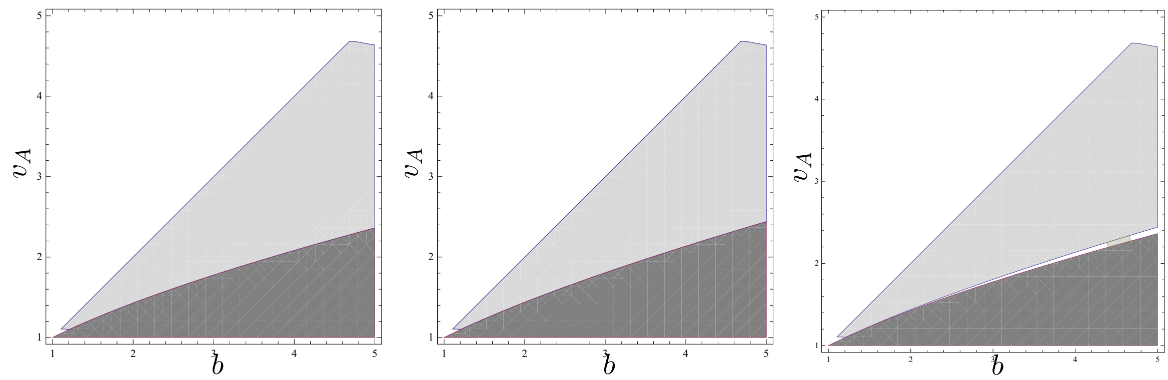
Now we consider the incentives to disclose or acquire information. The difference in utilities for A and B is equal to

$$\begin{aligned}
\Delta E[u_A] &= \frac{1}{b-a}\left(\frac{v_A^3}{v_A+a} - \frac{v_A^3}{v_A+b}\right) - \left(\frac{\frac{\left(2v_A\left(\sqrt{b}-\sqrt{a}\right)\right)^2}{v_A(\ln[b]-\ln[a])+(b-a)}}{b-a} - \left(\frac{2v_A\left(\sqrt{b}-\sqrt{a}\right)}{v_A\left(\ln[b]-\ln[a]\right)+(b-a)}\right)^2\right) \\
\Delta u_B &= \frac{v_B^3}{(v_B+v_A)^2} - v_B - 2\sqrt{\frac{2v_A\left(\sqrt{b}-\sqrt{a}\right)}{v_A\left(\ln[b]-\ln[a]\right)+(b-a)}}v_B + \left(\frac{2v_A\left(\sqrt{b}-\sqrt{a}\right)}{v_A\left(\ln[b]-\ln[a]\right)+(b-a)}\right)^2.
\end{aligned}$$

Ex-ante, before B knows its valuation the difference in expected utility is equal to

$$\begin{aligned}
\Delta E[u_B] &= \frac{\frac{v_A^3}{v_A+b} + 3v_A^2\ln[v_A+b] - 2v_A b + \frac{b^2}{2} - \left(\frac{v_A^3}{v_A+a} + 3v_A^2\ln[v_A+a] - 2v_A a + \frac{a^2}{2}\right)}{b-a} \\
&\quad -\frac{b-a}{2} + \frac{4}{3}\left(b^{\frac{3}{2}} - a^{\frac{3}{2}}\right)\sqrt{\frac{2v_A\left(\sqrt{b}-\sqrt{a}\right)}{v_A\left(\ln[b]-\ln[a]\right)+(b-a)}} - \left(\frac{2v_A\left(\sqrt{b}-\sqrt{a}\right)}{v_A\left(\ln[b]-\ln[a]\right)+(b-a)}\right)^2.
\end{aligned}$$

Normalizing the lowest valuation to one, $a = 1$, we illustrate the difference in utility in figure 2.4. b is plotted on the abscissa while v_A is on the ordinate. We plot only valuation pairs for which an interior solution exists. In the lightgray regions the lobbying groups prefer ignorance/non-disclosure, while in the darkgray region the lobbying groups prefer to acquire/disclose information. If A is relatively weak, information disclosure is favorable for both players while if A is relatively strong both players prefer asymmetric information.



panel a) panel b) panel c)

Figure 2.4: Difference in expected utility for lobbying group A (panel a)) and B (panel b)). Zone of agreement (panel c))

We find that players generally agree whether to disclose B's valuation. Only in a small region where A has an about average valuation, in other words v_A is close to $E[v_B]$, the players' preferences diverge. In these cases B prefers disclosure while A prefers to stay ignorant about B's value. This can be seen in figure 2.4 panel c).

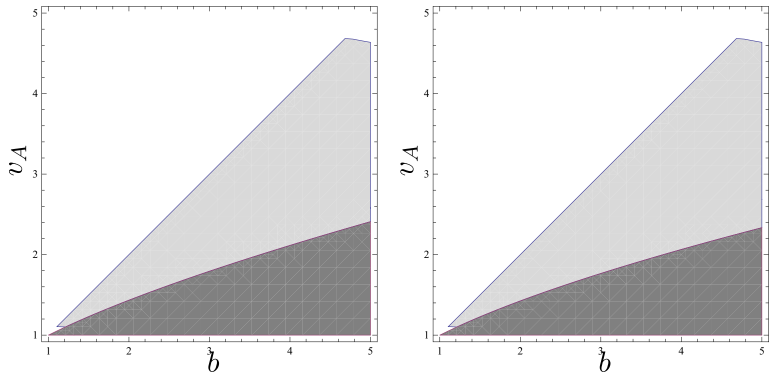
Expected aggregate effort under asymmetric and full-information is equal to

$$\begin{aligned} E[x^{AI}] &= \frac{\frac{2}{3}\left(b^{\frac{3}{2}} - a^{\frac{3}{2}}\right)}{b-a}\sqrt{\frac{2v_A\left(\sqrt{b}-\sqrt{a}\right)}{v_A\left(\ln[b]-\ln[a]\right)+(b-a)}} \\ E[x^{FI}] &= E\left[\frac{v_A^2 v_B}{(v_A+v_B)^2} + \frac{v_B^2 v_A}{(v_A+v_B)^2}\right] \end{aligned}$$

and their difference equals

$$\begin{aligned} E[x^{AI}] &= \frac{\frac{2}{3}\left(b^{\frac{3}{2}} - a^{\frac{3}{2}}\right)}{b-a}\sqrt{\frac{2v_A\left(\sqrt{b}-\sqrt{a}\right)}{v_A\left(\ln[b]-\ln[a]\right)+(b-a)}} \\ E[x^{FI}] &= E\left[\frac{v_A^2 v_B}{(v_A+v_B)^2} + \frac{v_B^2 v_A}{(v_A+v_B)^2}\right]. \end{aligned}$$

Figure 2.5, panel a) illustrates this difference. In the darkgray region disclosure leads to lower aggregate effort while in the lightgray region non-disclosure is preferable.



panel a) panel b)

Figure 2.5: Difference in aggregate effort (panel a)) and efficiency (panel b)).

Lastly, consider efficiency in figure 2.5, panel b). In the darkgray region disclosure leads to higher efficiency while in the lightgray region non-disclosure is preferable.

Overall we find that our results under a continuous uniform distribution are remarkably similar to the ones under only two types of player B, v_H and v_L.

Chapter 3

Does Entry Eliminate Economic Profit? An Experimental Study

Joint with John Morgan, Henrik Orzen and Martin Sefton

3.1 Introduction

The idea that free entry will drive out economic rents is a powerful insight from economic reasoning. The "proof" of this argument works by contradiction—were a situation of positive rents to persist, then individuals on the "outside" would enter to take advantage of the situation up to the point where the rents are competed away. A central implication of this reasoning is that the (risk-adjusted) returns on investment from any economic activity marked by free entry will dwindle to the point where they are just equal to the returns from the next best option.

While this all sounds quite reasonable, we ask a simple question in this chapter: Is it true? In the strategy literature, there are many cases which illustrate the possibility of earning substantial economic rents in industries where competitive forces make this unlikely. For instance, the famous Progressive Insurance case emphasizes how Progressive prospered in the extremely competitive non-standard auto insurance market (see Porter and Siggelkow (1997)). The apparent contradiction between Progressive's success and the forces of free entry is typically resolved by appealing to some inimitable element of Progressive's strategy that allows it to earn rents where others cannot.

But a simpler explanation might be this: Progressive spotted a market where there were rents to be had and, despite their success, this opportunity went unexploited by others. Of course, this explanation relies on the notion that, somehow, sophisticated firms like Allstate and State Farm consistently missed this opportunity despite it being self-evident that there were rents to be had. How plausible is this?

We study the power of free entry using controlled laboratory experiments. Our work builds on the considerable earlier literature on market entry games. In these games, players can decide to enter, in which case they receive a payoff that is decreasing in the number of entrants, or not enter, in which case they receive a fixed outside option. In the first experimental market entry games, Kahneman (1988) found that the number of entrants was very close to the number predicted by theory. Although subsequent experiments have found slight tendencies toward excess entry when equilibrium predicts few entrants and under-entry when equilibrium predicts many entrants, overall there is remarkable support for equilibrium predictions (see Camerer (2003) for a review). In other words, free entry competes away rents and equalizes returns exactly as theory predicts.

So why continue to pursue this apparently already settled question? An important difference between real world markets and the entry games conducted in the lab is that the payoff to a real-world entrant is obviously not a simple deterministic function of the number of rivals. Rather, competitive processes are shaped by both the number of rivals and, importantly, their post-entry strategies. In other words, payoffs depend crucially on what entrants do, not just on how many there are. We ask, does this make a difference?

There is an entry stage followed by a choice stage in our entry games. The choice stage consists of a Tullock rent-seeking contest. To start out, we replicate the earlier entry game experiments under a reduced form specification of the contest whereby payoffs only depend on the number of firms entering. We compare this to an "extensive form" Baseline treatment where players participate in the contest. Here, we find excess competition in the contest. The net effect is that entry payoffs are about 4.5% lower than the outside option. In other words, free entry fails to equalize payoffs across markets.

Next, we investigate the robustness of this effect. One key difference between the reduced form game and the extensive form contest is the volatility in contest payoffs. To determine whether this drives differences in behavior, we amend the contest so that each person earns their expected payoff in the contest given rent-seeking expenditures. Again the contest produces systematically lower payoffs in early rounds, but payoffs converge towards the end of the experiment. In another treatment we add volatility to the outside option so that, under equilibrium play, the contest and the outside option have the same variance. This treatment worsens matters considerably. There is now over-entry into the contest, and payoffs from the contest fall to 5% below the outside option. Finally, we investigate whether it is the contest itself that produces the effect by changing the outside option to another contest only with a larger prize. Once again, free entry predicts payoff equalization across the two contests. Instead, we observe dramatic differences. There is too little entry and too little investment in the contest with the large prize and the converse for the contest with the small prize. As a result the large prize contest systematically produces about 13% higher returns than does the small prize contest.

Our results lead to two important implications. First, free entry does not lead to payoff equalization—at least when there is a contest post-entry. Second, our results are not easily reconciled by amending the theory to account for risk preferences, love of winning, or mere enjoyment in participating in games. That leaves a puzzle—why and when does free entry fail?

The remainder of this chapter proceeds as follows: Section 2 reviews the extant experimental literature on free entry. Section 3 describes the experimental design and its theoretical properties. Section 4 describes our results on free entry and ends with a puzzle—payoffs are not equalized between the inside and outside options when there is a contest post-entry. Section 5 shows how introducing loss aversion into the model can rationalize many of our experimental findings. Finally, section 6 concludes.

3.2 Previous Experiments

The literature on experimental entry games dates back to Kahneman (1988) who was struck by the fact that entry closely coincided with the number of firms in the market such that profits from entering or not were equalized. Kahneman famously remarked, "To a psychologist, it looks like magic." Subsequent experiments found similar results.[1] While aggregate behavior appears consistent with equilibrium predictions, individual behavior is inconsistent with equilibrium. These discrepancies are well accounted for by learning and adaptation type models.[2]

[1]See, e.g., Ochs (1990), Rapoport (1995), Rapoport, Seale, and Ordonez (2002), and Sundali, Rapoport, and Seale (1995). Ochs (1998) offers an excellent survey.

[2]See, e.g., Meyer, Huyck, Battalio, and Saving (1992), Erev and Rapoport (1998), Rapoport, Seale, Erev, and Sundali (1998), and Duffy and Hopkins (2005). See also Goeree and Holt (2001) for QRE-based explanation of entry decisions.

Aggregate predictions diverge from equilibrium when payoffs are not simply a linear function of the number of entrants. Camerer and Lovallo (1999) use rank dependent payoffs and observe under-entry when rank is determined by luck and over-entry when it is determined by skill. Fischbacher and Thöni (2008) observe over entry in a winner take all market where the winner's payoffs are increasing in the number of entrants. Zwick and Rapoport (2002) vary the costs of over-entry and find that entrants earn significantly lower returns than those choosing to stay out.

Our experiments differ in two key respects from this extant literature. First, entry occurs in continuous time rather than simultaneously. In principle, this reduces the possibility for mis-coordination through mixed strategy play. Second, and more importantly, entrants' payoffs depend on their post-entry performance rather than on simply the number of entrants.

In this latter respect, our experiments are similar to Palfrey and Pevnitskaya (2008), who study entry in first-price auctions and Morgan, Orzen, and Sefton (2009), who study entry in contests. These papers are mainly concerned about whether endogenous selection "fixes" deviations from equilibrium in the component games while our focus is on entry and payoff equalization across options.

Finally, our work is somewhat related to the literature on endogenous selection into tournaments versus piece rate schemes. This literature is primarily concerned with how entry correlates with gender, ability at the task, and the prize structure of the tournament rather than on payoff equalization across options.[3]

3.3 Experimental Design and Procedures

The experiment was conducted in multiple sessions at the University of Nottingham. Subjects were recruited from a campus-wide distribution list of undergraduates, and no subject appeared in more than one session.

At the beginning of each session, subjects were seated at computer terminals and given a set of instructions which were read aloud. Any questions were dealt with in private by a monitor. No communication between subjects was permitted, and all choices and information were transmitted via the computer network. Before the decision-making part of the experiment began, groups of six subjects were randomly

[3] See, e.g., Niederle and Vesterlund (2007), Vandegrift, Yavas, and Brown (2007), as well as Cason, Masters, and Sheremeta (forthcoming).

formed and these remained fixed for the entire session. Subjects knew this but did not know which of the other people in the room were in their group.

The decision-making part of the session then consisted of fifty rounds. In each round, a subject was given 100 points and had to choose between two options, labeled "A" and "B". A timer was displayed on the subjects' screens, counting down 15 seconds. Subjects were informed that if they did not make a choice within the time limit the computer would make a choice for them at random.[4] During this time they could see how many members of their group had chosen A, how many had chosen B, and how many had not yet chosen. Once a subject had chosen option A or option B, he or she could not reverse that decision. The information on other group members' decisions was anonymous, in the sense that subjects could only see the number in each category and could not track who of the other group members were in each category from round to round. We incorporated this design choice to minimize the ability of subjects to build reputations.

The consequences from choosing A or B were varied across five experimental treatments. In each case the relevant consequences were carefully explained to subjects at the beginning of the session. In our *Baseline* treatment, anyone choosing option A received a fixed payment of 10 points on top of the initial 100 points. Those choosing option B competed for a prize, worth an additional 50 points. In the following we will refer to option A as the "outside option" and choosing B as the decision to "enter." An entrant's chances of receiving the 50-point prize were equal to the number of 'contest tokens' he or she bought divided by the total number of such tokens bought by all entrants in his or her group. Individuals decided independently and simultaneously how many contest tokens to buy, knowing how many other entrants there were in their group. Each contest token cost 1 point and contestants could spend up to 100 points on tokens.

The contest winner was determined using a computerized lottery wheel. All subjects in the group, whether they had entered or not, observed the purchase decisions and the lottery for the contest option. Again, this information was anonymous in the sense that subjects could see the purchase decisions but they could not associate them with specific group members. All subjects were also reminded of the fixed return from the outside option.

[4]Once the timer on the display had counted down from 15, the computer made the decision only after '0' had been displayed for one second. Thus, the effective time limit for subjects was, in fact, 16 seconds. About 3% of decisions were made by the computer. Our results are unaffected by the inclusion or exclusion of this data.

If less than two people chose to enter in a particular round, no contest was conducted in that round. If exactly one person chose to enter, that person received the prize automatically without having to purchase any contest tokens. With two or more entrants the contest took place as described above. Thus, contestant i's expected payoff is given by $\pi_i = w + (x_i/X)\,P - x_i$, where $w = 100$ is the initial endowment, x_i represents the contestant's investment, X is the total expenditure on contest tokens in the group and $P = 50$ is the value of the prize. In the symmetric Nash equilibrium with n risk-neutral players each contestant spends $50\frac{(n-1)}{n^2}$ points and expects to earn $100 + \frac{50}{n^2}$ points. The outside option, on the other hand, is worth 110 points.

The number of entrants is determined by the continuous time entry decisions of the subjects. A standard view of entry is that players will enter until the expected earnings in the contest equal the value of the outside option (modulo integer constraints on participation). Notice that the equilibrium payoff to an entrant is 112.5 points in a two-player contest and 105.56 points in a three player contest. Thus one might expect that entry would continue until two players have entered, and cease thereafter. See Morgan, et al. (2009) for a formal model of continuous time entry with individual specific delays that delivers this prediction. Of course, behavior in contests may deviate from equilibrium predictions for a variety of reasons, and if so this would affect contest payoffs. Nevertheless, if subjects are attempting to maximize own earnings, they will price contest payoffs into their entry decision and so entry decisions will equilibrate returns from the outside option and the contest.

Our second, *Shares*, treatment was completely identical to the *Baseline* treatment, except that a contestant now received a share of the 50-point prize corresponding to the proportion of contest tokens that he or she had bought relative to the number of contest tokens all contestants had bought in total. Thus, under risk-neutrality the standard-theoretical predictions are the same as in the *Baseline* treatment

In our *Reduced Form* treatment no contests were conducted. Instead, each entrant now simply received a payment equal to the expected profit a contestant would make in the first two treatments if all contestants played the symmetric equilibrium (for simplicity we rounded the relevant amounts to integers). Thus, if only one person entered, that person received 50 points. If two people entered, each of them received 13 points. With three entrants each entrant received 6 points, and so on (the analogous amounts for four, five and six entrants were 3, 2 and 1 points).

While the *Baseline*, *Shares* and *Reduced Form* treatments explore the effects of varying the consequences from choosing option B, our final two treatments study the effects of varying the consequences from choosing option A. In our *Coin Flip* treatment option A is no longer a safe alternative but involves a lottery in which the subject, with a 50-50 chance, either wins 35 points in addition to the initial endowment or loses 15 points. The outcome of this lottery was determined and visualized with a computerized coin toss. The contest in option B was identical to that in the *Baseline* treatment. As in the other treatments, all subjects in a group, whether they had chosen A or B, observed both the events in the contest and the outcome from the outside option (in this case: either +35 or −15). We picked the two coin-flip outcomes in such a way that the expected value is 10 points, the value of the outside option in the *Baseline* treatment, and that the variance of the coin-flip payoffs is identical to the variance of payoffs in the contest option if equilibrium is played (two entrants who each invest a quarter of the prize).

Finally, in the *Dual Contest* treatment the alternative option is *another contest*. The procedures we use for this second contest are identical to the original contest but the value of the prize is 200 points. As before, option B is the baseline contest with 50 points. Suppose the symmetric equilibrium is played in both subgames. If n players choose the 50-point contest their expected payoff is $100 + \frac{50}{n^2}$ points each, the expected payoff for the remaining $6 - n$ players in the 200-point contest is $100 + \frac{200}{(6-n)^2}$ points. The expected payoffs are equalized, and equal to 112.5, when $n = 2$. With two players in the 50-point contest and four in the 200-point contest switching to the other contest would leave any player worse off. Under any other distribution of players between the two contests, however, switching is always payoff-improving for one of the two groups.

At the end of the session, one round was chosen at random and subjects were paid in cash according to their point earnings from this selected round. An exchange rate of £0.10 per point was applied. Altogether 15 sessions with 18 subjects in each session were conducted, yielding a total of 270 participants.[5] Table 3.1 summarizes experimental design.

3.4 Results

We begin by presenting the results from our simplest treatment, the *Reduced Form* treatment. We then present the results from the other treatments, in which entrants

[5]In one of our Dual Contest sessions a technical problem resulted in our losing the last three rounds of data from one group and the last two rounds of data from the other two groups.

Treatment	Outside option ('Option A')	Entry option ('Option B')	Equilibrium no. of entrants	Number of subjects	Number of groups
Baseline	10 points	50-point winner-takes-all contest	2	54	9
Shares	10 points	50-point proportional-shares contest	2	54	9
Reduced Form	10 points	Fixed payments declining in the no. of entrants	2	54	9
Coin Flip	50-50 chance of +35 or -15 points	50-point winner-takes-all contest	2	54	9
Dual Contest	200-point winner-takes-all contest	50-point winner-takes-all contest	2	54	9

Table 3.1: Experimental treatments

compete with one another after entering, to show how enriching the post-entry decision environment affects entry and equilibration.

3.4.1 The *Reduced Form* Treatment

In each round of the *Reduced Form* treatment subjects receive a 100 point endowment and decide whether to enter or not. Those who do not enter receive an additional 10 points. Since two entrants earn an additional 13 points each and three entrants earn an additional 6 points each, one might expect that entry would cease after the second entrant, so that entrants would earn 3 points more than non-entrants. As shown in Figure 3.1 (see Appendix 3.A for all figures and tables), which displays a 10-round moving average of the difference between the observed payoffs from entering and the outside option, this is not the case. Entrants do not earn 3 points more than non-entrants, and in fact entrants earn slightly less than the outside option, but the gap narrows across rounds and in some of the last rounds payoffs from entering exceed those of the outside option. The average payoff from an entry decision in the experiment was 1.15 points lower than the outside option, just 0.46 points lower in the second half of the experiment and 0.45 points higher in the last ten rounds. Looking at only the last ten rounds, after subjects had ample time to experiment and learn how to play the game, we find that payoffs from the inside and outside option are not statistically different (one-sample permutation test on average payoff from entering, p-value=0.332).

[Figure 3.1 about here]

These earnings are a reflection of entry decisions. As shown in Figure 3.2, most entry games featured two entrants, but a substantial number of games, 193 of 450, or 43%, featured more than two entrants. One possibility is that subjects were attempting to use some form of repeated game strategy: by dissuading others from entering they might hope to guarantee their place as one of the two entrants in future rounds. However, if this were the explanation we would expect more aggressive entry in earlier rounds. As seen in Figure 3.2, we do not observe much change in entry behavior across rounds and even in the second half of the sessions 83/225 or 37% of games featured more than two entrants.

[Figure 3.2 about here]

A more likely reason for excess entry lies in the timing of decisions. Figure 3.3 displays a histogram of the delays between second and third entry decision. Of the 193 occasions when a third subject entered, 137 of them were less than a second after the second subject entered and the median delay between second and third entry time was 0.24 seconds. Thus second and third entry times are very close. Further, these entry times cluster near the beginning of the round: 124/193 or 64% of the third entry decisions took place within five seconds of the round beginning and as a result, entry games effectively ended quickly.

[Figure 3.3 about here]

Thus, it appears that, at least in some games, subjects were racing to be an entrant and ended up one of three entrants. Our interpretation of the results is that subjects recognized the potential three point advantage from being one of two entrants. However, since all subjects have an identical opportunity to enter, the identities of the entrants were determined by a race, and in racing subjects found a way to compete for the additional points. As seen in Figure 3.1, this resulted in the payoff difference between entering and staying out being "competed away".

The following dynamic entry game offers a formal model of this process. The game consists of an infinite sequence of periods. At the beginning of each period players are informed of the number of incumbents. Players that have waited in all previous periods choose simultaneously between IN, OUT and WAIT. A symmetric equilibrium of the game is for players to choose IN with probability p_0 and WAIT with probability $(1 - p_0)$ if there are no incumbents, to choose IN with probability p_1 and WAIT with probability $(1 - p_1)$ if there is one incumbent, and to choose OUT if there are two or more incumbents. If there are either 0 or 1 incumbents, it is clearly not an equilibrium for all active players to choose IN, or for all active players to WAIT. Instead players must mix so that the expected payoff from choosing IN equates to the expected payoff from choosing WAIT. The solution is that when there are no incumbents a player enters with probability $p_0 = 0.229$ and when there is one incumbent an active player enters with probability $p_1 = 0.119$. (Details are given in Appendix 3.C.)

What are the implications of this model? First, the model continues predict an expected number of entrants of 2.34 with a standard deviation of 0.59, not far from the observed average number of entrants of 2.51 with a standard deviation of 0.67. Moreover, in equilibrium the model has no entry games with less than two entrants and ninety-five percent of entry games feature either two or three entrants.[6] Most importantly, expected payoffs to an entrant are equal to the outside option, and in fact, we cannot reject equality of payoffs in our data in the last ten rounds (permutation test, p-value = 0.332). Our *Reduced Form* treatment is able to replicate the "magic" found by Kahneman (1988) and others.

3.4.2 Replacing the Reduced Form with a Contest: The *Baseline* Treatment

Entrants in the *Reduced Form* treatment receive a payoff which is a deterministic function of the number of entrants. Next we examine how well payoff-equalization works when these payoffs are replaced with a competitive game among the entrants.

In our *Baseline* treatment entrants take part in a standard Tullock rent-seeking contest. Entrants simultaneously choose contest investments and one of the entrants is awarded the prize, with the winner determined by a lottery in which each entrant's probability of winning equals his or her contest investment as a fraction of total contest investments. Under risk-neutrality, the Nash equilibrium of the n-player contest results in the same expected payoff to an entrant as in the *Reduced Form* treatment.

We begin with entry decisions. In contrast to the *Reduced Form* treatment there is little evidence of hurried decisions. Figure 3.4 presents histograms of the second entry time in each game, focusing on the second half of the experiment. The median time of the second entry decision in the *Baseline* treatment was 13.78 seconds, thus towards the end of the 15 second time limit, while in the *Reduced Form* treatment the median was

[6] An alternative static model of entry decisions where, in equilibrium, a player enters with probability 0.404 and stays out with probability 0.596 is not commensurate with our data because it implies too much variation in the number of entrants and in particular does not capture the concentration of the data on 2 or 3 entrants.

just 3.25 seconds. Thus, it appears that some subjects left it very late to enter, perhaps because they wanted to avoid getting into a contest with more than two players.

[Figure 3.4 about here]

Waiting to enter to avoid a crowded contest didn't always work: even when the second entrant entered after 13.78 seconds, a third entrant entered in 97 of 213 (45%) such games. In fact, although the timing of entry decisions was very different in our *Baseline* and *Reduced Form* treatments, Figure 3.5 shows that the distributions of the number of entrants were remarkably similar. In both treatments the modal number of contestants is two, although contests with three entrants are also commonly observed. Altogether, 88% of the contests involve either two or three entrants. Formal tests do not detect a significant difference in the mean number of entrants in the two treatments (two-sample Wilcoxon rank-sum test on independent groups, z=-0.222, p-value=0.8246).

[Figure 3.5 about here]

Given that the entry decisions are similar, entrant payoffs in the *Baseline* treatment should be similar to those in *Reduced Form* if subsequent contest investments are close to predictions. In fact this turns out not to be the case. Table 3.3 presents the total investment in a contest, averaged over contests, for each number of entrants. For now focus on the columns for investments in the *Baseline* treatment and the "Small Prize" theoretical prediction. Contest investments are 42% higher than in equilibrium in two-player contests and 24% higher than in equilibrium in three-player contests. Indeed, for any size of contest investment exceeds equilibrium levels.

[Table 3.3 about here]

A consequence of excessive investment is that contest payoffs are lower than the *Reduced Form* payoffs for any given number of entrants. Since the distributions of the number of entrants is similar across treatments this implies that entrants earn less than in the *Reduced Form* treatment. A two-sample permutation test reveals that this difference is significant, even in the last ten rounds (p-value=0.014). Figure 3.6 displays moving averages of payoff differentials in the two treatments. Payoffs are much lower than the outside option in the early rounds of the *Baseline* treatment, and increase over time. However payoffs level off somewhat short of the outside option. In the second half of the experiment, earnings per entry decision are 4.8 points below the outside option, in the last ten rounds even 5.58 points. While the permutation test on this difference turns out barely insignificant (p-value = 0.51), the magnitude of the loss is surprisingly high — entrants even in the last ten rounds earn 5% less than non-entrants.

Furthermore as we have shown, the difference to *Reduced Form* earnings, which closely approximate the outside option, is significant.

[Figure 3.6 about here]

3.4.3 Deterministic Contests: The *Shares* Treatment

Why do entrants persistently earn significantly less than the outside option in *Baseline*? Why don't they simply opt out? The probabilistic nature of success in baseline contests offers two possible explanations. First, if players are risk-seeking, then they will be willing to pay a risk premium to take part in the contest. Thus, even though the expected return from the contest is lower than the outside option, the variability in returns dissuades risk-seeking contestants from opting out. Second, even if a risk-neutral player would opt out if she knew the expected payoff from entry was less than the outside option, the expected payoff from the contest may not be transparent. For example, a player that attempts to estimate the expected value based on her own contest earnings will get at best a noisy estimate since her actual contest payoff will fluctuate dramatically depending on whether or not she wins the prize. Indeed, average payoffs may be a poor guide to expected payoffs even for a regular contestant.

To see whether persistent earnings differentials in *Baseline* are due to the probabilistic success function of the Tullock contest we ran a *Shares* treatment which replaces the probabilistic success function with a deterministic one. The contest in this treatment is the same as in *Baseline* except that each entrant receives a share of a prize of 50 points, where the share is determined by his or her own contest investment as a fraction of total contest investment. The Nash equilibrium of the share contest with n entrants is independent of risk preferences, and is the same as the Nash equilibrium of the *Baseline* contest under risk-neutrality. In particular, the Nash equilibrium of the n-player shares contest generates the same payoffs as when n subjects enter in the *Reduced Form* treatment.

We begin with entry decisions. Figure 3.7 compares the distributions of entry decisions in the *Baseline* and *Shares* treatments (based on data from rounds 26-50). As in the previous treatments discussed, the vast majority of games in the *Shares* treatment feature either two or three contestants. Here the modal number of two contestants is more pronounced than in *Baseline*, although formal tests also do not detect significant differences in the distributions of the number of entrants across treatments (two-sample Wilcoxon rank-sum test on independent groups, z=0.619, p-value=0.5361).

[Figure 3.7 about here]

Next we consider contest investments conditional on the number of entrants. As shown in Table 3, average investment in three-player shares contests is remarkably close to the equilibrium prediction (although note the variability of investments: contest investments range from 14 to 59 points). However, most contests involve two players, and in these average investments are more than 40% higher than in the Nash equilibrium. A consequence of this is that the expected payoff from a two-player shares contest will be lower than the reduced form payoff.

Figure 3.8 compares entrant earnings in the *Baseline* and *Shares* treatments. Qualitatively the moving averages are similar. Contest payoffs are substantially lower than the outside option in the early rounds, erode over the course of the session. But while they level off below the outside option in the *Baseline* treatment, we get payoff convergence in the *Shares* treatment. A permutation test does not detect any significant differences between returns to the inside and outside option in the last ten rounds (p-value = 0.832) and payoffs upon entry in *Baseline* and *Shares* are significantly different (p-value = 0.046). In fact, we find that payoffs from entering in the *Shares* and *Reduced Form* treatments in the last ten rounds are not significantly different (p-value = 0.537).

[Figure 3.8 about here]

3.4.4 Introducing Risk into the Outside Option: The *Coin Flip* Treatment

Up until now, we have considered the case where the payoffs from the outside option were fixed, and have examined the consequences of varying the nature of the 'inside option'. Suppose that the marginal individual is risk-seeking. In that case, entry would not lead to payoff equalization in the *Baseline* treatment. Instead, the risky contest would sell at a "discount" relative to the outside option. Thus, the difference between the long run expected payoff of the *Baseline* contest and the outside option may reflect a (negative) risk premium.

To address this possibility, in our *Coin Flip* treatment we modified the *Baseline* treatment by changing the risk of the outside option. Specifically, we set the expected value of the outside option equal to 10, as under the *Baseline* treatment, but we set the payoffs such that the variance in the outside option was equal to the variance of the inside option in the (risk-neutral) equilibrium of a two-player contest.

Entry decisions provide a striking contrast between the *Coin Flip* and other treatments. Figure 3.9 compares the distribution of the number of entrants in the second half of the *Coin Flip* and *Baseline* treatments. As the figure shows, adding risk to the outside option increases entry into the contest considerably. The mean number of entrants is 3.78 in *Coin Flip* compared with 2.47 in *Baseline*. Focusing on rounds 26-50 we use a rank-sum test to compare the distribution of entrants in each group across rounds under *Baseline* and *Coin Flip* treatments. The test rejects the hypothesis of equality of mean number of entrants (z = 3.226, p-value = 0.0013).

[Figure 3.9 about here]

Subsequent investment decisions suggest the reason for the higher entry rate. The contest investments displayed in Table 3.3 show that when there are 2 or 3 entrants, total investments in the *Coin Flip* treatment are close to equilibrium predictions while with 4 or more entrants (which now occurs in the majority of games), total investment is well below the equilibrium predictions. This is in sharp contrast to the *Baseline* treatment, which produced consistent overinvestment compared to equilibrium. At some level, the differences in investment behavior under the two treatments are surprising since, post-entry, the two games are exactly alike. However, the crucial difference is in how the nature of the outside option affects selection into the contest.

Obviously, the outside option under the *Coin Flip* treatment is riskier than it was under *Baseline*. Thus, one might conjecture that the force of selection would be to drive risk averse individuals into the contest. While the contest is, on average, a risky gamble, strategies are available to subjects that substantially reduce this risk. For instance, by opting into the contest and investing zero tokens, a subject removes all risk. In Table 3.4 we tabulate the percentage of low investment decisions in the second half of the experiment, where we define a low investment decision as investing 0 or 1 point. Such decisions account for about 20% of investment decisions in the *Coin Flip* treatment, whereas by comparison, in the *Baseline* treatment these low investment choices account for about 2% of investment decisions.

[Table 3.4 about here]

As table 3.4 shows, low investment decisions are also correlated with the number of contestants. In two-player contests players rarely make a low investment, while when there are four or more entrants, about one of these makes a low investment and effectively does not compete. Thus, the lower level of investment conditional on the number of entrants in *Coin Flip* relative to *Baseline* can in part be explained by the presence of "Passive Entrants" who enter the contest for an insurance motive and effectively do not compete.

However, this only explains part of the difference. In Table 3.5 we report contest investment conditional on the number of non-passive entrants, where a non-passive entrant is defined as one who invests more than one token. The table shows that even after accounting for passive entrants, investment conditional on the number of adjusted entrants is substantially lower in the *Coin Flip* than the *Baseline* treatment.

[Table 3.5 about here]

What can account for the rest of the difference between investment levels in the *Coin Flip* and *Baseline* treatments? One possible explanation is the following. Suppose that there are three types of subjects, gamblers, neutrals, and risk-averters. In the *Baseline* treatment, gamblers are attracted to the contest and invest overly aggressively. Naturally, neutrals and risk-averters stay out. When the outside option is a coin flip, gamblers are attracted to this gamble. Meanwhile risk-averters are attracted to the security of the contest. Neutrals, sensing an opportunity, are now also drawn to the contest. While this seems like a useful rationalization, as we will see in the next section, it fails as an explanation of behavior in the Dual Contest treatment.

Recall that in the *Baseline* treatment, returns from the inside option were systematically lower than the outside option owing to overly aggressive investment. While on the one hand investment behavior in the contest (once adjusted for passive entrants) is moderated and close to equilibrium predictions, on the other hand there is even more entry. What is the overall impact on payoff equalization? As shown in Figure 3.10 payoff differentials are lower in the early rounds of *Coin Flip*, but entry continues to produce systematically worse returns than the outside option. It appears that in the long run differentials in *Baseline* and *Coin Flip* are similar, although in the *Coin Flip* treatment the underperformance of the contest is driven mainly by excess entry rather than by overly aggressive contest investment. A two-sample permutation test cannot reject the hypothesis of equality of payoffs upon entry in the last ten rounds (p-value = 0.998) between *Baseline* and *Coin Flip* while it rejects the hypothesis that the payoff upon entry equals 110 in the last ten rounds (p-value=0.004).

[Figure 3.10 about here]

3.4.5 Introducing Strategic Behavior into the Outside Option: The Dual Contest Treatment

The previous section showed that adding risk to the outside option was not enough to achieve payoff equalization although it did lead to investment behavior closer to equilibrium predictions. The failure was mainly driven by excess entry. Perhaps subjects are willing to pay a premium to earn rewards from a competitive activity, rather than from a lottery. That is, the expected payoff differential may reflect a "fun and games" premium.

To examine this possibility, we change the outside option to another contest. If payoff differences merely reflected a "fun and games" premium, then they should vanish in this treatment since all subjects play a strategic game. Note that the expected payoff from the outside option now depends on entry decisions. In order to maintain comparability with the other treatments we kept option B exactly as in *Baseline*, and set the contest prize in option A to be 200 points. Assuming equilibrium contest investments, this implies that contest payoffs will be equalized when two players enter the small prize contest and four enter the large prize contest, and in this case the expected payoff from the small prize contest will be the same as in the other treatments.

Figure 3.11 displays the distribution of number of entrants in the small prize contest, together with the distribution from the *Baseline* treatment for comparison. Although the median and modal number of small prize contestants is three in the *Dual Contest* treatment versus two in the *Baseline* treatment, the mean number of entrants is very similar and formal tests do not detect significant differences in the distribution (Wilcoxon rank-sum test applied to independent groups: z = 0.619, p-value = 0.5361).

[Figure 3.11 about here]

Whether payoffs to entering the small prize contest differ across treatments will thus depend on investment behavior. In fact, investment in the small prize contest, conditional on the number of entrants, is very similar in *Baseline* and *Dual Contest* treatments (see Table 3.3). This is somewhat surprising since we saw that the contest offered a harbor for risk averse players when the outside option was a coin flip. Don't we observe similar levels of low investment in the small prize contest of the *Dual Contest* treatment? Table 3.6 displays the percentage of low investment decisions for each number of entrants and shows that the answer is 'No': passive entry is much less pronounced than in the *Coin Flip* treatment. One possibility is that the large prize contest is a more attractive harbor, offering at least the possibility of a windfall 200 points from investing a single point. But Table 3.6 also shows that the rate of passive entry is very similar across contests. Most contests feature two or three entrants (small prize) or three or four (large prize), and in these contests less than 7% of entrants are passive. As with the *Coin Flip* treatment, low investments mainly occur in contests with four or more entrants. However, there are far fewer small prize contests with four entrants in the *Dual Contest* treatment.

[Table 3.6 about here]

Because the distributions of entrants in the small prize contest and investments conditional on entering are similar to *Baseline*, the consequence is that the payoff to entering the small prize contest is similar: focusing on the last ten rounds of the sessions, entrants earned an additional 5.81 points on average in *Dual Contest* compared with 4.42 points in *Baseline*, and this difference is not significant (two-sample permutation test applied to independent groups: p-value = 0.032).

What about the large prize contest? Here the average number of entrants is 3.5, lower than predicted. Moreover, investment conditional on the number of entrants is lower than in equilibrium (see Table 3.3). A consequence is that entrants to the large prize contest earn more than predicted. Entrants to the large prize contest earned an additional 20.84 points on average in the last ten rounds of the experiment. To summarize, there is excess entry into, and excess investment in the small prize contest and under-entry into, and under-investment in the large prize contest relative to equilibrium predictions. What effect does this have on payoff differences between the two contests? Figure 3.12 plots the moving average of the payoff differential between the small and large prize contests. Expected payoffs are substantially, and persistently, lower in the small prize contest.

[Figure 3.12 about here]

3.4.6 Discussion of Results

Our results reproduce the qualitative finding from previous studies: in simple entry games where entrants payoffs depend only on the number of entrants we observe payoff equalization. But we also find that free entry does not lead to payoff equalization when there is a contest post-entry. This latter finding contrasts with the conventional view that free entry will lead to equalization of expected payoffs.

Our results are not easily reconciled by amending the theory to account for risk preferences. If players are risk seeking, then returns to the large prize contest should be below those of the small prize contest. Clearly this isn't the case. If, on the other hand, players are risk averse, then it is possible to account for higher returns to the large prize contest relative to the small prize contest (although in our view the size of the observed differential makes this rather implausible). But if players are risk averse the contest should trade at a premium in *Baseline*, which it doesn't.

Similarly, appealing to heterogeneity in risk preferences provides an explanation for some patterns but not others. Recall that investments in the *Coin Flip* treatment could be explained by the co-existence of risk-seeking, risk-neutral and risk-averse subjects. However, under that rationalization, we would expect risk-seekers to be attracted to the riskier large prize contest, risk averters would prefer the less risky small prize contest, while neutrals should select to whichever contest is yielding higher payoffs. That rationalization would suggest excess investment in the large prize and under-investment in the small prize contest. Instead, we observe just the opposite.

Could complexity be an issue? If the game is too complex and subjects do not understand their optimal strategy, we would expect payoff non-equalization. Even though we do not have a final answer to this question we believe that it is unlikely to be the reason for our results. First of all individuals have 50 rounds to learn about how to play the game, so there is ample time for experimentation. Second, in all but *Dual Contest* treatment we observe a significant improvement in equilibrium payoffs from the (more complex) inside option over time, which we would not expect if decisions were merely random. When looking at the characteristics of subjects entering in rounds 40-50 we see a similar pattern. Table 3.7 shows that entrants in the last ten rounds have on average (1) entered more often and (2) earned a higher payoff conditional upon entry in the first forty rounds, in every single treatment.

[Table 3.7 about here]

3.5 A Model of Loss Aversion

We are left with a puzzle. While standard explanations such as risk-preferences and love of winning can explain some of the failures of payoff equalization, they fail to account for the collection of results across treatments. In this section we go back to theory and explore in detail the effect of risk-preferences on entry and investment decisions - specifically in the framework of loss-aversion. We show how incorporating loss aversion into preferences can account for the bulk of our findings and suggest why some aspects of the data remain unexplained by this theory.

Recall that in our experiment, an endogenous number of players (say n) were competing in a contest for a prize worth R. When player i chooses contest effort x_i, her chance of winning the prize is simply $\frac{x_i}{\sum_{j=1}^{n} x_j}$. To allow for loss aversion requires that we add a reference point, r, to the model. Losses and gains are coded relative to the reference. For the moment, treat the reference point as exogenously given.

To close the model, we use the following procedure. First, fix the reference points under each option and let entry (n) adjust to the point where the expected gain/loss utilities equalize. Next, to determine the reference points, suppose that they are the same for the two options and let the reference point correspond to what the player *expects* to obtain from pursuing a given course of action. That is, under rational expectations, players repeatedly participating in a given gamble should not be systematically "surprised" by the outcome. Under this framework, the reference points should adjust such that the expected gain/loss utility under each option is equal to zero.[7] As we will show, this solution does not necessarily lead to payoff equalization.

Given the reference point r and an ending wealth state w, the payoffs of a player are

$$U = \begin{cases} \beta\,(w-r) & \text{if} \quad w > r \\ \alpha(w-r) & \text{if} \quad w \leq r \end{cases}$$

where $\beta \leq 1 \leq \alpha$. The parameter α represents the weighting on losses while β represents the weighting on gains relative to the reference point. As in Kahneman and Tversky (1979), we assume that losses are more painful than gains are pleasurable; hence the ordering on α and β.

To examine the equilibrium for this game, consider the contest stage of the game after n players have entered. We will compute a symmetric equilibrium for this game where each player undertakes equilibrium effort x^*. Temporarily assume that, in equilibrium $R - x^* > r$[8], then expected utility is equal to

$$EU = \frac{x_i}{\sum_{j=1}^{n} x_j}\beta\,(R - x_i - r) + \left(1 - \frac{x_i}{\sum_{j=1}^{n} x_j}\right)\alpha\,(-x_i - r)\,. \tag{3.1}$$

And, it is straightforward to show that:

[7] Köszegi and Rabin (2006) propose a model of an endogenous reference point in a decision theoretic framework with exogenous uncertainty. As in their work, the reference point in our model also adjusts to expected payoffs but in contrast to them it is not choice-acclimating. After entering the contest, and when deciding on contest investments, the reference point is assumed to be fixed.

[8] If this is not the case, the player evaluates winning and losing as a loss, and his behavior is equivalent to a risk-neutral player.

Proposition 11. *In the unique symmetric equilibrium with reference point* $0 \leq r \leq \frac{\beta n+\alpha(n-1)^2}{n(\beta+\alpha(n-1))}R$, *equilibrium effort is equal to*

$$x^* = (n-1)\frac{(\beta R + (\alpha-\beta)r)}{\beta(2n-1)+\alpha(n-1)^2}. \tag{3.2}$$

The proof is given in Appendix 3.D.

Proposition 11 shows that there is a unique symmetric equilibrium so long as the reference point is not too high relative to the size of the prize. While an exact inequality for this to hold is given in the proposition, it suffices that $r < \frac{n-1}{n}R$. Notice that, as the number of players in the contest gets large, this amounts to the condition that the reference point lies below the prize amount; that is, it cannot be the case that all players entering the contest expect to win it with certainty.

Free entry

Next we turn to entry decisions. Entry is determined by the expected payoff in the contest relative to the outside option. From proposition 11, the expected utility of a loss averse player with reference point at $0 \leq r \leq \frac{\beta n+\alpha(n-1)^2}{n(\beta+\alpha(n-1))}R$ when there are n entrants to the contest is equal to

$$EU = \frac{\beta^2 R - r\left((n-1)\alpha+\beta\right)^2}{\beta(2n-1)+\alpha(n-1)^2}, \tag{3.3}$$

when she enters the contest.

It remains to pin down the reference point. Recall that entrants to the contest could have opted for a (fixed) outside option, which we shall denote by z. Under rational expectations, the reference point should adjust such that the gain/loss utility is equal to zero; hence $r = z$. Finally, free entry will occur to the point where gain/loss utilities are equalized at zero for the two options. Thus, it must be that:

$$\frac{\beta^2 R - z\left((n-1)\alpha+\beta\right)^2}{\beta(2n-1)+\alpha(n-1)^2} = 0 \tag{3.4}$$

Solving for n, we obtain the unique feasible solution:

$$n^* = 1 + \frac{\beta}{\alpha}\left(\sqrt{\frac{R}{z}} - 1\right) \tag{3.5}$$

Thus, we have shown:

Proposition 12. *Suppose that the outside option is not too high. Then there is a unique symmetric equilibrium of the loss averse contest with entry. Formally, in equilibrium, n^* players enter defined by equation* (3.5)*.*

Notice that, under the loss averse model, utility payoffs equalize under free entry in the sense that the reference point reflects the value of the outside option and, moreover, the expected payoff (evaluated in gain/loss space) is equal to the reference point. That is to say, players have rational expectations. Nonetheless, monetary payoffs do not equalize nor is entry or investment behavior the same as under the risk-neutral prediction.

Coin Flip Treatment

Of course, in some of our treatments, the outside option was not fixed. Under the coin flip treatment, for instance, the outside option was a lottery. Intuitively, this should lead to a lower reference point since losses are more painful than gains are pleasurable. The next lemma shows this formally.

Lemma 4. *Suppose that utility equals zero under the outside option with reference point r'. Then the reference point under Coin Flip is smaller than that under the Baseline treatment.*

The proof is given in Appendix 3.E.

Thus, simply amend the model to reflect the diminished reference associated with the outside option and re-solve to obtain n^* and x^* under the *Coin Flip* treatment.

Dual Contest Contest

Finally, we turn to the *Dual Contest* treatment. In this treatment, there is no fixed outside option. Instead, we close the model by supposing that the reference points are the same and, in equilibrium, the gain/loss utility equals zero. Thus, the analysis is parallel to all the other treatments, this yields the system of equations

$$\frac{\beta^2 R_0 - r\left((n-1)\alpha + \beta\right)^2}{\beta(2n-1) + \alpha(n-1)^2} = \frac{\beta^2 R_1 - r\left((N-n-1)\alpha + \beta\right)^2}{\beta\left(2(N-n)-1\right) + \alpha\left((N-n)-1\right)^2} \tag{3.6}$$

$$\frac{\beta^2 R_0 - r\left((n-1)\alpha + \beta\right)^2}{\beta(2n-1) + \alpha(n-1)^2} = 0 \tag{3.7}$$

where $R_0 < R_1$ represents the prize in the small and large prize contests respectively and N denotes the total number of potential participants in either contest. Solving equations (3.7) and (3.6), we obtain equilibrium entry and reference point

$$\begin{aligned} n^* &= \frac{(R_0 + R_1)(\alpha - \beta) - R_0 \alpha N + ((N-2)\alpha + 2\beta)\sqrt{R_1 R_0}}{\alpha (R_1 - R_0)} \\ r^* &= \frac{R_0 \beta^2 (R_0 - R_1)^2}{\left(((N-2)\alpha + 2\beta)\left(\sqrt{R_0 R_1} - R_0\right)\right)^2}. \end{aligned}$$

Comparisons

We now use the loss averse model to make comparisons across treatments and between the loss averse model and the risk-neutral benchmark. We start by considering investment behavior in the contest.

First, let us examine how equilibrium investment varies with the outside option, which in equilibrium, is equal to the reference point. It may be readily seen by inspecting equation (3.2) that equilibrium effort is increasing in r. Thus, we observe:

Remark 1. *For a fixed number of contest entrants, the higher the value of the outside option (i.e. the reference point), the greater the contest expenditures.*

While the value of the outside option remains fixed under risk neutrality, this is not the case when players are loss averse. In particular, a fixed outside option is more valuable than a stochastic outside option. Thus, the comparison between the *Baseline* and *Coin Flip* treatments offers a direct test of the remark. As Table 3.3 and Table 3.5 show, for a given realization of n, investments are considerably lower under *Coin Flip* than under the *Baseline* treatment, which is consistent with the loss averse model.

How do contest expenditures under the loss averse model compare to the risk-neutral benchmark? Recall that under risk-neutrality, when there are n contest entrants, equilibrium investments are:

$$x_{RN} = (n-1)\frac{R}{(2n-1) + (n-1)^2} = \frac{(n-1)}{n^2}R$$

Comparing this to equation (3.2), one will observe higher expenditures under loss aversion if and only if

$$r > \left(\frac{n-1}{n}\right)^2 R. \tag{3.8}$$

Notice that this inequality depends only on the reference point, the number of contestants, and the size of the prize, the particular weights given to gains and losses do not figure into the comparison. It may be readily verified that for a dense set of parameter values in which Proposition 11 holds, the inequality given in equation (3.8) is also satisfied.[9] Since the reference point reflects the outside option, in equilibrium, then for a fixed reference point:

Remark 2. *The higher is the value of the prize, the greater the chance of underinvestment (relative to the risk-neutral prediction) in the loss averse model.*

A direct test of this implication of the model is a comparison of investment for a given number of contestants under a contest with a large prize versus a contest with a small prize. The loss averse model predicts more underinvestment in large prize contests. This is precisely what we observe in the *Dual Contest* treatment. Further experimental evidence consistent with this prediction may be seen in Morgan, Orzen, and Sefton (2009). Here, they compared a large prize contest to a fixed outside option as well as running our *Baseline* treatment. They found underinvestment in the large prize contest and overinvestment in the small prize contest.

Next, we turn to entry decisions under the loss averse model as compared to the risk-neutral model. Recall that the equilibrium number of entrants in the risk-neutral model (ignoring integer constraints) is

$$n_{RN} = \sqrt{\frac{R}{z}}$$

where the subscript RN denotes the fact that this is an equilibrium for the risk-neutral model. Differencing n^* and n_{RN} we find

$$\begin{aligned} n_{RN} - n^* &= \left(\sqrt{\frac{R}{z}} - 1\right)\left(\frac{\alpha - \beta}{\alpha}\right) \\ &> 0 \end{aligned} \tag{3.9}$$

since $\alpha > \beta$ and $R > z$. Thus, we may conclude

Remark 3. *In equilibrium, the loss averse model produces under-entry in the contest relative to the risk-neutral benchmark. Furthermore, the degree of under-entry increases with the size of the prize.*

This implication is consistent with the experimental findings of Morgan, Orzen, and Sefton (2009). They observed pronounced under-entry in the a contest with a large

[9]A sufficient condition is that $\frac{n}{n-1}r < R < \left(\frac{n}{n-1}\right)^2 r$

prize compared to a contest with a small prize for a fixed (and identical) outside option in each contest. Likewise, under the *Dual Contest* treatment, we observe significant under-entry in the large prize contest and (from the adding up condition) over-entry in the small prize contest. The loss averse model, however, predicts under-entry in the *Baseline* treatment relative to the risk-neutral prediction which is not supported in the data.

While equation (3.9) is appropriate when the outside option is fixed, it is not appropriate when the outside option is stochastic. In the *Coin Flip* treatment, for instance, the risk-neutral prediction is unchanged from the *Baseline* treatment. However, the randomness in the outside option depresses the value of z under loss aversion. Thus, we may conclude

Remark 4. *The loss averse model produces more entry (compared to the risk-neutral case) in the Coin Flip treatment than in the Baseline treatment.*

which is exactly what we find experimentally.

Let us turn to the *Dual Contest* treatment. Setting the payoffs to entry in the risk-neutral case equal

$$\frac{R_0}{n^2} = \frac{R_1}{(N-n)^2}$$

we get equilibrium entry

$$n = \frac{\left(\sqrt{R_0 R_1} - R_0\right) N}{(R_1 - R_0)}.$$

The difference in entry between risk-neutrality and loss-aversion is equal to

$$n_{RN} - n^* = \frac{(\alpha - \beta)}{\alpha (R_1 - R_0)} \left(2\sqrt{R_1 R_0} - (R_0 + R_1)\right)$$

Since

$$R_0 + R_1 > 2\sqrt{R_1 R_0}$$

(since the arithmetic mean is larger than the geometric mean), there is excess entry in the small contest and under-entry in the large contest under loss-aversion. Thus we have shown

Remark 5. *In the dual contest, there is under-entry in the large prize contest and over-entry in the small prize contest under loss aversion.*

Finally, we turn to monetary payoffs. Recall that under the benchmark model, expected monetary payoffs of the inside and outside options should equalize. Here, we show that this is not an implication of the loss averse model. In equilibrium, the expected monetary payoffs from entering the contest are:

$$\frac{1}{n^*}R-(n^*-1)\frac{(\beta R+(\alpha-\beta)z)}{\beta(2n^*-1)+\alpha(n^*-1)^2}$$

While those under the fixed outside option are simply z. Thus, the net payoff difference between the inside and outside options is

$$\Delta=\frac{1}{n^*}R-(n^*-1)\frac{(\beta R+(\alpha-\beta)z)}{\beta(2n^*-1)+\alpha(n^*-1)^2}-z$$

Substituting the value of n^* and simplifying, we obtain

$$\begin{aligned}\Delta &= \frac{z(\alpha-\beta)}{\beta\sqrt{z}\sqrt{R}+z(\alpha-\beta)}\left(\left(\sqrt{R}-\sqrt{z}\right)^2+\sqrt{z}\sqrt{R}-z\right)\\ &> 0\end{aligned}$$

since $\alpha>\beta$ and $R>z$. Thus, we have shown that, under the loss averse model, the monetary returns to the contest exceed the outside option. This is clearly at odds with the data. In *Baseline*, the inside option produced systematically *lower* monetary payoffs than the outside option. However, it does predict the shift in payoff differences with the size of the contest prize. Specifically, differentiating Δ with respect to R yields

$$\frac{\partial\Delta}{\partial R}\propto\beta\sqrt{z}R+2\sqrt{R}z(\alpha-\beta)-z^{\frac{3}{2}}(\alpha-\beta)$$

Next, notice that $\sqrt{R}>\sqrt{z}$ and hence $\frac{\partial\Delta}{\partial R}>0$. Thus, we have shown

Remark 6. *Under the loss averse model, as the size of the prize increases, so also does the difference between the returns from the inside option versus the outside option.*

The remark is consistent with observations from the Dual prize contest as well as the large prize treatment in Morgan, Orzen, and Sefton (2009). In both instances, the large prize contest produces disproportionately higher returns than the alternative.

In the *Coin Flip* treatment the reference is lower than the monetary value of the outside option, implying lower investments in the contest but also a higher number of entrants. The overall effect is ambiguous. It can easily be shown that for our parameter values the expected value of the lottery exceeds the monetary payoff from the contest which is consistent with our observations.

	Entry	**Investment**	**Payoff**
RF	as in RN	—	equalize
B	**underentry**, decreasing in R	ambiguous, decreasing in R	**higher payoff**, increasing in R
S	as in RN	as in RN	equalize
CF	more entry than B	for given n smaller than B	lower payoff
DC	overentry SPC	more underinvestment in LPC	higher payoff in LPC

Table 3.2: Predictions of the loss-averse model relative to risk-neutrality

Notice that, when there is no risk in the contest prize, as in the *Shares* and *Reduced form* treatments, then the loss averse model and the risk-neutral model coincide. Thus, the loss averse model can simultaneously explain the "success" of standard theory predictions for these treatments and the "failure" of payoff equalization in the other treatments. That being said, there is one key aspect of the data that is not rationalized by the loss aversion model, over-entry in the small contest. If players have rational expectations about reference points, then the loss averse model predict under-entry relative to the risk-neutral case and, despite aggressive investment behavior in the subsequent contest, players should not accumulate significant losses relative to the outside option. Of course, this is not what we observe in our experiments. Table 3.2 summarizes the predictions of the loss-averse model. Predictions in bold are not supported by the data.

One possible explanation for the observed overentry in *Baseline* is that relative preferences play an important role in contests where the equilibrium number of entrants is fewer than two. In the small prize contest, the risk-neutral equilibrium prediction is two entrants while the loss averse model predicts two or fewer entrants depending on the degree of loss aversion. But consider the situation when exactly one person enters. Absent additional entry, that person is assured of winning the contest with zero investment and thus stands to gain a significant payoff advantage over rivals staying out of the contest. If, however, rivals care about relative payoffs, there is a strong temptation to enter (even if this proves costly) and thereby lead to more equitable payoffs than to allow a single entrant to enjoy monopoly rents. Of course, there is a free-rider problem associated with this "kamikaze" entry strategy. It is better if someone else enters than that you enter. Thus, the small contest has aspects of a war of attrition whereby, once the first person has entered, all others have an incentive to wait for a rival to "volunteer" for the mission. Indeed, we see this behavior in the data. Moreover, by waiting until the end of the time to enter the contest, there is a risk that too many volunteers will enter at the last instant. Figure 3.13 displays the timing of the first and second entry decision in the *Baseline* treatment, given that there are two entrants. As the figure shows, initial entry occurs quite early, while the second entrant "snipes" in at the end of the time horizon. Thus, if we admit relative preferences as well as loss aversion, we can rationalize all aspects of the data.[10]

3.6 Conclusions

We study the power of free entry in a controlled laboratory experiment. On the one hand we replicate the "magic" found by Kahneman (1988) and others. When the value of entering only depends on how many others enter, returns equalize across the inside and outside option. On the other hand, when entrants face a strategic decision determining their payoffs post-entry, the results are not so clear. When entrants compete in a winner-take-all contest after entering, we observe excessive investments. Overall entrants earn about 5% less than non-entrants even towards the end of the experiment. In contrast, when entrants for a share of the prize, instead of subjecting them to chance, payoffs from inside and outside option equalize towards the end of the experiment. Adding a risky or a strategic outside option leads to even more entry in the contest and reduced payoffs from entering.

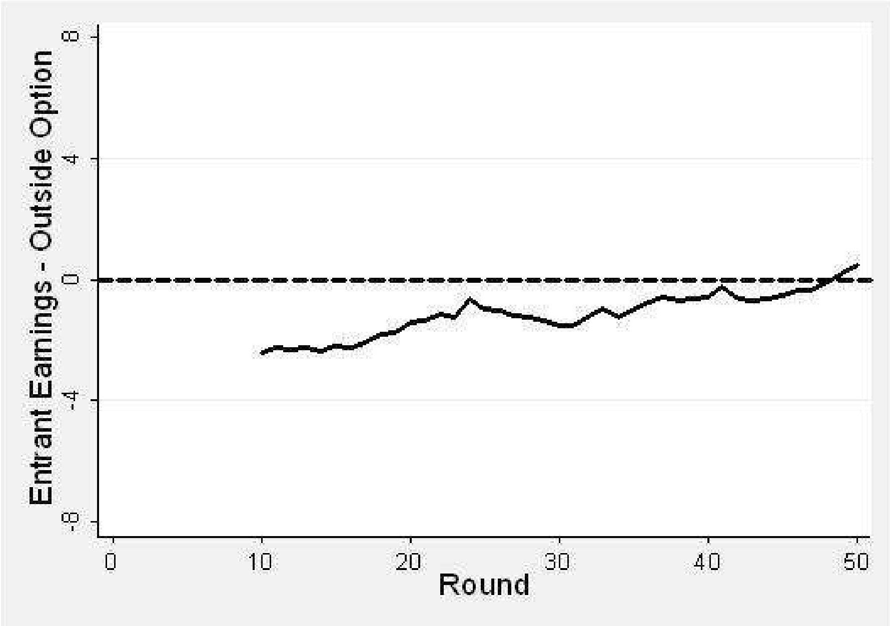
We discuss several explanations — risk-aversion, risk-seekingness, love of winning, heterogeneity and complexity. None of these are able to rationalize all of our treatments. We revert to theory to investigate in more detail the effect of risk-preferences, specifically in the form of loss-aversion, on entry and investment behavior. In the context of an entry game, a reference point becomes especially important. An entrant will enter only if he expects to gain at least the same as the outside option. Free entry should level gain expectations across inside and outside option. We show how loss-aversion with an endogenous reference point, together with the assumption that individuals put some weight on relative payoffs can rationalize our findings. Of course, this is merely a post hoc explanation of our data. We leave it to future research to fully resolve this puzzle. One possible future treatment is to lower the value of the fixed outside option. Then the loss averse model predicts: increased under-entry in the

[10] It is worth noting that we are not the first to incorporate loss aversion in a model of contests. Cornes and Hartley (2010) also study loss aversion in Tullock contests. They characterize equilibria with an exogenous number of loss-averse players and a zero reference point. With endogenous entry, our model departs significantly from their analysis. First, we allow the reference point to vary and endogenize it. Second, we endogenize the number of entrants in the contest.

contest and reduced investments for a given number of players in the contest relative to the *Baseline* treatment.

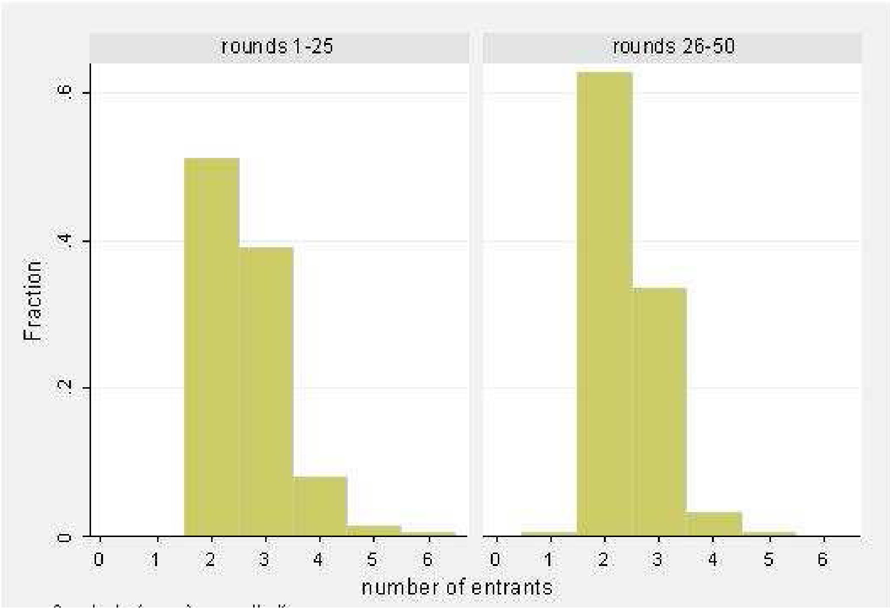
3.A Appendix A: Figures and Tables

Figure 3.1: Payoff differentials in the *Reduced Form* treatment[a]



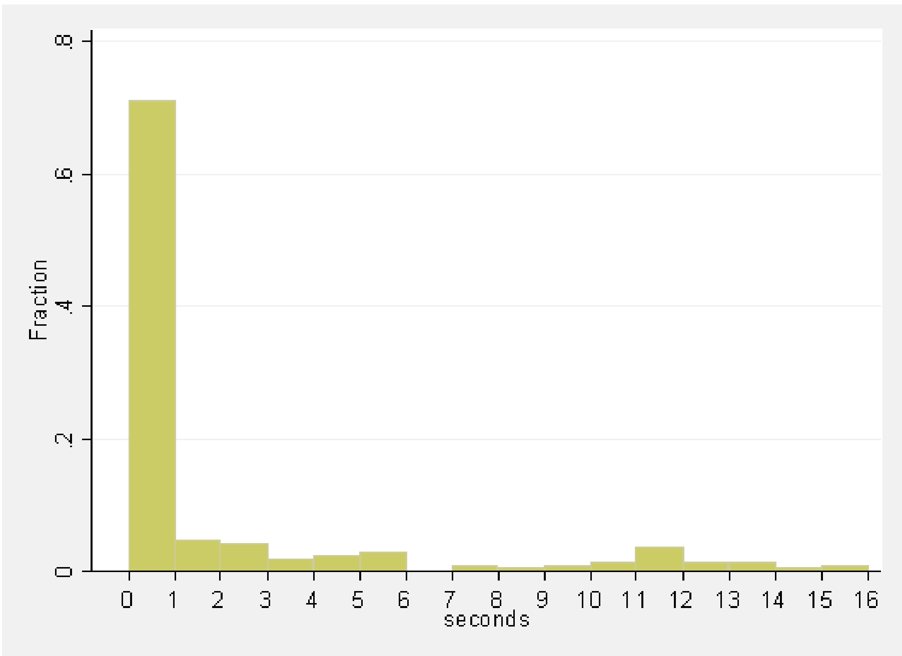
[a]Figure shows a ten-round moving average of earnings per entry decision

Figure 3.2: Number of entrants in *Reduced Form* treatment[a]



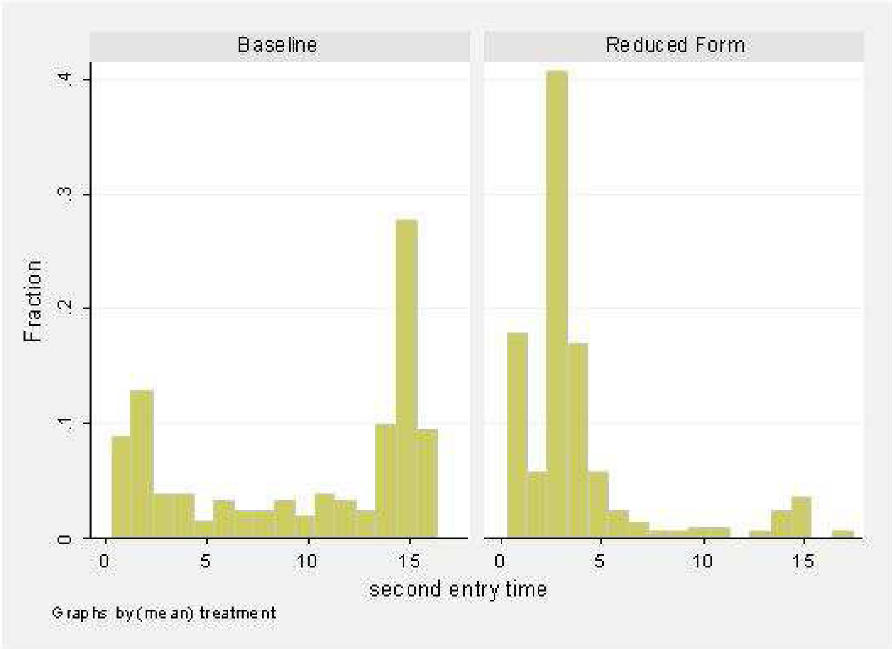
[a]rounds 1-25: avg=2.61, s.d.=0.73, obs=225; rounds 26-50: avg=2.4, s.d.=0.58, obs=225

Figure 3.3: Histogram of delay between second and third entry decisions[a]



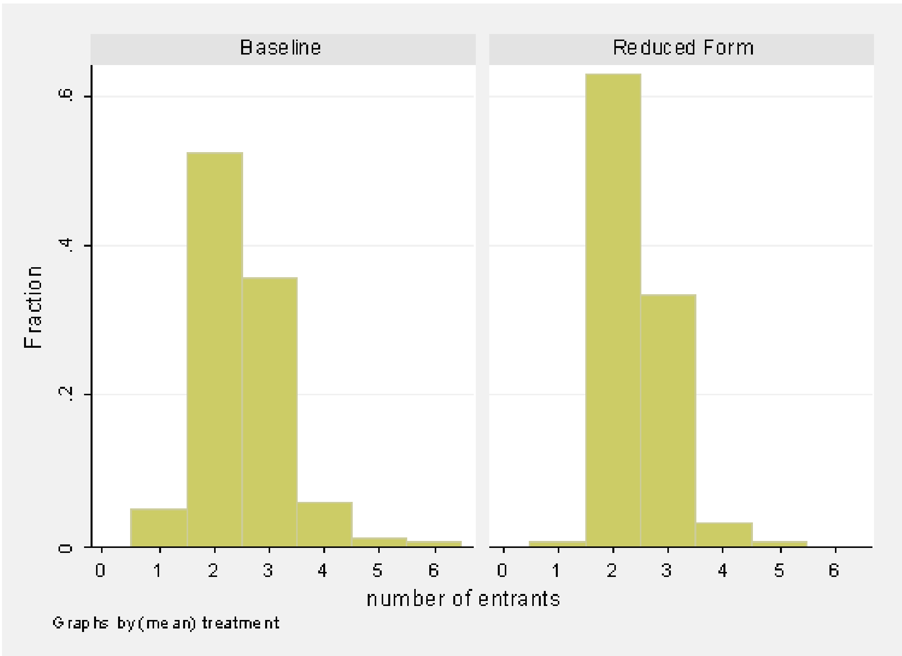
[a]avg = 2.12, s.d. = 3.89, obs = 193

Figure 3.4: Histograms of second entry times[a]



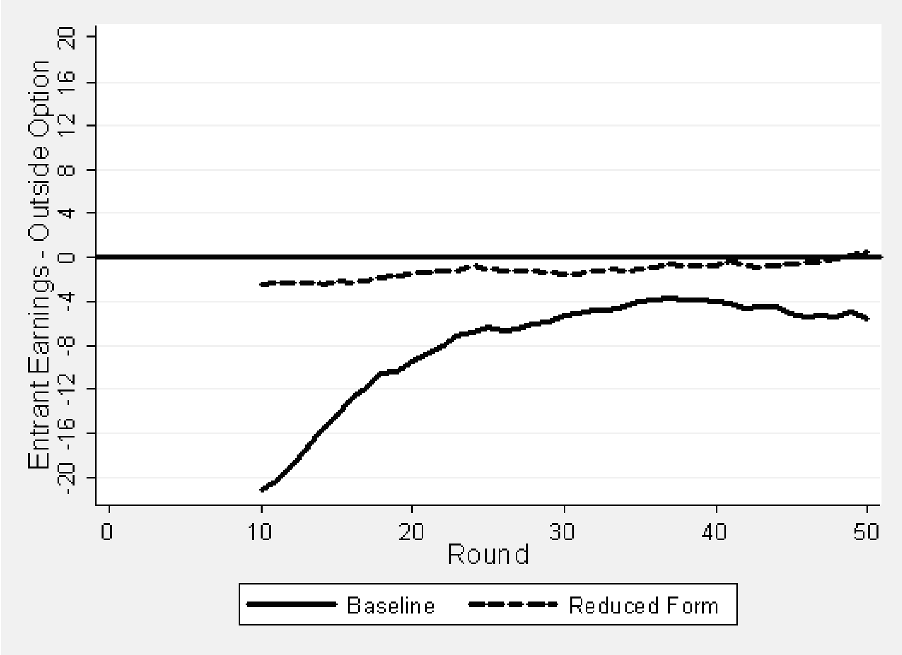
[a]Figures based on data from rounds 26-50.

Figure 3.5: Number of entrants in *Baseline* and *Reduced Form* treatments[a]



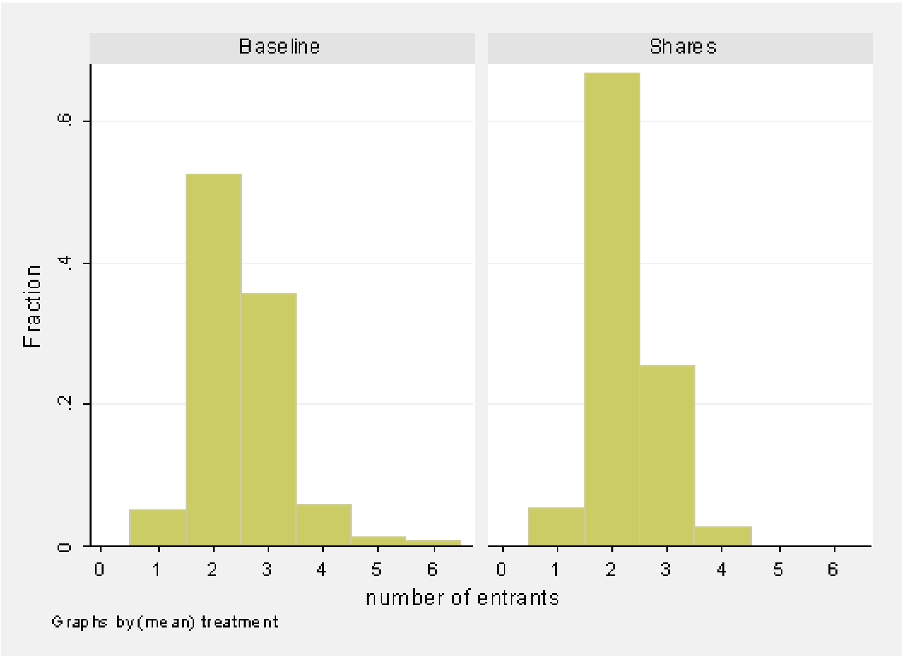
[a]Figure based on data from rounds 26-50. Baseline: avg = 2.47, s.d. = 0.76, obs = 225; Reduced Form: avg = 2.40, s.d. = 0.58, obs = 225

Figure 3.6: Payoff differentials in the *Baseline* and *Reduced Form* treatments[a]



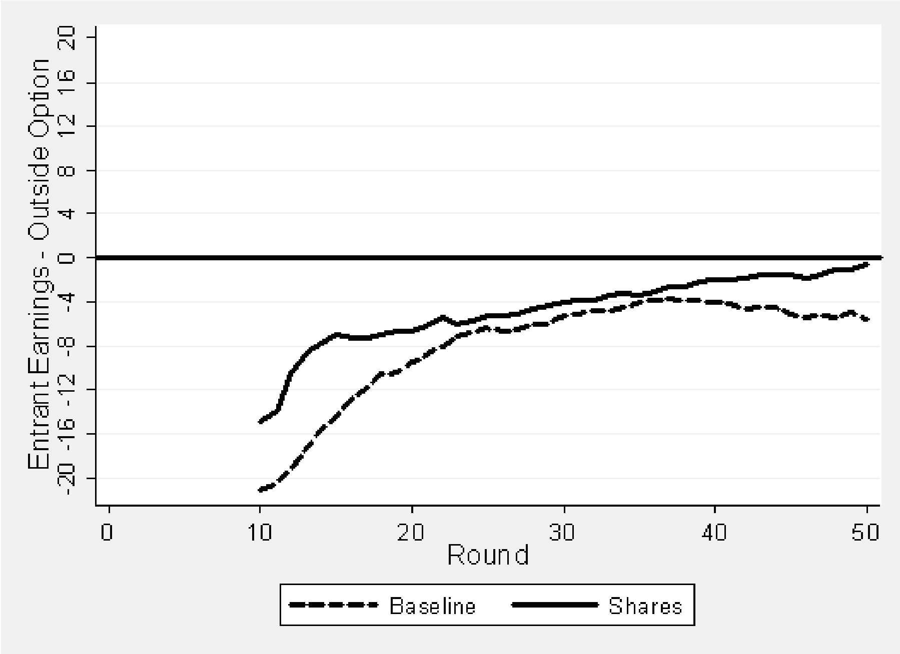
[a]Figure shows ten-round moving averages of earnings per entry decision.

Figure 3.7: Number of entrants in *Shares* and *Baseline* treatments[a]



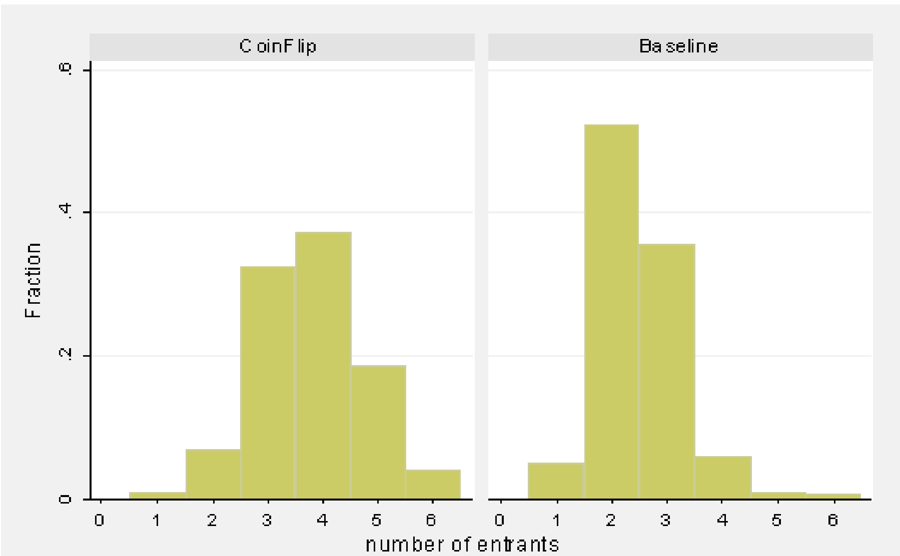
[a]Figure based on data from rounds 26-50. Baseline avg = 2.47, s.d. = 0.76, obs = 225; Shares: avg = 2.25, s.d. = 0.59, obs = 225.

Figure 3.8: Payoff differentials in the *Shares* and *Baseline* treatments[a]



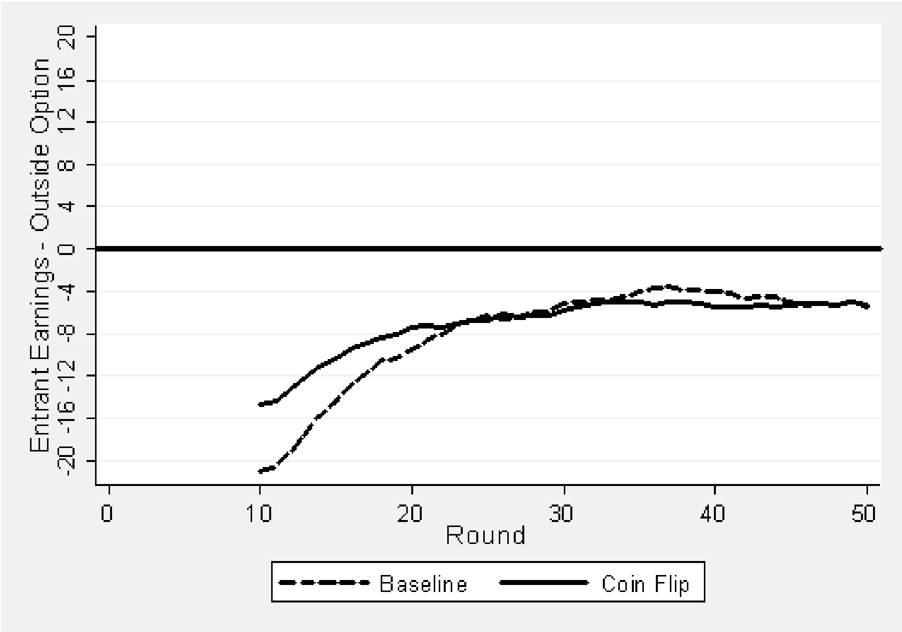
[a]Figure shows ten-round moving averages of earnings per entry decision.

Figure 3.9: Number of entrants in *Coin Flip* and *Baseline* treatments[a]



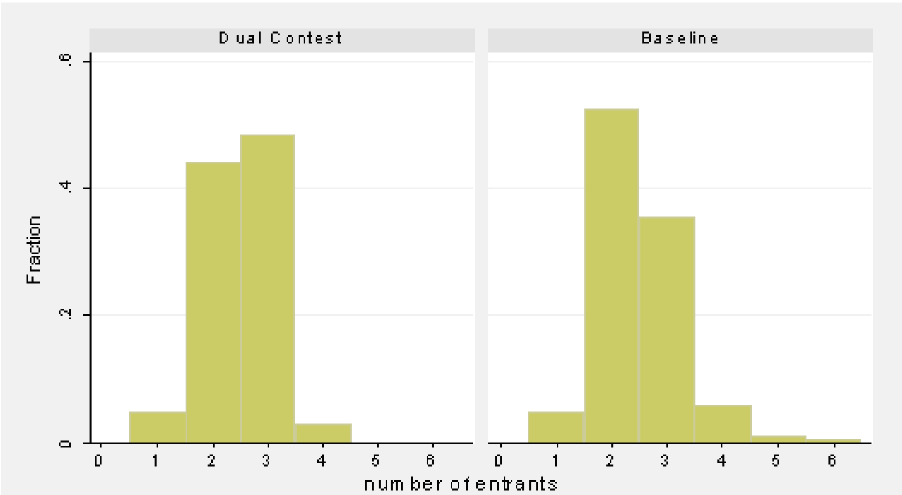
[a]Figure based on data from rounds 26-50. Coin Flip: avg = 3.78, s.d. = 0.99, obs = 225; Baseline: avg = 2.47, s.d. = 0.76, obs = 225.

Figure 3.10: Payoff differentials in the *Coin Flip* and *Baseline* treatments[a]



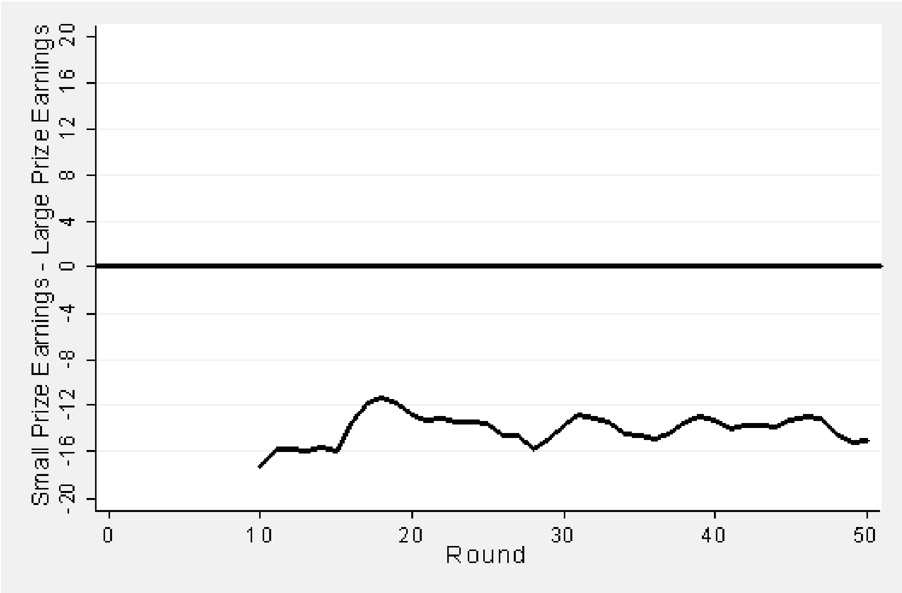
[a]Figure shows ten-round moving averages of earnings per entry decision.

Figure 3.11: Number of entrants in *Dual Contest* and *Baseline* treatments[a]



[a]Figure based on data from rounds 26-50. Dual Contest: avg = 2.50, s.d. = 0.63, obs = 218; Baseline: avg. = 2.47, s.d. = 0.76, obs = 225.

Figure 3.12: Payoff differential in the *Dual Contest* treatment[a]



[a]Figure shows ten-round moving averages of earnings per entry decision.

Figure 3.13: Timing of entry decisions for $n = 2$, *Baseline* treatment, rounds 26-50

First entrant Second entrant

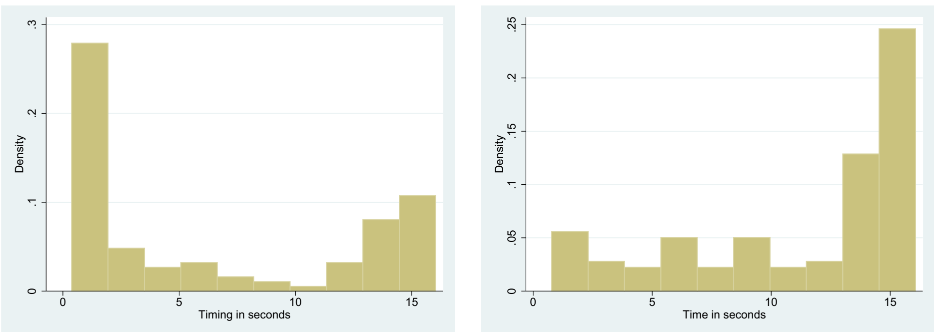


Table 3.3: Contest investment[1]

Entrants	Equilibrium		Treatment				
	SPC	LPC	*Baseline*	*Shares*	*Coin Flip*	*Dual Contest*	
1	0	0	0	0	0	0	–
			0	0	0	0	
			[11]	[12]	[2]	[10]	[0]
2	25	100	35.38	34.17	26.47	34.48	78.33
			(18.11)	(12.23)	(15.12)	(18.50)	(33.12)
			[118]	[150]	[15]	[96]	[6]
3	33.33	133.33	41.30	33.04	33.07	38.82	128.5
			(18.36)	(9.20)	(20.13)	(17.53)	(51.30)
			[80]	[57]	[73]	[106]	[106]
4	37.5	150	47.46	32.33	32.75	45.33	133.41
			(18.43)	(8.31)	(19.36)	(16.52)	(71.98)
			[13]	[6]	[84]	[6]	[96]
5	40	160	80	–	31.38	–	162.90
			0		(14.11)		(50.98)
			[2]	[0]	[42]	[0]	[10]
6	41.67	166.67	90	–	37.89	–	–
			0		(18.80)		
			[1]	[0]	[9]	[0]	[0]

[1] Table reports total contest investments per game, conditional on the number of contestants, with standard deviations in parentheses and number of games in square brackets, using data from rounds 26-50.

Table 3.4: "Low" contest investment[1]

Entrants	Percentage of Low Investors	
	Coin Flip	*Baseline*
2	6.67	1.27
	[30]	[236]
3	11.42	2.92
	[219]	[240]
4	21.73	1.92
	[336]	[52]
5	26.67	0
	[210]	[10]
6	25.93	0
	[54]	[6]
All games	20.02	2.02
	[849]	[544]

[1] Table shows percentage of entrants investing either 0 or 1 point in rounds 26-50. Number of decisions in square brackets.

Table 3.5: Contest investment adjusted for passive entrants[1]

Non-Passive Entrants	Equilibrium	Treatment	
		Coin Flip	*Baseline*
2	25	25.73 (12.46) [44]	36.11 (18.47) [122]
3	33.33	35.41 (20.97) [115]	40.93 (17.90) [74]
4	37.5	32.60 (13.88) [45]	49.08 (18.25) [12]
5	40	40.58 (13.61) [12]	80 0 [2]
6	41.67	– [0]	90 0 [1]

[1] Table reports contest investments conditional on number of non-passive entrants. Standard deviations in parentheses and number of games in square brackets. Based on rounds 26-50.

Table 3.6: "Low" investment decisions in the *Dual Contest* treatment[1]

Entrants	Percentage of Low Investors	
	Small Prize	*Large Prize*
2	3.13 [192]	0 [12]
3	09.12 [318]	5.66 [318]
4	12.50 [24]	08.07 [384]
5	– [0]	18.00 [50]
All games	7.12 [534]	7.59 [764]

[1] Table shows percentage of entrants investing either 0 or 1 point in rounds 26-50. Number of decisions in square brackets. Ten small prize contests with a single entrant not included.

Table 3.7: Characteristics of entrants in rounds 40-50[1]

		Red. F.	*Shares*	*Baseline*	*Coin Flip*	*Dual C.*
Cum. entry rounds 1-40	Entry	26.88	25.55	22.79	25.86	20.07
	No Entry	11.10	11.86	13.81	19.73	12.57
Av. payoff cond. on entry rounds 1-40	Entry	108.78	104.14	99.33	101.99	102.04
	No Entry	107.15	99.45	95.71	98.17	98.78

[1] Table shows average cumulative entry and average payoff conditional on entry in rounds 1-40 by entry decision in the last ten rounds.

3.B Appendix B: Instructions

Welcome! You are about to take part in an experiment in the economics of decision making. You will be paid in private and in cash at the end of the experiment. The amount you earn will depend on your decisions, so please follow the instructions carefully.

It is important that you do not talk to any of the other participants until the experiment is over. If you have a question at any time, raise your hand and someone will come to your desk to answer it.

The experiment will consist of fifty rounds. In each round you will be matched with the same five other participants, randomly selected from the people in this room. Together, the six of you form a group. Note that you will not learn who the other members of your group are, neither during nor after today's session.

Each round is identical. At the beginning of the round you will be given an initial point balance of 100 points. You will then have up to 15 seconds to decide between option A and option B. If, at the end of that time, you have not made a choice, then the computer will make a choice for you by selecting randomly between the two options. During the 15 seconds, your computer screen will keep you informed of how many group members have chosen each of the options so far, as well as the time remaining for you to make a choice. At the end of the 15 seconds the computer will display your choice and the number of group members choosing each option. Your final point earnings for the round will depend on your choice and the choices of other group members as described below.

At the end of the experiment one of the fifty rounds will be selected at random. Your earnings from the experiment will depend on your final point earnings in this randomly selected round. The final point earnings will be converted into cash at a rate of 10p per point.

Option A

[*Reduced Form, Shares, Baseline*:

If you select option A, 10 points will be added to your point balance. Your final point earnings for the round will be 110 points.]

[*Coin Flip:*

If you select option A, your final point earnings for the round will depend on the outcome of a computerized coin flip. The coin is equally likely to come up heads or tails. If the coin comes up heads 35 points will be added to your initial point balance and your final point earnings for the round will be 135 points; if the coin comes up tails 15 points will be subtracted from your initial point balance and your final point earnings for the round will be 85 points.]

[*Dual Contest:* If you select option A you will have a chance to win a prize of 200 points.

First, if you are the only group member to select option A, you will automatically win the prize, and 200 points will be added to your initial point balance. Your final point earnings for the round will be 300 points.

Second, if more than one group member selects option A there will be a contest among these group members to determine who wins the prize. In this contest the players first decide how many "contest tokens" to buy. Each contest token you buy reduces your point balance by 1 point. You can purchase up to 100 of these tokens. Everybody will be making this decision at the same time, so you will not know how many contest tokens the other players have bought when you make your choice. You will have 30 seconds to make a decision about how many contest tokens to buy. If you do not make a decision within this time limit the computer will make a choice for you by selecting zero tokens.

If nobody buys any tokens, nobody wins the prize. Otherwise, your chances of winning the prize will depend on how many contest tokens you buy and how many contest tokens the other players buy. This works as follows:

A computerized lottery wheel will be divided into shares with different colors. One share belongs to you and the other shares belong to each of the other players (a different color for each player). The size of your share on the lottery wheel is an exact representation of the number of contest tokens you bought relative to all contest tokens purchased. For instance, if you own just as many contest tokens as all the other players put together, your share will make up 50% of the lottery wheel. In another example, suppose that there are four players (including you) and that each of you owns the same number of contest tokens: in that case your share will make up 25% of the lottery wheel.

Once the shares of the lottery wheel have been determined, the wheel will start to rotate and after a short while it will stop at random. Just above the lottery wheel there is an indicator at the 12 o'clock position. The indicator will point at one of the shares, and the player owning that share will win the prize. Thus, your chances of winning the prize increase with the number of contest tokens you buy. Conversely, the more contest tokens the other players buy, the lower your chances of receiving the prize.

If you win the prize 200 points will be added to your point balance. Your final point earnings for the round will be (100 - the number of contest tokens you bought + 200) points.

If another player wins the prize zero points will be added to your point balance. Your final point earnings for the round will be (100 - the number of contest tokens you bought) points.]

Option B

[*Reduced Form:*

If you select option B you will receive some additional points depending on how many players choose option B.

If you are the only group member to select option B 50 points will be added to your initial point balance. Your final point earnings for the round will be 150 points.

If you and one other group member selects option B 13 points will be added to your initial point balance. Your final point earnings for the round will be 113 points.

If you and two other group members select option B 6 points will be added to your initial point balance. Your final point earnings for the round will be 106 points.

If you and three other group members select option B 3 points will be added to your initial point balance. Your final point earnings for the round will be 103 points.

If you and four other group member selects option B 2 points will be added to your initial point balance. Your final point earnings for the round will be 102 points.

If you and five other group member selects option B 1 point will be added to your initial point balance. Your final point earnings for the round will be 101 points.]

[*Shares:*

If you select option B you can receive a share of a prize of 50 points.

First, if you are the only group member to select option B, you will automatically receive all of the prize, and 50 points will be added to your initial point balance. Your final point earnings for the round will be 150 points.

Second, if more than one group member selects option B there will be a contest among these group members to determine how the prize is shared. In this contest the players first decide how many ”contest tokens” to buy. Each contest token you buy reduces your point balance by 1 point. You can purchase up to 100 of these tokens. Everybody will be making this decision at the same time, so you will not know how many contest tokens the other players have bought when you make your choice. You will have 30 seconds to make a decision about how many contest tokens to buy. If you do not make a decision within this time limit the computer will make a choice for you by selecting zero tokens.

If nobody buys any tokens, nobody receives any of the prize. Otherwise, your share of the prize will equal your share of all tokens bought times 50 points, rounded to the nearest point.

For example, if all players (including you) bought a total of 100 tokens and you bought 25 of these your share of all tokens bought is 25%. Your share of the prize is 25% of 50 points or 12.5 points, which is rounded to 13 points.

Thus, your share of the prize increases with the number of contest tokens you buy. Conversely, the more contest tokens the other players buy, the lower will be your share of the prize.

Your share of the prize will be added to your point balance. Your final point earnings for the round will be (100 - the number of contest tokens you bought + your share of the prize) points.]

[*Baseline, Coin Flip, Dual Contest:*

If you select option B you will have a chance to win a prize of 50 points.

First, if you are the only group member to select option B, you will automatically win the prize, and 50 points will be added to your initial point balance. Your final point earnings for the round will be 150 points.

Second, if more than one group member selects option B there will be a contest among these group members to determine who wins the prize. In this contest the players first decide how many "contest tokens" to buy. Each contest token you buy reduces your point balance by 1 point. You can purchase up to 100 of these tokens. Everybody will be making this decision at the same time, so you will not know how many contest tokens the other players have bought when you make your choice. You will have 30 seconds to make a decision about how many contest tokens to buy. If you do not make a decision within this time limit the computer will make a choice for you by selecting zero tokens.

If nobody buys any tokens, nobody wins the prize. Otherwise, your chances of winning the prize will depend on how many contest tokens you buy and how many contest tokens the other players buy. This works as follows:

A computerized lottery wheel will be divided into shares with different colors. One share belongs to you and the other shares belong to each of the other players (a different color for each player). The size of your share on the lottery wheel is an exact representation of the number of contest tokens you bought relative to all contest tokens purchased. For instance, if you own just as many contest tokens as all the other players put together, your share will make up 50% of the lottery wheel. In another example, suppose that there are four players (including you) and that each of you owns the same number of contest tokens: in that case your share will make up 25% of the lottery wheel.

Once the shares of the lottery wheel have been determined, the wheel will start to rotate and after a short while it will stop at random. Just above the lottery wheel there is an indicator at the 12 o'clock position. The indicator will point at one of the shares, and the player owning that share will win the prize. Thus, your chances of winning the prize increase with the number of contest tokens you buy. Conversely, the more contest tokens the other players buy, the lower your chances of receiving the prize.

If you win the prize 50 points will be added to your point balance. Your final point earnings for the round will be (100 - the number of contest tokens you bought + 50) points.]

Now, please look at your computer screen and begin making your decisions. If you have a question at any time please raise your hand and a monitor will come to your desk to answer it.

3.C Appendix C: Dynamic Entry Game

The game consists of an infinite number of periods. In each period active players simultaneously choose IN, OUT or WAIT. In the first period all players are active. IN/OUT choices are irrevocable, and so a player who choose IN or OUT becomes inactive in subsequent periods. There is no discounting.

A symmetric subgame perfect equilibrium of the game is:

- If there are already 2 or more incumbents an active player chooses OUT
- If there are $k < 2$ incumbents, an active player chooses IN with prob p_k, WAIT with prob. $1 - p_k$.

First, we identify p_1. Suppose there is one incumbent and denote

- A_1 = payoff to an active player when there is 1 incumbent

- E_1 = payoff to entering when there is one incumbent

$$\begin{aligned} &= p_1^4(1) + 4p_1^3(1-p_1)(2) + 6p_1^2(1-p_1)^2(3) \\ &\quad +4p_1(1-p_1)^3(6) + (1-p_1)^4(13) \end{aligned} \tag{3.10}$$

- W_1 = payoff to waiting when there is one incumbent

$$\begin{aligned} &= p_1^4(10) + 4p_1^3(1-p_1)(10) + 6p_1^2(1-p_1)^2(10) \\ &\quad +4p_1(1-p_1)^3(10) + (1-p_1)^4 A_1 \end{aligned} \tag{3.11}$$

In equilibrium $A_1 = E_1 = W_1$. Substituting $A_1 = W_1$ into 3.11

$$\begin{aligned} W_1 &= p_1^4(10) + 4p_1^3(1-p_1)(10) + 6p_1^2(1-p_1)^2(10) \\ &\quad +4p_1(1-p_1)^3(10) + (1-p_1)^4 W_1 \\ W_1 &= 10. \end{aligned} \tag{3.12}$$

Substituting $E_1 = W_1 = 10$ into 3.10

$$\begin{aligned} 10 &= p_1^4(1) + 4p_1^3(1-p_1)(2) + 6p_1^2(1-p_1)^2(3) \\ &\quad +4p_1(1-p_1)^3(6) + (1-p_1)^4(13) \\ p_1 &= 0.11875. \end{aligned} \tag{3.13}$$

Note that if there is a single incumbent in the current period his expected payoff is

$$\begin{aligned} Z &= p_1^5(1) + 5p_1^4(1-p_1)(2) + 10p_1^3(1-p_1)^2(3) + 10p_1^2(1-p_1)^3(6) \\ &\quad +5p_1(1-p_1)^4(13) + (1-p_1)^5 Z. \end{aligned} \tag{3.14}$$

Using $p_1 = 0.11875 \Rightarrow Z = 11.259$.

Next we identify p_0. Suppose there are no incumbents. The expected payoff from waiting is then

$$\begin{aligned} W_0 &= p_0^5(10) + 5p_0^4(1-p_0)(10) + 10p_0^3(1-p_0)^2(10) + 10p_0^2(1-p_0)^3(10) \\ &\quad +5p_0(1-p_0)^4(A_1) + (1-p_0)^5 A_0. \end{aligned} \tag{3.15}$$

Using $A_1 = 10$ this implies $W_0 = (1-(1-p_0)^5)(10) + (1-p_0)^5 A_0$. In equilibrium $A_0 = W_0 \Rightarrow A_0 = W_0 = 10$. The expected payoff from entering is

$$\begin{aligned} E_0 &= p_0^5(1) + 5p_0^4(1-p_0)(2) + 10p_0^3(1-p_0)^2(3) + 10p_0^2(1-p_0)^3(6) \\ &\quad +5p_0(1-p_0)^4(13) + (1-p_0)^5 Z. \end{aligned} \tag{3.16}$$

In equilibrium $E_0 = W_0 = 10$ and substituting this into the expression gives

$$\begin{aligned} 10 &= p_0^5(1) + 5p_0^4(1-p_0)(2) + 10p_0^3(1-p_0)^2(3) + 10p_0^2(1-p_0)^3(6) \\ &\quad +5p_0(1-p_0)^4(13) + (1-p_0)^5 Z. \end{aligned} \tag{3.17}$$

Using $Z = 11.259$ this yields the solution $p_0 = 0.22936$.

Next we compute the distribution of entrants. Note that no entry games will end with 0 or one entrant. To compute the probability of a game ending with $k > 1$ entrants note that in any given period with one incumbent

$$\begin{aligned} &\Pr(k \text{ entrants}|1 \text{ incumbent}) = \Pr(k-1 \text{ entrants this period}|1 \text{ incumbent}) \\ &\quad +\Pr(0 \text{ entrants this period}|1 \text{ incumbent}) \Pr(k \text{ entrants}|1 \text{ incumbent}). \end{aligned} \tag{3.18}$$

This can be re-arranged as

$\Pr(k \text{ entrants}|1 \text{ incumbent}) = C(k-1,5)p_1^{k-1}(1-p_1)^{6-k}/(1-(1-p_1)^5)$.

In any period with no incumbents

$$\begin{aligned} &\Pr(k \text{ entrants}|0 \text{ incumbents}) = \Pr(k \text{ entrants this period}) + \\ &\Pr(1 \text{ entrant this period}) \Pr(k \text{ entrants}|1 \text{ incumbent}) + \\ &\Pr(0 \text{ entrants this period}) \Pr(k \text{ entrants}|0 \text{ incumbents}). \end{aligned} \tag{3.19}$$

This can be re-arranged as

$$\begin{aligned} &\Pr(k \text{ entrants}|0 \text{ incumbents}) = \\ &[C(k,6)p_0^k(1-p_0)^{6-k} + 6p_0(1-p_0)^5 \Pr(k \text{ entrants}|1 \text{ incumbent})]/(1-(1-p_0)^6). \end{aligned} \tag{3.20}$$

n	0	1	2	3	4	5	6
$p(n)$	0	0	0.7137	0.2372	0.0443	0.0046	0.0002

Table 3.8: Distribution of entrants — theoretical prediction

Putting all this together:

$\Pr(k \text{ entrants}|0 \text{ incumbents}) = [C(k,6)p_0^k(1-p_0)^{6-k}+6p_0(1-p_0)^5 C(k-1,5)p_1^{k-1}(1-p_1)^{6-k}/(1-(1-p_1)^5)]/(1-(1-p_0)^6)$.

Since the game begins with no incumbents this is also the unconditional probability of the game ending with exactly $k > 1$ entrants. The distribution is tabulated below.

From the distribution it is straightforward to compute $E(n) = 2.34$; $V(n) = 0.3433$.

3.D Appendix D: Proof of Proposition 11

Proof. Differentiating equation (3.1) with respect to x_i yields the first-order condition:

$$\frac{\sum_{k\neq i} x_k}{\left(\sum_{j=1}^n x_j\right)^2}\left(\beta\left(R-x_i-r\right)\right)-\beta\frac{x_i}{\sum_{j=1}^n x_j}-\left(1-\frac{x_i}{\sum_{j=1}^n x_j}\right)\alpha+\frac{\sum_{k\neq i} x_k}{\left(\sum_{j=1}^n x_j\right)^2}\alpha\left(x_i+r\right)=0$$

It is routine to verify that equation (3.1) is strictly concave in x_i; hence the first-order condition is both necessary and sufficient.

Solving for a symmetric equilibrium, we have

$$\begin{aligned}\frac{(n-1)}{n^2 x}\left(\beta\left(R-x-r\right)\right)-\frac{\beta}{n}-\left(\frac{n-1}{n}\right)\alpha+\frac{(n-1)}{n^2 x}\alpha\left(x+r\right) &= 0\\ (n-1)\left(\beta\left(R-x-r\right)\right)-\beta n x-(n-1)\alpha n x+(n-1)\alpha\left(x+r\right) &= 0\end{aligned}$$

which yields

$$x=(n-1)\frac{(\beta R+(\alpha-\beta)r)}{\beta\left(2n-1\right)+\alpha\left(n-1\right)^2}$$

Need to check when my assumption that $R-x>r$ is satisfied.

$$R-(n-1)\frac{\beta R+r\left(\alpha-\beta\right)}{\beta\left(2n-1\right)+\alpha\left(n-1\right)^2}>r$$

Cross-multiplying

$$\begin{aligned}(R-r)\left(\beta\left(2n-1\right)+\alpha\left(n-1\right)^2\right)-(n-1)\left(\beta R+r\left(\alpha-\beta\right)\right) &> 0\\ R\left(\beta n+\alpha\left(n-1\right)^2\right)-r\left(\beta n+\alpha n\left(n-1\right)\right) &> 0\end{aligned}$$

Hence, we require that

$$r < \frac{R\left(\beta n + \alpha\left(n-1\right)^2\right)}{\left(\beta n + \alpha n\left(n-1\right)\right)}$$

□

3.E Appendix E: Proof of Lemma 4

Proof. Recall that, under the baseline treatment, there was a fixed outside option, z. Thus, the reference point equals this value. Now consider the coin flip treatment. Denote the reference point for this treatment by r'. Let z_1 denote the favorable outcome of the lottery and z_0 the unfavorable outcome (relative to the reference point). Suppose further that p_i denotes the probability of outcome z_i. Then, for zero expected gain/loss utility under the outside option, we have

$$r' = \frac{\beta p_1 z_1 + \alpha p_0 z_0}{p_1\beta + p_0\alpha}$$

Simplifying

$$r' = \frac{\beta z + \left(\alpha - \beta\right) p_0 z_0}{p_1\beta + p_0\alpha}$$

To see that this is smaller than z, notice that

$$\begin{aligned} r' = \tfrac{\beta z + (\alpha-\beta) p_0 z_0}{p_1\beta + p_0\alpha} &< z \\ \iff \quad \beta z + \left(\alpha-\beta\right) p_0 z_0 &< z\left(p_1\beta + p_0\beta + p_0\left(\alpha-\beta\right)\right) \\ \iff \quad \beta z + \left(\alpha-\beta\right) p_0 z_0 &< \beta z + z p_0\left(\alpha-\beta\right) \\ \iff \quad z_0 &< z \end{aligned}$$

which always holds. □

Conclusion

My dissertation consists of three chapters, all three devoted to incentives in contests and the role of heterogeneity.

Chapter 1 characterizes conditions under which introducing multiple prizes in a contest can be used to guarantee efficient incentives for the production of a public good when individuals are heterogeneous. With two types of individuals, efficiency can be guaranteed if: (i) the contest designer can use at least two prizes different from zero, (ii) there is a sufficient number of individuals of each type or types are sufficiently similar and (iii) the reservation utility of the individuals resulting from non-participation is sufficiently low. For a large class of problems the optimal prize structure is not monotonic.

Chapter 2 studies situations where two parties with differing valuations or abilities vie to capture some scarce resource. While one party's characteristics are common knowledge, the other's are private information. Is the right policy to mandate the disclosure of this information? When competition occurs via a noisy all-pay auction, the answer is no. Under mild conditions, decentralizing the disclosure decision produces less wasteful competition and more efficient outcomes than mandating disclosure. Our results have implications for transparency policy in lobbying, electoral competition and international relations among others.

Chapter 3 reports the results of laboratory experiments on market entry. Theory predicts that entry occurs up to the point where payoffs to inside and outside option equalize. Our findings are at odds with this prediction. In particular, entrants earn systematically less than those who stay out of the market. The payoff gap increases as a) the inside option becomes riskier; b) the outside option becomes riskier; c) the inside option becomes more strategic; and d) the outside option becomes more strategic. We discuss risk-preferences, complexity and heterogeneity as possible explanations of this puzzle.

All in all, this dissertation contributes to the understanding of heterogeneity in contests and draws practical implications for economic policy. However, a lot remains to be done. Extending the theoretical framework in interesting directions is a project I intend to pursue in the future. Also, empirical tests and experimental evidence on the theoretical analysis of chapters 1 and 2 could be particularly insightful. I can therefore only hope that my work has succeeded in awakening the interest of academic research on the topic.

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